QCD equation of state and compact stellar objects from Hard Thermal Loops

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- QPM with Im$\Pi_{\mathrm{i}} \neq 0$, plasmons and plasminos from HTL
- extrapolation of lattice QCD to large baryon densities, e.g. CBM@FAIR
- EOS for hydrodynamic phase of heavy-ion collisions (RHIC, LHC, SPS) and quark stars

From QCD to thermodynamics

QCD: $\mathcal{L}$

$\longrightarrow$ propagators

$\longrightarrow$ self-energies

thermodyn. potential Ω

$\longrightarrow$ state variables: p, s, nq,...

$\longrightarrow$ EOS $e = e(p)$

$\longrightarrow$ $T^{\mu\nu}$, hydrodynamics

$\longrightarrow$ TOV eq., quark stars

lattice QCD

$\longrightarrow$ available @ $\mu \simeq 0$

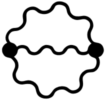
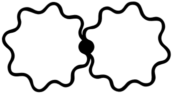
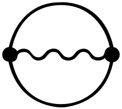
$\longrightarrow$ $\mu \neq 0$: sign problem

CJT formalism

- stationary effective action

$$\frac{\Omega}{V} = \mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4} n_{\mathrm{B}}(\omega)\,\mathrm{Im}\left(\ln D^{-1} - \Pi D\right) + 2\,\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4} n_{\mathrm{F}}(\omega)\,\mathrm{Im}\left(\ln S^{-1} - \Sigma S\right) - \frac{T}{V}\Gamma_2$$

- 2-loop Γ_2 functional

$\Gamma_2 = \frac{1}{12}$  $+ \frac{1}{8}$  $- \frac{1}{2}$ 

- Hard Thermal Loops $\longrightarrow$ gauge invariance

The model

- stationarity of Ω

$$s := -\frac{1}{V}\,\frac{\partial\Omega}{\partial T}\Big|_{\mu} = -\frac{1}{V}\Big(\frac{\partial\Omega}{\partial T}\Big|_{\text{expl.}} + \underbrace{\frac{\delta\Omega}{\delta D}}_{0}\,\frac{\partial D}{\partial T}\Big)_{\mu}$$

$$= s_{g,\mathrm{T}} + s_{g,\mathrm{L}} + s_{q,\mathrm{Pt}} + s_{q,\mathrm{Pl}} + s' \qquad s' = 0$$

Vanderheyden, Baym: JSP'98

- e.g. gluons:

$$s_{g,\mathrm{T}} \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\mathrm{B}}}{\partial T} \Big\{ \underbrace{\pi\varepsilon(\omega)\Theta\big(-\mathrm{Re}D_{\mathrm{T}}^{-1}\big)}_{\text{qp contribution}} + \underbrace{\mathrm{Re}D_{\mathrm{T}}\mathrm{Im}\Pi_{\mathrm{T}} - \arctan\Big(\frac{\mathrm{Im}\Pi_{\mathrm{T}}}{\mathrm{Re}D_{\mathrm{T}}^{-1}}\Big)}_{\text{damping terms}} \Big\}$$

Effective coupling

$$\boxed{g^2(x^2) = \frac{16\pi^2}{\beta_0 \ln(x^2)} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln[\ln(x^2)]}{\ln(x^2)}\right)}$$

- running coupling g^2 $\qquad x = \frac{\bar{\mu}}{\Lambda_{\mathrm{QCD}}}$ $\qquad \bar{\mu} \sim T$

$$\Big\downarrow \quad T > T_c,\ \mu = 0$$

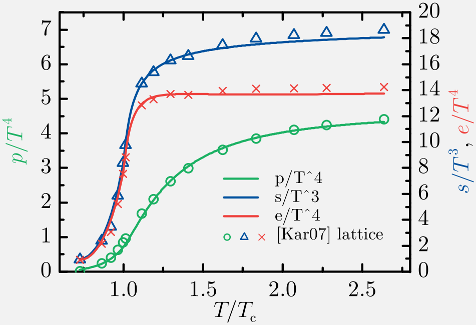
- effective coupling G^2 $\qquad x = \frac{(T\text{-}T_s)}{\lambda_{\mathrm{QCD}}}$

Adjustment @ $\mu = 0$

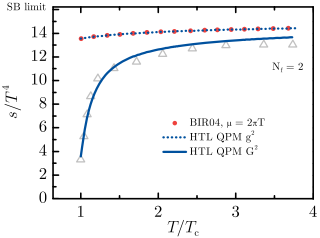
- $\mu = 0$: adjust to ℓQCD

  $\rightarrow \; T_s, \; \lambda$ fixed

  $\rightarrow \; G^2(T, \mu = 0)$



- $T_s = 0$:



state variables s, n, p, ...

↑

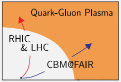
effective coupling G^2

↗ ↖

@ $\mu = 0$:
lattice data

@ $\mu \neq 0$:
??

Into the T-μ-plane



- $\mu > 0$: stationary potential, self-consistent model
  $\rightarrow$ impose Maxwell's relation

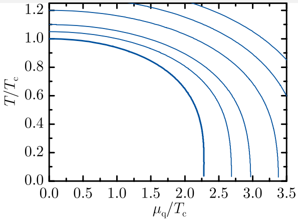
$$\frac{\partial s}{\partial \mu} = \frac{\partial n}{\partial T} \qquad \longrightarrow \qquad a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

Peshier, Kämpfer, Soff: PRC'00, PRD'02

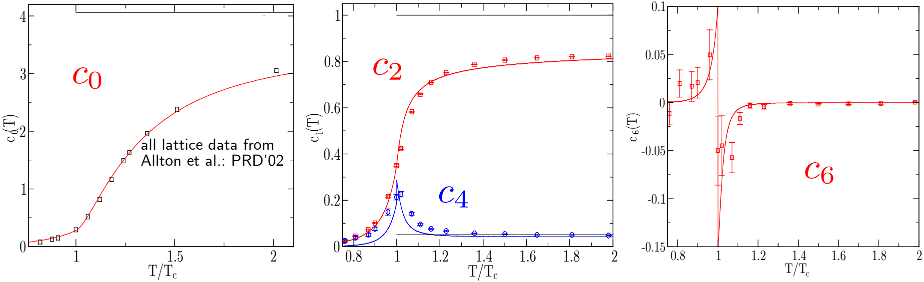
- quasilinear PDE for $G^2(T, \mu \neq 0)$: method of char.

Model tests

- anomaly-free characteristics
- thermodynamic laws obeyed
- pressure correction coefficients
  $p = T^4 \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n$



RS, Bluhm, Kämpfer: EPJ ST'08

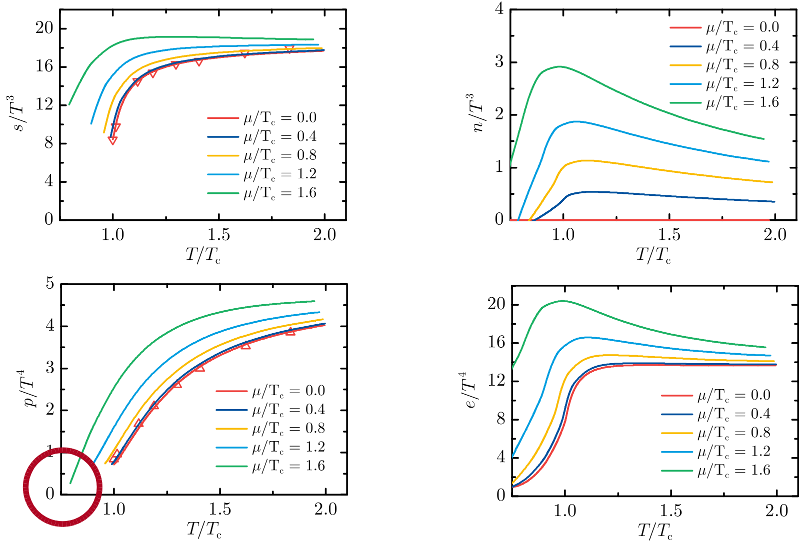


Bluhm, Kämpfer, Soff: PLB'05
Bluhm, Kämpfer, RS, Seipt: EPJC'07

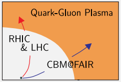
Thermodynamic bulk variables

RS, Bluhm, Kämpfer: PPNP'09

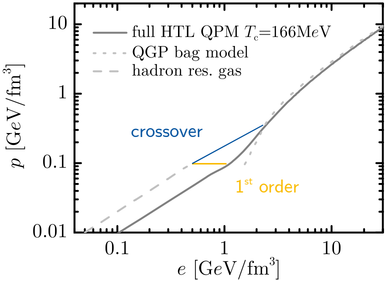
- evolution of state variables

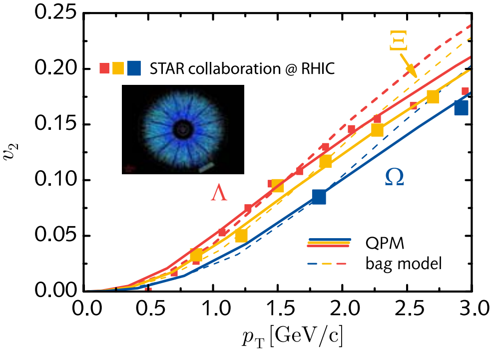


Heavy ion collisions



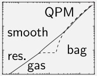
- model connection to hadron resonance gas
- successful application in hydro @ RHIC



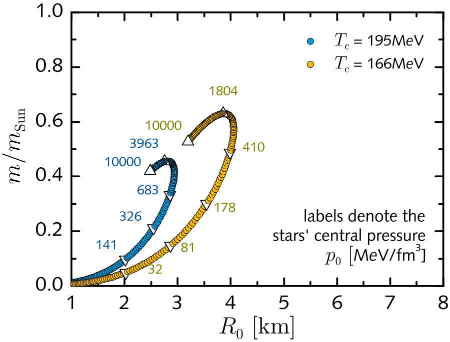


Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

Compact stellar matter



- Tolman-Oppenheimer-Volkov equations
- β-equilibrium by $d, s \leftrightarrow u, l, \nu_l$
  $\rightarrow \mu_l$ from charge neutrality
- compare with bag-like EOS $e = \alpha p + 4\tilde{B}$
  $\rightarrow \alpha \approx 4$
- strong dependence on critical temperature



Summary & Outlook

- 2-loop Γ_2 + eff. coupling $G^2 \to$ HTL QPM
- ℓQCD results describable; used as input
  $\to$ large μ accessible due to self-consistency
- EOS for heavy ion collision experiments available
- quark stars with even smaller radii than bag model

- outlook: hydro for SPS,FAIR
  critical endpoint

Kämpfer, Bluhm, RS, Seipt: NPA'06

Pressure

$$s \sim \left(\frac{\partial \Omega}{\partial T}\Big|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial T}\right)_\mu$$

$$s_i \sim \int_{\mathrm{d}^4k} \frac{\partial n_{\mathrm{B/F}}}{\partial T} \left\{\mathrm{qp} + \mathrm{damping}\right\}$$

- self-consistent formulation of the pressure

$$p = -\frac{\Omega}{V} := \sum_i p_i - B \qquad p_i \sim \int_{\mathrm{d}^4k} n_{\mathrm{B/F}} \left\{\mathrm{qp} + \mathrm{damping}\right\}$$

$$\frac{\partial B}{\partial T} := \sum_i \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} \qquad \left(\frac{\partial B}{\partial \mu} = \sum \frac{..}{..} \frac{..}{\partial \mu}\right)$$

- entropy density

$$s = \frac{\partial p}{\partial T} = \sum_i s_i + \frac{\partial p_i}{\partial \Pi_i} \frac{\partial \Pi_i}{\partial T} - \frac{\partial B}{\partial T} = \sum_i s_i$$

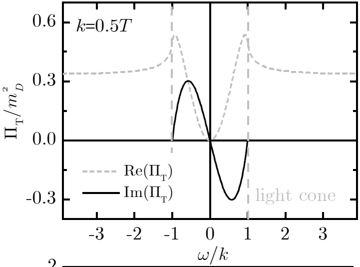
- net quark density

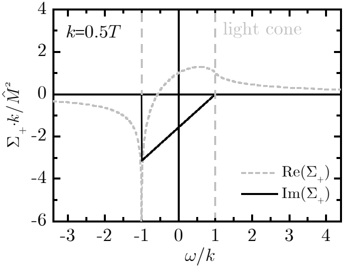
$$n \sim \frac{\partial \Omega}{\partial \mu}\Big|_{\text{expl.}} + \underbrace{\frac{\delta \Omega}{\delta D}}_{0} \frac{\partial D}{\partial \mu}$$

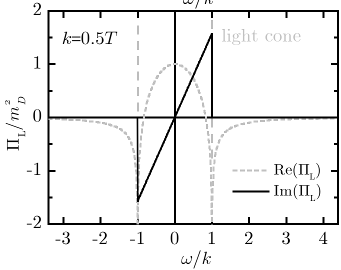
$$n_q = \frac{\partial p}{\partial \mu} \sim \int_{\mathrm{d}^4k} \left(\frac{\partial n_{\mathrm{F}}}{\partial \mu} + \frac{\partial n_{\mathrm{F}}^A}{\partial \mu}\right) \left\{\mathrm{qp} + \mathrm{damping}\right\}$$

HTL self-energies

- Im $\Pi \neq 0$ below the lightcone (solid lines)



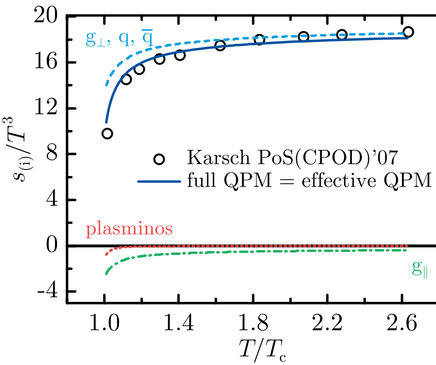




$\longrightarrow$ Landau damping

Influence of coll. modes + LD @ $\mu = 0$

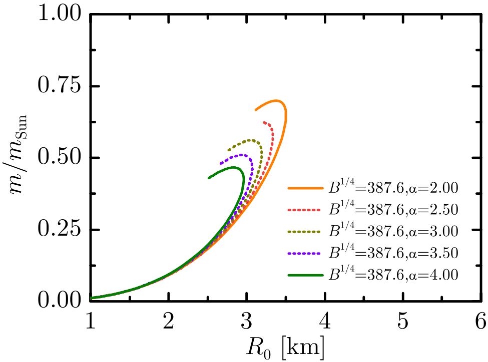
- individual entropy contributions



- Landau damping large close to T_c, decreases for higher temperatures

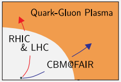
Backup

- influence of α

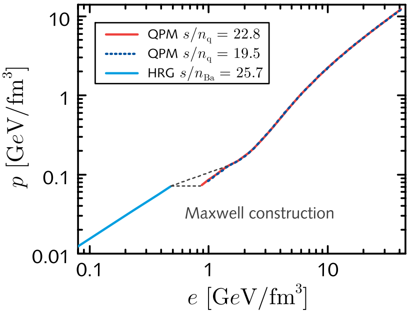


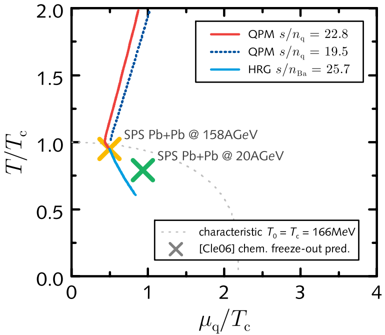
EOS for SPS

PRELIMINARY



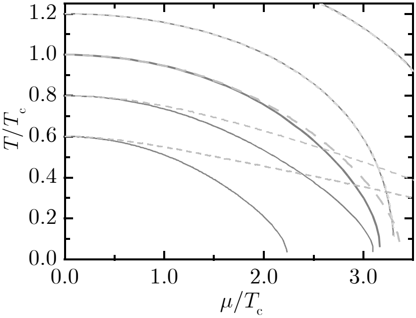
- SPS $s/n_q \approx 25.5\text{-}8.5$





More effects of collective excitations

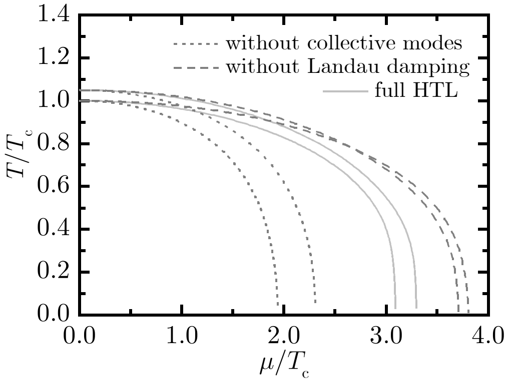
- collective modes
  $\longrightarrow$ neg. entropy contrib.



situation improves

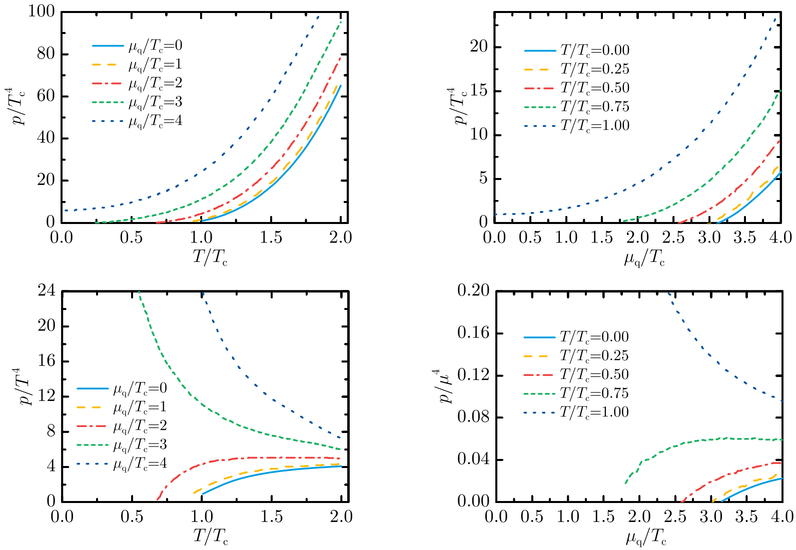
More effects of Landau damping

- only minor contribution at $\mu = 0$
- essential for $\mu > 0$



Results for the pressure (2)

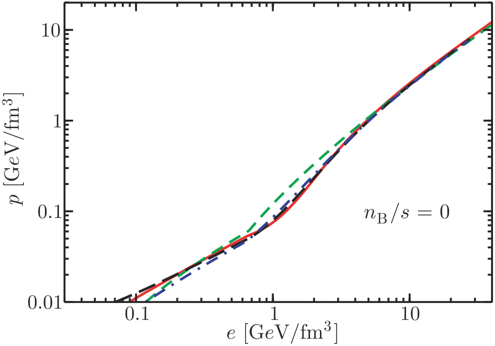
- pressure cuts

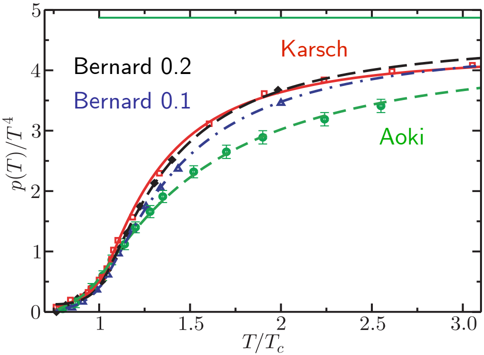


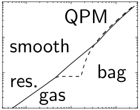
EOS for $N_f = 2+1$

- RHIC, LHC:

$$\mu = 0$$







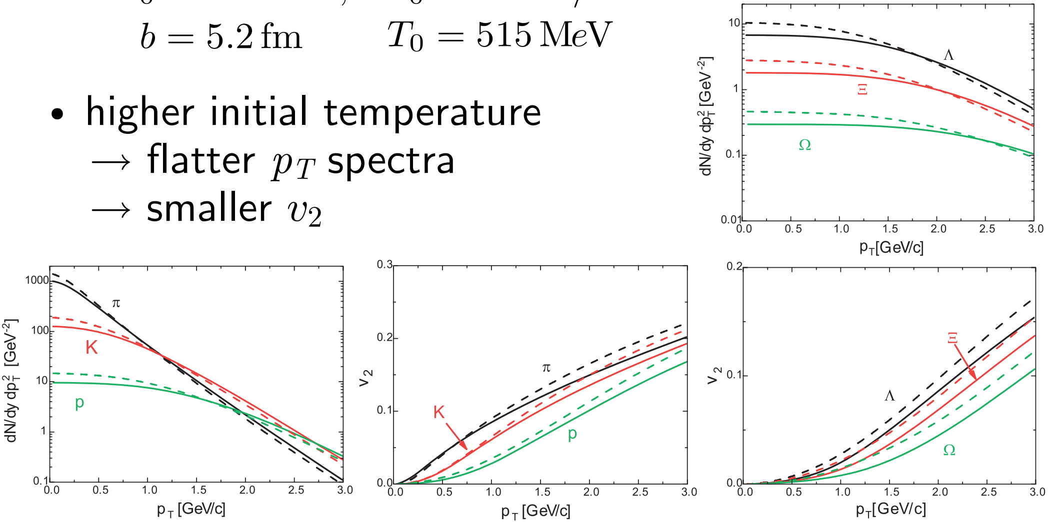
Kämpfer, Bluhm, RS, Seipt, Heinz: NPA'05
Bluhm, Kämpfer, RS, Seipt, Heinz: PRC'07

Predictions for LHC

- LHC Pb+Pb collisions - conservative guess:

$$s_0 = 330\,\mathrm{fm}^{-3}, \quad \tau_0 = 0.6\,\mathrm{fm}/c$$
$$b = 5.2\,\mathrm{fm} \quad T_0 = 515\,\mathrm{MeV}$$

- higher initial temperature
  $\rightarrow$ flatter p_T spectra
  $\rightarrow$ smaller v_2



More LHC predictions

- initial parameters translate to

$$e_0 = 127\,\mathrm{GeV},\ p_0 = 42\,\frac{\mathrm{GeV}}{\mathrm{fm}^3},\ T_0 = 515\,\mathrm{MeV}$$

- LHC: higher initial temperature $\rightarrow$ longer fireball lifetime $\rightarrow$ stronger radial flow $\rightarrow$ p_T spectra flat

More full HTL quasiparticle model

- now: $\boxed{\mathrm{Im}\,\Pi \neq 0}$ $+$ collective excitations

$$s = s_{qp} + s_{damp} = s_{qp} + (\tilde{s} - s_{qp})$$

$$\tilde{s} = \int d\omega \underbrace{\int dk\, \sigma(\omega, k)}_{\hat{=} s_{qp}(\omega)} \cdot F(\mathrm{Im}\Pi(\omega, k)) \qquad \xi := \frac{\mathrm{Im}\Pi}{\mathrm{Re}D^{-1}}$$

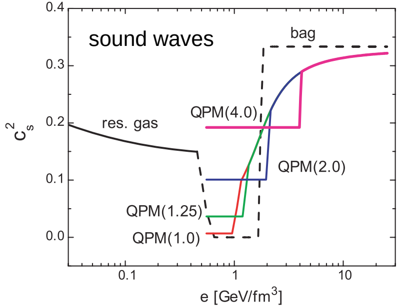
$$F := -\frac{1}{\pi}\left(\frac{\xi^2(\omega)}{(1+\xi^2(\omega))^2} + \frac{\xi^2(-\omega)}{(1+\xi^2(-\omega))^2}\right)\frac{\partial\xi}{\partial\omega}$$

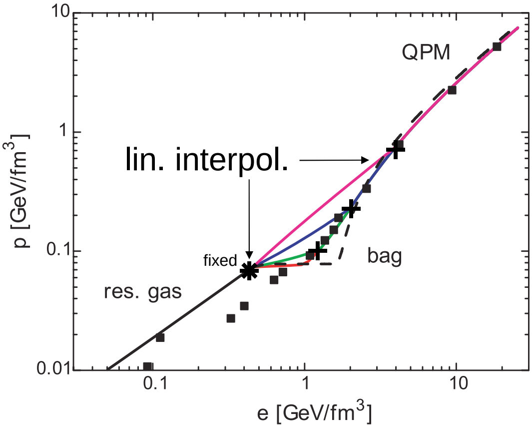
Backup

- model describes all available quantities:

A family of EOS's $\mu_B \ll T$

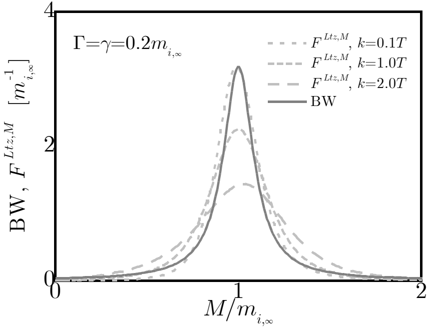
- interpolate between hadron gas and QPM description





Backup: Inclusion of widths

- Peshier: $\mathrm{Im}\,\Pi = 2\gamma\omega$

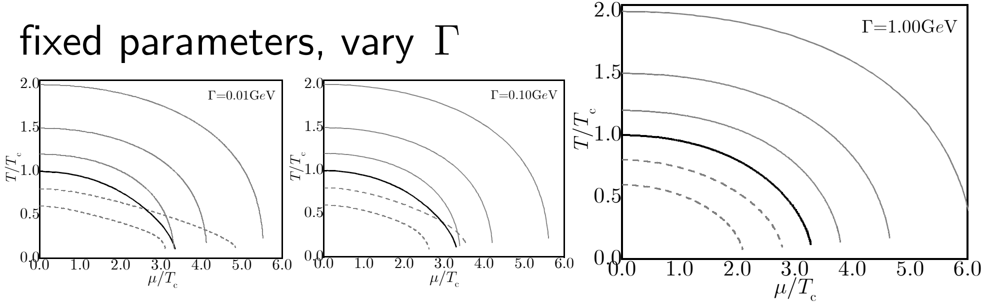


- ansatz $F(\omega,k) \rightarrow \mathrm{BW}(m)$

$$s(T) = \int \mathrm{d}M\, s_{qp}(T, M)\mathrm{BW}(m, M, \Gamma)$$

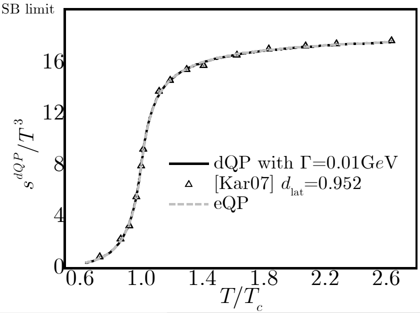
Backup: Distributed quasiparticle model

- fixed parameters, vary Γ



- adjustment to lattice

$$\Gamma = 0.01\,\mathrm{GeV}$$



Backup: Distributed quasiparticle model II

- bias adjustment $\Gamma \stackrel{!}{=} 1\,\mathrm{G}e\mathrm{V}$