

QCD equation of state and compact stellar objects from Hard Thermal Loops

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- QPM with $\text{Im}\Pi_i \neq 0$, plasmons and plasminos from HTL
- extrapolation of lattice QCD to large baryon densities, e.g. CBM@FAIR
- EOS for hydrodynamic phase of heavy-ion collisions (RHIC, LHC, SPS) and quark stars

From QCD to thermodynamics

QCD: \mathcal{L}

- propagators
- self-energies



Blaizot, Iancu, Rebhan: PRD'01

thermodyn. potential Ω

- state variables: p, s, nq, \dots
- EOS $e = e(p)$
- $T^{\mu\nu}$, hydrodynamics
- TOV eq., quark stars

lattice QCD

- available @ $\mu \simeq 0$
- $\mu \neq 0$: sign problem

CJT formalism

- effective action

$$\begin{aligned}\Gamma[D, S] = I - \frac{1}{2} & \left\{ \text{Tr} [\ln D^{-1}] + \text{Tr} [D_0^{-1} D - 1] \right\} \\ & + \left\{ \text{Tr} [\ln S^{-1}] + \text{Tr} [S_0^{-1} S - 1] \right\} + \Gamma_2[D, S]\end{aligned}$$

- translation-invariant systems, no broken symmetries

$$\begin{aligned}\frac{\Omega}{V} = & \text{tr} \int \frac{d^4 k}{(2\pi)^4} n_B(\omega) \text{Im} (\ln D^{-1} - \Pi D) \\ & + 2 \text{tr} \int \frac{d^4 k}{(2\pi)^4} n_F(\omega) \text{Im} (\ln S^{-1} - \Sigma S) - \frac{T}{V} \Gamma_2\end{aligned}$$

2-loop QCD thermodynamics

- truncate Γ_2 at 2-loop order

$$\Gamma_2 = \frac{1}{12} \text{ (one loop diagram)} + \frac{1}{8} \text{ (two loop diagram)} - \frac{1}{2} \text{ (circle diagram)}$$

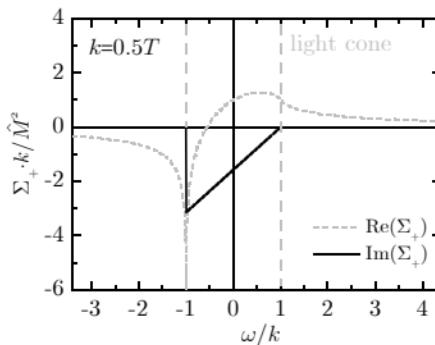
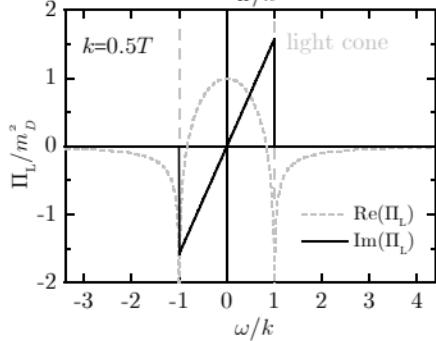
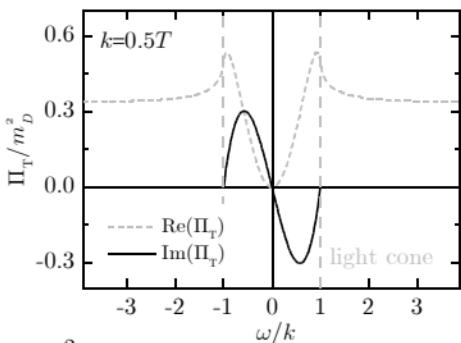
→ self-energies of 1-loop order

$$\Pi = \frac{1}{2} \text{ (one loop diagram)} + \frac{1}{2} \text{ (two loop diagram)} - \text{ (circle diagram)}$$

- gauge invariance: hard thermal loops (HTL)

HTL self-energies

- $\text{Im} \Pi \neq 0$ below the lightcone (solid lines)



→ Landau damping

Entropy

- stationarity of Ω

$$\begin{aligned}
 s &:= -\frac{1}{V} \left. \frac{\partial \Omega}{\partial T} \right|_{\mu} = -\frac{1}{V} \left(\left. \frac{\partial \Omega}{\partial T} \right|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D} \frac{\partial D}{\partial T}}_0 \right)_{\mu} \\
 &= s_{g,T} + s_{g,L} + s_{q,\text{Pt}} + s_{q,\text{Pl}} + s' \quad s' = 0
 \end{aligned}$$

Vanderheyden, Baym: JSP'98

- e.g. gluons:

$$s_{g,T} \sim \int d^4k \frac{\partial n_B}{\partial T} \left\{ \underbrace{\pi \varepsilon(\omega) \Theta(-\text{Re}D_T^{-1})}_{\text{qp contribution}} + \underbrace{\text{Re}D_T \text{Im} \Pi_T - \arctan\left(\frac{\text{Im} \Pi_T}{\text{Re}D_T^{-1}}\right)}_{\text{damping terms}} \right\}$$

Pressure

$$s \sim \left(\frac{\partial \Omega}{\partial T} \Big|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D} \frac{\partial D}{\partial T}}_0 \right)_\mu$$

$$s_i \sim \int_{\text{d}^4 k} \frac{\partial n_{\text{B/F}}}{\partial T} \left\{ \text{qp} + \text{damping} \right\}$$

- self-consistent formulation of the pressure

$$p = -\frac{\Omega}{V} := \sum_i p_i - B \quad p_i \sim \int_{\text{d}^4 k} n_{\text{B/F}} \left\{ \text{qp} + \text{damping} \right\}$$

$$\frac{\partial B}{\partial T} := \sum_i \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} \quad \left(\frac{\partial B}{\partial \mu} = \sum_i \frac{\partial p_i}{\partial \mu} \right)$$

- entropy density

$$s = \frac{\partial p}{\partial T} = \sum_i s_i + \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} - \frac{\partial B}{\partial T} = \sum_i s_i$$

- net quark density

$$n \sim \frac{\partial \Omega}{\partial \mu} \Big|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D} \frac{\partial D}{\partial \mu}}_0$$

$$n_q = \frac{\partial p}{\partial \mu} \sim \int_{\text{d}^4 k} \left(\frac{\partial n_{\text{F}}}{\partial \mu} + \frac{\partial n_{\text{F}}^A}{\partial \mu} \right) \left\{ \text{qp} + \text{damping} \right\}$$

Effective coupling

- fundamental parameter

$$g^2(x^2) = \frac{16\pi^2}{\beta_0 \ln(x^2)} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln[\ln(x^2)]}{\ln(x^2)} \right)$$

- running coupling g^2 $x = \frac{\bar{\mu}}{\Lambda_{\text{QCD}}}$ $\bar{\mu} \sim T$

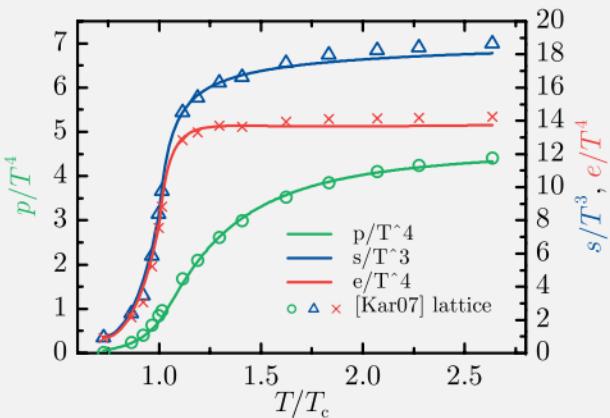


$$T > T_c, \mu = 0$$

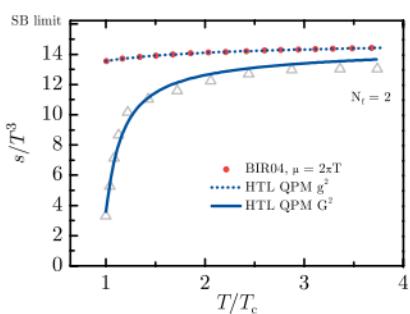
- effective coupling G^2 $x = \frac{(T-T_s)}{\lambda_{\text{QCD}}}$

Adjustment @ $\mu=0$

- $\mu=0$: adjust to ℓ QCD
 - T_s, λ fixed
 - $G^2(T, \mu=0)$

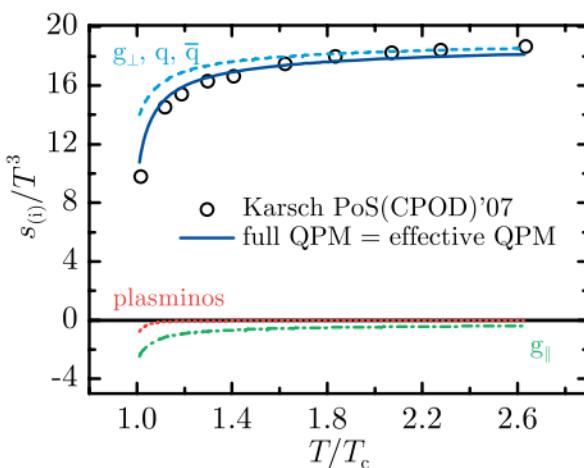


- $T_s=0$:

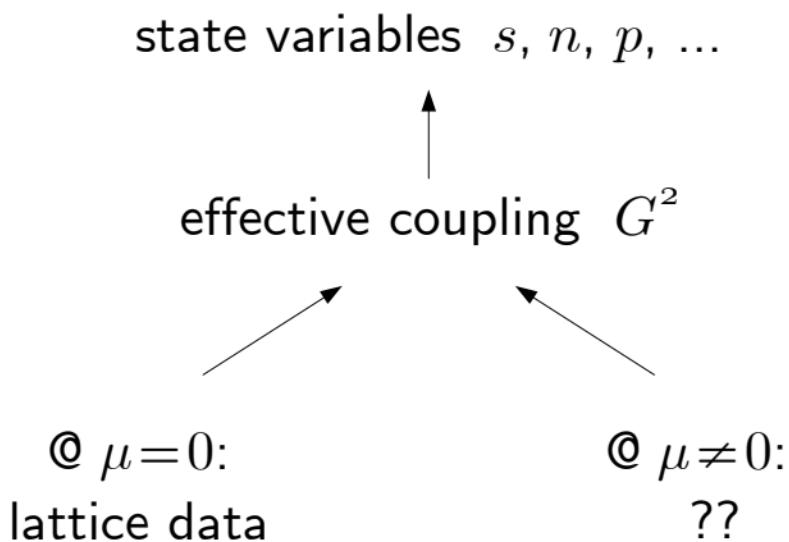


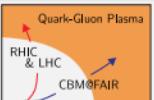
Influence of coll. modes + LD @ $\mu=0$

- individual entropy contributions



- Landau damping large close to T_c , decreases for higher temperatures





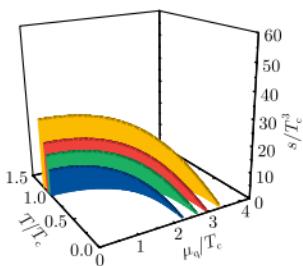
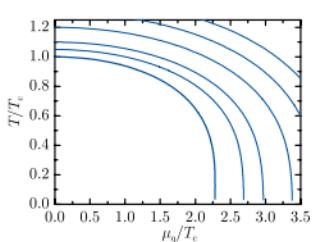
Into the T - μ -plane

- $\mu > 0$: stationary potential, self-consistent model
→ impose Maxwell's relation

$$\frac{\partial s}{\partial \mu} = \frac{\partial n}{\partial T} \quad \rightarrow \quad a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

Peshier, Kämpfer, Soff: PRC'00, PRD'02

- quasilinear PDE for $G^2(T, \mu \neq 0)$: method of char.

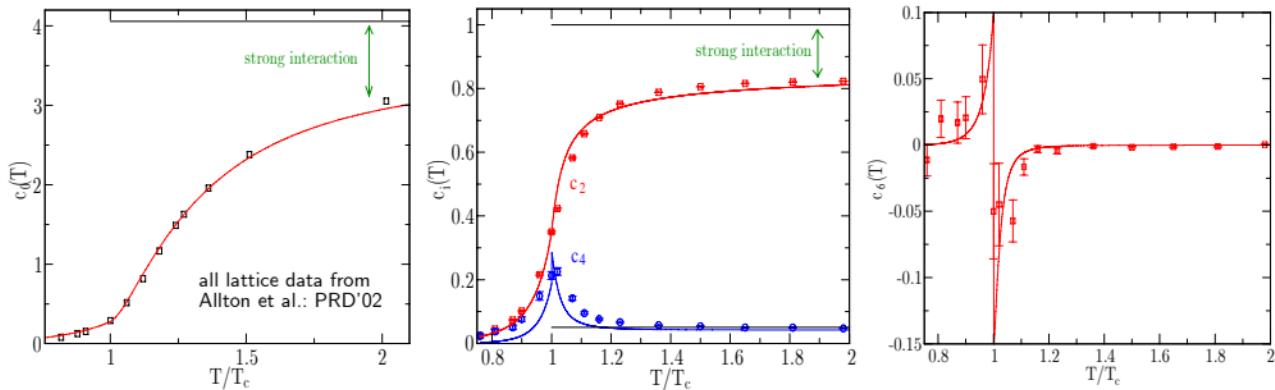


RS, Bluhm, Kämpfer: EPJ ST'08

Small chemical potential

- $p(T, \mu \gtrsim 0)$ lattice data

$$p = T^4 \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n \quad c_n(T) = \frac{1}{n!} \frac{\partial^n (p/T^4)}{\partial (\mu/T)^n}$$

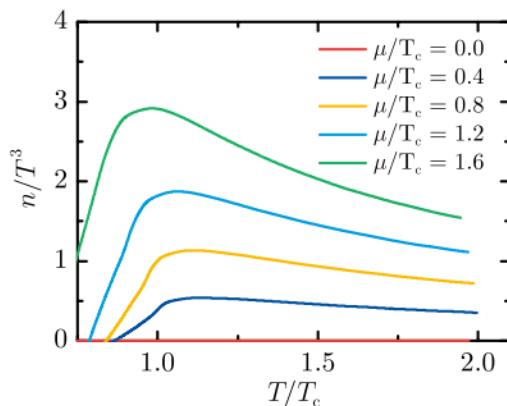
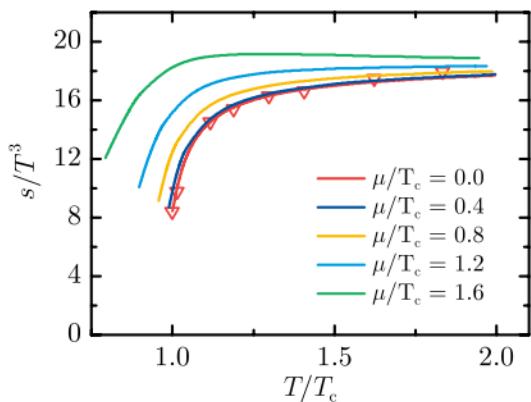


Bluhm, Kämpfer, Soff: PLB'05

Bluhm, Kämpfer, RS, Seipt: EPJC'07

Thermodynamic bulk variables

- entropy density and net quark density

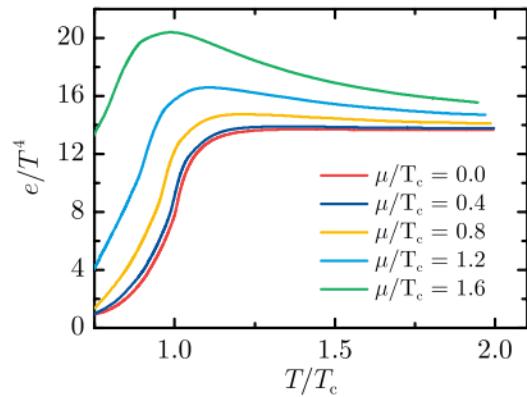
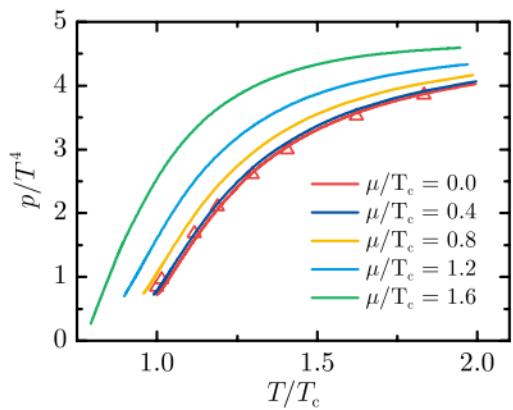


RS, Bluhm, Kämpfer: PPNP'09

- increase with chemical potential

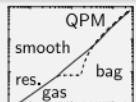
Thermodynamic bulk variables

- pressure and energy density



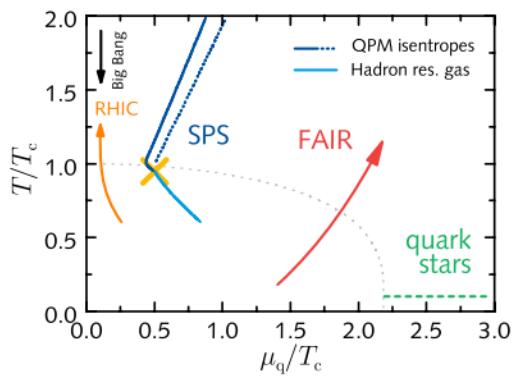
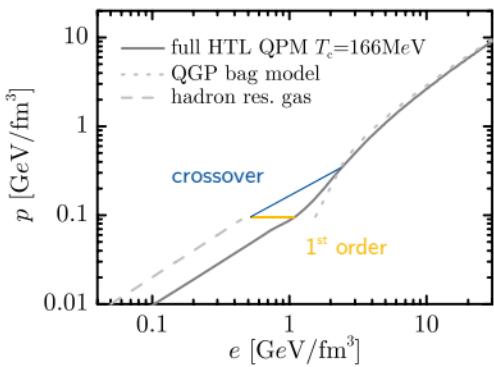
- small area of negative pressure
 → no problems for EOS @ RHIC, LHC, SPS, FAIR
 → natural limit of stability for quark stars (CFL?)

EOS for RHIC and LHC

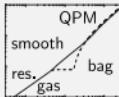


- EOS for LHC, RHIC

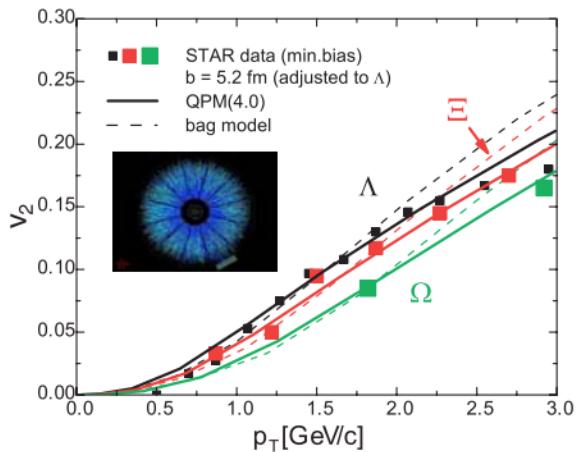
$$n_b/s \approx 0$$



Comparison with the experiment



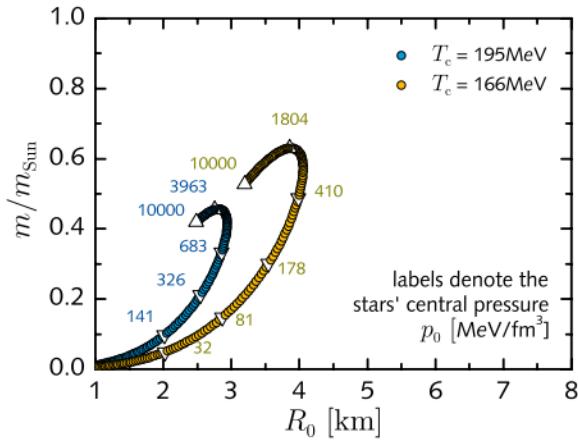
- calculate elliptic flow using relativistic hydro code
- compare with experimental data (RHIC)



Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

Compact stellar matter

- Tolman-Oppenheimer-Volkov equations
- β -equilibrium by $d, s \leftrightarrow u, l, \nu_l$
 $\rightarrow \mu_l$ from charge neutrality
- compare with bag-like EOS $e = \alpha p + 4\tilde{B}$
 $\rightarrow \alpha \approx 4$
- strong dependence on critical temperature



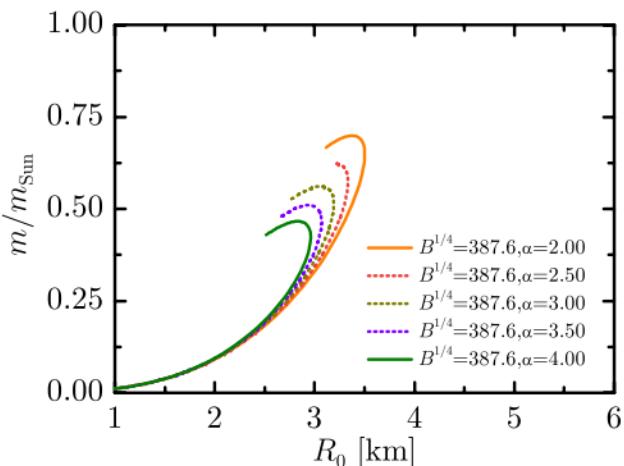
Summary & Outlook

- 2-loop $\Gamma_2 + \text{eff. coupling } G^2 \rightarrow \text{HTL QPM}$
- ℓQCD results describable; used as input
→ large μ accessible due to self-consistency
- EOS for heavy ion collision experiments available
- quark stars with even smaller radii than bag model
- outlook: hydro for SPS, FAIR
critical endpoint

Kämpfer, Bluhm, RS, Seipt: NPA'06

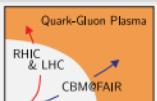
Backup

- influence of α

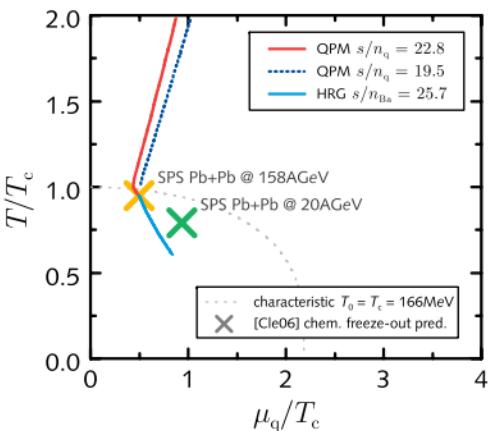
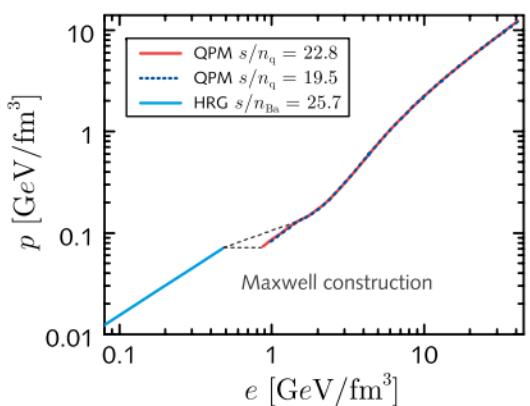


EOS for SPS

PRELIMINARY

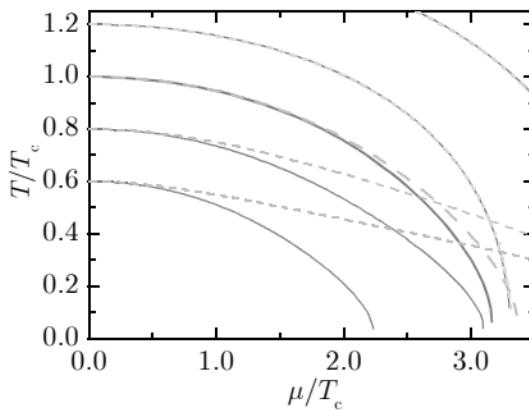


- SPS $s/n_q \approx 25.5-8.5$



More effects of collective excitations

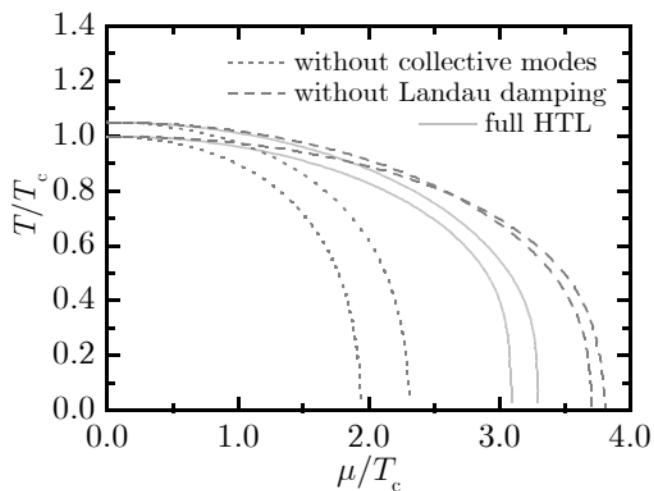
- collective modes
→ neg. entropy contrib.



situation improves

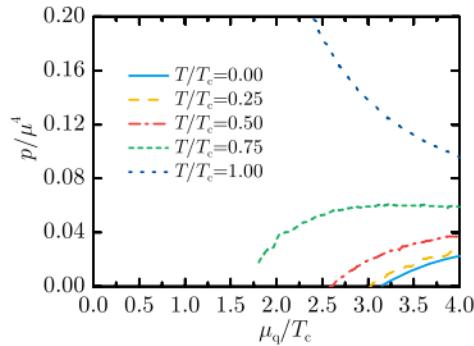
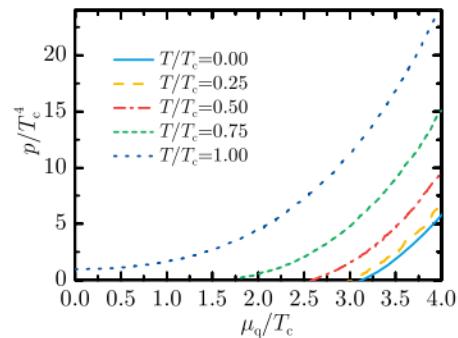
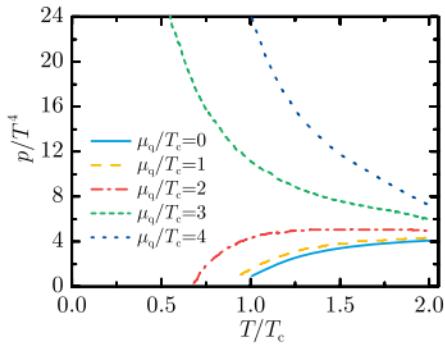
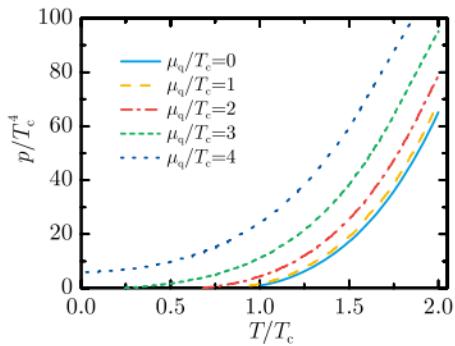
More effects of Landau damping

- only minor contribution at $\mu = 0$
- essential for $\mu > 0$



Results for the pressure (2)

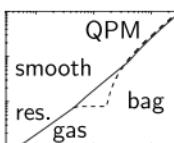
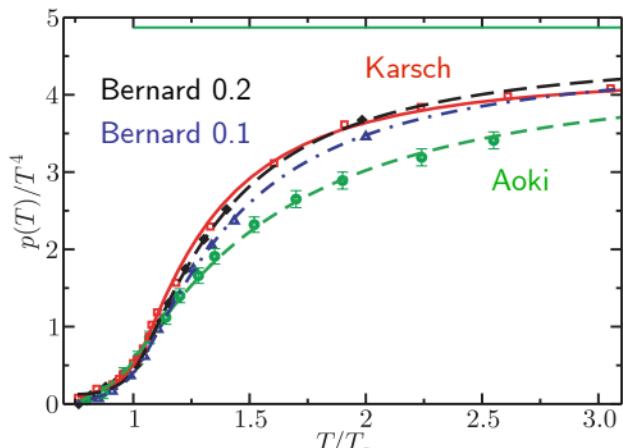
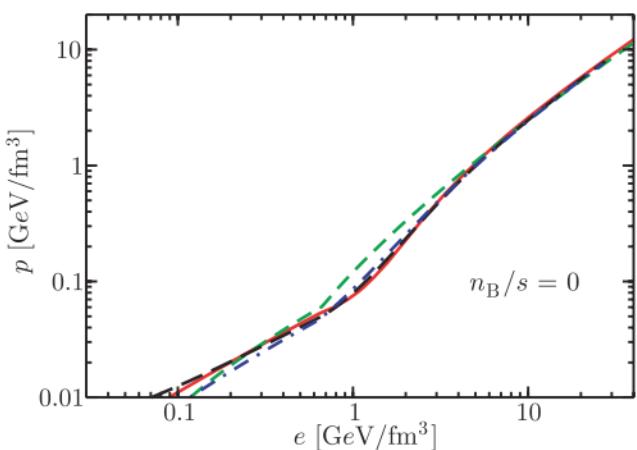
- pressure cuts



EOS for $N_f=2+1$

- RHIC, LHC:

$$\mu = 0$$



Kämpfer, Bluhm, RS, Seipt, Heinz: NPA'05
 Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

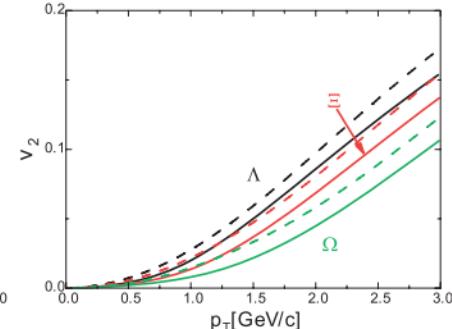
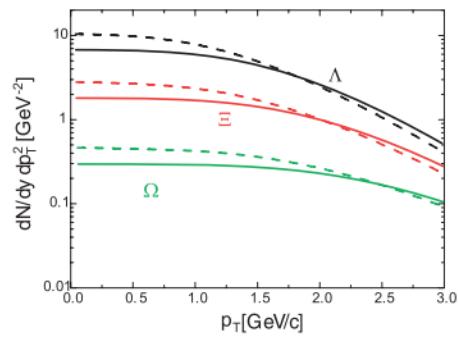
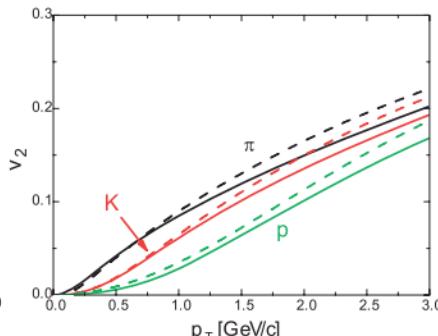
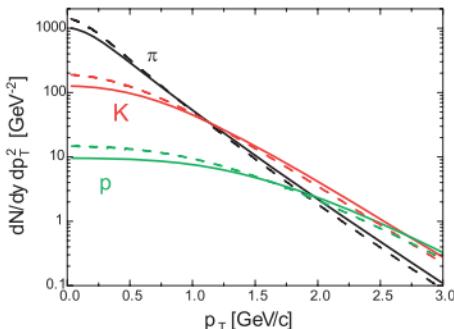
Predictions for LHC

- LHC Pb+Pb collisions - conservative guess:

$$s_0 = 330 \text{ fm}^{-3}, \quad \tau_0 = 0.6 \text{ fm}/c$$

$$b = 5.2 \text{ fm} \quad T_0 = 515 \text{ MeV}$$

- higher initial temperature
 → flatter p_T spectra
 → smaller v_2



More LHC predictions

- initial parameters translate to

$$e_0 = 127 \text{ GeV}, \quad p_0 = 42 \frac{\text{GeV}}{\text{fm}^3}, \quad T_0 = 515 \text{ MeV}$$

- LHC: higher initial temperature \rightarrow longer fireball lifetime \rightarrow stronger radial flow $\rightarrow p_T$ spectra flat

More full HTL quasiparticle model

- now: $\boxed{\text{Im}\Pi \neq 0}$ + collective excitations

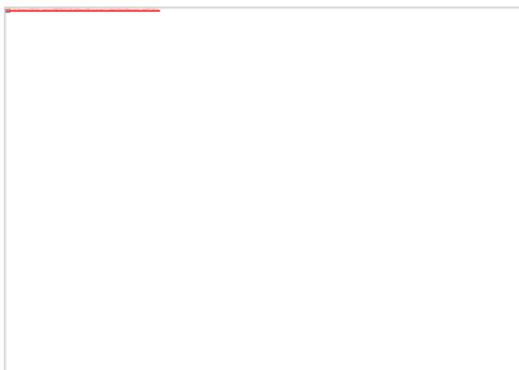
$$s = s_{qp} + s_{damp} = s_{qp} + (\tilde{s} - s_{qp})$$

$$\tilde{s} = \underbrace{\int d\omega \int dk \sigma(\omega, k) \cdot F(\text{Im}\Pi(\omega, k))}_{\hat{=} s_{qp}(\omega)} \quad F := -\frac{1}{\pi} \left(\frac{\xi^2(\omega)}{(1+\xi^2(\omega))^2} + \frac{\xi^2(-\omega)}{(1+\xi^2(-\omega))^2} \right) \frac{\partial \xi}{\partial \omega}$$

$$\xi := \frac{\text{Im}\Pi}{\text{Re}D^{-1}}$$

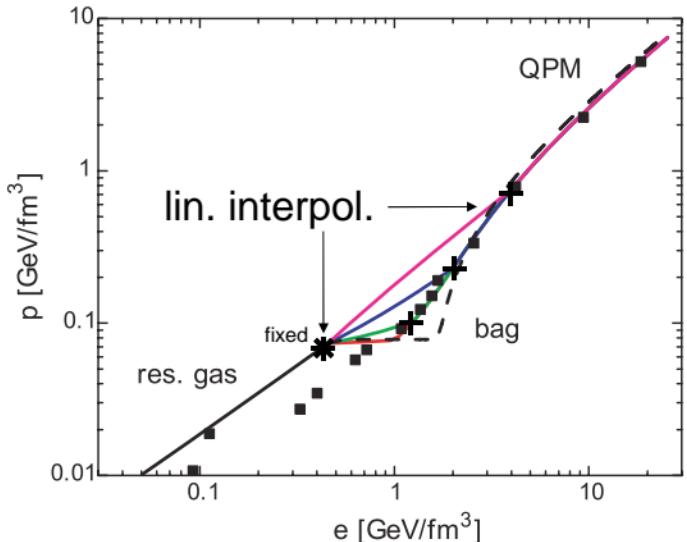
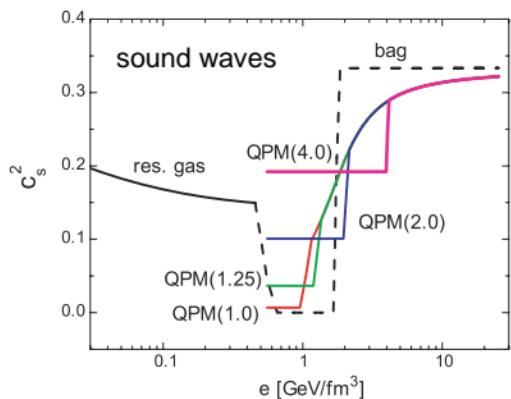
Backup

- model describes all available quantities:



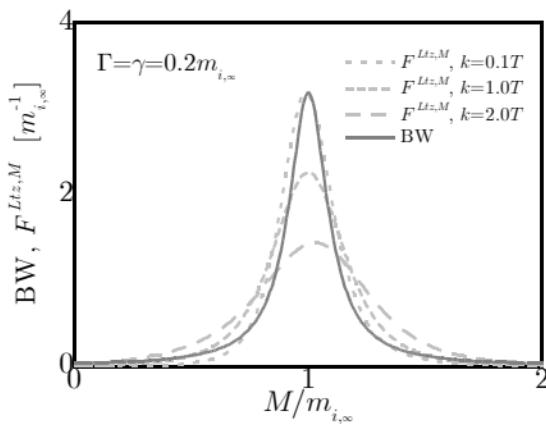
A family of EOS's $\mu_B \ll T$

- interpolate between hadron gas and QPM description



Backup: Inclusion of widths

- Peshier: $\text{Im } \Pi = 2\gamma\omega$

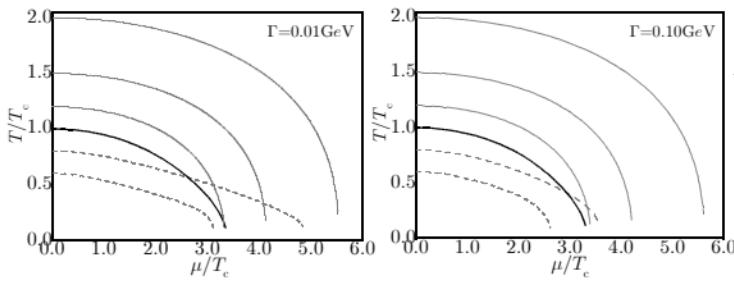


- ansatz $F(\omega, k) \rightarrow \text{BW}(m)$

$$s(T) = \int dM s_{qp}(T, M) \text{BW}(m, M, \Gamma)$$

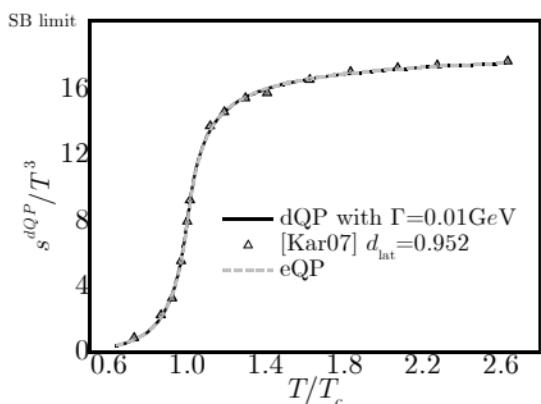
Backup: Distributed quasiparticle model

- fixed parameters, vary Γ



- adjustment to lattice

$$\Gamma = 0.01 \text{ GeV}$$



Backup: Distributed quasiparticle model II

- bias adjustment $\Gamma \stackrel{!}{=} 1 \text{ GeV}$

