Aspects of confinement and dynamical chiral symmetry breaking in Landau gauge QCD

Christian S. Fischer

TU-Darmstadt

C.F., J.Phys.G32:R253-R291,2006, [hep-ph/0605173]

C.F., P. Watson, W. Cassing, PRD **72** (2005) 094025, [hep-ph/0509213]

R. Alkofer, C.F., F. Llanes-Estrada, hep-ph/0607293

C.F., J. M. Pawlowski, hep-th/0609009

Outline

- **Introduction**
- Infrared fixed points in SU(N) Yang-Mills theory
- Infrared Slavery in quenched QCD
- Light mesons: unquenching effects

Motivation

QCD Green's functions

- are connected to confinement:
  - Kugo-Ojima confinement criterion
  - Positivity
  - Quark-antiquark potential
- encode DχSB
- are ingredients for hadron phenomenology
  - Bound state equations:
    Bethe–Salpeter equation / Faddeev equation

The Goal:

Ab initio description of hadrons as bound states
in terms of underlying substructure

Propagators of QCD: Covariant Gauge

★ Faddeev-Popov method: quark, gluon,ghost

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[A,c,\Psi] \exp\left\{ \int \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{\left(\partial^\mu A^a_\mu\right)^2}{2\xi} + \bar{c}^a(-\partial^\mu D_\mu)c^a \right\}$$

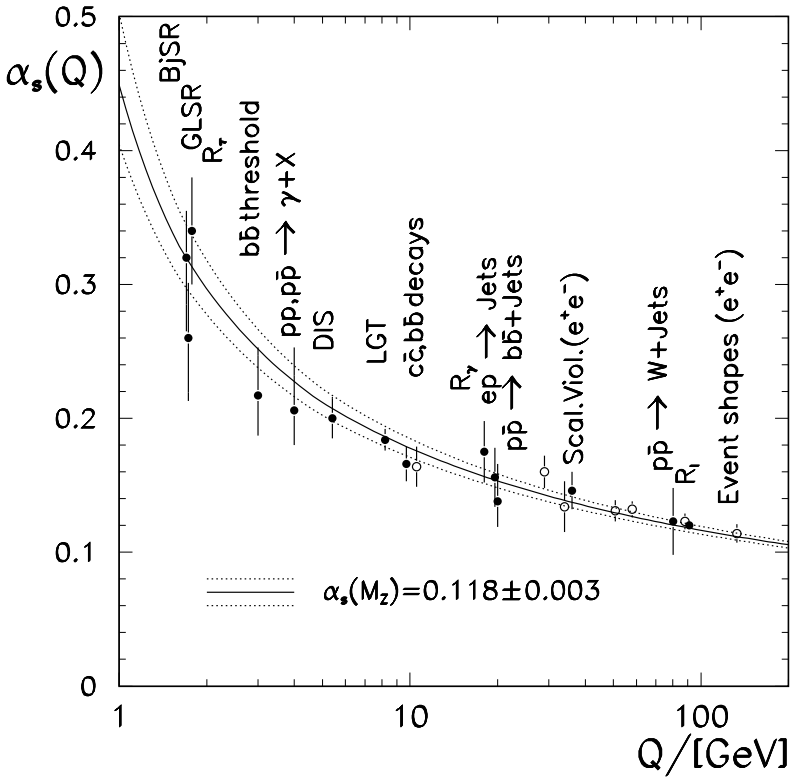
★ Landau gauge propagators in momentum space,

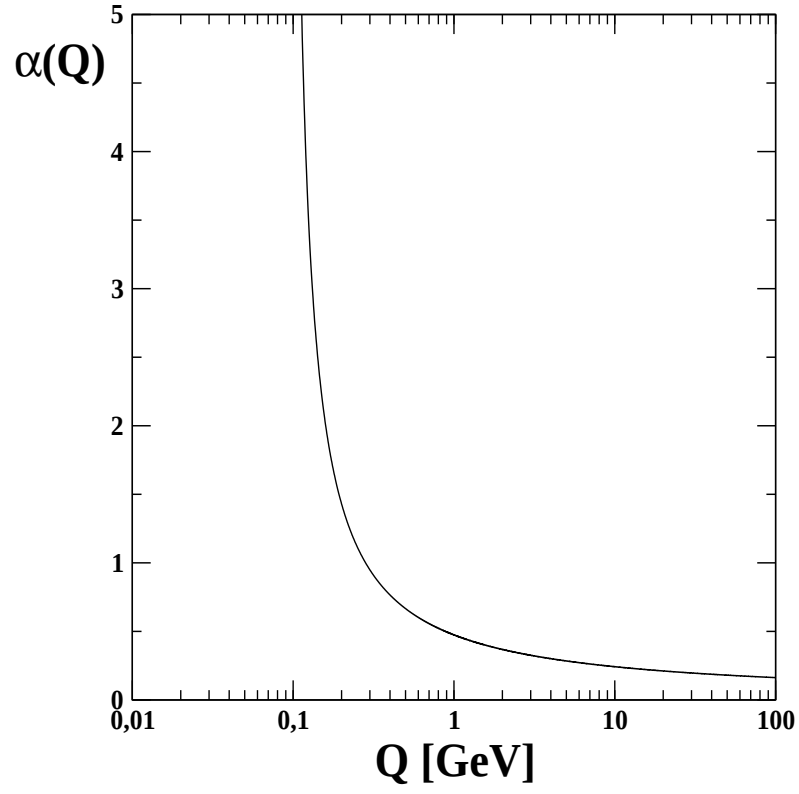
$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$D^{\text{Ghost}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

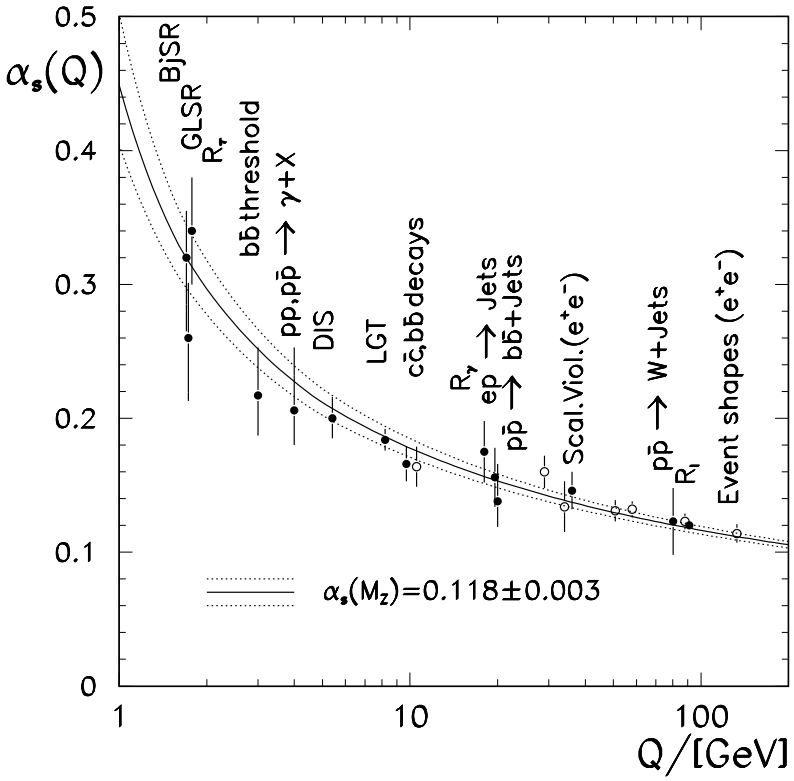
Running Coupling I

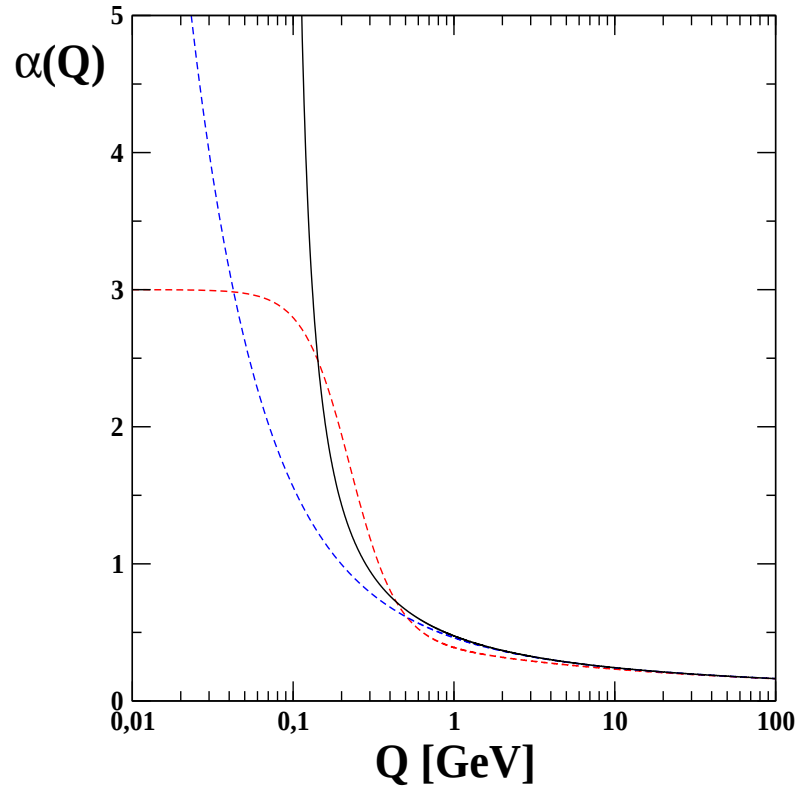




- Perturbation theory: Landau pole

Running Coupling I



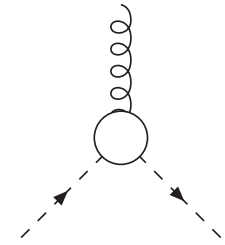


D. J. Gross and F. Wilczek, Phys. Rev. **D8** (1973) 3633–3652.

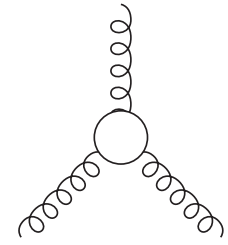
- Nonperturbative definition of running coupling needed!

Running Coupling II (Nonpert. definition)

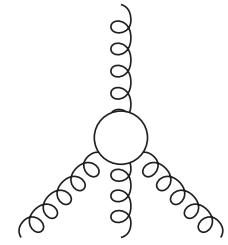
$$\alpha^{gh-gl}(p^2) = \alpha_\mu \, G^2(p^2) \, Z(p^2)$$



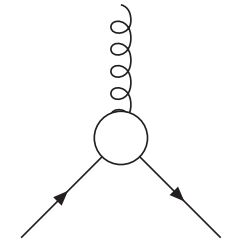
$$\alpha^{3g}(p^2) = \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2)$$



$$\alpha^{4g}(p^2) = \alpha_\mu \, [\Gamma^{4g}(p^2)] \, Z^2(p^2)$$



$$\alpha^{qg}(p^2) = \alpha_\mu \, [\Gamma^{qg}(p^2)]^2 \, Z_f^2(p^2) \, Z(p^2)$$



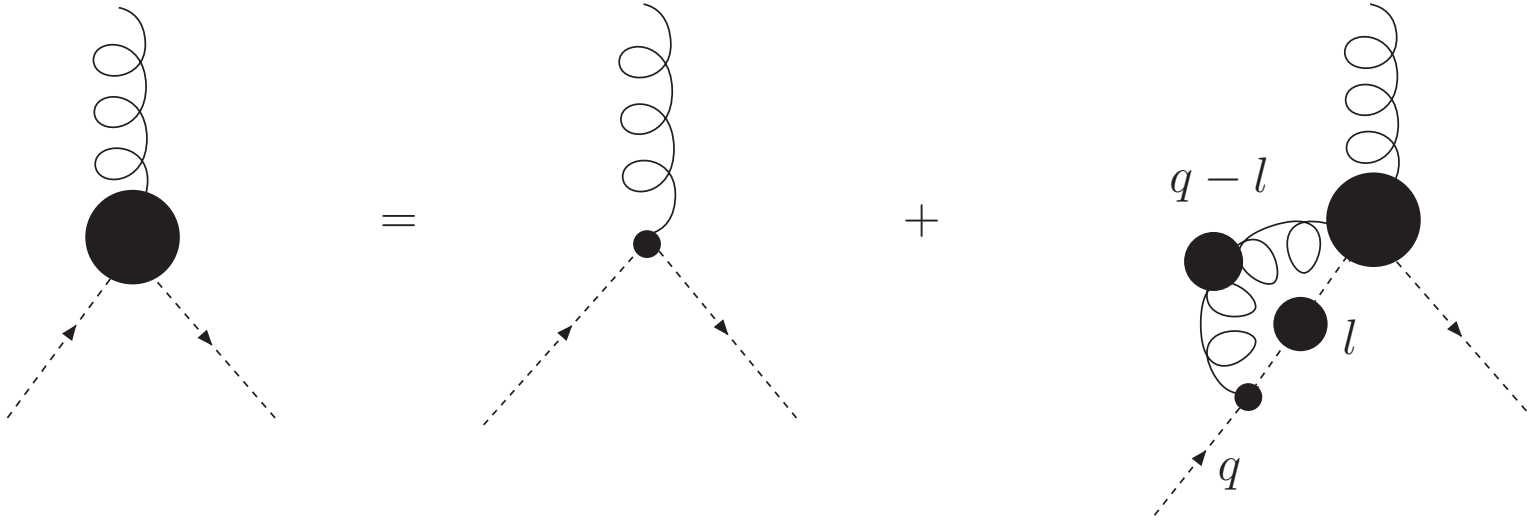
Definitions based on Slavnov-Taylor identities

Outline

- Introduction
- **Infrared fixed points in SU(N) Yang-Mills theory**
- Infrared Slavery in quenched QCD
- Light mesons: unquenching effects

Infrared Structure of YM-theory I

- Starting point: Ghost-Gluon-Vertex



  - $l_\mu D_{\mu\nu}(l-q) = q_\mu D_{\mu\nu}(l-q) \Rightarrow$ Bare Vertex for $q_\mu \to 0$
  - No anomalous dimensions in the IR

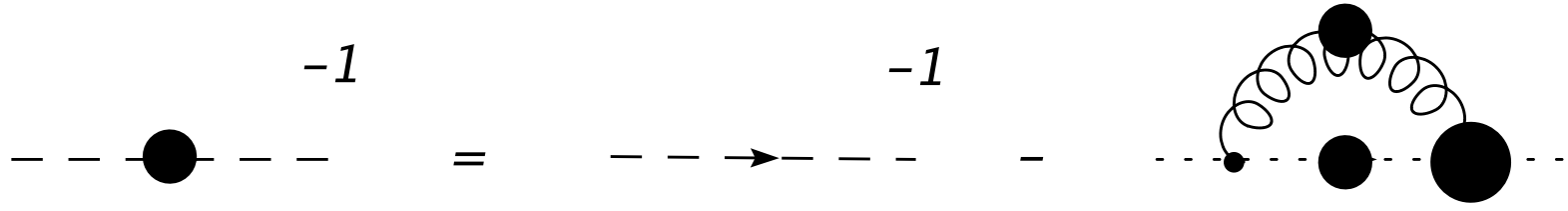
J. C. Taylor, Nucl. Phys. B **33** (1971) 436.

A. Cucchieri, T. Mendes and A. Mihara, JHEP 0412:012 (2004)

W. Schleifenbaum, A. Maas, J. Wambach and R. Alkofer, Phys.Rev.D72 (2005) 014017

Infrared Structure of YM-theory II

- DSE for the ghost-propagator



- Selfconsistency: $Z(p^2) \sim (p^2)^{2\kappa}$, $G(p^2) \sim (p^2)^{-\kappa}$

L. v. Smekal, A. Hauck, R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591

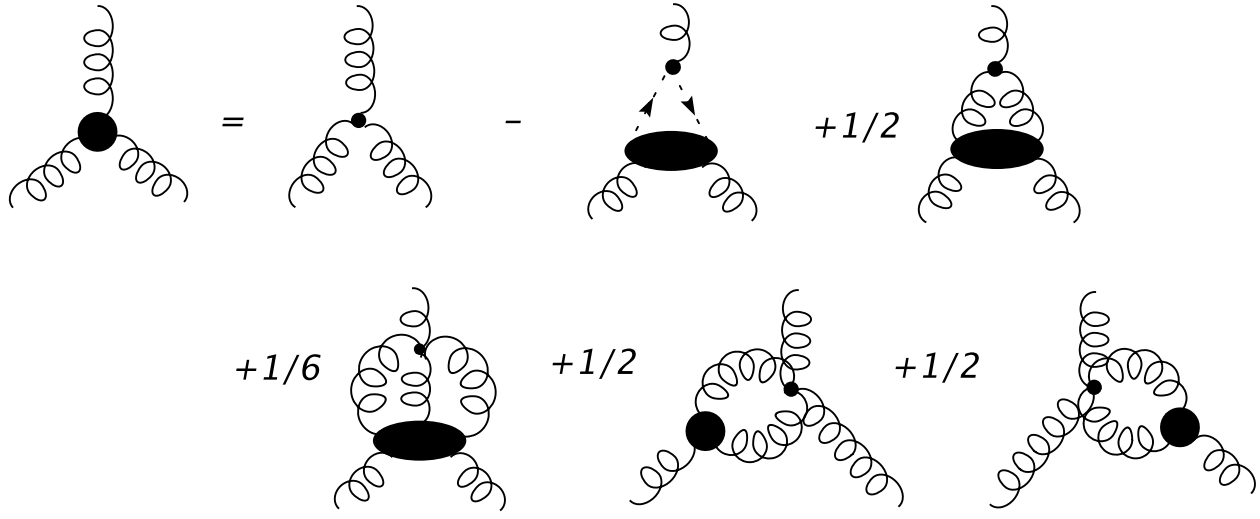
- $\kappa > 0 \quad \Rightarrow$ **Well defined global colour charge**

P. Watson and R. Alkofer, Phys. Rev. Lett. **86** (2001) 5239

C. Lerche and L. v. Smekal, Phys. Rev. D **65** (2002) 125006

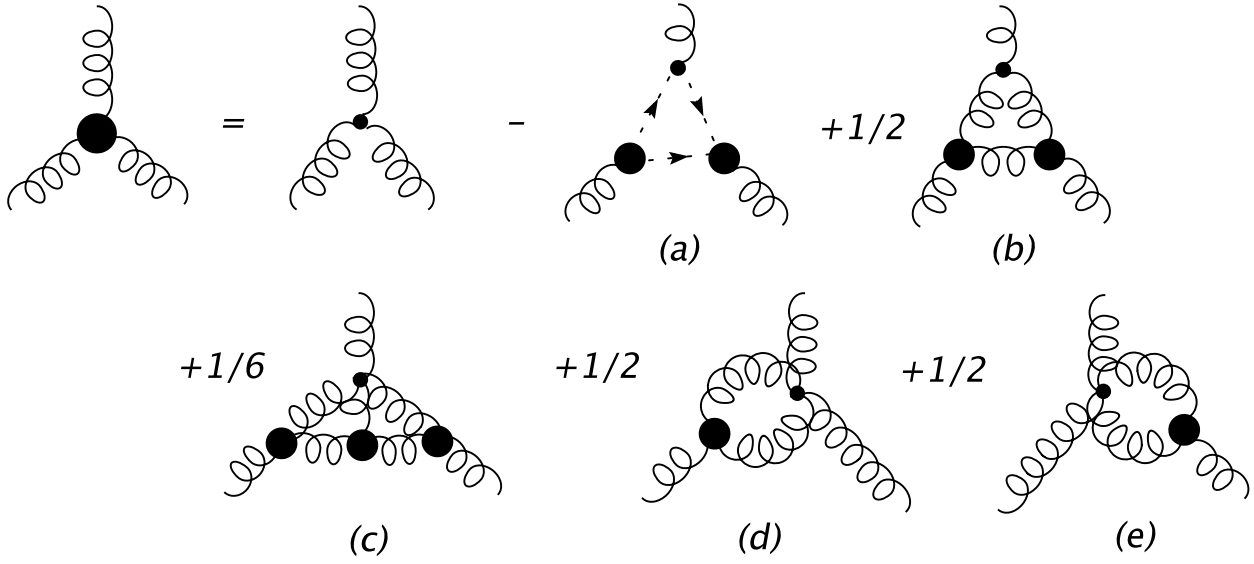
Infrared Structure of YM-theory III

- Three-gluon vertex:



Infrared Structure of YM-theory III

- Three-gluon vertex: **lowest order** in skeleton expansion



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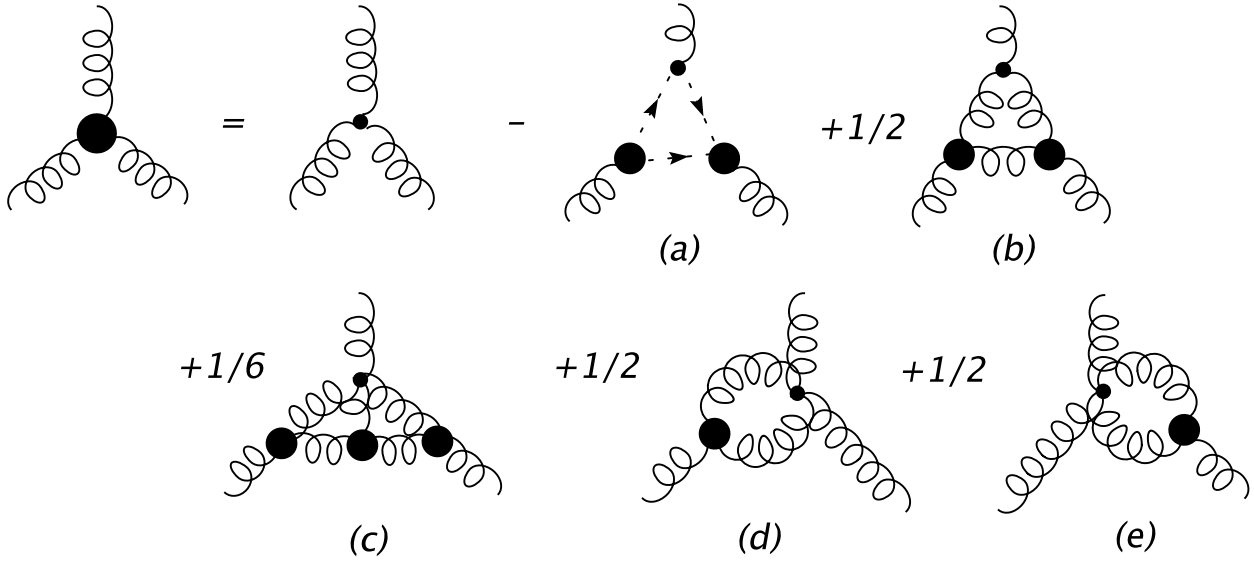


- Selfconsistency: (a) is leading diagram in IR
- $Z(p^2) \sim (p^2)^{2\kappa}$, $G(p^2) \sim (p^2)^{-\kappa} \Rightarrow \Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}$
- **Works to all orders in skeleton expansion !**

General Infrared Structure

- Greens function with
  2n external ghost legs,
  m external gluon legs:

$$\Gamma^{2n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Solves DSEs and STIs

- Ghost sector of YM-theory dominates IR

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

Uniqueness of IR-solution I

- DSEs from:

$$\frac{\delta\Gamma}{\delta\phi}[\phi] = \frac{\delta S_{\rm cl}}{\delta\phi}\left[\left(\frac{\delta^2\Gamma}{\delta\phi\delta\phi_i}\right)^{-1}\frac{\delta}{\delta\phi_i} + \phi\right],$$

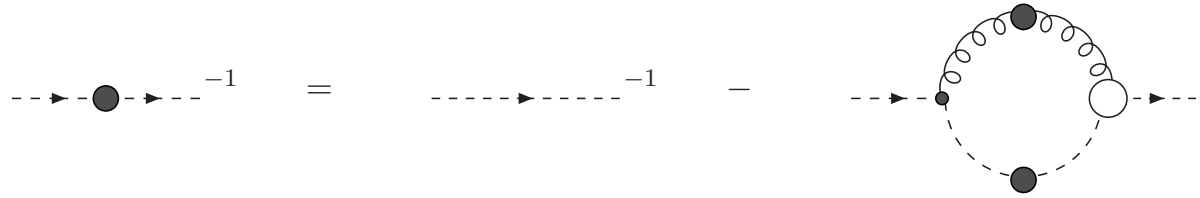
- ERGEs from:

$$\partial_k\Gamma_k[\phi] = \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\partial_k R_k(q^2)\left(\frac{\delta^2\Gamma_k}{\delta\phi(q)\delta\phi(-q)} + R_k(q^2)\right)^{-1}$$

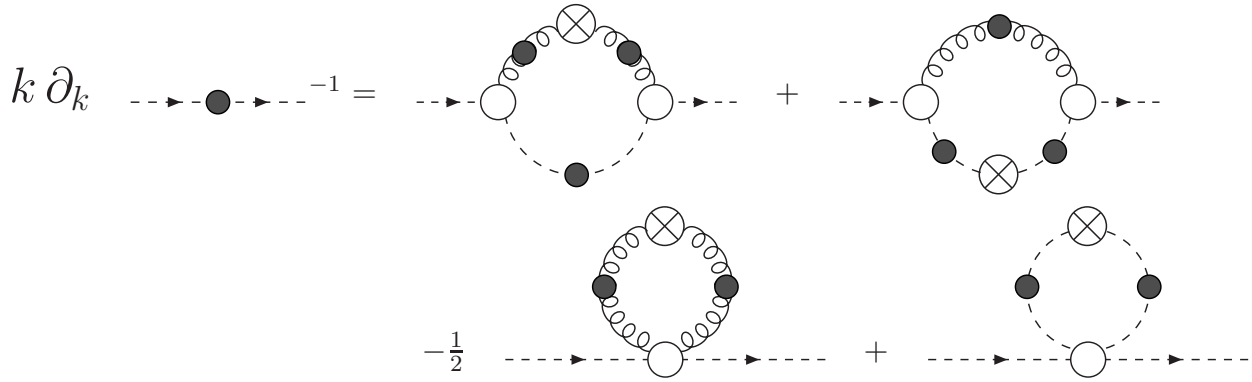
$\longrightarrow$ Two different towers of equations for Green functions

Uniqueness of IR-solution II

Ghost-DSE:



Ghost-ERGE:



IR-Analysis of whole tower of equations $\Rightarrow$

$$\Gamma^{2n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

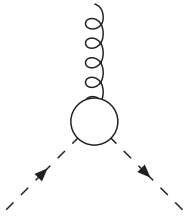
is unique!

C.F., J. M. Pawlowski, hep-th/0609009

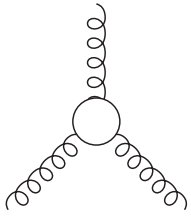
Running Coupling: IR-Universality

$$G(p^2) \sim (p^2)^{-\kappa} \,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

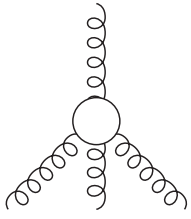
$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa} \,, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$



$$\alpha^{gh-gl}(p^2) \;=\; \alpha_\mu \, G^2(p^2) \, Z(p^2) \sim \frac{const_{gh-gl}}{N_c}$$

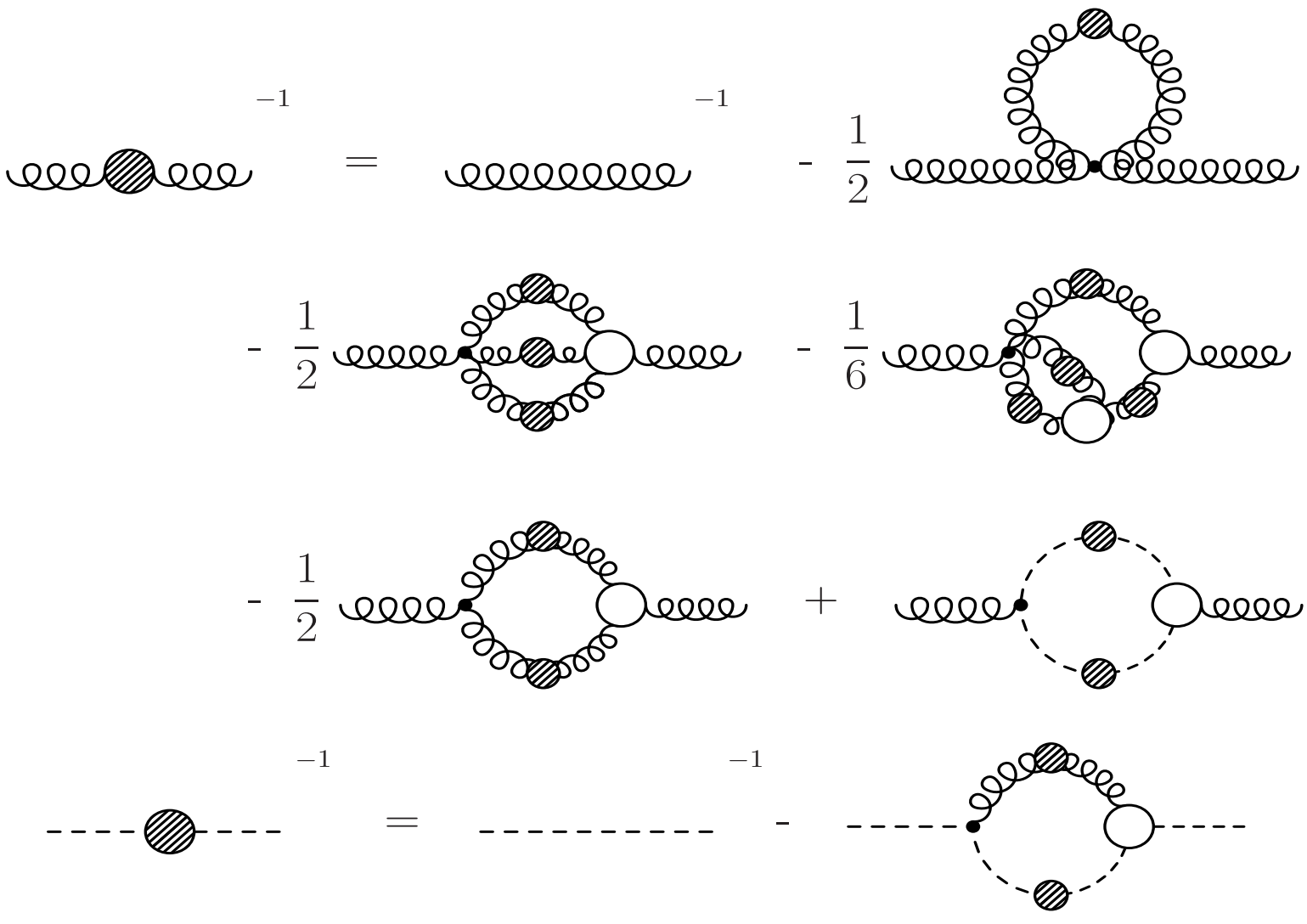


$$\alpha^{3g}(p^2) \;=\; \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2) \sim \frac{const_{3g}}{N_c}$$

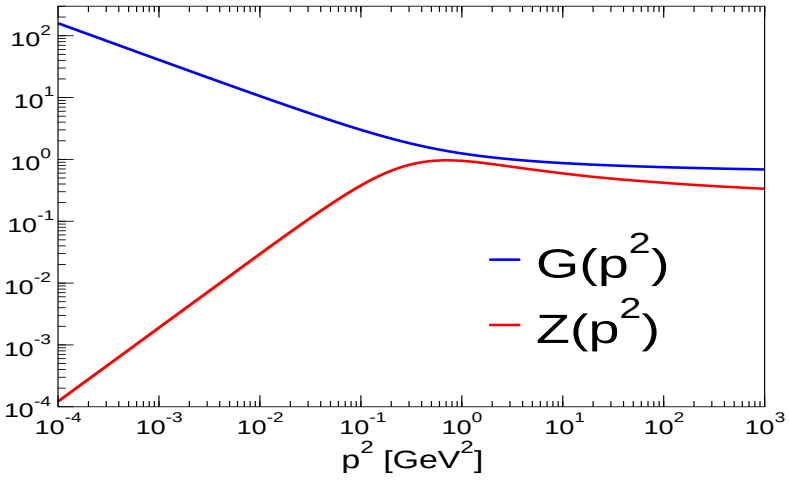


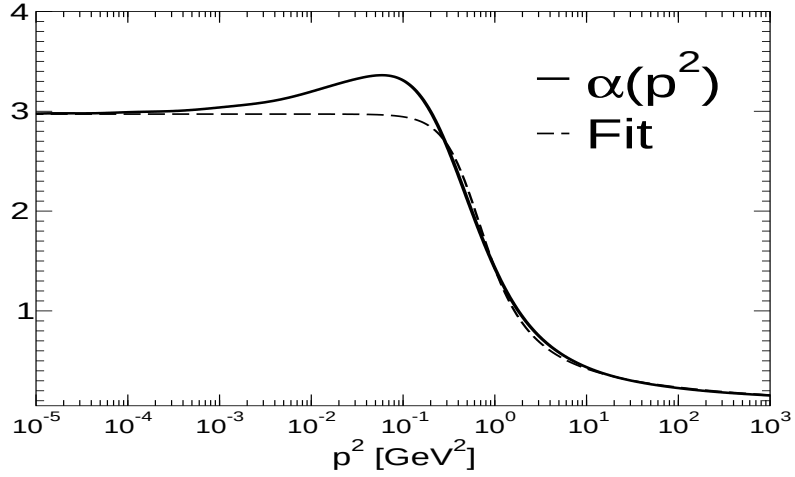
$$\alpha^{4g}(p^2) \;=\; \alpha_\mu \, [\Gamma^{4g}(p^2)] \, Z^2(p^2) \sim \frac{const_{4g}}{N_c}$$

DSEs for Ghost and Glue



Quenched Ghost and Glue I





C. F., R. Alkofer, PLB **536** (2002) 177

- IR fixed point: $\alpha(0) = 8.915/N_c$

  C. Lerche and L. v. Smekal, PRD **65** (2002) 125006.

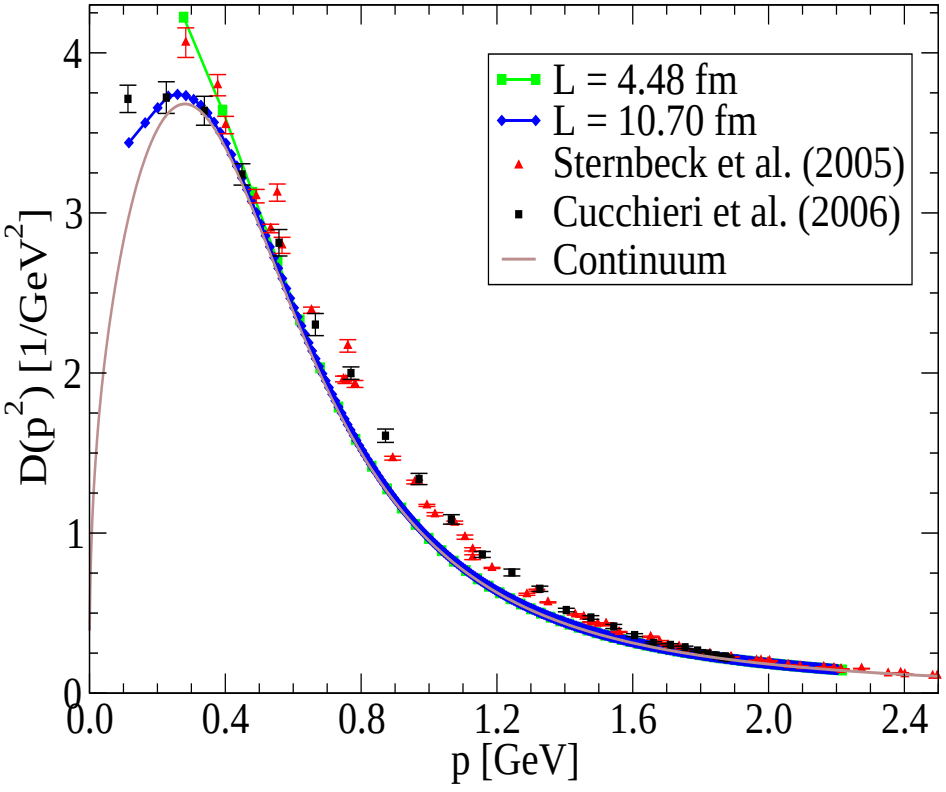
- Confirmed in ERGE-formalism

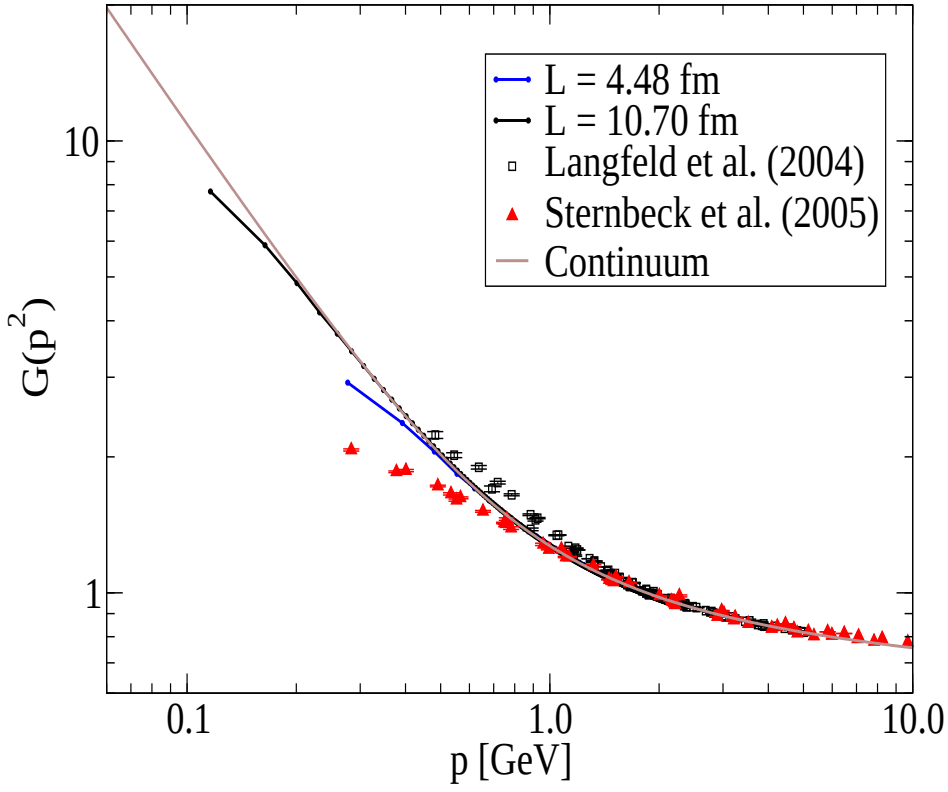
  J. M. Pawlowski *et al*, PRL **93** (2004) 152002, C. F. and H. Gies, JHEP **0410** (2004) 048

- Fixed point in all transverse gauges (incl. Coulomb gauge)

  C. F. and D. Zwanziger, PRD **72** (2005) 054005.

Quenched Ghost and Glue II





C.F., A. Maas, J. M. Pawlowski, L. v. Smekal, in preparation

Outline

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Dynamical Chiral Symmetry Breaking

$$S^{-1}(p) = \left[i\not{p} + M(p^2) \right] / Z_f(p^2)$$

- **Explicit** breaking of chiral symmetry ($m_{u,d,s,\ldots} \neq 0$)
- **Dynamical** breaking of chiral symmetry:

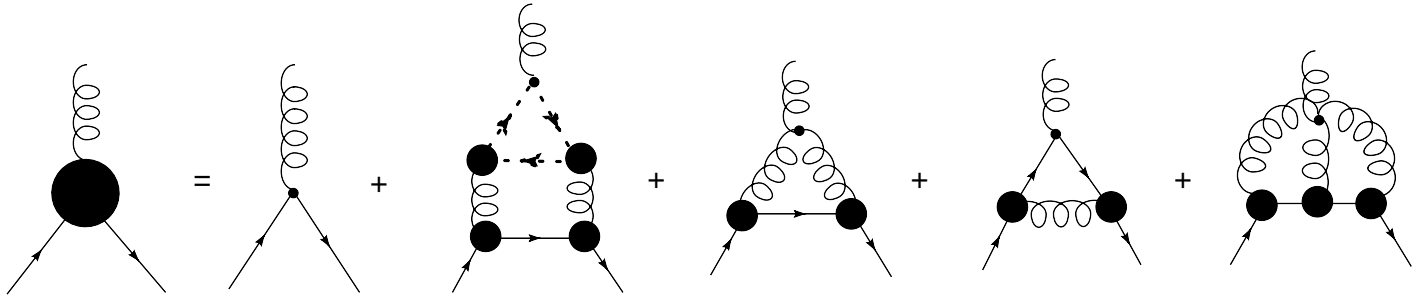
  $M(p^2) \neq 0 \qquad (m_{u,d,s,\ldots} = 0)$

  Nonperturbative effect!
- Chiral condensate $\langle \bar{\Psi}\Psi \rangle$:

  $$M(p^2) \overset{p^2 \to \infty}{\longrightarrow} \frac{-\langle \bar{\Psi}\Psi \rangle}{p^2 \left[\log(p^2/\Lambda^2) \right]^{1-\gamma}} \qquad (m_{u,d,s,\ldots} = 0)$$

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion



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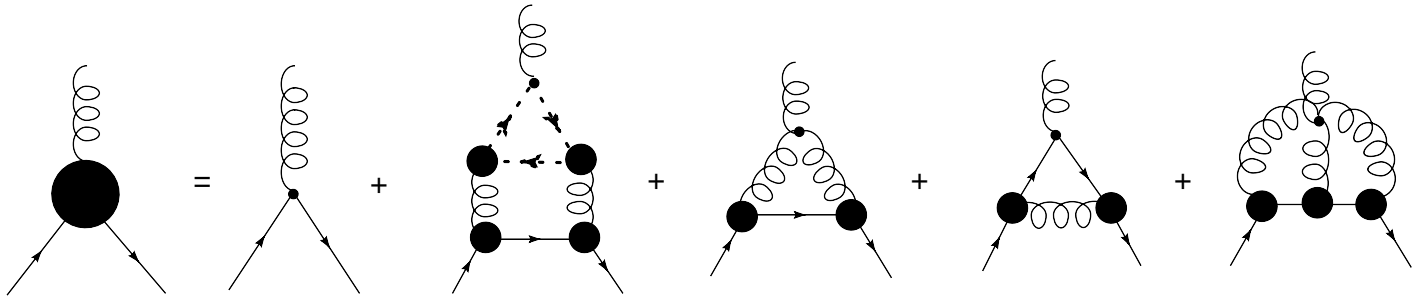

$$S(p) = \frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \rightarrow \frac{Z_f \not{p}}{M^2} + \frac{Z_f}{M}$$

Infrared Structure of QCD II

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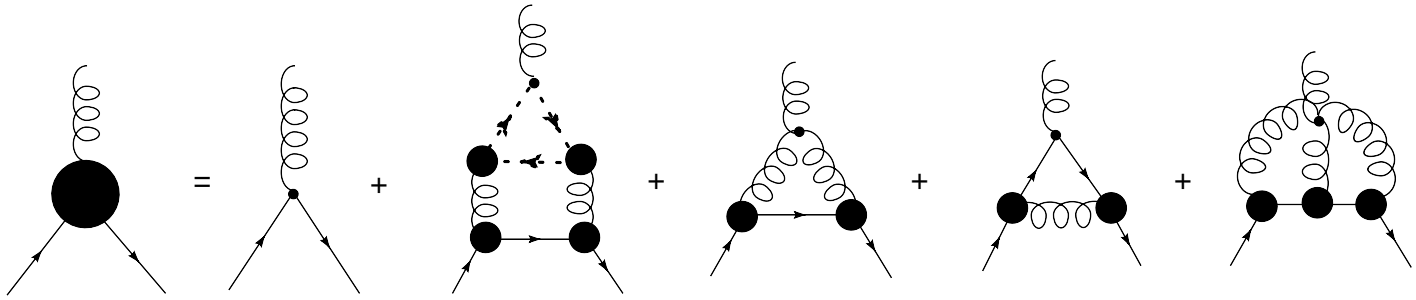

$$S(p) = \frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \rightarrow \frac{Z_f \not{p}}{M^2} + \frac{Z_f}{M}$$

$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G_\mu^i \,, \quad G_\mu^1 = \gamma_\mu \,, \; G_\mu^2 = \hat{p}_\mu \,, \; G_\mu^3 = \hat{\not{p}}\, \hat{p}_\mu \,, \; G_\mu^4 = \hat{\not{p}}\, \gamma_\mu$$

Infrared Structure of QCD II

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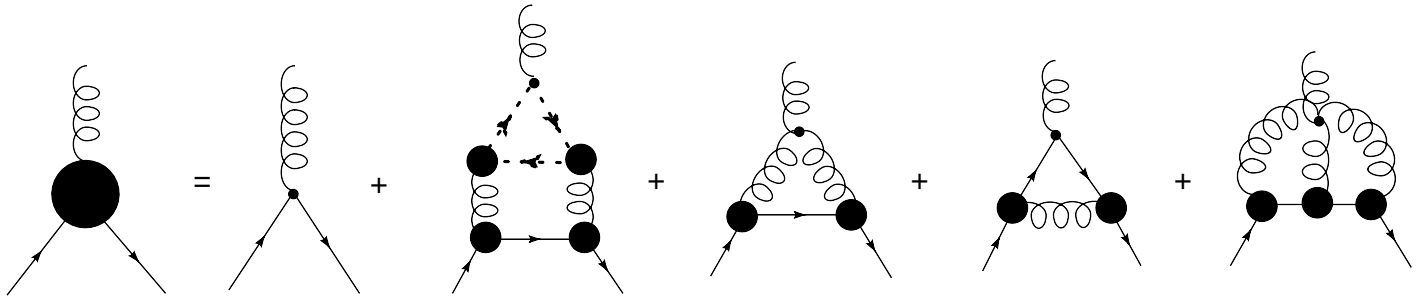


$$S(p) = \frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \to \frac{Z_f \not{p}}{M^2} + \frac{Z_f}{M}$$

$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G_\mu^i \,, \quad G_\mu^1 = \gamma_\mu \,, \; G_\mu^2 = \hat{p}_\mu \,, \; G_\mu^3 = \hat{\not{p}}\, \hat{p}_\mu \,, \; G_\mu^4 = \hat{\not{p}}\, \gamma_\mu$$

$$\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa}$$

General Infrared Structure

- Greens function with 2n ghost legs, m gluon legs, 2l quark legs (solves DSEs and STIs):

$$\Gamma^{2n,m,2l}(p^2) \sim (p^2)^{(n-m)\kappa - l/2}$$

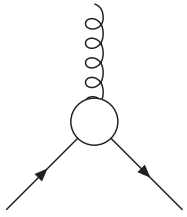
R. Alkofer, C. F., F. Llanes-Estrada, hep-ph/0607293

- Ghost sector generates IR-singularities

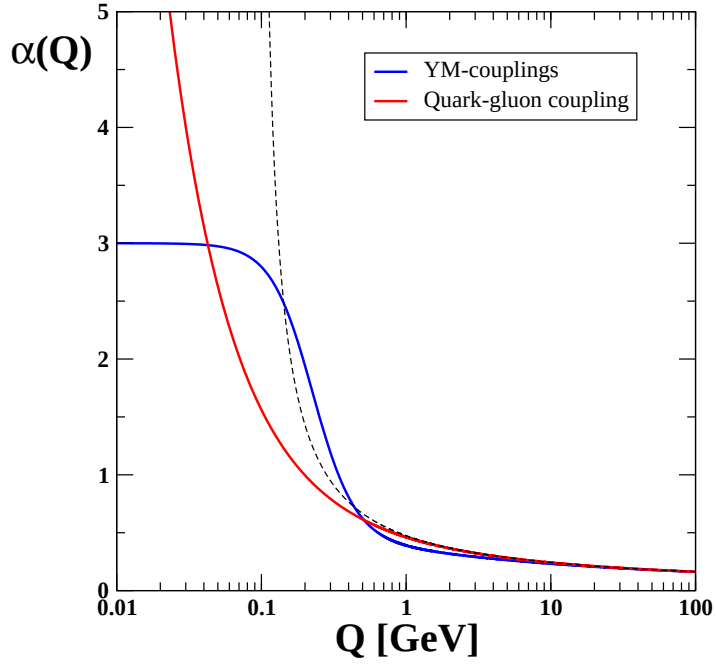
- Caveat: Uniqueness remains to be shown!

Running Coupling: IR-slavery

$$\Gamma^{qg}(p^2) \sim (p^2)^{-1/2-\kappa}\,, \quad Z_f(p^2) \sim const\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

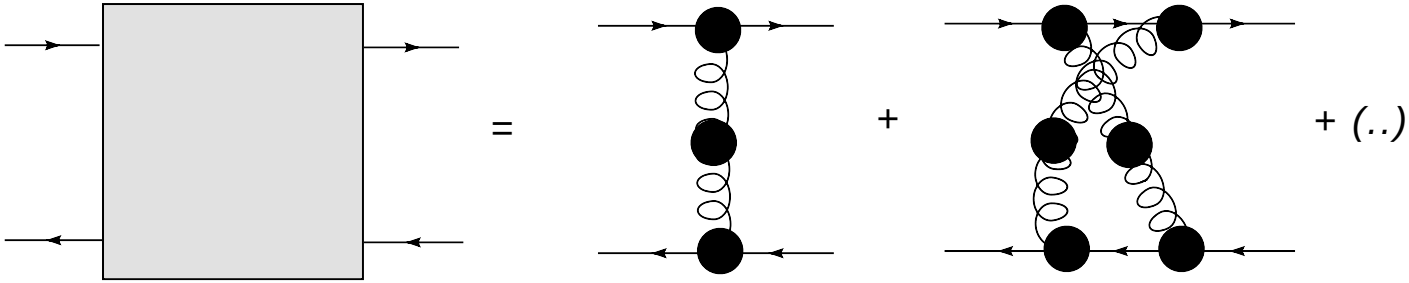


$$\alpha^{qg}(p^2) = \alpha_\mu \left[\Gamma^{qg}(p^2)\right]^2 \left[Z_f(p^2)\right]^2 Z(p^2) \sim \frac{const_{qg}}{N_c}\frac{1}{p^2}$$



The quark-antiquark potential

- quenched QCD



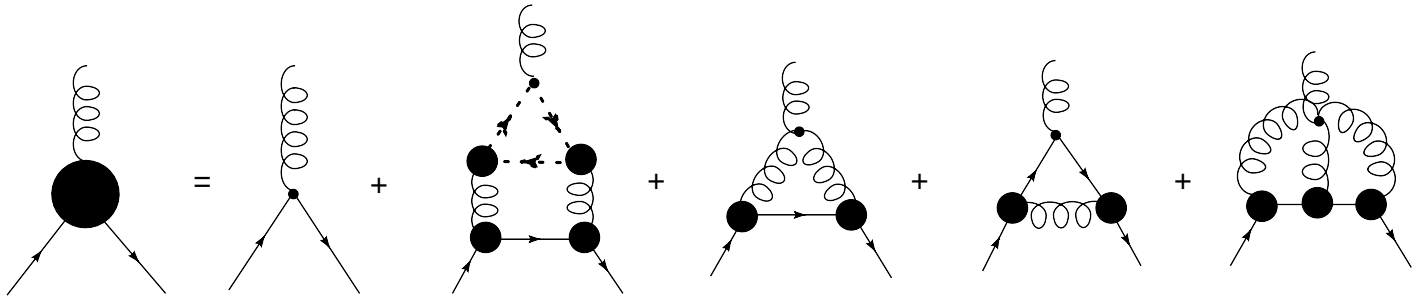
$$\Gamma^{0,0,2}(p^2) \sim (p^2)^{-1}$$

$$V(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3p \, e^{i\mathbf{pr}} \frac{(p^2)^{-1}}{p^2} \quad \sim \quad |\mathbf{r}|$$

Quark confinement

Chiral symmetry restoration I

- Quark-gluon vertex: **lowest order** in skeleton expansion



Chiral symmetry restoration I

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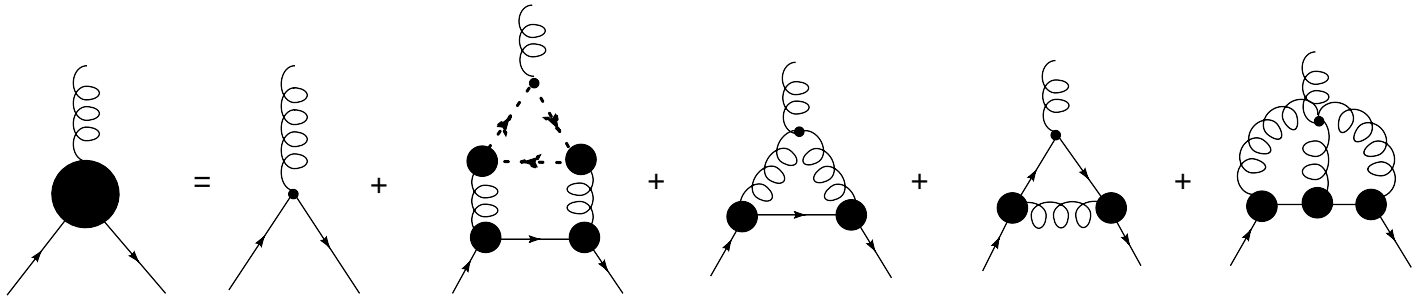
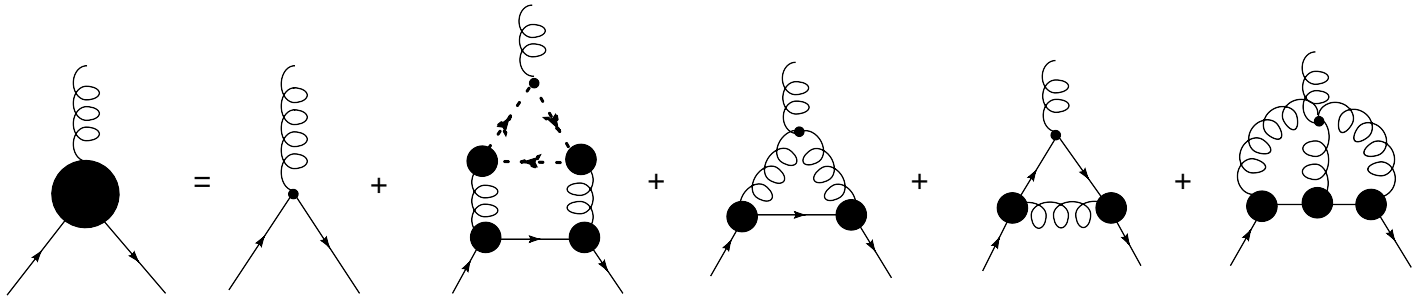


$$S(p) = \frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \rightarrow \frac{Z_f \not{p}}{p^2}$$

Chiral symmetry restoration I

- Quark-gluon vertex: **lowest order** in skeleton expansion



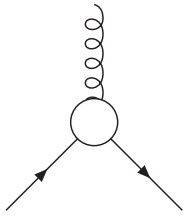
$$S(p) = \frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \to \frac{Z_f \not{p}}{p^2}$$

$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G_\mu^i \,, \quad G_\mu^1 = \gamma_\mu \,, \;\; G_\mu^2 = \hat{p}_\mu \,, \;\; G_\mu^3 = \hat{\not{p}}\, \hat{p}_\mu \,, \;\; G_\mu^4 = \hat{\not{p}}\, \gamma_\mu \,,$$

$$\lambda_1 \sim (p^2)^{-\kappa} \,, \;\; \lambda_2 = 0 \,, \;\; \lambda_3 \sim (p^2)^{-\kappa} \,, \;\; \lambda_4 = 0$$

Chiral symmetry restoration II

- Running coupling: Restoration of fixed point



$$\alpha^{qg}(p^2) = \alpha_\mu \left[\Gamma^{qg}(p^2)\right]^2 \left[Z_f(p^2)\right]^2 Z(p^2) \sim const_{qg}$$

- Quark-antiquark potential: Deconfinement

$$\Gamma^{0,0,2}(p^2) \sim const.$$

$$V(\mathbf{r}) \sim \frac{1}{(2\pi)^3} \int \frac{1}{p^2} e^{i\mathbf{pr}} d^3p \quad \sim \quad \frac{\mathbf{1}}{|\mathbf{r}|}$$

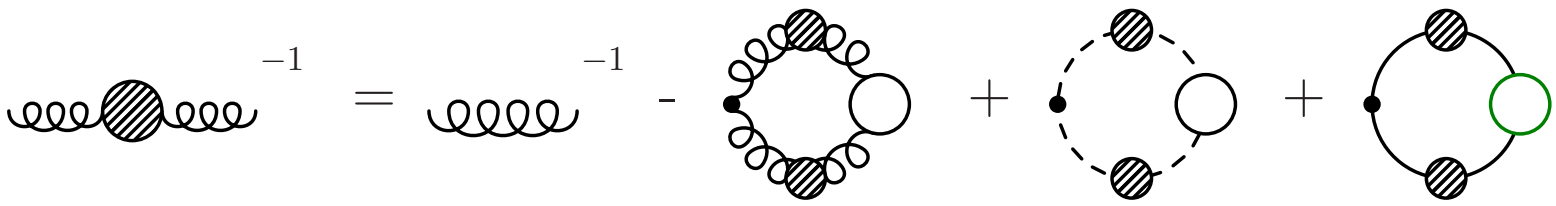
Chiral symmetry breaking $\leftrightarrow$ Quark confinement

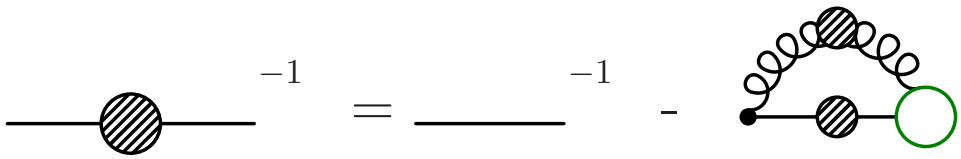
R. Alkofer, C. F., F. Llanes-Estrada, hep-ph/0607293

Outline

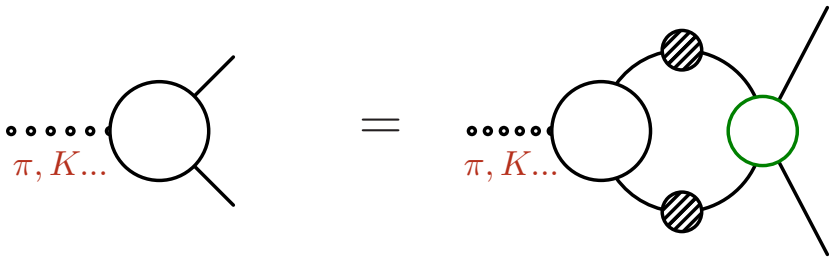
- Introduction
- Infrared fixed points in SU(N) Yang-Mills theory
- Infrared Slavery in quenched QCD
- **Light mesons: unquenching effects**

DSEs and BSE

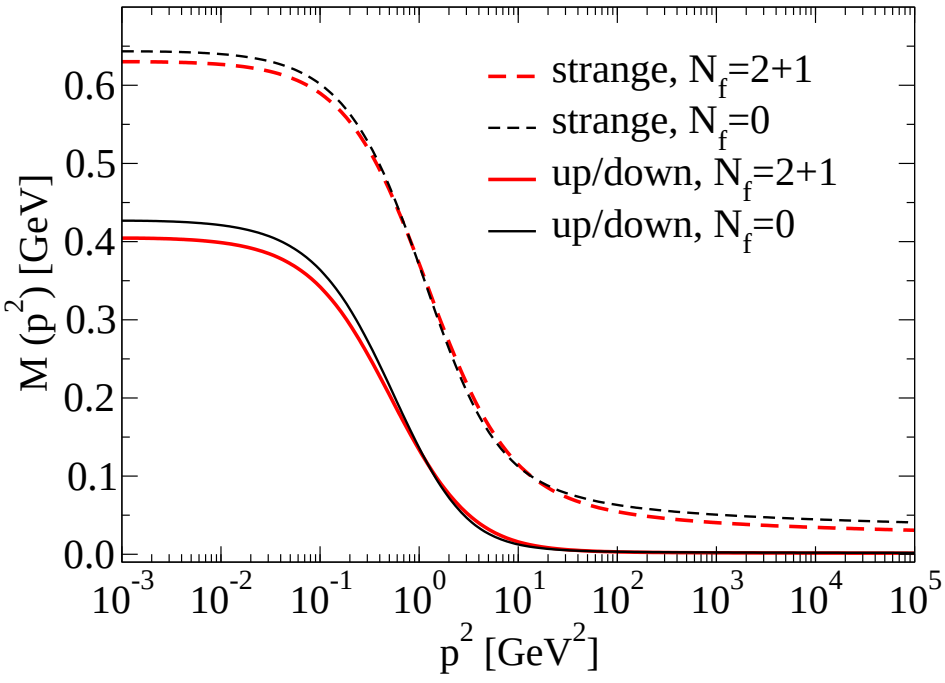


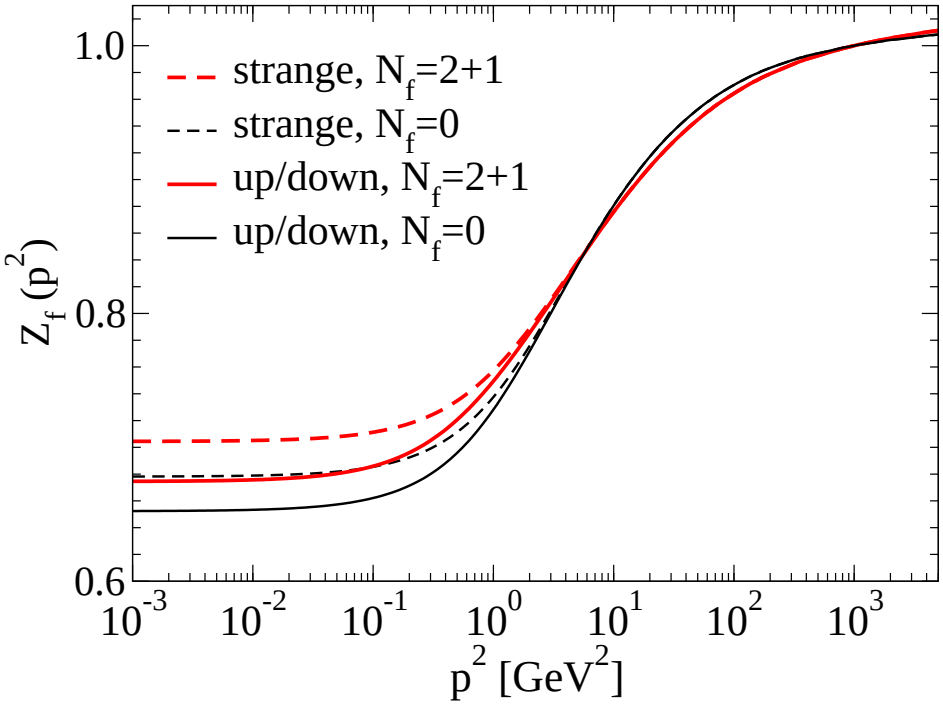


$$\Gamma_\mu(p,k) \;\sim\; \gamma_\mu \; G^2(k) \; A(k)$$



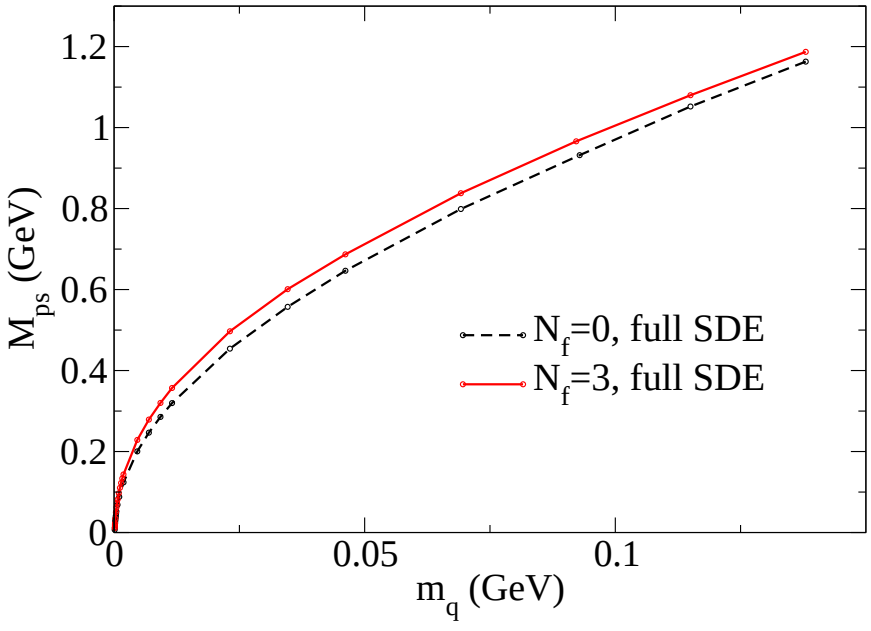
Unquenched Quark

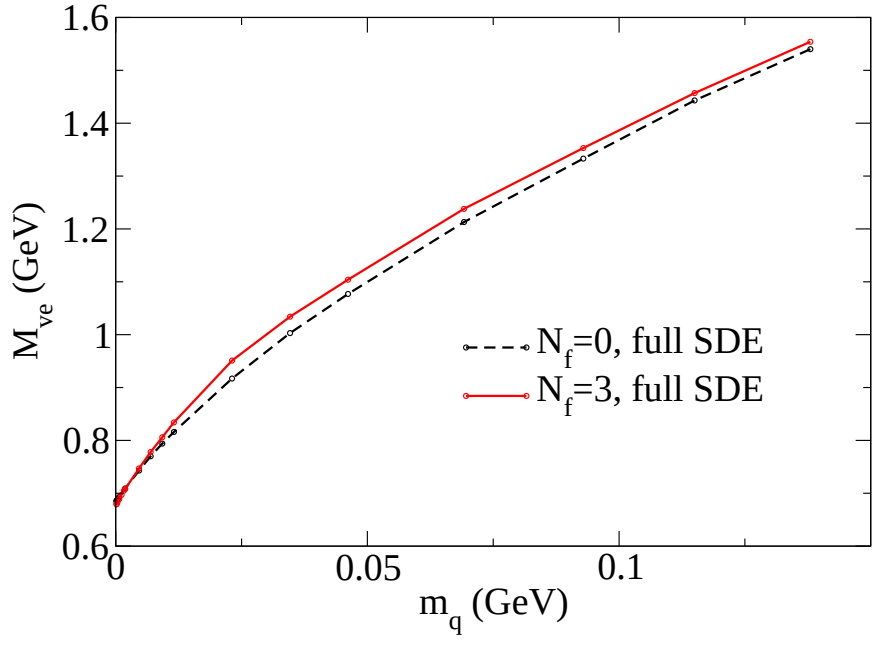




	$-(<\overline{q}q>^0)^{1/3}$ (MeV)	$M_{ch}(p^2=0)$ (MeV)
$N_f=0$	266	416
$N_f=2+1$	271	388

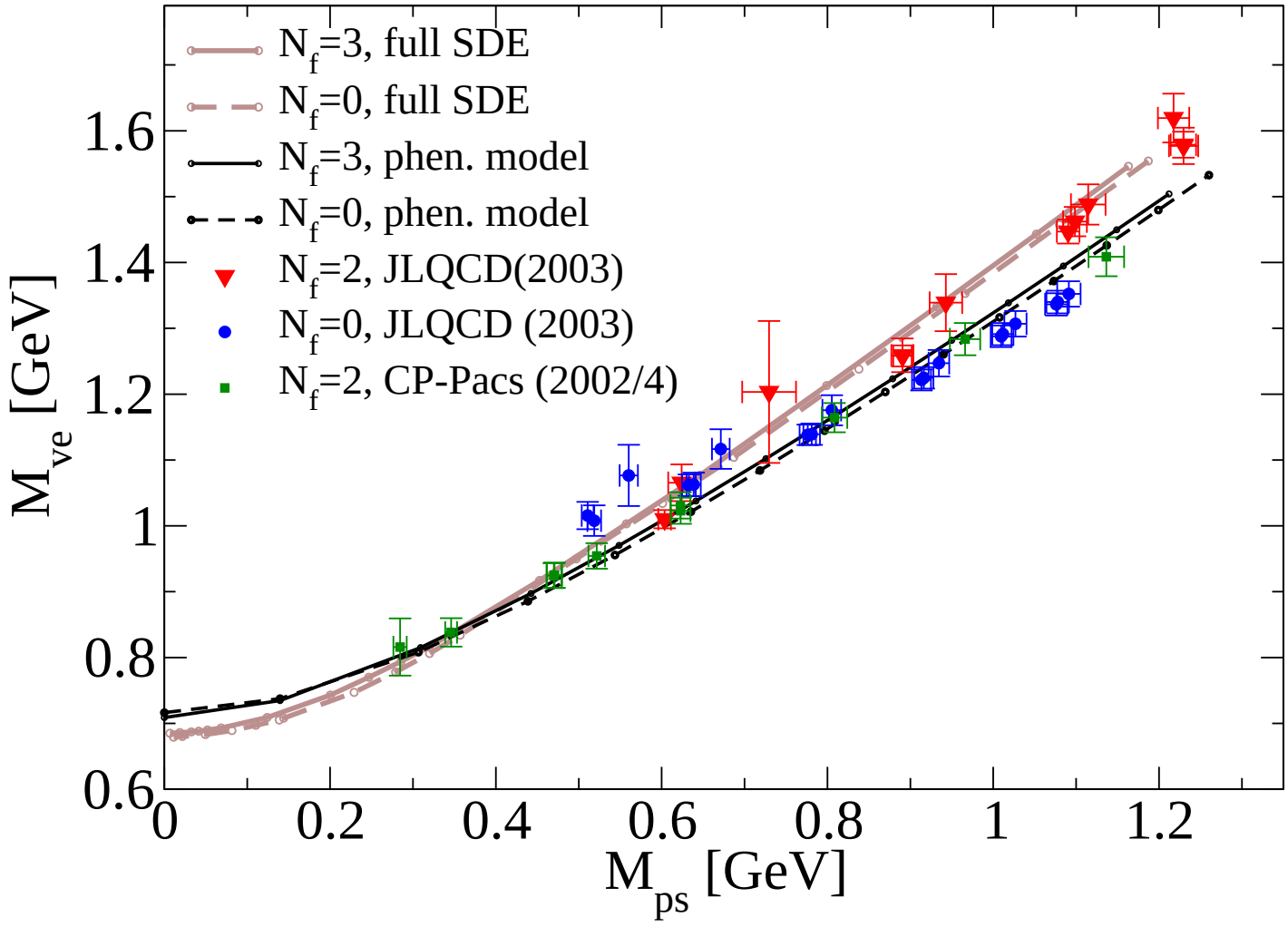
Unquenching: Light Mesons I





	m_u	m_s	m_π	f_π	m_K	f_K	m_ρ
$N_f = 0$	4.17	88.2	139.7	130.9	494.5	165.6	708.0
$N_f = 2 + 1$	4.06	86.0	140.0	131.1	493.3	169.5	695.2
$N_f = 3$	4.06		139.7	130.8			690.0
PDG	3.0-5.5	80-130	139.6	130.7	493.7	160.0	770

Unquenching: Light Mesons II



C. F., P. Watson and W. Cassing, Phys. Rev. D **72** (2005) 094025

Quenched QCD with DχSB

- 1PI-Greens functions (Landau gauge):

$$\Gamma^{2n,m,l}(p^2) \sim (p^2)^{(n-m)\kappa - l/2}$$

- YM-couplings: Universal IR-fixed point
  - Unique
  - Present in transverse gauges (incl. Coulomb gauge)
- Quark-gluon-couplings: Infrared slavery
- Quark-quark-potential: Linearly rising

Quenched QCD without DχSB

- 1PI-Greens functions (Landau gauge):

$$\Gamma^{2n,m,l}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Quark-gluon-couplings: IR-fixed point
- Quark-quark-potential: Deconfinement

Chiral phase transition $\leftrightarrow$ Deconfinement transition

- Caveat: Uniqueness of IR-solution in quark sector remains to be shown