

Stabilization of internal spaces in multidimensional cosmology

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Effective 4-dimensional theories are investigated which were obtained under dimensional reduction of multidimensional cosmological models with a minimal coupled scalar field as a matter source. Conditions for the internal space stabilization are considered and the possibility for inflation in the external space is discussed. The electroweak as well as the Planck fundamental scale approaches are investigated and compared with each other. It is shown that there exists a rescaling for the effective cosmological constant as well as for gravitational exciton masses in the different approaches.

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I. INTRODUCTION

Stabilization of additional dimensions near their present day values (dilaton and geometrical moduli stabilization) is one of the main problems for any multidimensional theory because a dynamical behavior of the internal spaces would result in a variation of the fundamental physical constants. Observations show that internal spaces should be static or nearly static at least from the time of recombination (in some papers arguments are given in favor of the assumption that variations of the fundamental constants are absent from the time of primordial nucleosynthesis [1]).

Observations further indicated that standard model (SM) matter cannot propagate a large distance in extra dimensions. This allowed for two classes of model building implications.

The first class consists of models with extra spacetime dimensions compactified at scales less the Fermi length $L_F \sim 10^{-17}$ cm as the characteristic scale of the experimentally tested electroweak interaction $L_F \sim M_{EW}^{-1} \sim 1$ TeV $^{-1}$. Up until the early 1990s this was a standard assumption in string phenomenology with the string scale slightly below the 4-dimensional Planck scale $M_s \sim 10^{16}$ GeV [2]. The question about a concrete mechanism for the stabilization of the compactification scales (moduli stabilization) remained open in this discussion [2,3].

The second class of models starts from the assumptions that observable SM matter is confined to a 3-brane located in a higher dimensional bulk spacetime and that gravitational interactions can propagate in the whole bulk spacetime provided that a mechanism exists which ensures the usual Newton r^{-2} law at distances ≥ 1 cm accessible to present gravitational tests. The thickness of the 3-brane in this case should be of order of the Fermi length L_F . The additional bulk dimensions can be compactified or noncompact.

Historically, the first proposal for an interpretation of our

apparently 4-dimensional Universe as a submanifold embedded into a noncompact higher dimensional bulk space dates back to the 1983 work of Rubakov and Shaposhnikov [4] and Akama [5] (still without accounting for gravitational interactions) and Visser's consideration from 1985 [6] (studying the localization (trapping) of particles via gravity to a 4-dimensional submanifold of a 5-dimensional "real" world).

Within the framework of superstring theory and M theory new arguments have been given for a self-content embedding of the 4-dimensional $SU(3) \times SU(2) \times U(1)$ standard model of strong and electroweak interactions into a fundamentally higher dimensional spacetime manifold. For example, in Horava-Witten theory [7,8] one starts from the strongly coupled regime of $E_8 \times E_8$ heterotic string theory and interprets it as M theory on an orbifold $\mathbb{R}^{10} \times S^1/\mathbb{Z}_2$. After compactification on a Calabi-Yau threefold one arrives at solutions which may be considered as a pair of parallel 3-branes with opposite tension, and location at the orbifold planes.

After 1995 it became clear from investigations in type I string theory that due to compactified higher dimensions the string scale M_s can be much smaller than the 4-dimensional Planck scale $M_{Pl(4)} = 1.22 \times 10^{19}$ GeV and that it is bounded from below only experimentally by the scale of electroweak interaction $M_{EW} \lesssim M_s \lesssim M_{Pl(4)}$ [8–13]. As suggested by Arkani-Hamed *et al.* [10,11,14] it is even possible to lower the fundamental Planck scale $M_{Pl(4+D')}$ of the $(4+D')$ -dimensional theory down to the SM electroweak scale $M_{Pl(4+D')} \sim M_{EW} \sim 1$ TeV. This allows for a solution of the hierarchy problem not relying on supersymmetry or technicolor. In this approach gravity can propagate in all multidimensional bulk space whereas ordinary SM fields are localized on a 3-brane with thickness in the extra dimensions of order the Fermi length L_F . As a result, the 4-dimensional Planck scale of the external space is connected with the electroweak scale by the relation

$$M_{Pl(4)}^2 \sim V_{D'} M_{EW}^{(2+D')}, \quad (1)$$

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where $V_{D'}$ is the volume of the internal spaces. Thus, the scale of the internal space compactification is of order

$$a \sim V_{D'}^{1/D'} \sim 10^{32/D' - 17} \text{ cm.} \quad (2)$$

In this model physically acceptable values correspond to $D' \geq 2$, e.g., for $D' = 2$ the internal space scale of compactification is $a \sim 10^{-1}$ cm.

The stabilization of extra dimensions (geometrical moduli stabilization) in models with submillimeter internal spaces was considered in Refs. [14,15] where the dynamics of the conformal excitations of the internal spaces near minima of an effective potential have been investigated. Due to the product topology of the $(4+D')$ -dimensional bulk space-time constructed from Einstein spaces with scale (warp) factors depending only on the coordinates of the external 4-dimensional component, the conformal excitations have the form of massive scalar fields living in the external space. Within the framework of multidimensional cosmological models (MCMs) we investigated such excitations in [16–19] and called them gravitational excitons. Later, since the submillimeter weak-scale compactification approach these geometrical moduli excitations are also known as radions [14,15].

Recently Randall and Sundrum [20] proposed an interesting construction for the solution of the hierarchy problem localizing low energy SM matter as well as low energy gravity on a 3-brane in a slice of anti-de Sitter space AdS_5 . Subsequently, it has been shown that such a localization of low energy physics can be also achieved at the intersection of a system of $(n+2)$ -branes in AdS_{4+n} [21] allowing for an interpretation of our observable universe, e.g., as a defect in a higher dimensional brane crystal [22]. But the RS proposal and its generalizations are not considered in the present paper.

The main goal of our present comments consists in a clarification of conditions which ensure the stabilization of the internal spaces in multidimensional models with a minimal coupled scalar field as a matter source (Sec. II). A general method for the solution of such problems in models with an arbitrary number of internal spaces was proposed in Ref. [16]. There it was shown that the problem of the internal space stabilization can be solved most easily in the Einstein frame (although it is clear that if stabilization takes place in the Einstein frame it will also take place in the Brans-Dicke frame and vice versa). Our investigations (see also [17–19]) show that inflation of the external space which was maintained for some models in earlier references (see e.g. [23,24]) is destroyed by a required stabilization of the internal spaces.

On the other hand, there are also papers devoted to inflation where stabilization of the internal spaces was supposed *a priori* (see e.g. [25–27]). We would like to stress here that it is necessary to be rather careful in this case because stabilization can destroy inflation. For example, the appearance of a negative effective cosmological constant, which in some models is a necessary condition for the internal spaces stabilization, can either destroy inflation at all or make problematic its successful completion. This situation occurs, e.g., in the simple toy model which we consider in Sec. III of the present paper. We use this model in order to show exactly

under which conditions stabilization takes place in multidimensional cosmological models with a minimal coupled scalar field and to discuss briefly a possibility for inflation in these models.

In the present paper most of the calculations are performed in the electroweak fundamental scale approach. In the conclusion Sec. IV we compare the corresponding results with those for the Planck fundamental scale approach and show that the transition from one approach to the other results in a rescaling of the effective cosmological constant Λ_{eff} as well as of gravitational exciton masses m_i . The corresponding rescaling prefactors which appear due to the transition [see Eq. (38)] lead to a different functional dependence of Λ_{eff} and m_i on the compactification sizes of the internal spaces in the two approaches. As result, in the Planck fundamental scale approach the values of Λ_{eff} and m_i can be much smaller than in the electroweak approach. Finally, we discuss some bounds on the parameters of the model which follow from observable cosmological data. These bounds strongly depend on the details of the behavior of the inflaton and gravitational exciton fields after inflation, e.g., on the times of their reheating and decay.

II. STABILIZATION OF THE INTERNAL SPACES

We consider a cosmological model with metric

$$g = g^{(0)} + \sum_{i=1}^n e^{2\beta^i(x)} g^{(i)}, \quad (3)$$

which is defined on a manifold with product topology

$$M = M_0 \times M_1 \times \dots \times M_n, \quad (4)$$

where x are some coordinates of the $D_0 = (d_0 + 1)$ -dimensional manifold M_0 and

$$g^{(0)} = g_{\mu\nu}^{(0)}(x) dx^\mu \otimes dx^\nu. \quad (5)$$

Let manifolds M_i be d_i -dimensional Einstein spaces with metric $g^{(i)}$, i.e.,

$$R_{mn}[g^{(i)}] = \lambda^i g_{mn}^{(i)}, \quad m, n = 1, \dots, d_i \quad (6)$$

and

$$R[g^{(i)}] = \lambda^i d_i \equiv R_i. \quad (7)$$

In the case of constant curvature spaces parameters λ^i are normalized as $\lambda^i = k_i(d_i - 1)$ with $k_i = \pm 1, 0$. Later on we shall not specify the structure of the spaces M_i . We require only M_i to be compact spaces with arbitrary sign of curvature.

With total dimension $D = D_0 + \sum_{i=1}^n d_i$, κ_D^2 a D -dimensional gravitational constant, Λ a D -dimensional cosmological constant, and S_{YGH} the standard York-Gibbons-Hawking boundary term [28,29], we consider an action of the form

$$\begin{aligned}
 S = & \frac{1}{2\kappa_D^2} \int_M d^D x \sqrt{|g|} \{R[g] - 2\Lambda\} \\
 & - \frac{1}{2} \int_M d^D x \sqrt{|g|} (g^{MN} \partial_M \Phi \partial_N \Phi + 2U(\Phi)) + S_{YGH},
 \end{aligned} \tag{8}$$

where the minimal coupled scalar field Φ with an arbitrary potential $U(\Phi)$ depends on the external coordinates x only. This field can be understood as a zero mode of a bulk field. Such a scalar field can naturally originate also in nonlinear D -dimensional theories [30] where metric ansatz (3) ensures its dependence on x only.

Let β_0^i be the scale of compactification of the internal spaces at the present time and

$$V_{D'} \equiv V_I \times V_0 \equiv \prod_{i=1}^n \int_{M_i} d^{d_i} y \sqrt{|g^{(i)}|} \times \prod_{i=1}^n e^{d_i \beta_0^i} \tag{9}$$

the corresponding total volume of the internal spaces ($[V_{D'}] = \text{cm}^{D'}$, $[V_I] = 1$, where $D' = D - D_0$ is the number of extra dimensions). Instead of β^i it is convenient to introduce a shifted quantity:

$$\tilde{\beta}^i = \beta^i - \beta_0^i. \tag{10}$$

Then, after dimensional reduction action (8) reads

$$\begin{aligned}
 S = & \frac{1}{2\kappa_0^2} \int_{M_0} d^{D_0} x \sqrt{|g^{(0)}|} \prod_{i=1}^n e^{d_i \tilde{\beta}^i} \\
 & \times \left\{ R[g^{(0)}] - G_{ij} g^{(0)\mu\nu} \partial_\mu \tilde{\beta}^i \partial_\nu \tilde{\beta}^j + \sum_{i=1}^n \tilde{R}_i e^{-2\tilde{\beta}^i} - 2\Lambda \right. \\
 & \left. - g^{(0)\mu\nu} \kappa_D^2 \partial_\mu \Phi \partial_\nu \Phi - 2\kappa_D^2 U(\Phi) \right\},
 \end{aligned} \tag{11}$$

where $\tilde{R}_i := R_i e^{-2\beta_0^i}$, $G_{ij} = d_i \delta_{ij} - d_i d_j$ ($i, j = 1, \dots, n$) is the midisuperspace metric [31,32] and

$$\kappa_0^2 := \frac{\kappa_D^2}{V_{D'}} \tag{12}$$

is the D_0 -dimensional (4-dimensional) gravitational constant. If we take the electroweak scale M_{EW} and the Planck scale M_{Pl} as fundamental ones for D -dimensional and 4-dimensional spacetimes respectively:

$$\begin{aligned}
 \kappa_D^2 &= \frac{8\pi}{M_{EW}^{2+D'}}, \\
 \kappa_0^2 &= \frac{8\pi}{M_{Pl}^2},
 \end{aligned} \tag{13}$$

then we reproduce Eqs. (1) and (2).

Action (11) describes a generalized σ -model with target space metric G_{ij} where the scale factors β^i play the role of scalar fields. The problem of the internal space stabilization is reduced now to the investigation of the dynamics of these fields. Most easily this can be done in the Einstein frame. For this purpose we perform a conformal transformation

$$g_{\mu\nu}^{(0)} = \Omega^2 \tilde{g}_{\mu\nu}^{(0)} := \left(\prod_{i=1}^n e^{d_i \tilde{\beta}^i} \right)^{-2/(D_0-2)} \tilde{g}_{\mu\nu}^{(0)}, \tag{14}$$

which yields [16]

$$\begin{aligned}
 S = & \frac{1}{2\kappa_0^2} \int_{M_0} d^{D_0} x \sqrt{|\tilde{g}^{(0)}|} \{ \tilde{R}[\tilde{g}^{(0)}] - \tilde{G}_{ij} \tilde{g}^{(0)\mu\nu} \partial_\mu \tilde{\beta}^i \partial_\nu \tilde{\beta}^j \\
 & - \tilde{g}^{(0)\mu\nu} \kappa_D^2 \partial_\mu \Phi \partial_\nu \Phi - 2U_{eff} \},
 \end{aligned} \tag{15}$$

where $\tilde{G}_{ij} = d_i \delta_{ij} + [1/(D_0-2)] d_i d_j$ and

$$\begin{aligned}
 U_{eff}[\tilde{\beta}, \Phi] = & \left(\prod_{i=1}^n e^{d_i \tilde{\beta}^i} \right)^{-2/(D_0-2)} \\
 & \times \left[-\frac{1}{2} \sum_{i=1}^n \tilde{R}_i e^{-2\tilde{\beta}^i} + \Lambda + \kappa_D^2 U(\Phi) \right]
 \end{aligned} \tag{16}$$

is the effective potential.

With the help of a regular coordinate transformation $\varphi = Q\beta$, $\beta = Q^{-1}\varphi$ midisuperspace metric (target space metric) \tilde{G} can be transformed to a pure Euclidean form: $\tilde{G}_{ij} d\beta^i \otimes d\beta^j = \sigma_{ij} d\varphi^i \otimes d\varphi^j = \sum_{i=1}^n d\varphi^i \otimes d\varphi^i$, $\sigma = \text{diag}(+1, +1, \dots, +1)$. An appropriate transformation $Q: \beta^i \rightarrow \varphi^i = Q^i_j \beta^j$ can be found, e.g., in [16]. We note that in the case of one internal space ($n=1$) this transformation is reduced to a simple redefinition

$$\varphi \equiv \varphi^1 := \pm \sqrt{\frac{d_1(D-2)}{D_0-2}} \tilde{\beta}^1, \tag{17}$$

which yields

$$\begin{aligned}
 U_{eff}[\varphi, \Phi] = & e^{2\varphi \sqrt{d_1/(D-2)(D_0-2)}} \\
 & \times \left[-\frac{1}{2} \tilde{R}_1 e^{2\varphi \sqrt{(D_0-2)d_1/(D-2)}} + \Lambda + \kappa_D^2 U(\Phi) \right].
 \end{aligned} \tag{18}$$

[For definiteness we use the minus sign in Eq. (17).]

It is clear now that stabilization of the internal spaces can be achieved if the effective potential U_{eff} has a minimum with respect to fields $\tilde{\beta}^i$ (or fields φ^i). In general it is possible for potential U_{eff} to have more than one extremum. But it can be easily seen that for the model under consideration we can get one extremum only. Let us find conditions which ensure a minimum at $\tilde{\beta} = 0$.

The extremum condition yields

$$\frac{\partial U_{eff}}{\partial \tilde{\beta}^k} \Big|_{\tilde{\beta}=0} = 0 \Rightarrow \tilde{R}_k = -\frac{d_k}{D_0-2} \left(\sum_{i=1}^n \tilde{R}_i - 2(\Lambda + \kappa_D^2 U(\Phi)) \right). \quad (19)$$

The left-hand side of this equation is a constant but the right-hand side is a dynamical function. Thus, stabilization of the internal spaces in such type of models is possible only when the effective potential has also a minimum with respect to the scalar field Φ (in Ref. [33] it was proved that for this model the only possible solutions with static internal spaces correspond to the case when the minimal coupled scalar field is in its extremum position too). Let Φ_0 be the minimum position for field Φ . From the structure of the effective potential (16) it is clear that minimum positions of the potentials $U_{eff}[\tilde{\beta}, \Phi]$ and $U(\Phi)$ with respect to field Φ coincide with each other:

$$\frac{\partial U_{eff}}{\partial \Phi} \Big|_{\Phi_0} = 0 \Leftrightarrow \frac{\partial U(\Phi)}{\partial \Phi} \Big|_{\Phi_0} = 0. \quad (20)$$

Hence, we should look for parameters which ensure a minimum of U_{eff} at the point $\tilde{\beta}^i=0, \Phi=\Phi_0$. Equations (19) show that there exists a fine tuning condition for the scalar curvatures of the internal spaces:

$$\frac{\tilde{R}_k}{d_k} = \frac{\tilde{R}_i}{d_i} \quad (i, k = 1, \dots, n). \quad (21)$$

Introducing the auxiliary quantity

$$\tilde{\Lambda} \equiv \Lambda + \kappa_D^2 U(\Phi) \Big|_{\Phi_0}, \quad (22)$$

we get the useful relations

$$\Lambda_{eff} := U_{eff} \Big|_{\tilde{\beta}^i=0, \Phi=\Phi_0} = \frac{D_0-2}{D-2} \tilde{\Lambda} = \frac{D_0-2}{2} \frac{\tilde{R}_k}{d_k}, \quad (23)$$

which show that $\text{sgn } \Lambda_{eff} = \text{sgn } \tilde{\Lambda} = \text{sgn } R_k$. It is clear that Λ_{eff} plays the role of an effective cosmological constant in the external spacetime. For the masses of the normal mode excitations of the internal spaces (gravitational excitons) and of the scalar field near the extremum position we obtain respectively [16]

$$m_1^2 = \dots = m_n^2 = -\frac{4\Lambda_{eff}}{D_0-2} = -2\frac{\tilde{R}_k}{d_k} > 0, \quad (24)$$

$$m_\Phi^2 := \frac{\partial^2 U(\Phi)}{\partial \Phi^2} \Big|_{\Phi_0}.$$

These equations show that for our specific model a global minimum can only exist in the case of compact internal spaces with negative curvature $R_k < 0$ ($k = 1, \dots, n$). The effective cosmological constant is negative also: $\Lambda_{eff} < 0$. Obviously, in this model it is impossible to trap the internal spaces at a minimum of U_{eff} if they are tori ($\tilde{R}_i = 0$) because

for Ricci-flat internal spaces the effective potential has no minimum at all. Equations (23) and (24) show also that a stabilization by trapping takes place only for $\tilde{\Lambda} < 0$.¹ This means that the minimum of the scalar field potential should be negative $U(\Phi_0) < 0$ for non-negative bare cosmological constant $\Lambda \geq 0$ or it should satisfy inequality $\kappa_D^2 U(\Phi_0) < |\Lambda|$ for $\Lambda < 0$.

For small fluctuations of the normal modes in the vicinity of the minima of the effective potential action (15) reads

$$S = \frac{1}{2\kappa_0^2} \int_{M_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \{ \tilde{R}[\tilde{g}^{(0)}] - 2\Lambda_{eff} \} - \frac{1}{2} \int_{M_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \sum_{i=1}^n (\tilde{g}^{(0)\mu\nu} \psi_{,\mu}^i \psi_{,\nu}^i + m_i^2 \psi^i \psi^i) + \tilde{g}^{(0)\mu\nu} \phi_{,\mu} \phi_{,\nu} + m_\phi^2 \phi \phi \right\}. \quad (25)$$

[For convenience we use here the normalizations: $\kappa_0^{-1} \tilde{\beta} \rightarrow \tilde{\beta}$ and $\sqrt{V_{D'}}(\Phi - \Phi_0) \rightarrow \phi$.] Thus, conformal excitations of the metric of the internal spaces behave as massive scalar fields developing on the background of the external spacetime. In analogy with excitons in solid state physics where they are excitations of the electronic subsystem of a crystal, we called the excitations of the subsystem of internal spaces gravitational excitons [16]. Later, since [14,15] these particles are also known as radions.

III. INFLATION OF THE EXTERNAL SPACE

In this section we discuss briefly the possibility for inflation in the external space of our model. We perform the analysis in the Einstein frame where the effective theory is described by action (15) and inflation depends on the form of potential (16).

For simplicity we consider a model with only one internal space and an effective potential given by Eq. (18). All our conclusions can be easily generalized to a model with n internal spaces.

First, we consider region

$$e^{2\varphi\sqrt{(D_0-2)/d_1(D-2)}} \gg |\Lambda + \kappa_D^2 U(\Phi)|, \quad (26)$$

where the effective potential reads

¹An interesting scenario for a dynamical stabilization of the internal space was proposed in Ref. [34] for a model with $\tilde{\Lambda} = \tilde{R}_i = 0$. If, at some stage of the Universe evolution, the inflaton field Φ reached its zero minimum and was frozen out, then there exists a solution $\tilde{\beta} \rightarrow 0$ for times $t \rightarrow \infty$ which corresponds to a dynamical stabilization of the internal space. However, the inflaton field is never frozen out completely and its dynamics can destabilize the internal space. An investigation of this problem in collaboration with Anupam Mazumdar will be presented soon in a common paper.

$$U_{eff} \approx \frac{1}{2} |\bar{R}_1| e^{2\varphi \sqrt{(D-2)/d_1(D_0-2)}}. \quad (27)$$

It is well known [35] that for models with potential $U(\varphi) \sim A e^{\lambda\varphi}$ the scale factor behaves as $\tilde{a} \sim \tilde{t}^{2/\lambda^2}$ and power law inflation takes place if $\lambda^2 < 2$. In our case we have

$$\lambda^2 = \frac{4(D-2)}{d_1(D_0-2)} \Big|_{D_0=4} = 2 \left(1 + \frac{2}{d_1} \right) > 2 \quad (28)$$

and power law inflation is impossible in this region of the model. For the model with n internal spaces the assisted inflation proposed in Ref. [36] is impossible also in this region because of the form of the effective potential (it is impossible to split the effective potential into a sum of n terms where each of them depends on one scalar field only).

Second, we consider the region near the minimum of the effective potential. In the scenario of assisted chaotic inflation [25–27] with a sufficiently large number of scalar fields ψ^i inflation occurs at scales much less than Planck scale: $|\psi^i| \ll 1$. In our model the effective action for these fields is given by Eq. (25) and it has the typical form of an action allowing for this type of inflation. Therefore, it is of interest to investigate the possibility for assisted chaotic inflation here. Unfortunately, for our particular model the internal space stabilization takes place only for negative effective cosmological constant. This destroys inflation because, as it follows from Eq. (24) $m_i^2 \sim |\Lambda_{eff}|$, the energy density of the potential U_{eff} is not sufficient for inflation. There is also another drawback of theories with negative cosmological constant. Even if they have a period of inflation there is a problem of successful completion of it. We shall return to this problem in the next section.

Third, we consider the region

$$\kappa_D^2 U(\Phi) \gg \Lambda + \frac{1}{2} |\bar{R}_1| e^{2\varphi \sqrt{(D_0-2)/d_1(D-2)}}, \quad (29)$$

where the effective potential reads

$$U_{eff} \approx e^{2p\varphi} \kappa_D^2 U(\Phi), \quad p := \sqrt{\frac{d_1}{(D-2)(D_0-2)}}. \quad (30)$$

For models with $n+1$ scalar fields the slow roll conditions are [30]

$$\epsilon \approx \frac{1}{2U_{eff}^2} \sum_{i=1}^{n+1} \left(\frac{\partial U_{eff}}{\partial \varphi^i} \right)^2 \quad (31)$$

and

$$\eta_i \approx -\epsilon + \frac{1}{U_{eff}} \sum_{j=1}^{n+1} \frac{\partial^2 U_{eff}}{\partial \varphi^i \partial \varphi^j} \left(\frac{\partial U_{eff}}{\partial \varphi^j} \Big/ \frac{\partial U_{eff}}{\partial \varphi^i} \right), \quad (32)$$

$$i = 1, \dots, n+1.$$

Inflation is possible if these parameters are small: $\epsilon, |\eta_i| < 1$. For potential (30) we get

$$\epsilon \approx \eta_1 \approx 2p^2 + \epsilon_\Phi,$$

$$\eta_2 \approx 2p^2 + \eta_\Phi, \quad (33)$$

where $\epsilon_\Phi := \frac{1}{2} (U'(\Phi)/U(\Phi))^2$ and $\eta_\Phi := -\epsilon_\Phi + U''(\Phi)/U(\Phi)$. Because of

$$2p^2|_{D_0=4} = 1 - \frac{2}{d_1+2} < 1, \quad (34)$$

inflation is possible in this region if

$$\epsilon_\Phi, \eta_\Phi \ll 1. \quad (35)$$

Thus, the scalar field Φ can act as inflaton and drive the inflation of the external space if its potential in region (29) satisfies conditions (35). It is clear that estimates (34) and (35) are rather crude and they show only the principal possibility for inflation to occur. For each particular form of $U(\Phi)$ a detailed analysis of the dynamical behavior of the fields in this region should be performed to confirm inflation. Obviously, if the inflation in our model is realized it takes place before the stabilization of the internal spaces. In the case of constant scalar field $\Phi = \text{const}$ or its absence inflation of the external space in our model is not possible at all.

IV. DISCUSSION AND CONCLUSIONS

In the present paper we considered the possibility for stabilization of the internal space and inflation in the external space using as example a multidimensional cosmological toy model with minimal coupled scalar field as matter source. The calculations above were performed in a model with the electroweak scale M_{EW} as fundamental scale of the D -dimensional theory [see Eq. (13)]. Clearly, it is also possible to choose the Planck scale as the fundamental scale.

For this purpose we will not fix the compactification scale of the internal spaces at the present time. We consider them as free parameters of the model and demand only that $L_{Pl} < a_{(0)i} = e^{\beta_0^i} < L_F$. So, we shall not transform β^i to $\tilde{\beta}^i$. In this case, after dimensional reduction of action (8) the effective D_0 -dimensional gravitational constant κ_0^2 is defined as

$$\frac{1}{\kappa_0^2} = V_I \frac{(L_{Pl})^{D'}}{\kappa_D^2}. \quad (36)$$

At the other hand there holds $\kappa_0^2 = 8\pi/M_{Pl}^2$ (for $D_0=4$). Thus, $\kappa_D^2 = 8\pi V_I/M_{Pl}^{(2+D')}$, so that the Planck scale becomes the fundamental scale of the D -dimensional theory. In this approach Eqs. (11), (14)–(18) preserve their form with only substitutions $\tilde{\beta} \rightarrow \beta$ and $\tilde{R}_i \rightarrow R_i$.

The analysis of the internal space stabilization shows that the fine tuning condition (21) is not changed:

$$\frac{R_k}{d_k} e^{-2\beta_0^k} = \frac{R_i}{d_i} e^{-2\beta_0^i}, \quad i, k = 1, \dots, n \quad (37)$$

and the masses squared of the gravitational excitons and the effective cosmological constant are shifted by the same prefactor:

$$\begin{aligned} m_i^2 &\rightarrow \left(\prod_{i=1}^n e^{d_i \beta_0^i} \right)^{-2/(D_0-2)} m_i^2 \\ &= -2 \left(\prod_{i=1}^n e^{d_i \beta_0^i} \right)^{-2/(D_0-2)} \frac{R_i}{d_i}, \\ \Lambda_{eff} &\rightarrow \left(\prod_{i=1}^n e^{d_i \beta_0^i} \right)^{-2/(D_0-2)} \Lambda_{eff} \\ &= \frac{D_0-2}{2} \left(\prod_{i=1}^n e^{d_i \beta_0^i} \right)^{-2/(D_0-2)} \frac{R_i}{d_i}. \end{aligned} \quad (38)$$

For example, in the one-internal-space case we get the estimate² [16]

$$|\Lambda_{eff}| \sim m_1^2 \sim (e^{-\beta_0})^{2[(D-2)/(D_0-2)]} |_{D_0=4} = (a_{(0)1})^{-(D-2)}. \quad (39)$$

This expression shows that due to the power $(2-D)$ the effective cosmological constant and the masses of the gravitational excitons can be very far from the planckian values even for scales of compactification of the internal spaces close to the Planck length.

Another important note consists in the observation that the Einstein frame metrics of the external spacetime in both approaches are equivalent to each other up to a numerical prefactor:

$$\tilde{g}_{\mu\nu}^{(0)}|_{EW} = v_0^{-2/(D_0-2)} \tilde{g}_{\mu\nu}^{(0)}|_{Pl}. \quad (40)$$

Equation (14) shows that in the electroweak approach the Brans-Dicke and Einstein scales coincide with each other at the point of stabilization: $\tilde{\beta}^i = 0 \Rightarrow \Omega = 1$. In the Planck fundamental scale approach this has place when the internal scale factors are equal to the Planck length: $\beta^i = 0 \Rightarrow \Omega = 1$. This does not mean that in the latter approach the stabilization of the internal space takes place at the Planck length. Depending on the concrete form of the effective potential U_{eff} its minimum position (stabilization point) β_0^i can be located at much larger scales $L_{Pl} \ll a_{(0)i} = e^{\beta_0^i} L_{Pl}$. Generally speaking, we should not exclude also a possibility for internal spaces to change very slowly with time. In this case β_0^i is not so strictly defined as for models with the internal space stabilization in minima of the effective potential.

Let us return to the comparison of the electroweak and the Planck scale approaches. From Eqs. (38) it is clear that the reason for the rescaling (lightening) of the effective cosmological constant as well as of the gravitational exciton

masses in the Planck scale approach consists in the prefactor $(\prod_{i=1}^n e^{d_i \beta_0^i})^{-2/(D_0-2)}$. In spite of the smallness of the internal space sizes in the Planck fundamental scale approach ($L_{Pl} < a_{(0)1} < L_F$) in comparison with the sizes in the electroweak fundamental scale approach ($a_{(0)1} \sim 10^{-1}$ cm for $D'=2$ and $a_{(0)1} \rightarrow 10^{-17}$ cm for $D' \rightarrow \infty$), the prefactors in Eqs. (38) can considerably reduce the values of Λ_{eff} and m_i making them much smaller than in the electroweak approach.

Let us compare now some estimates following from the electroweak as well as from the Planck fundamental scale approaches. (We use the obvious subscripts *EW* and *Pl*, respectively.)

In the first case, the scale of the internal space compactification is given by formula (2). We take for definiteness the total number of dimensions $D=6$ and $D=10$ and obtain respectively the following scales of compactification: $a_{(0)1} \sim 10^{-1}$ cm for $D=6$ and $a_{(0)1} \sim 10^{-9}$ cm for $D=10$. Then, from Eqs. (23) and (24) we get

$$|\Lambda_{eff}| |_{EW} \sim \frac{1}{a_{(0)1}^2} \sim \begin{cases} 10^2 \text{ cm}^{-2} \sim 10^{-64} \Lambda_{Pl}, & D=6, \\ 10^{18} \text{ cm}^{-2} \sim 10^{-48} \Lambda_{Pl}, & D=10, \end{cases} \quad (41)$$

and

$$m_1 |_{EW} \sim \frac{1}{a_{(0)1}} \sim \begin{cases} 10^{-32} M_{Pl} \sim 10^{-4} \text{ eV}, & D=6, \\ 10^{-24} M_{Pl} \sim 10^4 \text{ eV}, & D=10. \end{cases} \quad (42)$$

In the second case, the scale of compactification is not fixed, but a free parameter. We demand only that it should be smaller than the Fermi length. For definiteness let us use $a_{(0)1} \sim 10^{-18}$ cm. Then, from Eq. (39) we obtain

$$|\Lambda_{eff}| |_{Pl} \sim a_{(0)1}^{-(D-2)} \sim \begin{cases} 10^6 \text{ cm}^{-2} \sim 10^{-60} \Lambda_{Pl}, & D=6, \\ 10^{-54} \text{ cm}^{-2} \sim 10^{-120} \Lambda_{Pl}, & D=10, \end{cases} \quad (43)$$

and

$$m_1 |_{Pl} \sim a_{(0)1}^{-(D-2)/2} \sim \begin{cases} 10^{-30} M_{Pl} \sim 10^{-2} \text{ eV}, & D=6, \\ 10^{-60} M_{Pl} \sim 10^{-32} \text{ eV}, & D=10. \end{cases} \quad (44)$$

Estimates (41) and (43) show that for the electroweak scale the effective cosmological constant is much greater than the present day observable limit $\Lambda \leq 10^{-122} \Lambda_{Pl} \sim 10^{-57} \text{ cm}^{-2}$ (for our model $|\Lambda_{eff}| |_{EW} \geq 10^2 \text{ cm}^{-2}$), whereas in the Planck scale approach we can satisfy this limit even for very small compactification scales. For example, if we demand in accordance with observations $|\Lambda_{eff}| \sim 10^{-122} \Lambda_{Pl}$ then Eq. (39) gives a compactification scale $a_{(0)1} \sim 10^{122/(D-2)} L_{Pl}$. Thus, $a_{(0)1} \sim 10^{15} L_{Pl} \sim 10^{-18}$ cm for $D=10$ and $a_{(0)1} \sim 10^5 L_{Pl} \sim 10^{-28}$ cm for $D=26$, which does not contradict to observations because for this approach the scales of compactification should be $a_{(0)1} \leq 10^{-17}$ cm. Assuming an estimate $\Lambda_{eff} \sim 10^{-122} L_{Pl}$, we automatically get from Eq. (39) the value of the gravitational exciton mass: $m_1 \sim 10^{-61} M_{Pl} \sim 10^{-33} \text{ eV} \sim 10^{-66} g$ which is

²We use standard Planck length unit conventions with $[m] = \text{cm}^{-1}$ and the corresponding shorthand, e.g., $m_1^2 \sim (a_{(0)1})^{-(D-2)} \equiv (a_{(0)1}/L_{Pl})^{-(D-2)} L_{Pl}^{-2}$.

extremely light. Nevertheless such light particles are not in contradiction with the observable Universe, as we shall show below.

Similar to the Polonyi fields in spontaneously broken supergravity [37,38] or moduli fields in the hidden sector of SUSY [3,39–41] the gravitational excitons are weakly interacting massive particles (WIMPs) [42] because their coupling to the observable matter is suppressed by powers of the Planck scale. In Ref. [43] we show that the decay rate of the gravitational excitons with mass m_φ is $\Gamma \sim m_\varphi^3/M_{Pl}^2$ as for Polonyi and moduli fields.

Let us assume for a moment that after inflation the inflaton field ϕ has already decayed and produced the main reheating of the Universe. For our model it may happen if $m_\phi \gg m_\varphi$ and the inflaton field starts to oscillate and decay much earlier than the φ field (coherent oscillations of field ϕ with mass m_ϕ usually start when the Hubble constant $H \lesssim m_\phi$). The Universe is radiation dominated in this period and the Hubble constant is defined by $H \sim T^2/M_{Pl}$. After the temperature is fallen to the value $T_{in} \sim \sqrt{m_\phi M_{Pl}}$ the scalar field³ φ begins to oscillate coherently around the minimum and its density evolves as T^3 [38,44]:

$$\rho_\varphi(T) = \rho_\varphi(T_{in})(T/T_{in})^3 = m_\varphi^2 \varphi_{in}^2(T_{in})(T/T_{in})^3, \quad (45)$$

where $\varphi_{in} := (\varphi - \varphi_0)_{in}$ is the amplitude of initial oscillations of the field φ near the minimum position. It is clear that for the extremely light particles we can neglect their decay ($\Gamma_\varphi \approx 0$). Then, because the ratio ρ_φ/ρ_{rad} increases as $1/T$, at some temperature the Universe will be dominated (up to present time) by the energy density of the coherent oscillations. We can easily estimate the mass of the gravitational excitons which overclose the Universe. Assuming that at present time $\rho_\varphi \lesssim \rho_c$, where ρ_c is the critical density of the present day Universe, we obtain⁴

$$m_\varphi \lesssim 10^{-56} M_{Pl} \left(\frac{M_{Pl}}{\varphi_{in}} \right)^4. \quad (46)$$

Usually, it is assumed that $\varphi_{in} \sim O(M_{Pl})$ although it depends on the form of U_{eff} and can be considerably less than M_{Pl} . If we put $\varphi_{in} \sim O(M_{Pl})$ then excitons with masses $m_\varphi \lesssim 10^{-28}$ eV will not overclose the Universe [38,40]. If $\varphi_{in} \ll M_{Pl}$ this estimate will be not so severe. We see that our mass $m_\varphi \sim 10^{-33}$ eV satisfies the most severe estimate. It can be considered as hot dark matter which negligibly contributes to the total amount of dark matter and does not contradict to the model of cold dark matter.

Of course, as it follows from Eqs. (39) and (44) the mass m_φ could be considerably heavier than 10^{-33} eV but as result we would arrive at an effective cosmological constant

³Here, $\varphi = \pm (M_{Pl}/\sqrt{8\pi}) \sqrt{d_1(D-2)/(D_0-2)} \beta^1$ where β^1 is the logarithm of the internal space scale factor: $a_1 = e^{\beta^1} L_{Pl}$. If stabilization occurs at $a_{(0)1} \sim 10^{-n} L_{Pl}$ ($0 < n < 18$), then it corresponds to the minimum position $\varphi_0 = \mp (n \ln 10 / \sqrt{8\pi}) \sqrt{d_1(D-2)/(D_0-2)} M_{Pl}$.

⁴See also *Note added*.

greater than the observable one [see Eqs. (43) and (44) for $D=6$ and $a_{(0)1} \sim 10^{-18}$ cm] and we would need a mechanism for its reduction to the observable value. An example for such a reduction of the cosmological constant was proposed in [41] for SUSY breaking models with moduli masses $m \sim 10^{-2} - 10^{-3}$ eV. Such masses we get also in our model if we take for the Planck scale approach $D=6$ and $a_{(0)1} \sim 10^{-18}$ cm [see Eq. (44)]. For these particles we cannot neglect the decay rate Γ_φ which results in converting of the coherent oscillations into radiation. In this case the Universe has a further reheating to the temperature [38,39]

$$T_{RH} \sim \sqrt{\frac{m_\varphi^3}{M_{Pl}}}. \quad (47)$$

For $m_\varphi \sim 10^{-2}$ eV the reheating temperature $T_{RH} \sim 10^{-23}$ MeV $\ll 1$ MeV is much less than the temperature $T \sim 1$ MeV at which the nucleosynthesis begins. Thus, either decaying particles should have masses $m_\varphi > 10^4$ GeV to get $T_{RH} > 1$ MeV or we should get rid off such particles before nucleosynthesis. The latter can be achieved if the decay rate becomes larger. In [41] it was proposed that at a very early stage of the Universe evolution (after inflation) WIMPs collapse into stars (e.g. modular stars) where their field strength could be very large and leads to a substantial enhancement of the decay into ordinary particles. For example, in Ref. [43] it is shown that gravexcitons have a coupling to photons of the form $(\varphi/M_{Pl}) F_{\mu\nu}^2$. In the core of such stars the gravexciton amplitude φ might be much larger than M_{Pl} , enhancing the coupling of this field to photons and leading to explosions of these stars into bursts of photons.

As it follows from Eqs. (24) and (42), in the electroweak approach gravexciton masses should satisfy the inequality $m_\varphi \gtrsim 10^{-4}$ eV. If the above mentioned mechanism of the gravexciton energy dilution due to modular star explosions or due to some other reasons⁵ does not work, the bound $m_\varphi \gtrsim 10^4$ GeV is valid and leads to the large D' limit ($D' \gg 30$) with a scale of compactification $a_{(0)1} \sim \sqrt{D'} m_\varphi^{-1}$ [see Eq. (24)]. Thus for $D' \sim 100$ and $m_\varphi \sim 10^4$ GeV we get $a_{(0)1} \sim 10^{-17}$ cm which is not in strong contradiction with the value $a_{(0)1} \sim 10^{-16.7}$ cm which follows from Eq. (2). It is clear that in this approach an increasing of the mass by one order requires an increasing of the number of internal dimensions by two orders.

Above, we considered the case $m_\phi \gg m_\varphi$ when the inflaton field starts to oscillate coherently much earlier than the scale factors of the internal spaces. Let us suppose now that $m_\phi \sim m_\varphi \equiv m$. Thus, the inflaton ϕ and gravexciton φ fields start to oscillate coherently at the same time t_{in} with approximately the same initial amplitude $\phi_{in} \sim \varphi_{in}$. At this time the Universe becomes matter dominated with $\rho_\varphi \sim \rho_\phi \sim 1/\tilde{a}^3$

⁵For example, in [15] for this purpose a short period of late inflation was proposed which should be followed by a reheating. However, it is necessary to be rather careful to avoid the generation of quantum fluctuations of gravexcitons during inflation again [45].

where \tilde{a} is the scale factor of the external space-time. We assume also that the inflaton ϕ is not a WIMP and its decay rate $\Gamma_\phi \sim \alpha_\phi^2 m \gg \Gamma_\varphi \sim m^3/M_{Pl}^2$. Thus the effective coupling α_ϕ of the inflaton field ϕ satisfies: $\alpha_\phi \gg m/M_{Pl}$. Because $m \ll M_{Pl}$ the effective coupling α_ϕ still may be much less than 1.

First, we consider the case when the gravexciton decay rate is negligibly small: $\Gamma_\varphi \approx 0$. Let t_{RH} be the time of reheating due to inflaton decay and let us suppose that all the inflaton energy is converted into radiation ($\rho_\phi(t_{RH}) \sim \rho_{rad|_{RH}} \sim T_{RH}^4$). It can be easily seen that for $t > t_{RH}$ the relative contribution of φ to the energy density starts to increase as

$$\frac{\rho_\varphi(T)}{\rho_{rad}(T)} = \frac{T_{RH}}{T}. \quad (48)$$

Here, in the sudden decay approximation the reheating temperature, T_{RH} , is defined by equating the Hubble constant with the rate of decay: $H(t_D) \sim \Gamma_\phi \sim \alpha_\phi^2 m$, where $t_D \sim t_{RH}$ is the decay time. Because $H^2(t_D) \sim M_{Pl}^{-2} \rho_\phi|_{t_D} \sim T_{RH}^4/M_{Pl}^2$ we get

$$m \sim \frac{1}{\alpha_\phi^2} \frac{T_{RH}^2}{M_{Pl}}. \quad (49)$$

This formula shows that to get the temperature $T_{RH} > 1$ MeV, which is necessary for the nucleosynthesis, the mass should satisfy the inequality

$$m \geq \frac{1}{\alpha_\phi^2} 10^{-16} \text{ eV}. \quad (50)$$

At the other hand, at present time (which we denote by a subscript 0) the condition that gravexcitons do not overclose the Universe reads $\rho_{\phi_0} = (T_{RH}/T_0) \rho_{rad|_0} \leq \rho_c$ and gives a second limit for the mass:

$$m \leq \frac{1}{\alpha_\phi^2} \left(\frac{\rho_c}{\rho_{rad|_0}} \right)^2 \frac{T_0^2}{M_{Pl}}. \quad (51)$$

Inserting into this formula the present day values for the temperature T_0 and the critical energy density ρ_c we obtain

$$m \leq \frac{1}{\alpha_\phi^2} 10^{-26} \text{ eV}, \quad (52)$$

which obviously is in contradiction to the previous estimate (50).

Second, to solve this problem we consider the possibility of a further reheating due to gravexciton decay: $\Gamma_\varphi \neq 0$. In order to estimate the temperature at which this decay occurs we should take into account that after the first reheating [with the temperature defined by Eq. (49)] the Universe is matter dominated because $\rho_\varphi/\rho_{rad} = T_{RH}/T > 1$ for $T < T_{RH}$ and for the Hubble constant holds $H^2 \sim \rho_\varphi/M_{Pl}^2$. Thus,

equating the Hubble constant with the decay rate: $H(T'_D) \sim \Gamma_\varphi \sim m^3/M_{Pl}^2$ we obtain the temperature of the gravexciton decay:

$$(T'_D)^3 \sim \frac{1}{\alpha_\phi} \frac{m^{11/2}}{M_{Pl}^{5/2}}. \quad (53)$$

In this scenario the temperatures of the gravexciton decay and the reheating are denoted by a prime to distinguish them from the corresponding temperatures of inflaton decay and reheating. In the sudden decay approximation the temperature of the second reheating is obtained by equating the squared decay rate and the radiation energy density just after reheating [because $H^2(T'_D) \sim \Gamma_\varphi^2 \sim \rho_{rad}/M_{Pl}^2$] which obviously leads again to Eq. (47) (where T_{RH} should be replaced by T'_{RH}). Again, for a successful nucleosynthesis with $T'_{RH} > 1$ MeV the mass should be $m \geq 10^4$ GeV. The reheating from T'_D to T'_{RH} produces an entropy increase given by

$$\Delta = \left(\frac{T'_{RH}}{T'_D} \right)^3 \sim \alpha_\phi \frac{M_{Pl}}{m} \gg 1, \quad (54)$$

which is much greater than 1 because $\alpha_\phi \gg m/M_{Pl}$. However, it may be much less (not so severe) than the usual estimate [38]: $\Delta \sim M_{Pl}/m$ because α_ϕ may be much less than 1. If we require $\Delta \leq 10^5$, as a maximal permissible factor for the dilution of the high-temperature baryogenesis, we obtain the bound $m \geq \alpha_\phi 10^{-5} M_{Pl}$ and for $\alpha_\phi \leq 1$ this bound is not so strong as the usual one: $m \geq 10^{14}$ GeV.

Summarizing the discussion we see that in models, where the coherent oscillation of gravitational excitons starts in the radiation dominated era, the gravexcitons should be either extremely light [see Eq. (46)] or very heavy particles ($m_\varphi > 10^4$ GeV for a successful nucleosynthesis; in case that the hot baryogenesis is taken into account: $m_\varphi > 10^{14}$ GeV). In models, where inflaton and gravitational exciton start their coherent oscillation at the same time, extremely light excitons are forbidden. Heavier excitons with masses $m_\varphi \geq \alpha_\phi 10^{-5} M_{Pl}$ are allowed (for a successful nucleosynthesis and high-temperature baryogenesis).

As conclusion we would like to note that in our toy model the stabilization of the internal spaces is realized only when the effective cosmological constant is negative (for both fundamental scale approaches). It is well known that for such models inflation is never successfully completed [41], because in this case our (external) space has a turning point at its maximal scale factor where it stops to expand and begins to contract. If the spatial curvature of our Universe is non-negative (according to the latest observational data it is zero), then the internal scale factors cannot be frozen because solutions $\beta^i = \beta_0^i = \text{const}$ correspond to a negative squared Hubble constant: $H^2 = \frac{1}{3} [\frac{1}{2} \bar{G}_{ij} \dot{\beta}^i \dot{\beta}^j + U_{eff}] \Rightarrow \frac{1}{3} \Lambda_{eff} < 0$. To describe the postinflationary stage for such models we should extend our consideration including, e.g., additional perfect fluid terms into the action functional which correspond to usual matter in the universe (see [19] for the details of this method). Another possible generalization con-

sists in an inclusion of additional terms which result in a positive effective cosmological constant in accordance with recent observational data [46]. This can be achieved, e.g., with the help of antisymmetric form-fields [47]. For these models the gravexciton masses and the (positive) effective cosmological constant are defined by equations similar to Eq. (39). Such models can solve the following three important problems simultaneously: they yield stabilization of the internal spaces, allow for inflation of the external space, and lead to a positive observable effective cosmological constant. In these models the mechanism of lightening of the effective cosmological constant as well as the gravitational exciton masses will work also in the Planck fundamental scale approach because Eqs. (38) are general for this type of models.

Note added. Alexander Sakharov informed us about another upper bound on m_φ following from isocurvature gravexciton fluctuations if $m_\phi \gg m_\varphi$ because in this case gravexcitons on the stage of inflation can be considered as massless particles. These isocurvature fluctuations result in a CMBR anisotropy $\delta T/T$. The amplitude of these fluctuations can be estimated as $\delta\varphi \approx H_{inf}/2\pi$ and is connected with $\delta T/T$ as follows: $\delta T/T \approx (\rho_\varphi/\rho_c)(\delta\varphi/\varphi_{in}) \approx (\rho_\varphi/\rho_c)(H_{inf}/2\pi\varphi_{in})$, where H_{inf} is the Hubble constant at the inflation stage. According to COBE data, $\delta T/T$

$\lesssim 10^{-5}$ and $H_{inf} \approx 10^{-5} M_{Pl}$. Thus, we get the following limitation on the gravexciton energy density at the present time: $\rho_\varphi \lesssim 2\pi\rho_c\varphi_{in}/M_{Pl}$. Substitution of Eq. (45) into this limitation gives

$$m_\varphi \lesssim 10^{-55} M_{Pl} \left(\frac{M_{Pl}}{\varphi_{in}} \right)^2. \quad (55)$$

So, if $\varphi_{in} \sim O(M_{Pl})$ then both Eqs. (46) and (55) give close limitations on m_φ . However, for $\varphi_{in} \gg M_{Pl}$ we should use Eqs. (46) and (55) correspondingly. In the case of decaying gravexcitons the CMBR anisotropy due to gravexciton isocurvature fluctuations is washed out.

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