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GRAVITATIONAL EXCITONS – FLUCTUATING PARTICLES FROM EXTRA DIMENSIONS

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We show that for warped product space-times the conformal (geometrical moduli) excitations of the internal compactified factor spaces should be observable as massive scalar fields in the external space-time. These scalar fields (gravitational excitons) describe weakly interacting particles and can be considered as dark matter component. On the other hand, they provide possible values for the effective cosmological constant.

1 Introduction

The multi-dimensionality of our Universe is one of the basic assumptions in modern theories beyond the $SU(3) \times SU(2) \times U(1)$ Standard Model of electroweak and strong interactions. Superstring theory and M-theory use this concept as basic assumptions and have a consistent formulation in space-times with total dimension $D = 10$ and $D = 11$.

The fundamental constants in these theories are related to the vacuum expectation values of the dilaton and moduli fields, and variations of these fields would result in variations of the constants. In the context of standard

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Kaluza-Klein models, the moduli are defined by the shape and size of the internal spaces (geometrical moduli). Up to now, there are no experiments which show a time variation of fundamental constants. This means, that according to observations, the internal spaces should be static or nearly static at least from the time of recombination (in some papers, arguments are given in favor of the assumption that a variation of the fundamental constants is absent from the time of primordial nucleosynthesis). Therefore, a mechanism for moduli stabilization should be part of any realistic multi-dimensional model.

Within multi-dimensional cosmological models of the standard Kaluza-Klein type[a] such a stabilization is achieved, e.g. via trapping of the geometrical moduli fields by effective potentials of dimensionally reduced models. On the other hand, it is important to consider possible observable consequences of various stabilization mechanisms. We show that fluctuations of the multi-dimensional geometry near minima of corresponding effective potentials should be observable as fluctuating scalar fields in our Universe, i.e. as scalar particles [1].

2 Conformal Fluctuations of Internal Spaces

We consider a cosmological model with the metric

$$g = g^{(0)} + \sum_{i=1}^{n} e^{2\beta^i(x)} g^{(i)}, \tag{1}$$

which is defined on a manifold with warped product topology

$$M = M_0 \times M_1 \times \cdots \times M_n, \tag{2}$$

where x are some coordinates of the $D_0 = (d_0 + 1)-$dimensional external space-time manifold M_0 and

$$g^{(0)} = g^{(0)}_{\mu\nu}(x) dx^\mu \otimes dx^\nu. \tag{3}$$

Let the manifolds M_i be d_i-dimensional Einstein spaces with metric $g^{(i)}$, i.e.

$$R_{mn}\left[g^{(i)}\right] = \lambda^i g^{(i)}_{mn}, \quad m,n = 1,\ldots,d_i, \quad R\left[g^{(i)}\right] = \lambda^i d_i \equiv R_i. \tag{4}$$

[a]Here, it is assumed that the internal spaces are compactified at sizes somewhere between the Planck scale $L_{\rm Pl} \sim 10^{-33}$cm and the Fermi scale $L_F \sim 10^{-17}$cm to make them unobservable.

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In the case of constant curvature spaces, the parameters λ^i are normalized as $\lambda^i = k_i(d_i - 1)$ with $k_i = \pm 1, 0$. Later on, we shall not specify the structure of the spaces M_i. We require only that M_i are compact spaces with arbitrary sign of curvature.

With total dimension $D = D_0 + \sum_{i=1}^n d_i$, κ_D^2 being a $D-$dimensional gravitational constant and Λ a $D-$dimensional cosmological constant, we consider an action functional of the form

$$S = \frac{1}{2\kappa_D^2} \int\limits_M d^D x \sqrt{|g|} \left\{ R[g] - 2\Lambda \right\} + S_m \, , \tag{5}$$

where S_m is a non-specified action term which takes into account additional matter fields. To illustrate the natural origin of gravitational excitons (gravexcitons for short) we shall consider a pure geometrical model: $S_m \equiv 0$. The generalization to models with explicit matter-terms is obvious. As an illustration we consider such a term (resulting from the Casimir effect) at the end of this section.

Let β_0^i be the scale of compactification of the internal spaces at the present time. Instead of β^i it is convenient to introduce a shifted quantity: $\tilde{\beta}^i = \beta^i - \beta_0^i$.

Then, after dimensional reduction and conformal transformation

$$g_{\mu\nu}^{(0)} = \Omega^2 \tilde{g}_{\mu\nu}^{(0)} := \left(\prod_{i=1}^n e^{d_i \tilde{\beta}^i} \right)^{-\frac{2}{D_0 - 2}} \tilde{g}_{\mu\nu}^{(0)} \tag{6}$$

action (5) reads

$$S = \frac{1}{2\kappa_0^2} \int\limits_{M_0} d^{D_0} x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - \bar{G}_{ij} \tilde{g}^{(0)\mu\nu} \partial_\mu \tilde{\beta}^i \, \partial_\nu \tilde{\beta}^j - 2U_{\text{eff}} \right\} , \tag{7}$$

where $\tilde{R}_i := R_i e^{-2\beta_0^i}$. $\bar{G}_{ij} = d_i \delta_{ij} + d_i d_j/(D_0 - 2)$ is the midisuperspace metric and $\kappa_0^2 := \kappa_D^2 / V_{D'}$ denotes the D_0-dimensional gravitational constant ($V_{D'}$ is the total volume of the internal space). If we take the TeV scale [2,3] M_{TeV} and the Planck scale M_{Pl} as fundamental ones for $D-$dimensional space-time and the 4-dimensional large-scale space-time, respectively: $\kappa_D^2 = 8\pi / M_{\text{TeV}}^{2+D'}$, and $\kappa_0^2 = 8\pi / M_{\text{Pl}}^2$, then we reproduce the well-known relation [2,3]: $M_{\text{Pl}}^2 = V_{D'} M_{\text{TeV}}^{(2+D')}$. This implies that the scale of the internal space compactification is fixed and of the order

$$a \sim V_{D'}^{1/D'} \sim 10^{32/D' - 17} \text{cm} \, . \tag{8}$$

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The effective potential in (7) reads

$$U_{\text{eff}}[\tilde{\beta}] = \left(\prod_{i=1}^{n} e^{d_i \tilde{\beta}^i}\right)^{-\frac{2}{D_0-2}} \left[-\frac{1}{2}\sum_{i=1}^{n} \tilde{R}_i e^{-2\tilde{\beta}^i} + \Lambda\right] . \tag{9}$$

With the help of a regular coordinate transformation $\varphi = Q\tilde{\beta}, \quad \tilde{\beta} = Q^{-1}\varphi$, the midisuperspace metric (target space metric) $\bar{G}$ can be transformed to a pure Euclidean form: $\bar{G}_{ij} d\tilde{\beta}^i \otimes d\tilde{\beta}^j = \sigma_{ij} d\varphi^i \otimes d\varphi^j = \sum_{i=1}^{n} d\varphi^i \otimes d\varphi^i, \quad \sigma =$ diag $(+1, +1, \ldots, +1)$ (see e.g. Ref. [1]).

It is clear that a stabilization of the internal spaces can be achieved, if the effective potential U_{eff} has a minimum with respect to fields $\tilde{\beta}^i$ (or fields φ^i). In general, it is possible that the potential U_{eff} has more than one extremum. But it can be easily seen that, for the pure geometrical model under consideration, we can get one extremum only. This corresponds to $\tilde{\beta}^i = 0$. For the masses of the normal mode excitations of the internal spaces (gravitational excitons) around the minimum position we obtain:

$$m_1^2 = \cdots = m_n^2 = -\frac{4\Lambda_{\text{eff}}}{D_0 - 2} = -2\frac{\tilde{R}_k}{d_k} > 0\,, \tag{10}$$

where

$$\Lambda_{\text{eff}} := U_{\text{eff}}\Big|_{\tilde{\beta}^i = 0} \tag{11}$$

plays the role of an effective cosmological constant in the external space-time. These equations show that a global minimum can only exist for our specific model in the case of compact internal spaces with negative curvature $R_k < 0$ $(k = 1, \ldots, n)$. The effective cosmological constant is also negative: $\Lambda_{\text{eff}} < 0$. Models which include matter can have minima for internal spaces of positive curvature. Usually, the effective cosmological constant is positive in this case.

For small fluctuations of the normal modes in the vicinity of the minima of the effective potential action Eq. (7) reads

$$\begin{aligned} S = {} & \frac{1}{2\kappa_0^2} \int\limits_{M_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - 2\Lambda_{\text{eff}} \right\} \\ & - \frac{1}{2} \int\limits_{M_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \sum_{i=1}^{n} \left(\tilde{g}^{(0)\mu\nu} \psi^i_{,\mu} \psi^i_{,\nu} + m_i^2 \psi^i \psi^i \right) \right\} \end{aligned} \tag{12}$$

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(for convenience we use here the normalizations: $\kappa_0^{-1}\tilde{\beta} \to \tilde{\beta}$). Thus, conformal excitations of the metric of the internal spaces behave as massive scalar fields developing on the background of the external space-time. In analogy to excitons in solid state physics, where they are excitations of the electronic subsystem of a crystal, we called the excitations of the subsystem of internal spaces gravitational excitons [1]. Later these particles have also become known as radions [2,3].

From Eq. (10) follows that

$$|\Lambda_{\text{eff}}| \sim m_i^2 \sim a_{(0)i}^{-2}\,, \tag{13}$$

where $a_{(0)i} = \exp \beta_0^i$ are the scale factors of the stabilized internal spaces.

The calculations above were performed for a model with the TeV scale M_{TeV} as fundamental scale of the $D-$dimensional theory (see Eq. (8)). Clearly, it is also possible to choose the Planck scale as the fundamental scale.

For this purpose we do not fix the compactification scale of the internal spaces at the present time. We consider them as free model parameters and demand only that $L_{\text{Pl}} < a_{(0)i} = e^{\beta_0^i} < L_F \sim 10^{-17}$cm. So, we shall not transform β^i to $\tilde{\beta}^i$. In this case, $\kappa_D^2 \sim M_{\text{Pl}}^{-(2+D')}$, so that the Planck scale becomes the fundamental scale of the $D-$dimensional theory. In this approach, Eqs. (6), (7), and (9) preserve their form with the only substitutions $\tilde{\beta} \longrightarrow \beta$ and $\tilde{R}_i \longrightarrow R_i$. The Einstein frame metrics of the external space-time in both approaches are equivalent to each other up to a numerical prefactor:

$$\tilde{g}^{(0)}_{\mu\nu}\Big|_{\text{TeV}} = v_0^{-2/(D_0-2)}\ \tilde{g}^{(0)}_{\mu\nu}\Big|_{\text{Pl}}\,, \tag{14}$$

where $v_0 = \prod_{i=1}^n \exp(d_1\beta_0^i)$. Obviously, the same rescaling takes place for the squared masses of the gravitational excitons and the effective cosmological constant: $m_i^2 \longrightarrow (v_0)^{-2/(D_0-2)}\, m_i^2$ and $\Lambda_{\text{eff}} \longrightarrow (v_0)^{-2/(D_0-2)}\,\Lambda_{\text{eff}}$. Thus, in the latter approach we get, instead of (13), the relation:

$$|\Lambda_{\text{eff}}| \sim m_i^2 \sim \left(a_{(0)i}\right)^{-(D-2)}\,, \tag{15}$$

where we set $D_0 = 4$. This relation shows that, due to the power $(2-D)$, the effective cosmological constant and the masses of the gravitational excitons can be very far from Planckian values even for scales of compactification of the internal spaces close to the Planck length.

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Let us return to the comparison of the TeV scale and the Planck scale approaches. If we set, e.g. $6 \le D < \infty$ in the TeV-scale approach, then the internal space scale factors, the gravexciton masses and the effective cosmological constant run correspondingly as: $10^{-1}\text{cm} \le a_{(0)i} < 10^{-17}\text{cm}$, $10^{-4}\text{eV} \le m_i < 1\text{TeV}$ and $10^{-64}\Lambda_{\text{Pl}} \le |\Lambda_{\text{eff}}| \le 10^{-32}\Lambda_{\text{Pl}}$. For this approach, the internal space scale factors are defined by Eq. (8) due to the demand that the $D-$dimensional gravitational constant is of order of the TeV scale. In the Planck scale approach such a condition is absent and the scale factors $a_{(0)i}$ are free parameters. Let us take, e.g. $a_{(0)i} \sim 10^{-18}$ cm. Then, in the Planck scale approach for $6 \le D \le 10$, the gravexciton masses and effective cosmological constant run correspondingly as $10^{-2}\text{eV} \le m_i \le 10^{-32}\text{eV}$ and $10^{-60}\Lambda_{\text{Pl}} \le |\Lambda_{\text{eff}}| \le 10^{-120}\Lambda_{\text{Pl}}$.

These estimates show that, within the TeV scale approach, the effective cosmological constant is much greater than the present day observable limit $\Lambda \le 10^{-122}\Lambda_{\text{Pl}} \sim 10^{-57}\text{cm}^{-2}$ (for our model $|\Lambda_{\text{eff}}|_{\text{TeV}} \ge 10^2\,\text{cm}^{-2}$), whereas in the Planck scale approach we can satisfy this limit even for very small compactification scales. For example, if we require $|\Lambda_{\text{eff}}| \sim 10^{-122}\Lambda_{\text{Pl}}$ in accordance with observations, then Eq. (15) gives a compactification scale $a_{(0)1} \sim 10^{122/(D-2)}L_{\text{Pl}}$. Thus, $a_{(0)1} \sim 10^{15}L_{\text{Pl}} \sim 10^{-18}\text{cm}$ for $D = 10$ and $a_{(0)1} \sim 10^{5}L_{\text{Pl}} \sim 10^{-28}\text{cm}$ for $D = 26$, which does not contradict observations because, for this approach, the scales of compactification should be $a_{(0)1} \le 10^{-17}\text{cm}$. Assuming an estimate $\Lambda_{\text{eff}} \sim 10^{-122}L_{\text{Pl}}$, we automatically get from Eq. (15) the value of the gravitational exciton mass: $m_1 \sim 10^{-61}M_{\text{Pl}} \sim 10^{-33}eV \sim 10^{-66}\text{g}$, i.e. the gravexcitons are in this case extremely light particles. Nevertheless, such light particles are not in contradiction with the observable Universe, because they do not overclose the Universe [4].

As shown above, the effective cosmological constant is negative in the pure geometrical case. However, according to the modern observation $\Lambda_{\text{eff}} > 0$. This problem can be solved if we take into account matter fields. In many important cases matter can be described phenomenologically via a perfect fluid ansatz. In this case the effective potential reads [1]

$$U_{\text{eff}}[\tilde{\beta}] = \left(\prod_{i=1}^{n} e^{d_i\tilde{\beta}^i}\right)^{-\frac{2}{D_0-2}} \left[-\frac{1}{2}\sum_{i=1}^{n}\tilde{R}_i e^{-2\tilde{\beta}^i} + \Lambda + \kappa_D^2\rho\right], \qquad (16)$$

where ρ is the energy density of the perfect fluid. A number of effective potentials of this type ensuring stability were described in Refs. [1,4–6]. Among

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them, the Casimir potential is one of the most important [5]. The Casimir effect is connected with the vacuum polarization of quantized fields due to the non-trivial topology of the background space or the presence of boundaries in the space. As a result, one obtains a nonvanishing energy density of the quantized fields in the vacuum state. In our case, this phenomenon should take place due to the compactness of the internal spaces. For one compact internal space ($n = 1$) with compactification scale $a_1 \ll a_0$, the Casimir energy density reads

$$\rho = C \exp\left(-D\beta^1\right) \equiv \tilde{C} \exp\left(-D\tilde{\beta}^1\right) . \tag{17}$$

Here C is a constant which strongly depends on the topology of the model. The equations of state in the external and internal spaces read, respectively [7]:

$$P_0 = -\rho\,, \quad P_1 = \frac{D_0}{d_1}\rho\,. \tag{18}$$

It can easily be seen that the effective potential (16) with the Casimir energy density (17) can have a non-negative minimum (non-negative effective cosmological constant) [5].

Conclusion

From the geometrical point of view it is clear that gravitational excitons are an inevitable consequence of the existence of extra dimensions. For any theory with compactified internal spaces, conformal excitations (fluctuations) of the internal space metric will result in gravitational excitons in the external space-time. The form of the effective potential as well as masses of gravitational excitons and the value of the effective cosmological constant are strongly model dependent. Gravexcitons may play an important role in cosmology. On the one hand, they are connected with the observed cosmological constant via the effective potential. On the other hand, the interaction between gravexcitons and usual matter is Planck-scale suppressed and they can give a significant contribution to Dark Matter [8].

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