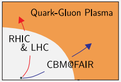


Equation of state for strongly interacting matter within a HTL quasiparticle model

Robert Schulze

collaborators: M. Bluhm, B. Kämpfer
TU Dresden, FZ Dresden-Rossendorf

- QPM with Im$\Pi_{\mathrm{i}} \neq 0$, plasmons and plasminos from HTL
- extrapolation of lattice QCD to large baryon densities, e.g. CBM@FAIR
- EOS for hydrodynamic phase of heavy-ion collisions (SPS, RHIC, LHC)

From QCD to thermodynamics

QCD: $\mathcal{L}$
$\rightarrow$ propagators
$\rightarrow$ self-energies

Blaizot, Rebhan, Iancu

thermodyn. potential Ω
$\rightarrow$ state variables: p, s, n_q,...
$\rightarrow$ EOS $e = e(p)$
$\rightarrow$ $T^{\mu\nu}$, hydrodynamics

lattice QCD
$\rightarrow$ available @ $\mu \simeq 0$
$\rightarrow$ $\mu \neq 0$: sign problem

CJT formalism

- effective action

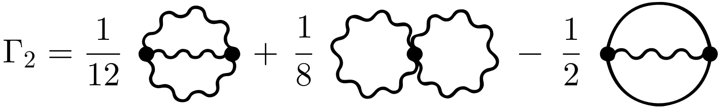
$$\Gamma[D,S] = I - \frac{1}{2}\left\{\mathrm{Tr}\left[\ln D^{-1}\right] + \mathrm{Tr}\left[D_0^{-1}D - 1\right]\right\} + \left\{\mathrm{Tr}\left[\ln S^{-1}\right] + \mathrm{Tr}\left[S_0^{-1}S - 1\right]\right\} + \Gamma_2[D,S]$$

- translation-invariant systems, no broken symmetries

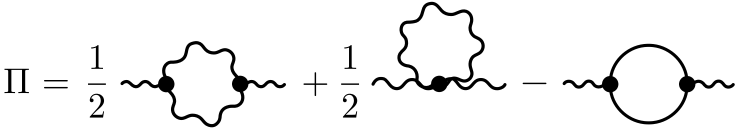
$$\frac{\Omega}{V} = \mathrm{tr}\int\frac{\mathrm{d}^4k}{(2\pi)^4}n_\mathrm{B}(\omega)\,\mathrm{Im}\left(\ln D^{-1} - \Pi D\right) + 2\,\mathrm{tr}\int\frac{\mathrm{d}^4k}{(2\pi)^4}n_\mathrm{F}(\omega)\,\mathrm{Im}\left(\ln S^{-1} - \Sigma S\right) - \frac{T}{V}\Gamma_2$$

2-loop QCD thermodynamics

- truncate Γ_2 at 2-loop order



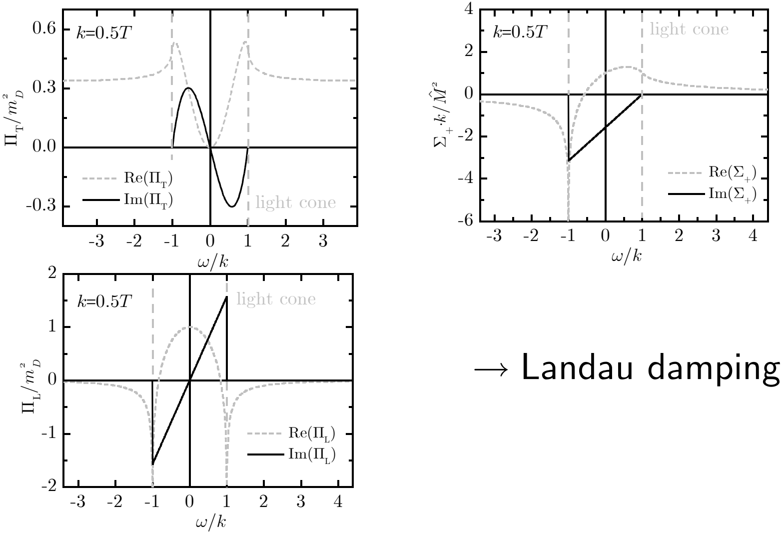
$\longrightarrow$ self-energies of 1-loop order



- gauge invariance: HTL self-energies instead

HTL self-energies

- $\mathrm{Im}\,\Pi \neq 0$ below the lightcone (solid lines)



$\longrightarrow$ Landau damping

Entropy

- stationarity of Ω

$$s := -\frac{1}{V}\,\frac{\partial\Omega}{\partial T}\bigg|_{\mu} = -\frac{1}{V}\left(\frac{\partial\Omega}{\partial T}\bigg|_{\mathrm{expl.}} + \underbrace{\frac{\delta\Omega}{\delta D}}_{0}\,\frac{\partial D}{\partial T}\right)_{\mu}$$

$$= s_{g,\mathrm{T}} + s_{g,\mathrm{L}} + s_{q,\mathrm{Pt}} + s_{q,\mathrm{Pl}} + s' \qquad\qquad s' = 0$$

Vanderheyden, Baym: JSP'98

- e.g. gluons:

$$s_{g,\mathrm{T}} \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\mathrm{B}}}{\partial T} \left\{ \underbrace{\pi\varepsilon(\omega)\Theta\left(-\mathrm{Re}D_{\mathrm{T}}^{-1}\right)}_{\text{qp contribution}} + \underbrace{\mathrm{Re}D_{\mathrm{T}}\mathrm{Im}\Pi_{\mathrm{T}} - \arctan\left(\frac{\mathrm{Im}\Pi_{\mathrm{T}}}{\mathrm{Re}D_{\mathrm{T}}^{-1}}\right)}_{\text{damping terms}} \right\}$$

- HTL QPM model $\Longleftrightarrow$ effective QPM

~~coll. modes~~, ~~damping~~, asympt. disp. rel.

Blaizot, Iancu, Rebhan: PRD'01
Rebhan, Romatschke: PRD'03

Bluhm, Kämpfer, RS, Seipt: EPJC'07

Pressure

$$s \sim \left(\left.\frac{\partial \Omega}{\partial T}\right|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial T} \right)_{\mu}$$

$$s_i \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\mathrm{B/F}}}{\partial T} \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

- self-consistent formulation of the pressure

$$p = -\frac{\Omega}{V} := \sum_i p_i - B$$

$$p_i \sim \int_{\mathrm{d}^4k} n_{\mathrm{B/F}} \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

$$\frac{\partial B}{\partial T} := \sum_i \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} \qquad \left(\frac{\partial B}{\partial \mu} = \sum \frac{\because}{\because} \frac{\because}{\partial \mu} \right)$$

- entropy density

$$s = \frac{\partial p}{\partial T} = \sum_i s_i + \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} - \frac{\partial B}{\partial T} = \sum_i s_i$$

- net quark density

$$n \sim \left.\frac{\partial \Omega}{\partial \mu}\right|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial \mu}$$

$$n_q = \frac{\partial p}{\partial \mu} \sim \int_{\mathrm{d}^4k} \left(\frac{\partial n_{\mathrm{F}}}{\partial \mu} + \frac{\partial n_{\mathrm{F}}^{A}}{\partial \mu} \right) \left\{ \mathrm{qp} + \mathrm{damping} \right\}$$

Effective coupling

- fundamental parameter

$$g^2(x^2) = \frac{16\pi^2}{\beta_0 \ln(x^2)} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln[\ln(x^2)]}{\ln(x^2)}\right)$$

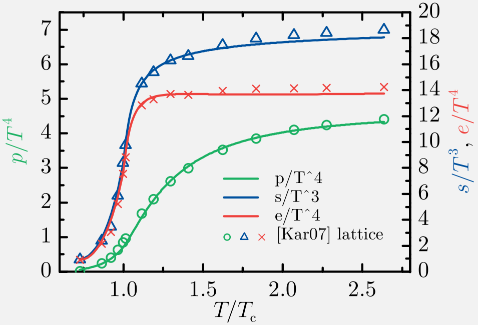
- running coupling g^2 $\qquad x = \frac{\bar{\mu}}{\Lambda_{\mathrm{QCD}}}$ $\qquad \bar{\mu} \sim T$

$$\Big\downarrow \quad T > T_c,\ \mu = 0$$

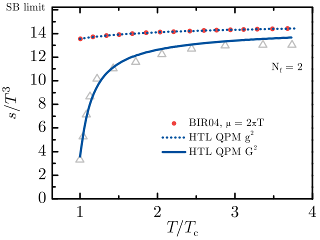
- effective coupling G^2 $\qquad x = \frac{\lambda}{T_c}(T - T_s)$

Adjustment @ $\mu = 0$

- $\mu = 0$: adjust to ℓQCD
  - $\rightarrow \; T_s, \; \lambda$ fixed
  - $\rightarrow \; G^2(T, \mu = 0)$

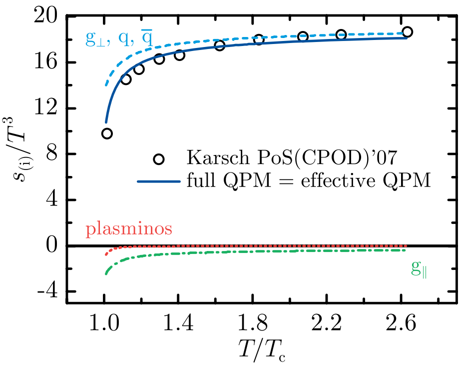


- $T_s = 0$:



Influence of coll. modes + LD @ $\mu = 0$

- individual entropy contributions



- Landau damping large close to T_c, decreases for higher temperatures state variables s, n, p, ...

effective coupling G^2

@ $\mu = 0$:
lattice data

@ $\mu \neq 0$:
??

Into the T-μ-plane

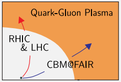
- $\mu > 0$: stationary potential, self-consistent model
  $\rightarrow$ impose Maxwell's relation

$$\frac{\partial s}{\partial \mu} = \frac{\partial n}{\partial T} \qquad \longrightarrow \qquad a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

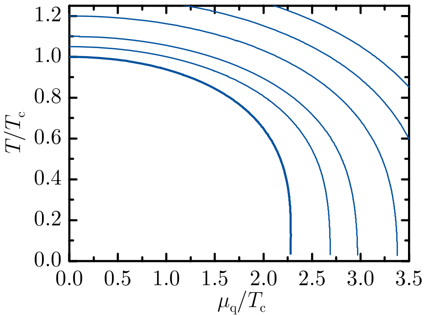
Peshier, Kämpfer, Soff: PRC'00, PRD'02

- solve quasilinear PDE for $G^2(T, \mu \neq 0)$ using method of characteristics
- test with lattice data for $\mu \simeq 0$: successful (eQPM)

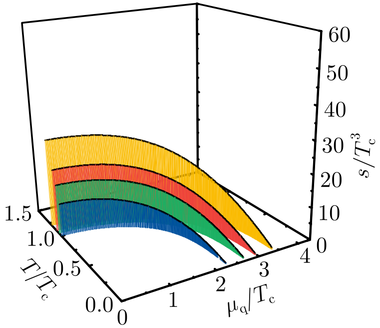
Larger chemical potential



- full HTL QPM:
  stable characteristics



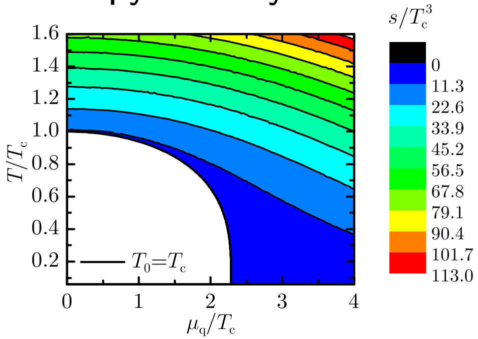
- entropy density

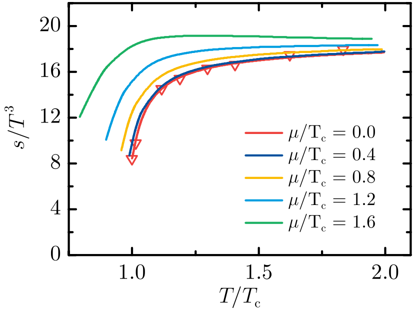


RS, Bluhm, Kämpfer: EPJ ST'08

Results (1)

- entropy density



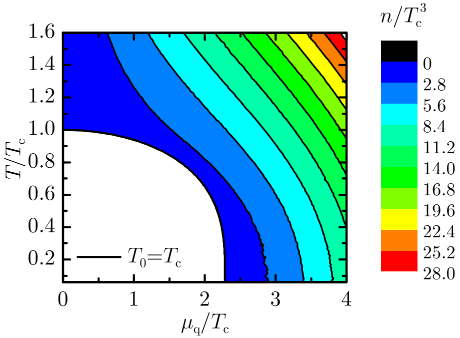


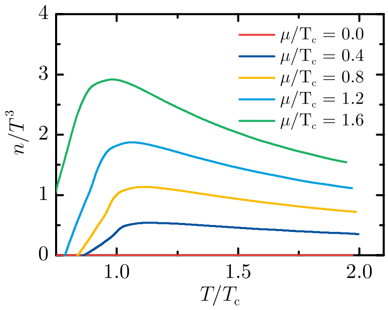
RS, Bluhm, Kämpfer: arXiv:0803.1571

- increases with temperature and chemical potential

Results (2)

- net quark density

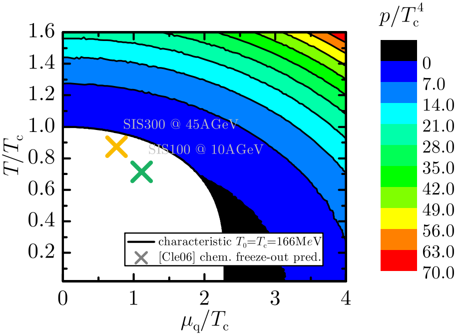


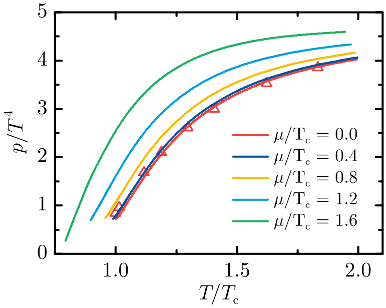


- small area of negative net quark density below transition line

Results (3)

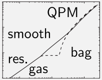
- pressure also increases with T and μ





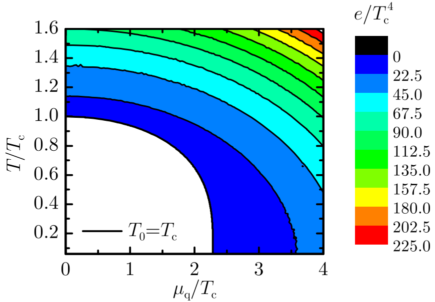
- small area of negative pressure also above transition line
  $\longrightarrow$ no problems for EoS @ RHIC, LHC, SPS, FAIR

EOS for RHIC and LHC



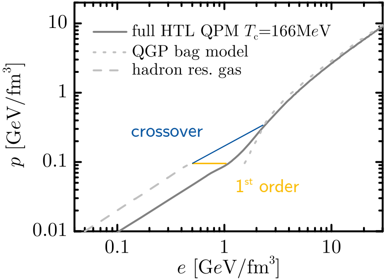
- energy density

$$e = -p + sT + \mu n$$



- EOS for LHC, RHIC

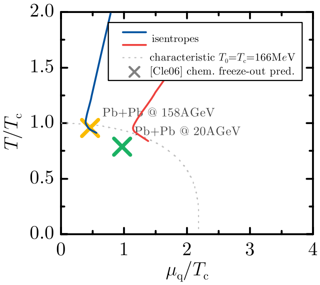
$$n_\mathrm{b}/s \approx 0$$

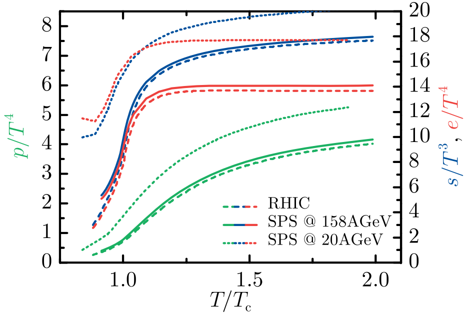


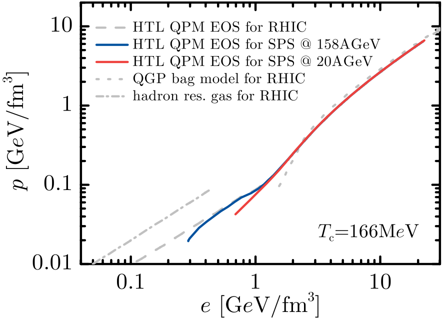
EOS for SPS

PRELIMINARY

- SPS







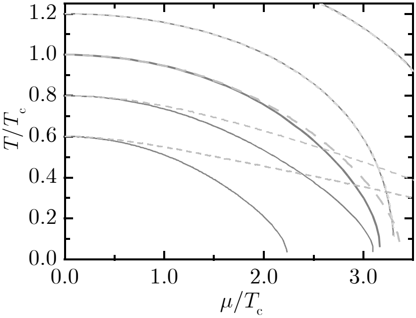
Summary & Outlook

- 2-loop Γ_2 + eff. coupling G^2 $\rightarrow$ HTL QPM
- Landau damping + collective modes
  $\rightarrow$ large μ accessible
- limitation: negative pressure for small T and μ
  $\rightarrow$ however deconfinement region @ LHC, RHIC, FAIR, SPS accessible
- outlook: EOS for FAIR, critical endpoint

Kämpfer, Bluhm, RS, Seipt: NPA'06

More effects of collective excitations

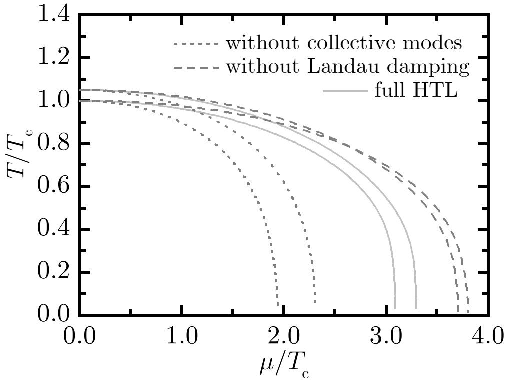
- collective modes
  $\longrightarrow$ neg. entropy contrib.



situation improves

More effects of Landau damping

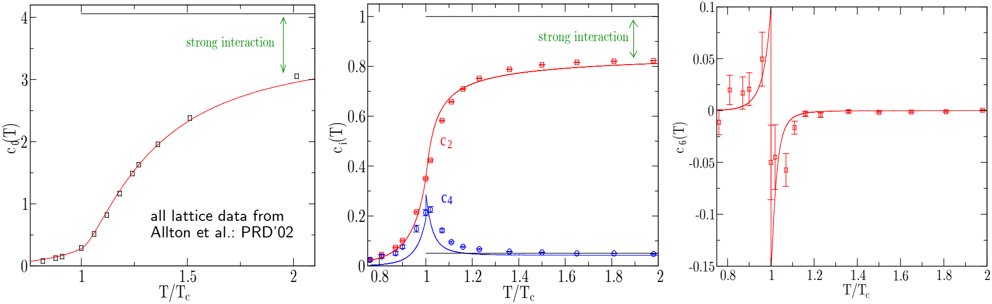
- only minor contribution at $\mu = 0$
- essential for $\mu > 0$



Small chemical potential

- $p(T,\mu \gtrsim 0)$ lattice data

$$p = T^4 \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n \qquad c_n(T) = \frac{1}{n!} \frac{\partial^n (p/T^4)}{\partial(\mu/T)^n}$$

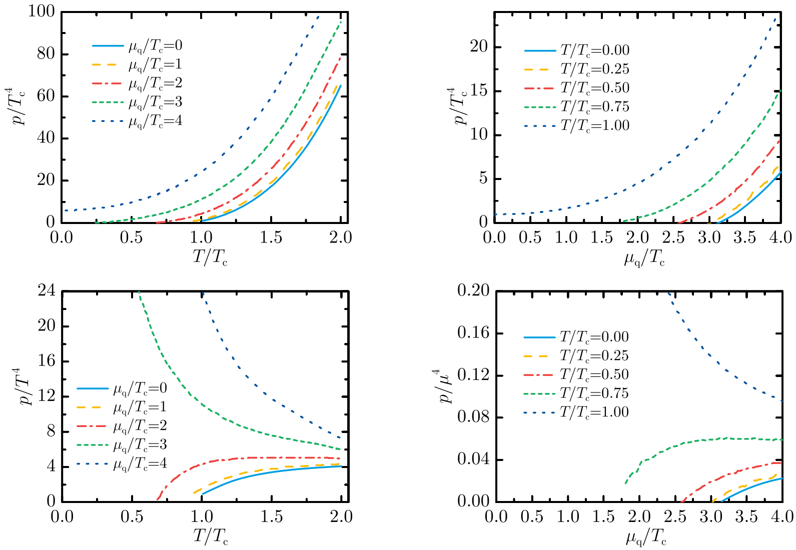


Bluhm, Kämpfer, Soff: PLB'05

- small $\mu \longrightarrow$ effective QPM suffices

Results for the pressure (2)

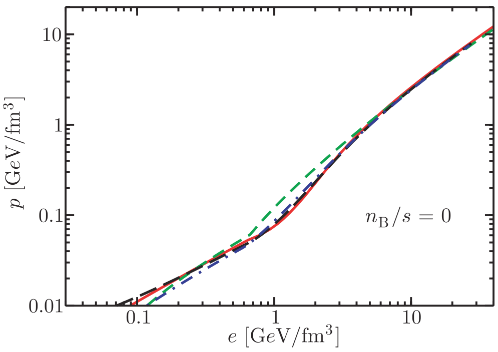
- pressure cuts

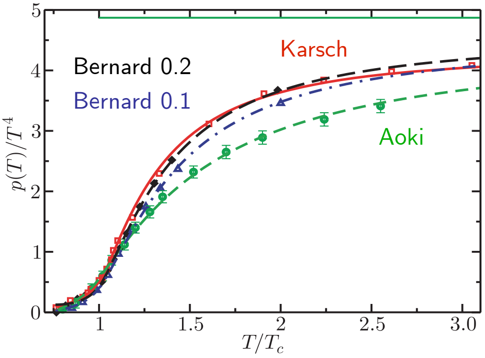


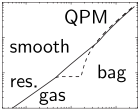
EOS for $N_f = 2+1$

- RHIC, LHC:

$$\mu = 0$$



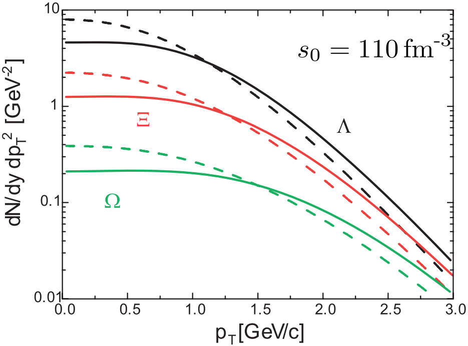


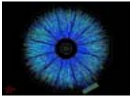


Kämpfer, Bluhm, RS, Seipt, Heinz: NPA'05
Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

First test

- calculate elliptic flow using relativistic hydro code
- compare with experimental data (RHIC)



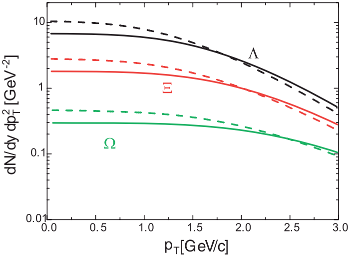


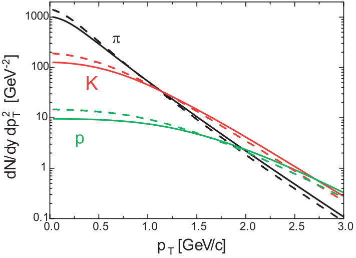
Predictions for LHC

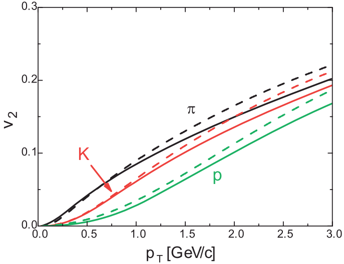
- LHC Pb+Pb collisions - conservative guess:

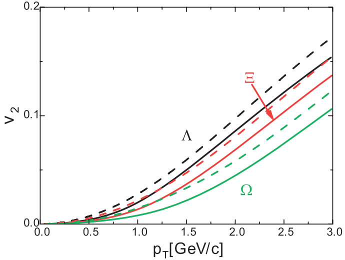
$$s_0 = 330\,\mathrm{fm}^{-3}, \quad \tau_0 = 0.6\,\mathrm{fm}/c$$
$$b = 5.2\,\mathrm{fm} \quad T_0 = 515\,\mathrm{MeV}$$

- higher initial temperature
  $\longrightarrow$ flatter p_T spectra
  $\longrightarrow$ smaller v_2









More LHC predictions

- initial parameters translate to

$$e_0 = 127\,\mathrm{GeV},\ p_0 = 42\,\frac{\mathrm{GeV}}{\mathrm{fm}^3},\ T_0 = 515\,\mathrm{MeV}$$

- LHC: higher initial temperature $\rightarrow$ longer fireball lifetime $\rightarrow$ stronger radial flow $\rightarrow$ p_T spectra flat

More full HTL quasiparticle model

- now: $\boxed{\mathrm{Im}\,\Pi \neq 0}$ $+$ collective excitations

$$s = s_{qp} + s_{damp} = s_{qp} + (\tilde{s} - s_{qp})$$

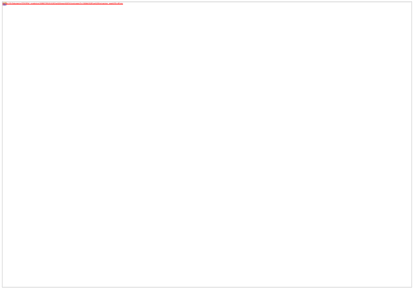
$$\xi := \frac{\mathrm{Im}\pi}{\mathrm{Re}D^{-1}}$$

$$\tilde{s} = \int d\omega \underbrace{\int dk\, \sigma(\omega, k)}_{\hat{=} s_{qp}(\omega)} \cdot F(\mathrm{Im}\Pi(\omega, k))$$

$$F := -\frac{1}{\pi}\left(\frac{\xi^2(\omega)}{(1+\xi^2(\omega))^2} + \frac{\xi^2(-\omega)}{(1+\xi^2(-\omega))^2}\right)\frac{\partial\xi}{\partial\omega}$$

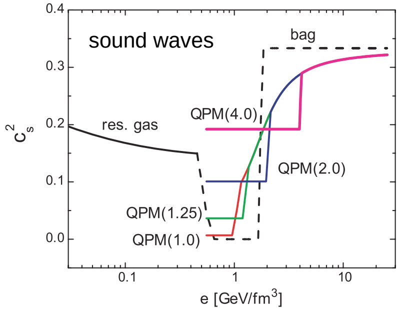
Backup

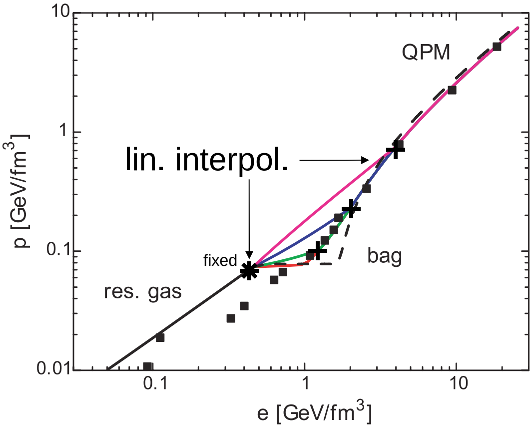
- model describes all available quantities:



A family of EOS's $\mu_B \ll T$

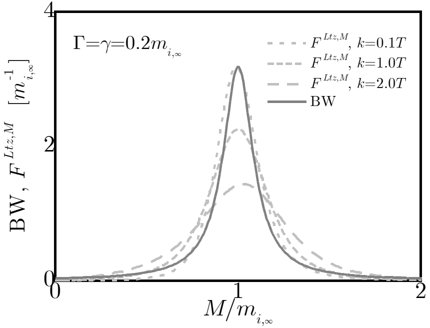
- interpolate between hadron gas and QPM description





Backup: Inclusion of widths

- Peshier: $\mathrm{Im}\,\Pi = 2\gamma\omega$

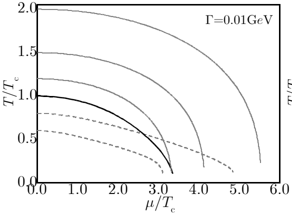


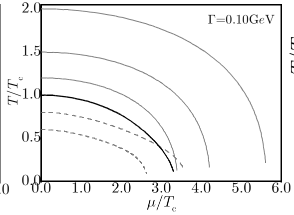
- ansatz $F(\omega,k) \rightarrow \mathrm{BW}(m)$

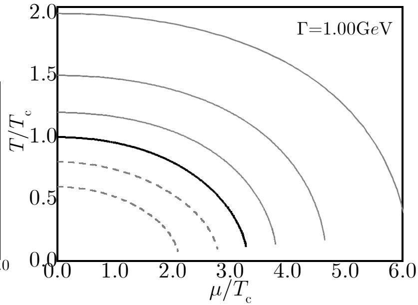
$$s(T) = \int \mathrm{d}M\, s_{qp}(T,M)\mathrm{BW}(m,M,\Gamma)$$

Backup: Distributed quasiparticle model

- fixed parameters, vary Γ

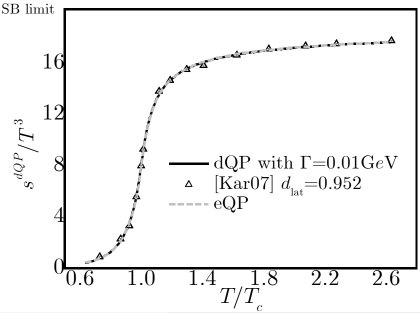






- adjustment to lattice

$$\Gamma = 0.01\,\mathrm{GeV}$$



Backup: Distributed quasiparticle model II

- bias adjustment $\Gamma \stackrel{!}{=} 1\,\mathrm{G}e\mathrm{V}$