Are there plasminos in superconductors?

Barbara Betz and Dirk-H. Rischke

Institut für Theoretische Physik

Johann Wolfgang Goethe-Universität

Frankfurt am Main

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  - ◇ J.-P. Blaizot and J.-Y. Ollitrault, Phys.Rev.D 48,1390 (1993)
  - ◇ R.D.Pisarski, Nucl.Phys. A 498,423c (1989)
  - ◇ R.D.Pisarski, D.H. Rischke, Phys.Rev.D 60,094013 (1999)

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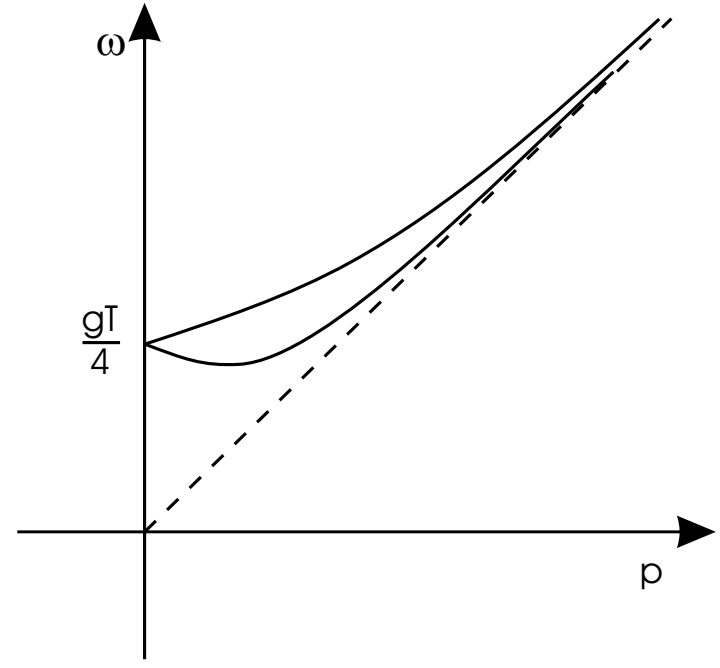
  $\longrightarrow$ 2 types of excitations



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Phys. Rev. D 46 (1992) 4043

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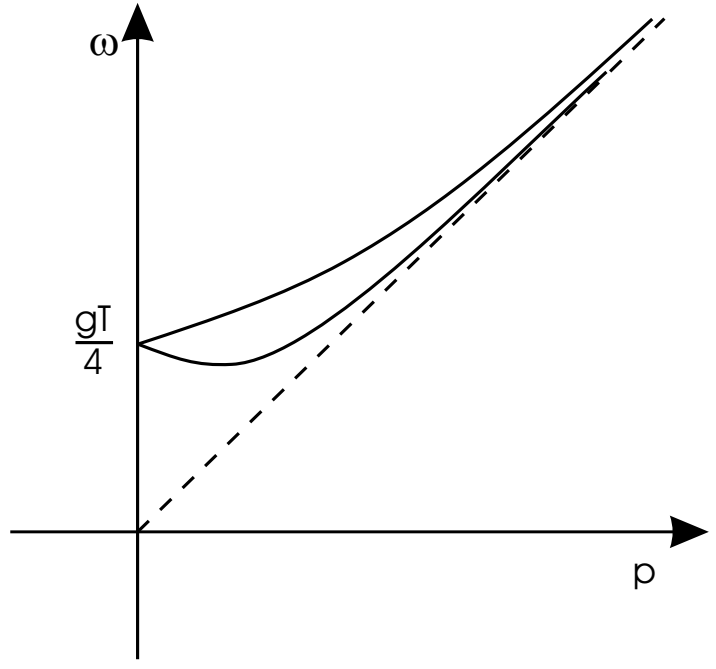

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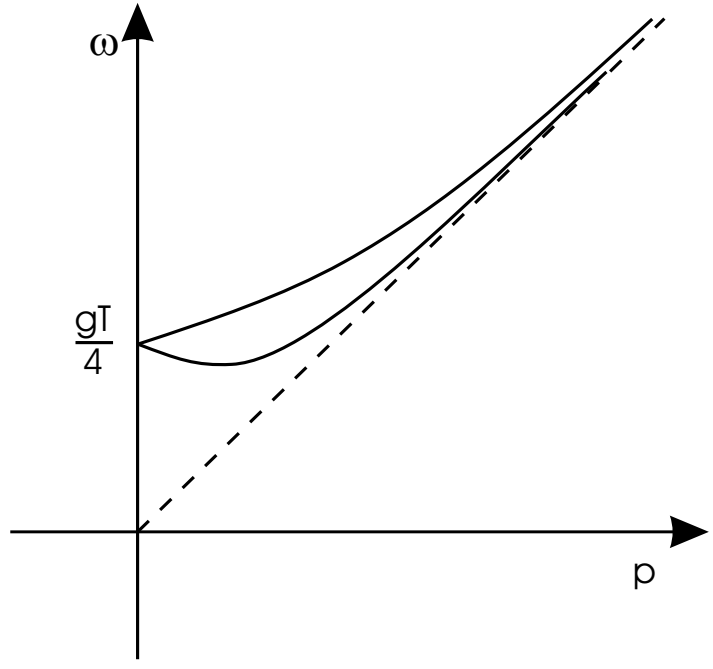

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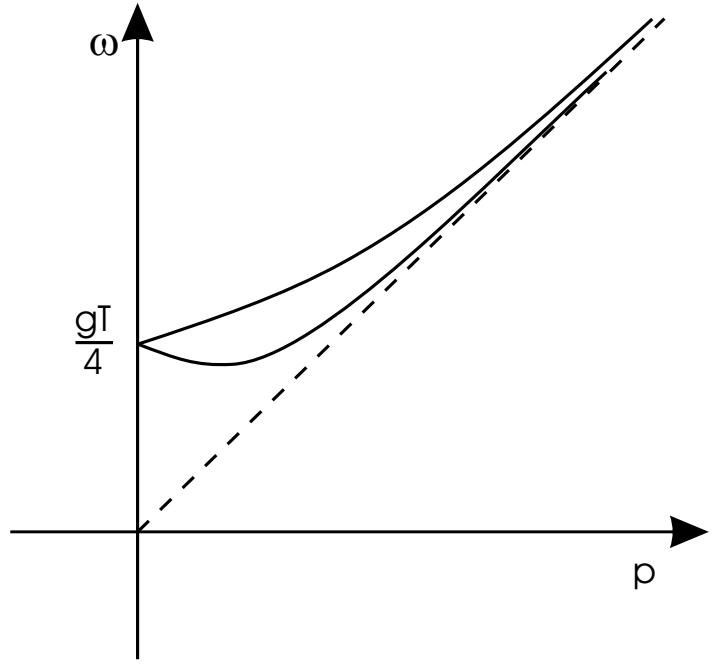
  $\Rightarrow$ plasmino and anti-plasmino



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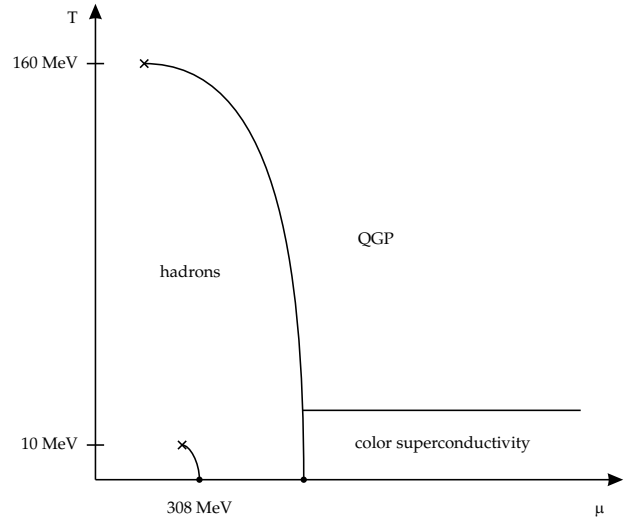

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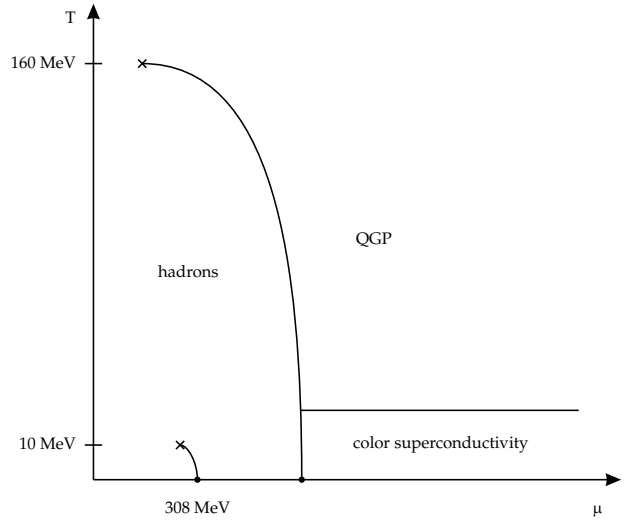

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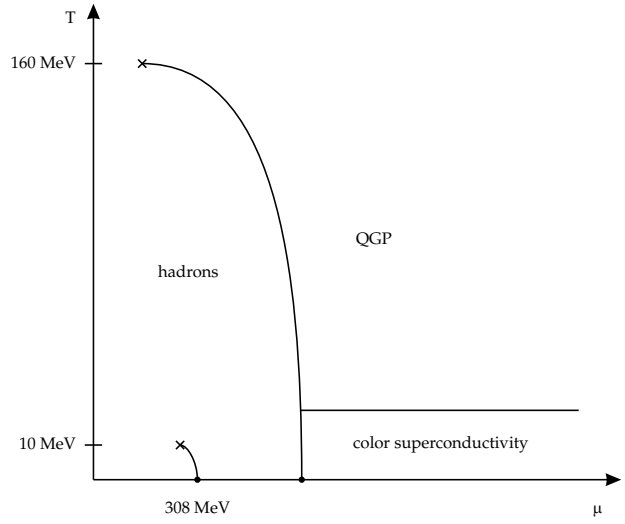

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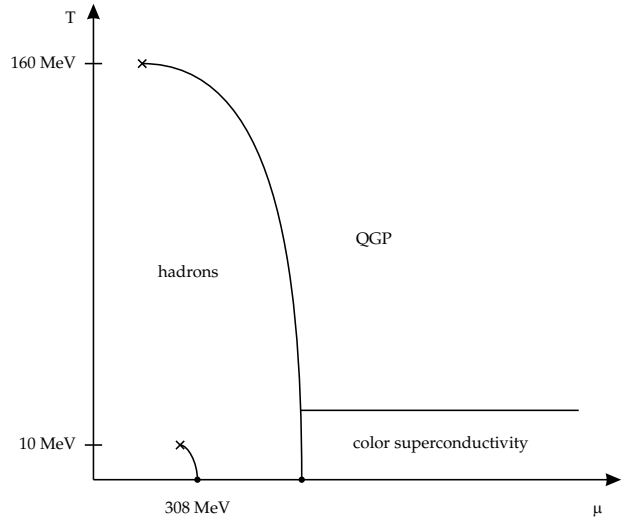

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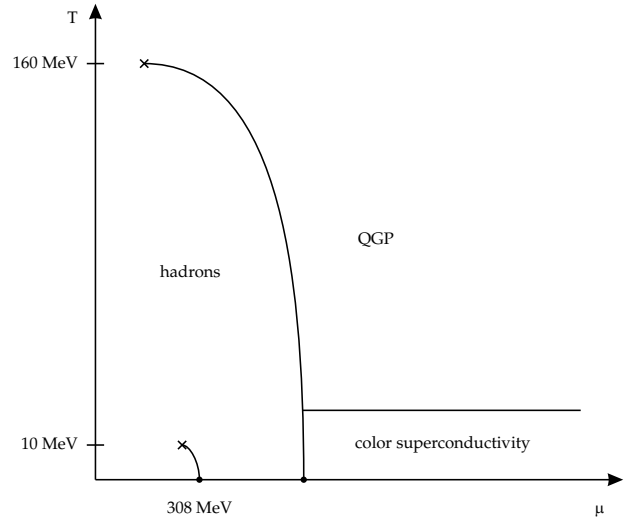

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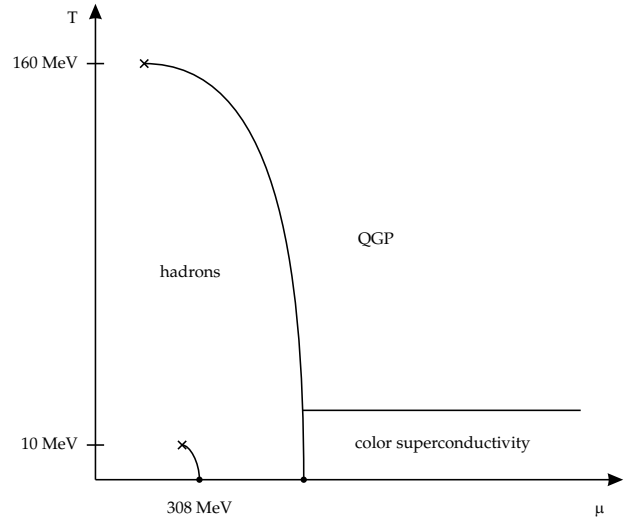

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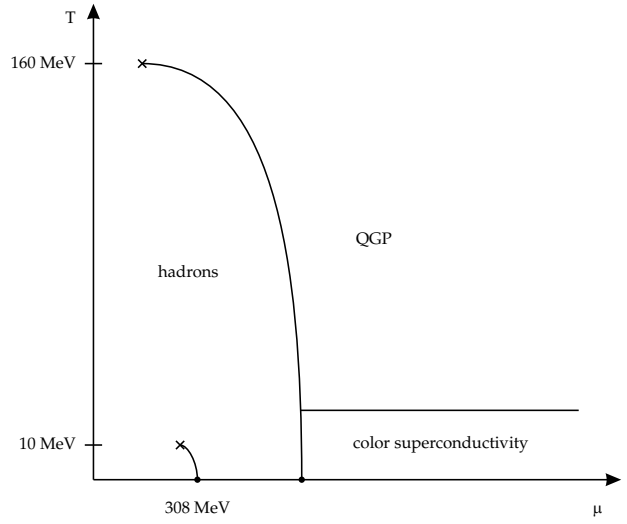

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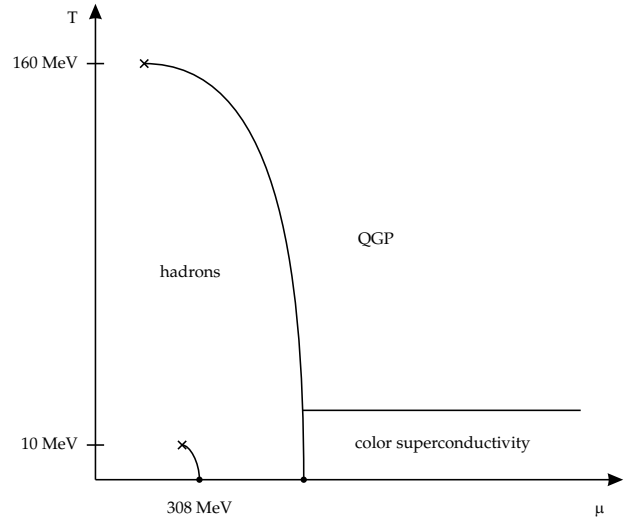

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- Do plasminos occur in cold and superconducting matter?
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  - ◇ in the limit of $T \longrightarrow 0$

Normal-conducting fermions – Self-energy

- fermions interacting with scalar bosons

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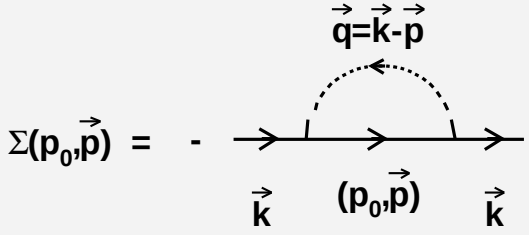

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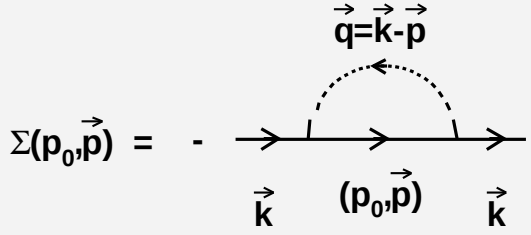

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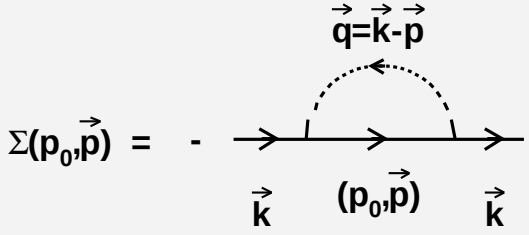

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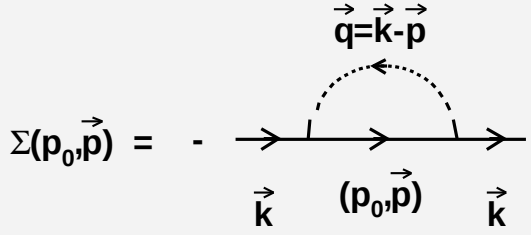

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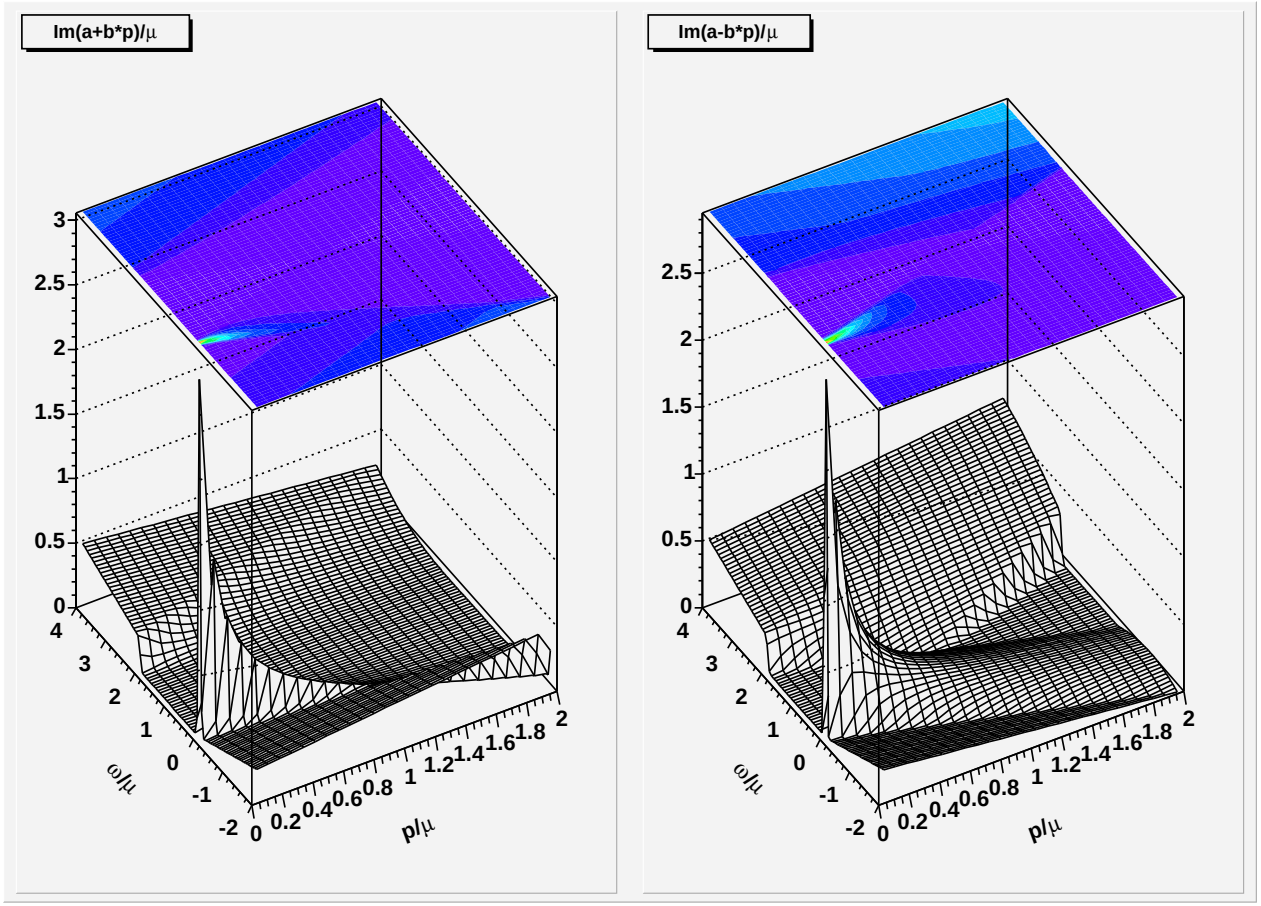
Normal-conducting fermions – Imaginary part of the self-energy



massless fermions (left panel) and antifermions (right panel)

for $g^2/4\pi = 1$, at $T = 0$

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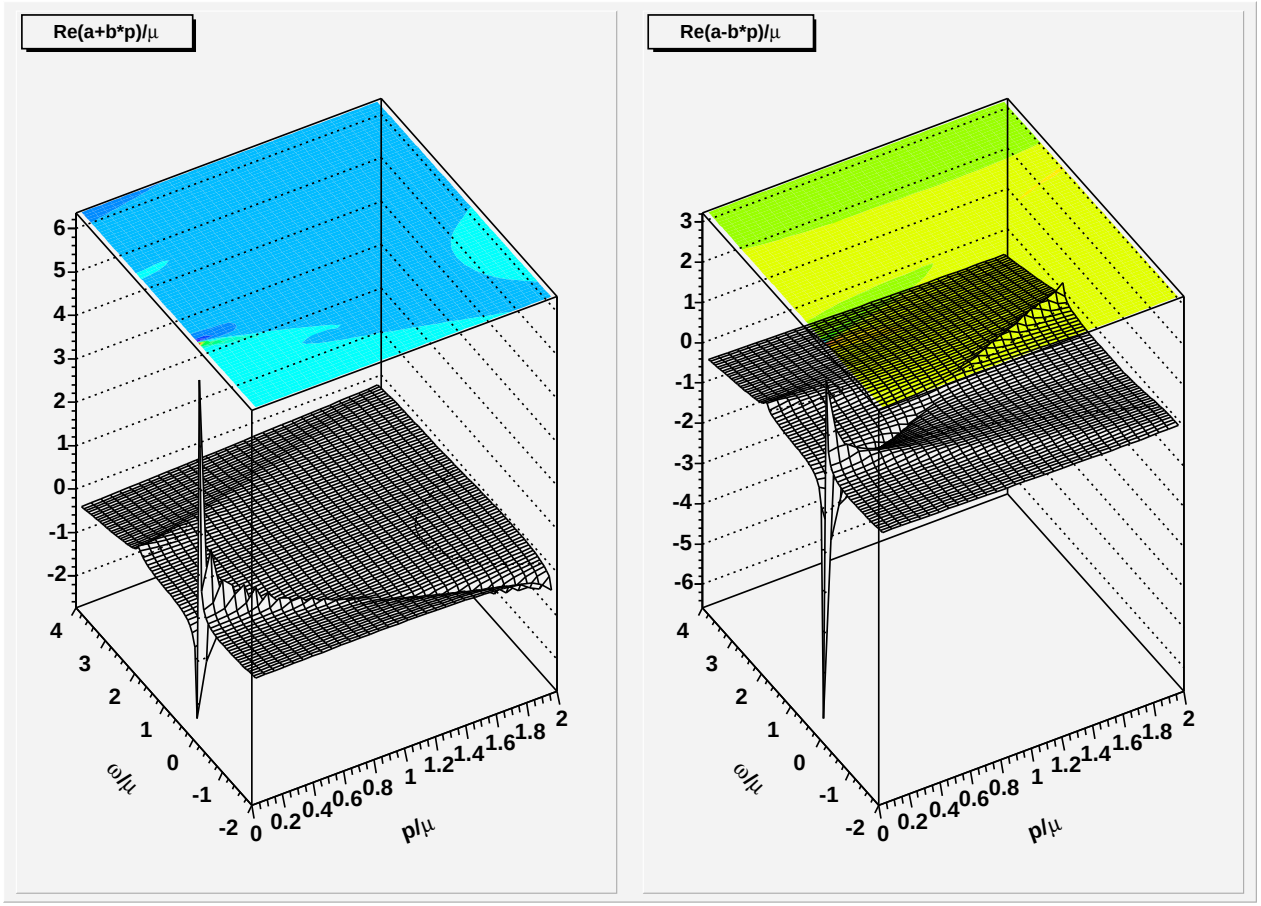

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- is given by the roots of the full inverse propagator
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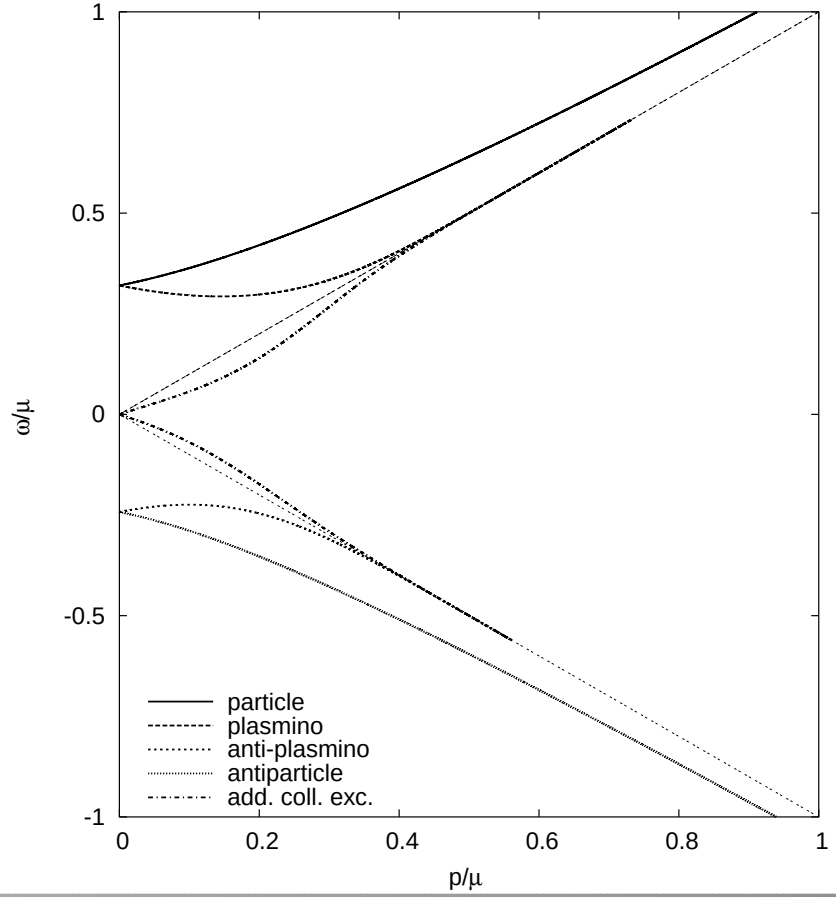

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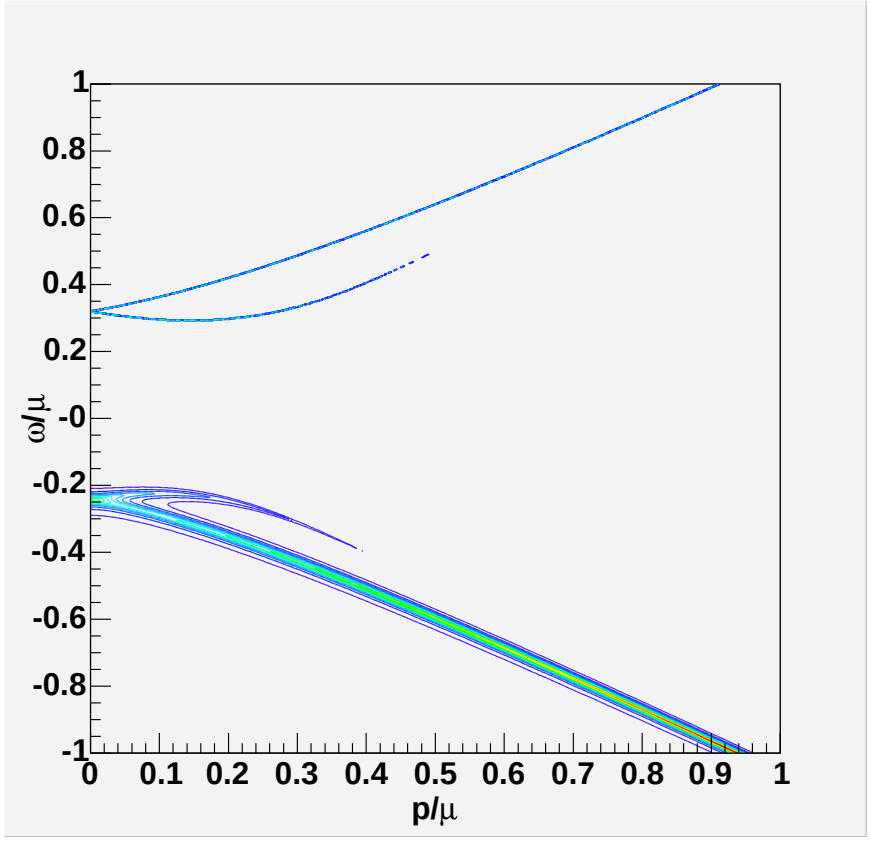

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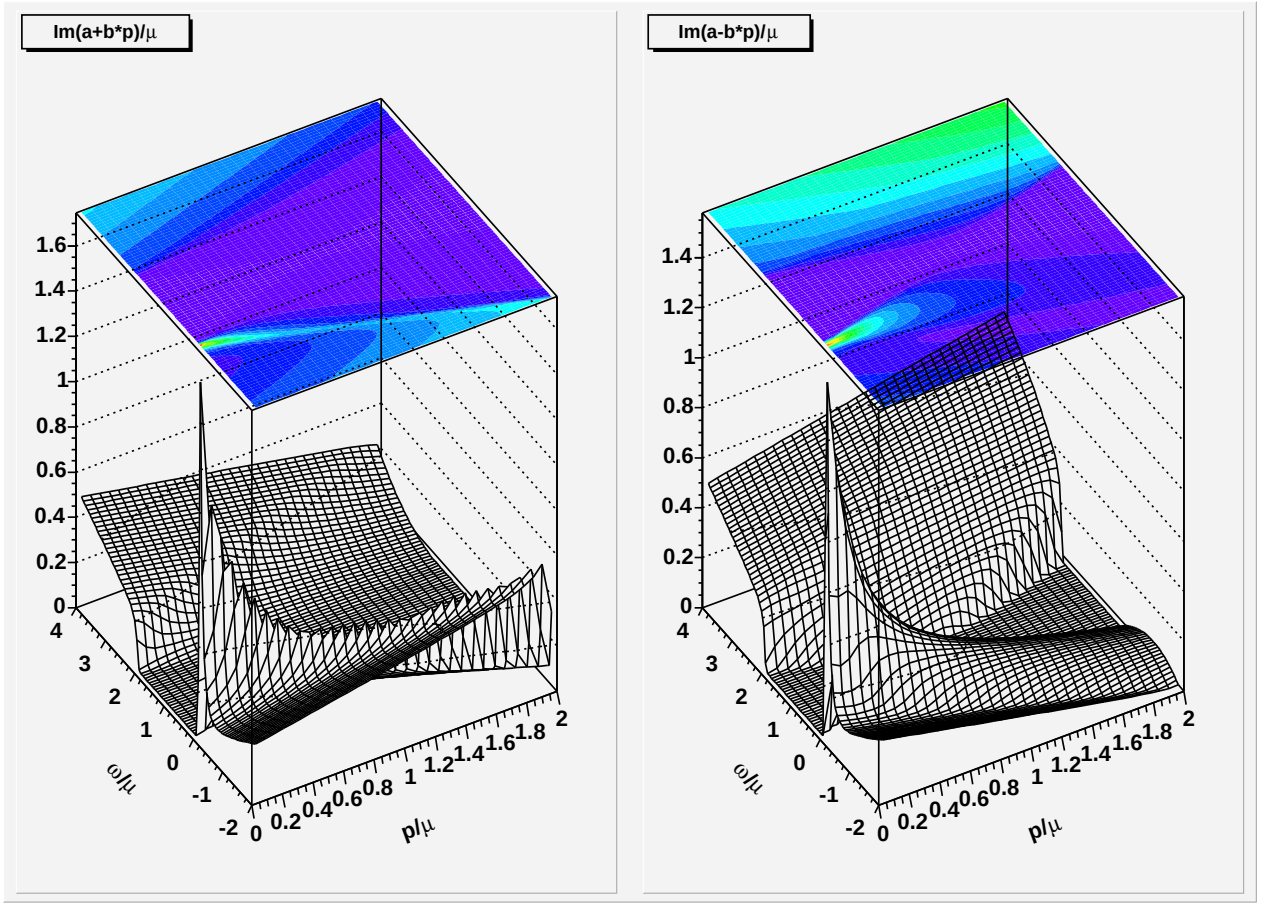
  - ◇ gap function of particles $\phi_+ = 0.25\,\mu$ and
  - ◇ gap function of antiparticles $\phi_- = 0$ leads to ...

Superconducting fermions – Imaginary part of the self-energy



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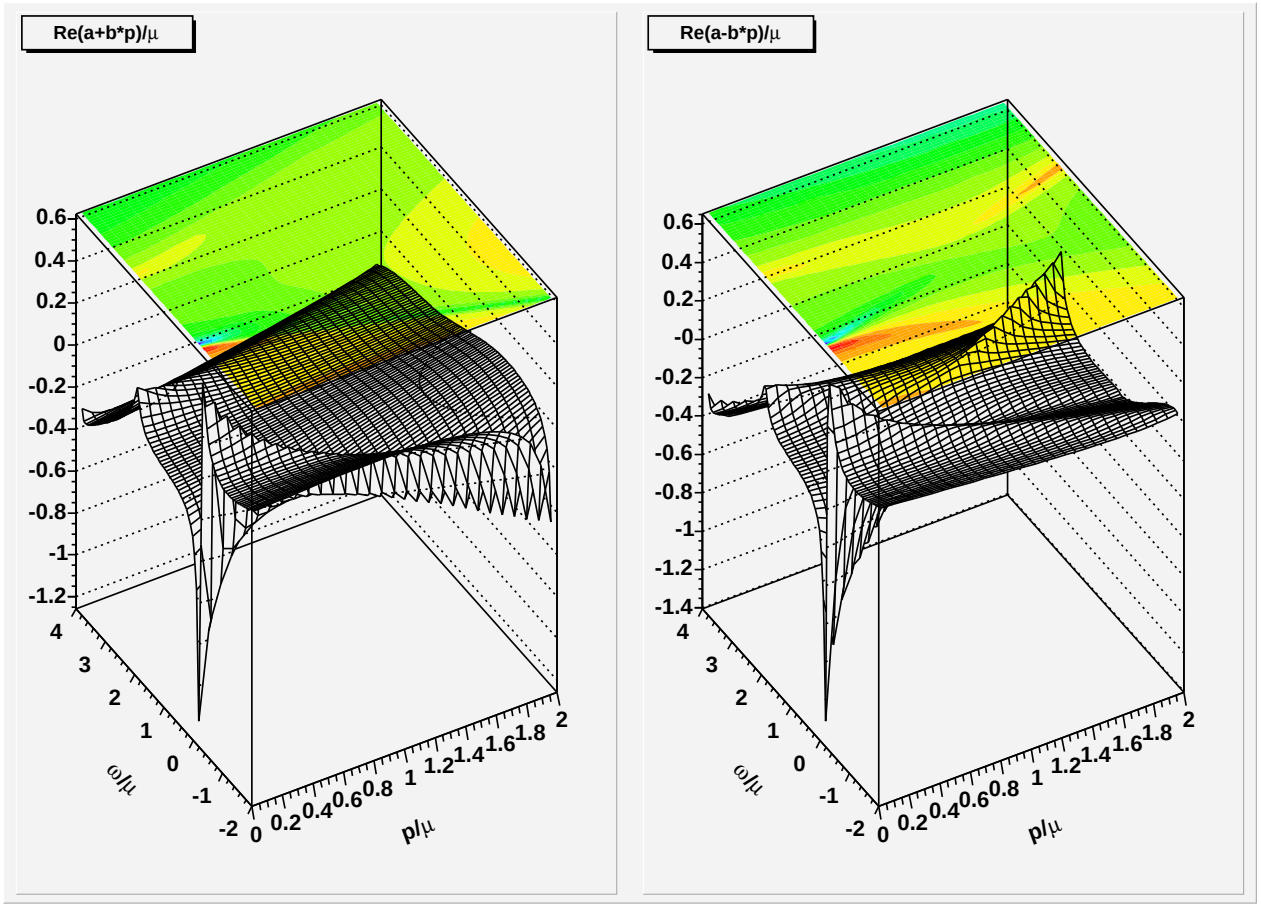

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$$\Sigma^{-}(P) = -\gamma_0 \sum_{e=\pm} \Sigma_e(-P) \, \Lambda^{-e}_{\vec{p}}$$

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$$\Rightarrow\ \mathcal{G}_+^+(P) = \frac{p_0-\mu+p-\Sigma_+(-P)}{[p_0+\mu-p+\Sigma_+(P)][p_0-\mu+p-\Sigma_+(-P)]-|\phi_+(P)|^2}$$

$$\mathcal{G}_-^+(P) = \frac{1}{p_0+\mu+p+\Sigma_-(P)}$$

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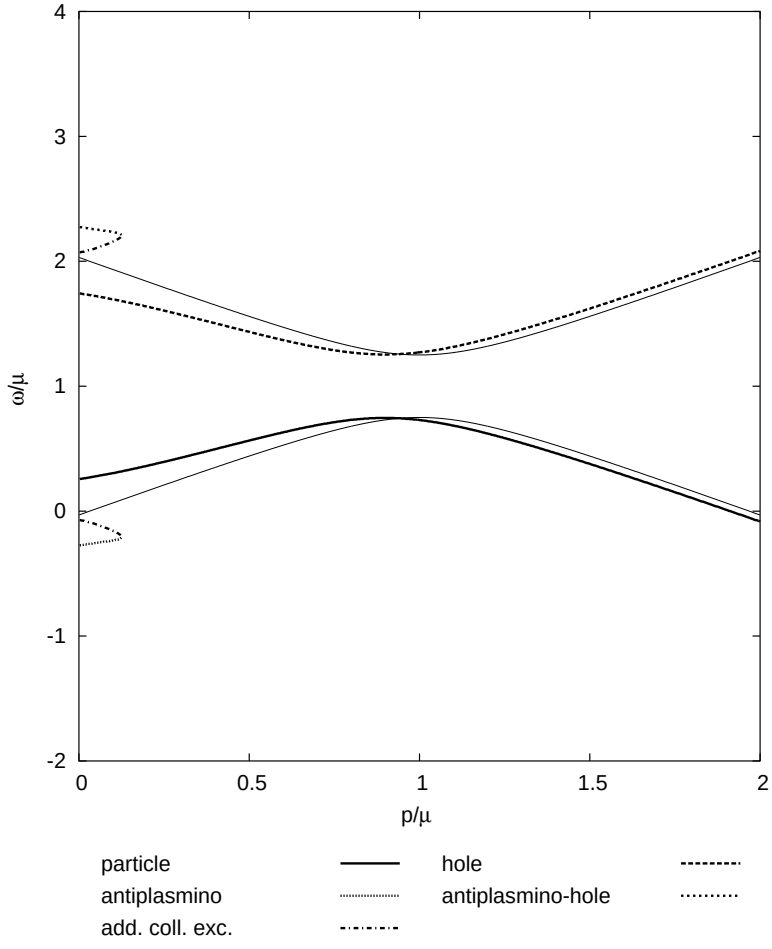

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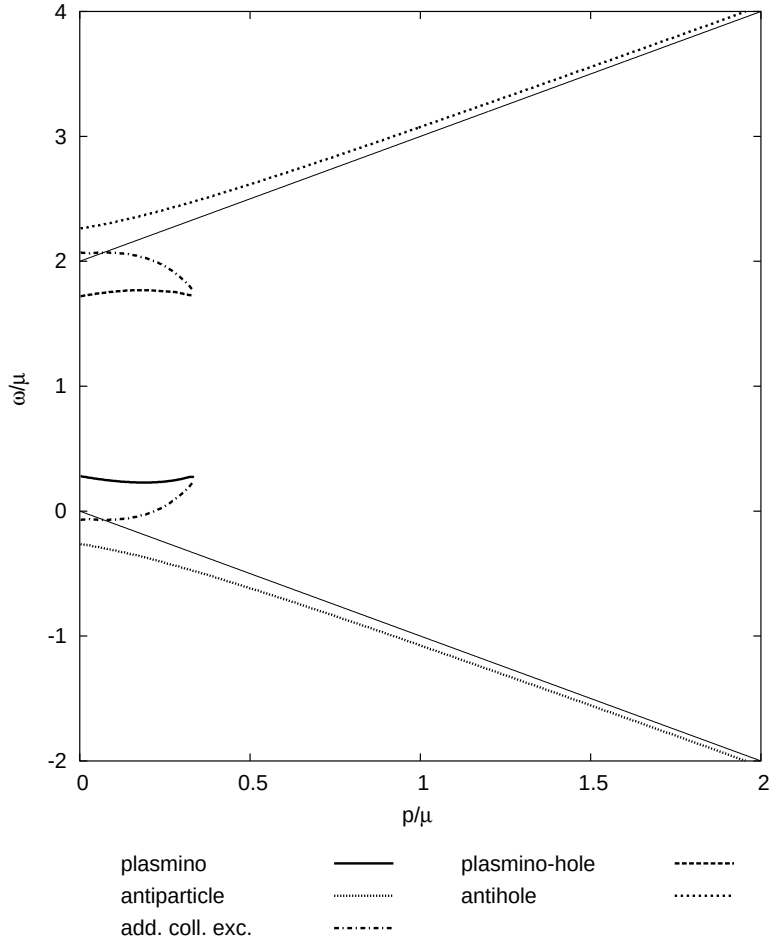

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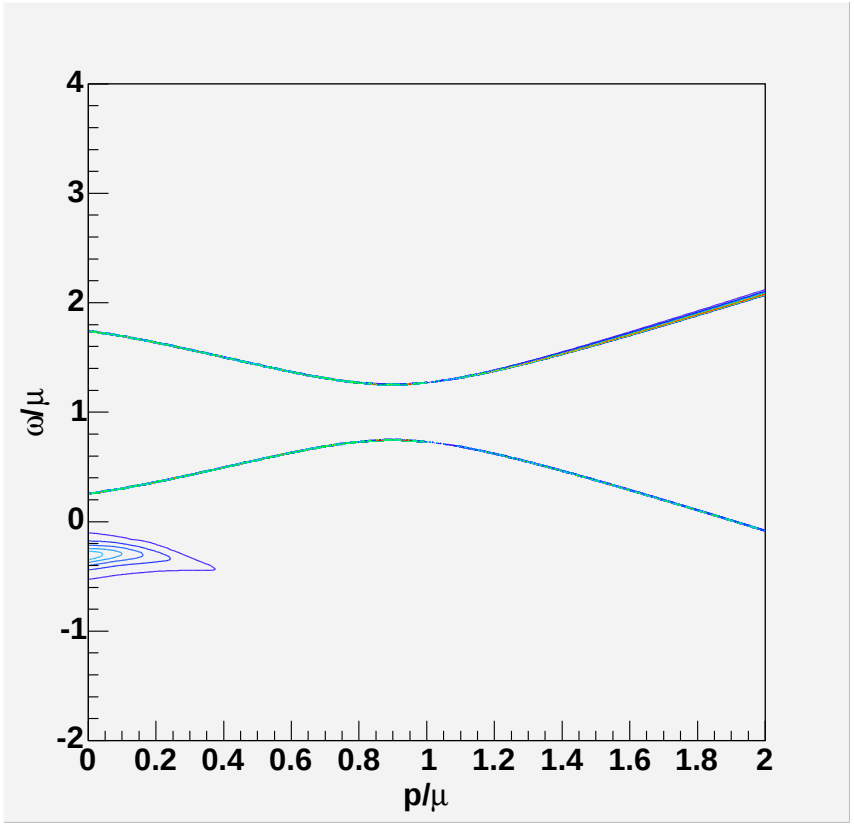
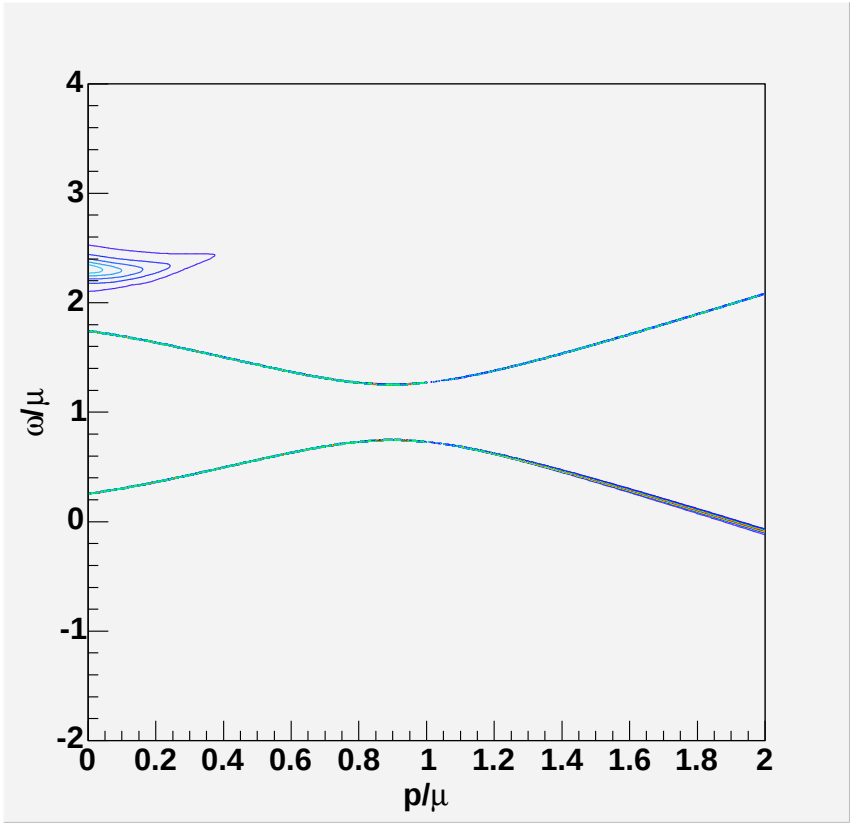


spectral density ρ_+^+ for $g^2/(4\pi) = 1$, $T = 0$



spectral density ρ_-^- for $g^2/(4\pi) = 1$, $T = 0$

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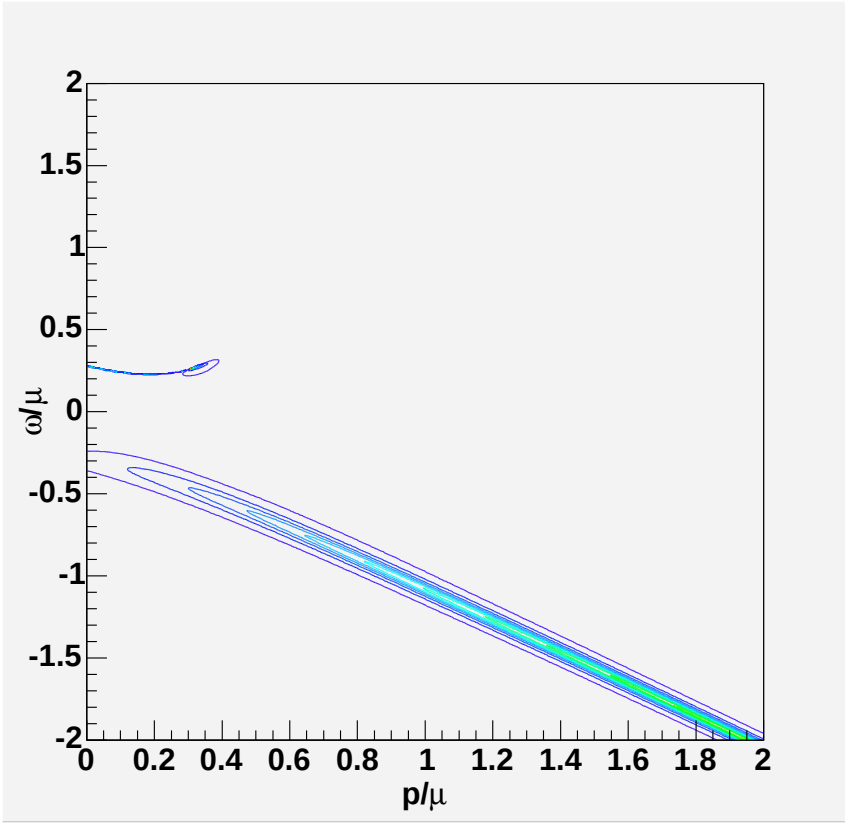

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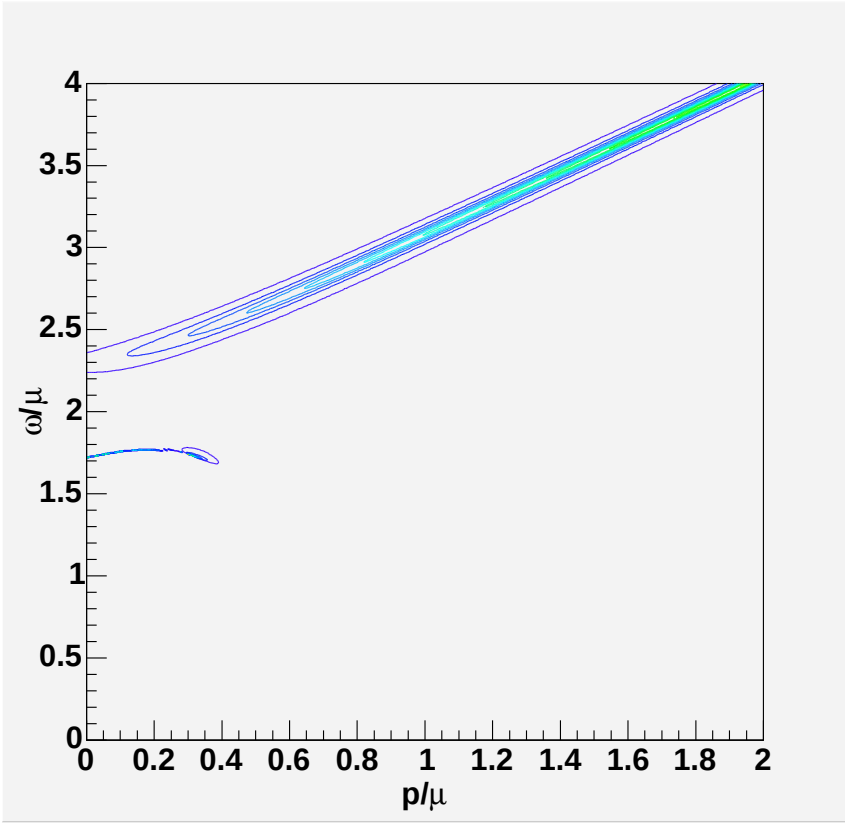

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- Superconducting excitation spectrum is the extension of the normal-conducting one.

Superconducting fermions – Dispersion relation

- is given by the poles of the propagator

$$\mathcal{G}^{\pm} = \left([G_0^{\pm}]^{-1} + \Sigma^{\pm} - \Phi^{\mp} \left\{ [G_0^{\mp}]^{-1} + \Sigma^{\mp} \right\}^{-1} \Phi^{\pm} \right)^{-1}$$

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- for quasiparticles

$$\mathcal{G}^{+} = \left([G_0^{-}]^{-1} + \Sigma^{-} \right) \left\{ \left([G_0^{+}]^{-1} + \Sigma^{+} \right) \left([G_0^{-}]^{-1} + \Sigma^{-} \right) - \Phi^{-} \left([G_0^{-}]^{-1} + \Sigma^{-} \right)^{-1} \Phi^{+} \left([G_0^{-}]^{-1} + \Sigma^{-} \right) \right\}^{-1}$$

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- 2SC-Phase

$$\Phi^- \left([G_0^-]^{-1} + \Sigma^- \right)^{-1} \Phi^+ \left([G_0^-]^{-1} + \Sigma^- \right) = \sum_{e=\pm} |\phi^e(P)|^2 L_{\vec{p}} \Lambda^{-e}_{\vec{p}}$$

$$\star\ L_{\vec{p}} = (J_3)^2 (\tau_2)^2 = (\delta_{ij} - \delta_{i3}\delta_{j3})\delta_{fg}$$