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MASSIVE SCALAR FIELDS IN THE EARLY UNIVERSE

ALEXANDER ZHUK*

Departamento de Física, Universidade Federal da Paraíba,
CEP 58059-970 João Pessoa, PB, Brazil
zhuk@paco.net

UWE GÜNTHER†

Institut für Mathematik, Universität Potsdam,
D-14415 Potsdam, Germany
u.guenther@fz-rossendorf.de

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We discuss the role of gravitational excitons/radions in different cosmological scenarios. Gravitational excitons are massive moduli fields which describe conformal excitations of the internal spaces and which, due to their Planck-scale suppressed coupling to matter fields, are WIMPs. It is demonstrated that, depending on the concrete scenario, observational cosmological data set strong restrictions on the allowed masses and initial oscillation amplitudes of these particles.

Keywords: Scalar fields; early Universe; excitons; radions.

1. Introduction

The concept of scalar fields is widely used in cosmology. Such fields can have different origin, e.g. they can be Higgs fields in 4D GUT, dilaton fields in string theoretic models, geometrical moduli fields in different multidimensional setups. They can play an important role in the early Universe because all of them, in principle, can be considered as inflaton. On the other hand, the late time acceleration of the Universe can also be explained by the presence of such fields (in quintessential models). Additionally, they will be connected with various observable phenomena so that cosmological and astrophysical data will strongly constrain the parameters of these fields. In the present paper, we shall mainly concentrate on gravitational excitons (for short gravexcitons)/radions ψ which are conformal (geometrical moduli)

*Address on leave: Department of Physics, University of Odessa, 2 Dvoryanskaya St., Odessa 65100, Ukraine.
†Present address: Research Center Rossendorf, D-01314 Dresden, Germany.

excitations of the internal spaces in multidimensional models.[1] We suppose that due to the stabilization of the internal spaces, the gravexcitons acquire a mass m_ψ. The characteristic feature of these particles is their Planck scale suppressed coupling $1/M_{\rm Pl}$ to standard matter (SM) fields. As a result, the decay rate for their decay into observable matter is small: $\Gamma \sim m_\psi^3/M_{\rm Pl}^2 \ll m_\psi$. From this point of view, gravexcitons are WIMPs (Weakly-Interacting Massive Particles[2]). It is worth noting, that a similar Planck scale suppressed interaction and corresponding WIMP-like behavior holds for Polonyi particles in spontaneously broken supergravity, scalarons in the $(R + R^2)$ fourth order theory of gravity or moduli fields in the hidden sector of SUSY. In the present work we demonstrate how cosmological data can constrain gravexciton-related parameters for different possible scenarios. In particular, we show how restrictions can be derived on the masses of the gravexcitons as well as on the amplitudes of their initial oscillations.

2. Effective Equation of Motion

The effective equation of motion for massive gravexcitons is (for details see Ref. 3):

$$\ddot{\psi} + (3H + \Gamma)\dot{\psi} + m_\psi^2 \psi = 0\,, \tag{1}$$

where (by analogy with Ref. 4) we took into account the effective decay of gravexcitons into ordinary matter, e.g. into 4D photons:

$$\Gamma \sim m_\psi \left(\frac{m_\psi}{M_{\rm Pl}}\right)^2 \ll m_\psi\,. \tag{2}$$

In Refs. 3 and 5 it was shown that the gravexciton production due to interactions with matter fields is negligible for the models under consideration. A corresponding source term on the RHS of Eq. (1) can, hence, be omitted.

The investigation of Eq. (1) is most conveniently started by a substitution

$$\psi := B(t)u(t) := M_{\rm Pl} e^{-\frac{1}{2}\int(3H+\Gamma)dt} u(t)\,, \tag{3}$$

which, for constant Γ, leads to the following differential equation (DE) for the auxiliary function $u(t)$:

$$\ddot{u} + \left[m_\psi^2 - \frac{1}{4}(3H + \Gamma)^2 - \frac{3}{2}\dot{H}\right]u = 0\,. \tag{4}$$

This equation shows that at times when the Hubble parameter $H = s/t \sim 1/t$ is less than the mass

$$H < m_\psi \quad \Longrightarrow \quad t > t_{\rm in} \sim H_{\rm in}^{-1} \sim \frac{1}{m_\psi}\,, \tag{5}$$

the scalar field oscillates

$$\psi \approx CB(t)\cos(m_\psi t + \delta)\,. \tag{6}$$

The time $t_{\rm in}$ roughly indicates the beginning of the oscillations. Substituting the Hubble parameter $H = s/t$ into the definition of $B(t)$ we obtain

$$B(t) = M_{\rm Pl}\, e^{-\frac{1}{2}\Gamma t} \frac{1}{(M_{\rm Pl} t)^{3s/2}}\,, \tag{7}$$

where $s = 1/2,\ 2/3$ for radiation dominated (RD) and matter dominated (MD) stages, respectively. The parameter C in Eq. (6) can be obtained from the amplitude of the initial oscillation $\psi_{\rm in}$:

$$\psi_{\rm in} \sim CB(t_{\rm in}) \quad\Longrightarrow\quad C_r \sim \frac{\psi_{\rm in}}{M_{\rm Pl}}\left(\frac{M_{\rm Pl}}{m_\psi}\right)^{3/4}, \quad C_m \sim \frac{\psi_{\rm in}}{m_\psi}\,. \tag{8}$$

C_r and C_m correspond to particles which start to oscillate during the RD and MD stages. Additionally, we took into account that $\Gamma t_{\rm in} \sim \Gamma/m_\psi \ll 1$.

Further useful differential relations are those for $B(t)$ in Eqs. (3) and (7), and for the energy density ρ_ψ and the number density n_ψ of the gravexcitons. It can be easily seen from the definition of $B(t)$ that this function satisfies the DE

$$\frac{d}{dt}(a^3 B^2) = -\Gamma a^3 B^2\,, \tag{9}$$

with $a(t) \sim t^s$ as scale factor of the Friedmann Universe. The energy density of the gravexciton field and the corresponding number density, which can be approximated as

$$\rho_\psi = \frac{1}{2}\dot{\psi}^2 + \frac{1}{2}m_\psi^2\,\psi^2 \approx \frac{1}{2}C^2B^2m_\psi^2\,, \quad n_\psi \approx \frac{1}{2}C^2B^2m_\psi\,, \tag{10}$$

satisfy the DEs

$$\frac{d}{dt}(a^3\rho_\psi) = -\Gamma a^3\rho_\psi \quad\text{and}\quad \frac{d}{dt}(a^3 n_\psi) = -\Gamma a^3 n_\psi \tag{11}$$

with solutions

$$\rho_\psi \sim e^{-\Gamma t}a^{-3} \quad\text{and}\quad n_\psi \sim e^{-\Gamma t}a^{-3}\,. \tag{12}$$

This means that during the stage $m_\psi > H \gg \Gamma$ the gravexcitons perform damped oscillations and their energy density behaves like a red-shifted dust-like perfect fluid ($\rho_\psi \sim a^{-3}$) with slow decay $\sim e^{-\Gamma t} \sim 1$:

$$\rho_\psi \approx \psi_{\rm in}^2 m_\psi^2\left(\frac{T}{T_{\rm in}}\right)^3. \tag{13}$$

$T_{\rm in}$ denotes the temperature of the Universe when the gravexcitons started to oscillate. According to the Friedmann equation (the 00-component of the Einstein equation), the Hubble parameter and the energy density (which defines the dynamics of the Universe) are connected (for flat spatial sections) by the relation

$$H(t)M_{\rm Pl} = \sqrt{\frac{8\pi}{3}\,\rho(t)} \sim \sqrt{\rho(t)}\,. \tag{14}$$

During the RD stage we have $\rho(t) \sim T^4$, and hence, $H^2 \sim T^4/M_{\rm Pl}^2$. For gravexcitons which start their oscillations during this stage, the temperature $T_{\rm in}$ is now easily estimated as

$$H_{\rm in} \sim m_\psi \sim \frac{T_{\rm in}^2}{M_{\rm Pl}} \quad \Longrightarrow \quad T_{\rm in} \sim \sqrt{m_\psi M_{\rm Pl}}\,. \tag{15}$$

If there is no broad parametric resonance ("preheating"),[4] then the decay plays the essential role when $H \lesssim \Gamma$ and the evolution of the energy density of the gravexcitons is dominated by an exponential decrease. The most effective decay takes place at times

$$t_{\rm D} \sim H_{\rm D}^{-1} \sim \Gamma^{-1} \sim \left(\frac{M_{\rm Pl}}{m_\psi}\right)^2 m_\psi^{-1}\,. \tag{16}$$

3. Light and Ultra-Light Gravexcitons: $m_\psi \leq 10^{-2}$ GeV

If the decay time $t_{\rm D}$ of the gravexcitons exceeds the age of the Universe $t_{\rm univ} \sim 10^{18}$ sec then the decay can be neglected. Equation (16) shows that this is the case for particles with masses $m_\psi \leq 10^{21} M_{\rm Pl} \sim 10^{-2}$ GeV $\sim 20 m_e$ (where m_e is the electron mass).

Subsequently, we split our analysis, considering separately particles which start to oscillate before matter/radiation equality $t_{\rm eq} \sim H_{\rm eq}^{-1}$ (i.e. during the RD stage) and after $t_{\rm eq}$ (i.e. during the MD stage). According to present WMAP data for the ΛCDM model, the following holds:

$$H_{\rm eq} \equiv m_{\rm eq} \sim 10^{-56} M_{\rm Pl} \sim 10^{-28}\ {\rm eV}\,. \tag{17}$$

An obvious requirement is that gravexciton should not over-close the observable Universe. This means that for particles with masses $m_\psi > m_{\rm eq}$ the energy density at the time $t_{\rm eq}$ should not exceed the critical density:[a]

$$\sqrt{\rho_\psi}\big|_{t_{\rm eq} \sim H_{\rm eq}^{-1}} \lesssim H_{\rm eq} M_{\rm Pl} \quad \Longrightarrow \quad m_\psi \lesssim m_{\rm eq} \left(\frac{M_{\rm Pl}}{\psi_{\rm in}}\right)^4 . \tag{18}$$

Here we used the estimate $\rho_\psi \sim (\psi_{\rm in}/t)^2 (m_\psi t)^{1/2}$ which follows from Eqs. (7), (8) and (10). For particles with masses $m_\psi \gtrsim m_{\rm eq}$, relation (18) implies the additional consistency condition $\psi_{\rm in} \lesssim M_{\rm Pl}$, or more exactly

$$\psi_{\rm in} \lesssim \left(\frac{m_{\rm eq}}{m_\psi}\right)^{1/4} M_{\rm Pl} \lesssim M_{\rm Pl}\,. \tag{19}$$

[a] It is clear that the ratio between the energy densities of gravexciton and matter becomes fixed after $t_{\rm eq}$. If ρ_ψ is less than the critical density at this moment then it will remain under-critical forever.

Let us now consider particles with masses $m_\psi \lesssim m_{\rm eq}$ which start to oscillate during the MD stage. From Eqs. (7), (8) and (10) one finds for these particles $\rho_\psi \sim (\psi_{\rm in}/t)^2$ so that the inequality

$$\sqrt{\rho_\psi}|_{t_{\rm in}\sim H_{\rm in}^{-1}} \lesssim H_{\rm in} M_{\rm Pl} \quad \Longrightarrow \quad \psi_{\rm in} \lesssim M_{\rm Pl} \tag{20}$$

ensures under-criticality of the energy density with respect to over-closure of the Universe.

It is worth noting that the combination $-(9/4)H^2 - (3/2)\dot{H}$ in DE (4) vanishes for the MD stage because of $H = 2/(3t)$. Hence, for times $t \geq t_{\rm eq} \sim 1/m_{\rm eq}$ the solutions of DE (4) have an oscillating behavior (provided that $m_\psi > \Gamma$) with a period of oscillations $t_{\rm osc} \sim 1/m_\psi$. For light particles with masses $m_\psi \leq m_{\rm eq}$ this implies that the initial oscillations start at $t_{\rm in} \sim 1/m_\psi$.

Finally, it should be noted that light gravexcitons can lead to the appearance of a fifth force with characteristic length scale $\lambda \sim 1/m_\psi$. Recent gravitational (Cavendish-type) experiments (see e.g. Ref. 6) exclude fifth force particles with masses $m_\psi \lesssim 1/(10^{-2}$ cm$) \sim 10^{-3}$ eV. This sets an additional restriction on the allowed mass region of gravexcitons. Furthermore, such gravexcitons will lead to a temporal variability of the fine structure constant above the experimentally established value.[3] Thus, physically sensible models should allow for parameter configurations which exclude such ultra-light gravexcitons.

4. Heavy Gravexcitons: $m_\psi \geq 10^{-2}$ GeV

This section is devoted to the investigation of gravexcitons/radions with masses $m_\psi \geq 10^{-2}$ GeV for which the decay plays an important role. Because of $m_\psi \gg m_{\rm eq}$, the corresponding modes begin to oscillate during the RD stage. We consider two scenarios separately. The first one contains an evolutionary stage with transient gravexciton dominance (ψ-dominance), whereas in the second one gravexcitons remain always sub-dominant.

4.1. *The transiently ψ-dominated Universe*

In this subsection we consider a scenario where the Universe is already at the RD stage when the gravexcitons begin their oscillations. The initial heating could be induced, e.g. by the decay of some additional very massive (inflaton) scalar field. We assume that the Hubble parameter at this stage is defined by the energy density of the radiation. The gravexcitons start their oscillations when the radiation cools down to the temperature $T_{\rm in} \sim \sqrt{m_\psi M_{\rm Pl}}$ [see Eq. (15)]. From the dust-like red-shifting of the energy density ρ_ψ [see Eq. (13)] follows that the ratio $\rho_\psi/\rho_{\rm rad}$ increases like $1/T$ when T decreases. At some critical temperature $T_{\rm crit}$ this ratio reaches ~ 1 and the Universe becomes ψ-dominated:

$$m_\psi^2 \psi_{\rm in}^2 \left(\frac{T}{T_{\rm in}}\right)^3 \sim T^4 \quad \Longrightarrow \quad T_{\rm crit} \sim T_{\rm in}\left(\frac{\psi_{\rm in}}{M_{\rm Pl}}\right)^2 . \tag{21}$$

After that the Hubble parameter is defined by the energy density of the gravexcitons: $H^2 M_{\rm Pl}^2 \sim \rho_\psi$ (with ρ_ψ from Eq. (13)). This stage is transient and ends when the gravexcitons decay at the temperature $T_{\rm D}$:

$$H_{\rm D}^2 M_{\rm Pl}^2 \sim \Gamma^2 M_{\rm Pl}^2 \sim m_\psi^2 \psi_{\rm in}^2 \left(\frac{T_{\rm D}}{T_{\rm in}}\right)^3 \quad \Longrightarrow \quad T_{\rm D} \sim T_{\rm in} \left(\frac{M_{\rm Pl}}{\psi_{\rm in}}\right)^{2/3} \left(\frac{m_\psi}{M_{\rm Pl}}\right)^{4/3} . \tag{22}$$

We assume that, due to the decay, all the energy of the gravexcitons is converted into radiation and that a reheating occurs. The corresponding reheating temperature can be estimated as:

$$H_{\rm D}^2 M_{\rm Pl}^2 \sim \Gamma^2 M_{\rm Pl}^2 \sim T_{\rm RH}^4 \quad \Longrightarrow \quad T_{\rm RH} \sim \sqrt{\frac{m_\psi^3}{M_{\rm Pl}}} . \tag{23}$$

Because the Universe before the gravexciton decay was gravexciton-dominated, it is clear that the reheating temperature $T_{\rm RH}$ should be higher than the decay temperature $T_{\rm D}$. This provides a lower bound for $\psi_{\rm in}$:

$$T_{\rm RH} \geq T_{\rm D} \quad \Longrightarrow \quad \psi_{\rm in} \geq \sqrt{m_\psi M_{\rm Pl}} \sim T_{\rm in} . \tag{24}$$

Substitution of this estimate into Eq. (21) shows that the minimal critical temperature (at which the Universe becomes ψ-dominated) is equal to the reheating temperature: $T_{\rm crit(min)} \sim T_{\rm RH}$.

If we additionally assume the natural initial condition $\psi_{\rm in} \sim M_{\rm Pl}$, then it holds $T_{\rm crit} \sim T_{\rm in}$ and the Universe will be ψ-dominated from the very beginning of the gravexciton oscillations. The upper bound on $\psi_{\rm in}$ is set by the exclusion of quantum gravity effects: $m_\psi^2 \psi_{\rm in}^2 \leq M_{\rm Pl}^4$. Hence, in the considered scenario, it should hold

$$T_{\rm in} \leq \psi_{\rm in} \leq M_{\rm Pl} \left(\frac{M_{\rm Pl}}{m_\psi}\right) . \tag{25}$$

A successful nucleosynthesis requires a temperature $T \gtrsim 1$ MeV during the RD stage. If we assume that this lower bound is fulfilled for the reheating temperature (23), then we find the lower bound on the gravexciton mass

$$m_\psi \gtrsim 10^4 \ {\rm GeV} . \tag{26}$$

It is also possible to consider a scenario where the ψ-field acts as inflaton itself. In such a scenario, the Universe is ψ-dominated from the very beginning and for the amplitude of the initial oscillations one obtains:

$$H M_{\rm Pl} \sim \sqrt{\rho_\psi} \quad \Longrightarrow \quad H_{\rm in} M_{\rm Pl} \sim m_\psi M_{\rm Pl} \sim \psi_{\rm in} m_\psi \quad \Longrightarrow \quad \psi_{\rm in} \sim M_{\rm Pl} . \tag{27}$$

The reheating temperature is then again given by the estimate (23) and the gravexciton masses should also fulfill the requirement (26).

4.2. *Sub-dominant gravexcitons*

In this subsection we consider a scenario where the ψ-field undergoes a decay, but the gravexcitons never dominate the dynamics of the Universe. The Hubble parameter of the Universe is then defined by the energy density of other (matter) fields which behave as radiation for $t \leq t_{\rm eq}$ and as dust for $t \geq t_{\rm eq}$. The energy density ρ_ψ is always much less than the total energy density of the other fields.

4.2.1. *Decay during the RD stage*

In this subsection, we analyze the behavior of ψ-particles that decay during the RD stage, when $H^2 \sim T^4/M_{\rm Pl}^2$. Again we will clarify for which masses m_ψ and initial oscillation amplitudes $\psi_{\rm in}$ such a scenario can hold. A decay during RD implies that the decay temperature $T_{\rm D}$, estimated as

$$\Gamma \sim H_{\rm D} \sim \frac{T_{\rm D}^2}{M_{\rm Pl}} \quad \Longrightarrow \quad T_{\rm D} \sim \sqrt{\frac{m_\psi^3}{M_{\rm Pl}}}\,, \tag{28}$$

should be higher than the temperature

$$T_{\rm eq} \sim \sqrt{H_{\rm eq} M_{\rm Pl}} \sim 1 \text{ eV} \tag{29}$$

of the matter/radiation equality. This yields the following restriction on the gravexciton masses:

$$T_{\rm D} \gtrsim T_{\rm eq} \quad \Longrightarrow \quad m_\psi \gtrsim M_{\rm Pl}\left(\frac{T_{\rm eq}}{M_{\rm Pl}}\right)^{2/3} \equiv m_d \sim 1 \text{ GeV}\,. \tag{30}$$

The mass parameter m_d corresponds to particles which decay at the moment $t_{\rm eq}$: $\Gamma \sim H_{\rm eq}$. The bound on the initial oscillation amplitude $\psi_{\rm in}$ can be found from the energy sub-dominance condition for the gravexcitons at the moment of their decay $t_{\rm D}$

$$\rho_\psi|_{t_{\rm D}} \approx \psi_{\rm in}^2 m_\psi^2 \left(\frac{T_{\rm D}}{T_{\rm in}}\right)^3 < T_{\rm D}^4\,. \tag{31}$$

It reads

$$\psi_{\rm in} < \sqrt{m_\psi M_{\rm Pl}} \sim T_{\rm in}\,, \tag{32}$$

where $T_{\rm in}$ is defined by Eq. (15). It can be easily seen that condition (32) is supplementary to the condition (24). During the decay, the energy of the gravexcitons is converted into radiation: $\rho_\psi \to \rho_{r,2}$ with temperature $T_r^4 \sim \rho_\psi|_{t_{\rm D}}$. For a scenario with $\rho_\psi|_{t_{\rm D}} \ll T_{\rm D}^4$, and hence $T_r \ll T_{\rm D}$, the energy density $\rho_{r,2}$ contributes only negligibly to the total energy density and the gravexciton decay does not spoil the standard picture of a hot Universe with successful big bang nucleosynthesis (BBN).

4.2.2. *Gravexciton decay during the MD stage*

At the MD stage (for $t > t_{\rm eq}$) the Hubble parameter reads (see e.g. Ref. 2, page 504)

$$t \sim H^{-1} \sim \frac{M_{\rm Pl}}{T^{3/2}T_{\rm eq}^{1/2}} \quad \Longrightarrow \quad HM_{\rm Pl} \sim T^{3/2}T_{\rm eq}^{1/2}\,, \tag{33}$$

and the decay temperature $T_{\rm D}$ of the gravexcitons can be estimated as

$$\Gamma \sim H_{\rm D} \sim \frac{T_{\rm D}^{3/2}T_{\rm eq}^{1/2}}{M_{\rm Pl}} \quad \Longrightarrow \quad T_{\rm D}^3 \sim \frac{(\Gamma M_{\rm Pl})^2}{T_{\rm eq}} \sim T_{\rm eq}^3\left(\frac{m_\psi}{M_{\rm Pl}}\right)^4\left(\frac{m_\psi}{m_{\rm eq}}\right)^2 . \tag{34}$$

For a decay during MD this decay temperature should be less than $T_{\rm eq}$, and as implication a restriction on the mass of the ψ-field can be obtained

$$T_{\rm D} < T_{\rm eq} \quad \Longrightarrow \quad m_\psi < M_{\rm Pl}\left(\frac{T_{\rm eq}}{M_{\rm Pl}}\right)^{2/3} = m_d\,, \tag{35}$$

which is supplementary to the inequality (30). The restriction on the initial amplitude $\psi_{\rm in}$ can be found from the condition of matter dominance and the fact that heavy gravexcitons begin to oscillate at the RD stage when $T_{\rm in} \sim \sqrt{m_\psi M_{\rm Pl}}$

$$\psi_{\rm in}^2 m_\psi^2\left(\frac{T_{\rm D}}{T_{\rm in}}\right)^3 \sim T_r^4 < H_{\rm D}^2 M_{\rm Pl}^2 \sim \frac{T_{\rm D}^3 T_{\rm eq}}{M_{\rm Pl}^2}M_{\rm Pl}^2$$
$$\Longrightarrow \quad \psi_{\rm in} < M_{\rm Pl}\left(\frac{T_{\rm eq}}{T_{\rm in}}\right)^{1/2} \sim M_{\rm Pl}\left(\frac{H_{\rm eq}}{m_\psi}\right)^{1/4} \ll M_{\rm Pl}\,. \tag{36}$$

Condition (36) guarantees that there is no additional reheating and the BBN is not spoilt. We discussed different cosmological scenarios affected by the dynamics of gravitational excitons/radions. These massive moduli fields describe the conformal excitations of the internal spaces in higher dimensional models and are WIMPs in the external spacetime. We demonstrated that observable cosmological data set strong constraints on the gravexciton masses and the amplitudes of their initial oscillations.

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