QCD quasi-particle model with widths and Landau damping

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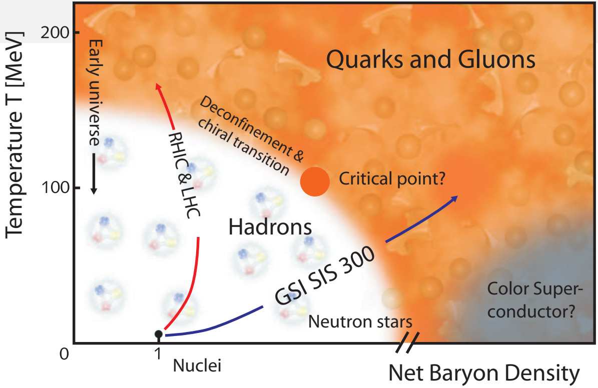
Motivation



Lagrangian $\mathcal{L}_{QCD}$
$\rightarrow$ Feynman rules
$\rightarrow$ propagators
$\rightarrow$ self energies

$\Leftrightarrow$

thermodyn. potential Ω
$\rightarrow$ state variables: p, s, n_q, etc.
$\rightarrow$ EOS $e = e(p)$
$\rightarrow$ $T^{\mu\nu}$, hydrodynamics

CJT formalism

- require stationarity of the *effective action*

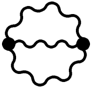
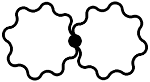
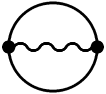
$$
\begin{aligned}
\Gamma[D,S] \;=\;& I - \frac{1}{2}\left\{\mathrm{Tr}\left[\ln D^{-1}\right] + \mathrm{Tr}\left[D_0^{-1}D - 1\right]\right\} \\
&+ \left\{\mathrm{Tr}\left[\ln S^{-1}\right] + \mathrm{Tr}\left[S_0^{-1}S - 1\right]\right\} + \Gamma_2[D,S]
\end{aligned}
$$

- For translation invariant systems without broken symmetries at the stationary point

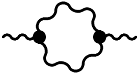
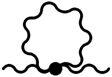
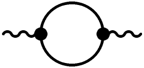
$$
\begin{aligned}
\frac{\Omega}{V} \;=\;& \mathrm{tr}\int \frac{d^4k}{(2\pi)^4} n_B(\omega)\cdot \mathrm{Im}\big(\ln D^{-1} - \Pi D\big) \\
&+ 2\cdot \mathrm{tr}\int \frac{d^4k}{(2\pi)^4} n_F(\omega)\cdot \mathrm{Im}\big(\ln S^{-1} - \Sigma S\big) - \frac{T}{V}\Gamma_2
\end{aligned}
$$

2-loop QCD thermodynamics

- truncate Γ_2 at *2-loop* order

$\Gamma_2 = \frac{1}{12}$  $+ \frac{1}{8}$  $- \frac{1}{2}$ 

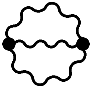
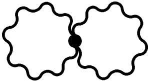
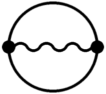
$\longrightarrow$ self-energies of *1-loop* order

$\Pi = \frac{1}{2}$  $+ \frac{1}{2}$  $-$ 

- to ensure gauge invariance: additional HTL ($p \sim gT$) approximation ($\longrightarrow$ Mr. Seipt, HK 20.2 @ 5.15pm)

2-loop QCD thermodynamics

- truncate Γ_2 at *2-loop* order

$\Gamma_2 = \frac{1}{12}$  $+ \frac{1}{8}$  $- \frac{1}{2}$ 

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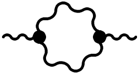
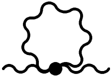
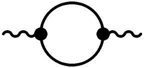
$\Pi = \frac{1}{2}$  $+ \frac{1}{2}$  $-$ 

- to ensure gauge invariance: additional HTL $(p \sim gT)$ approximation ($\longrightarrow$ Mr. Seipt, HK 20.2 @ 5.15pm)

Effective coupling

- severe approximation → introduce flexibility to accomodate further non-perturbative effects
- parametrize running coupling g^2
  - $g^2(\bar{\mu}) = \frac{16\pi^2}{\beta_0 \ln(\bar{\mu}^2/\Lambda^2)} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln[\ln(\bar{\mu}^2/\Lambda^2)]}{\ln(\bar{\mu}^2/\Lambda^2)}\right)$ at 2-loop order

  ↓ substitute by effective coupling ↓

  - $G^2(T, \mu = 0) = \frac{16\pi^2}{\beta_0 \ln\left(\frac{T - T_s}{T_0/\lambda}\right)^2} \left(1 - \frac{4\beta_1}{\beta_0^2} \frac{\ln\left[\ln\left(\frac{T - T_s}{T_0/\lambda}\right)^2\right]}{\ln\left(\frac{T - T_s}{T_0/\lambda}\right)^2}\right)$

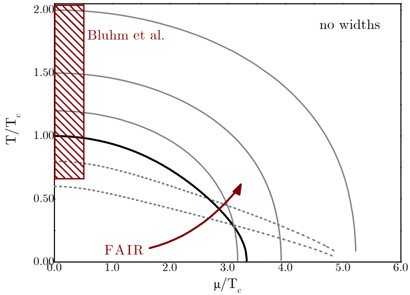
Model outline

- entropy $s := -\frac{1}{V}\left.\frac{\partial\Omega}{\partial T}\right|_\mu = s_g + s_q + s'$ with $s' = 0$ [Van.+Baym '98]

$$s_{g,T} \sim \int\frac{d^4k}{(2\pi)^4}\frac{\partial n_B}{\partial T}\Bigg\{\underbrace{\pi\varepsilon(\omega)\Theta\left(\text{-Re}D_T^{-1}\right)}_{s_{g,qp}} + \underbrace{\text{Re}D_T\text{Im}\Pi_T - \text{atan}\left(\frac{\text{Im}\Pi_T}{\text{Re}D_T^{-1}}\right)}_{s_{g,damp}}\Bigg\}$$

- lattice data available for vanishing and small chemical potential only $\longrightarrow$ self-consist. extrapolation to $\mu > 0$ using Maxwell eq. [Peshier '00]

$$\frac{\partial s}{\partial\mu} = \frac{\partial n}{\partial T} \qquad \Longrightarrow$$



Motivation + Outline

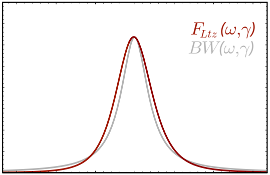
investigation: $s = s_{qp} + s_{damp} = s_{qp} + (\tilde{s} - s_{qp})$

$$\tilde{s} = \int d\omega \underbrace{\int dk\, \sigma(\omega,k)}_{\hat{=} s_{qp}(\omega)} \cdot F(\mathrm{Im}\Pi(\omega,k))$$

$$F := -\frac{1}{\pi}\left(\frac{\xi^2(\omega)}{(1+\xi^2(\omega))^2} + \frac{\xi^2(-\omega)}{(1+\xi^2(-\omega))^2}\right)\frac{\partial\xi}{\partial\omega}$$

$$\xi := \frac{\mathrm{Im}\Pi}{\mathrm{Re}D^{-1}}$$

1. limit $\mathrm{Im}\Pi \to 0$: $F \to \delta(\omega - \omega_{k,T})$
   $\tilde{s} \to s_{qp}$

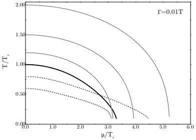


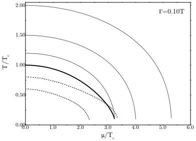
2. $\mathrm{Im}\Pi_{Ltz} = 2\gamma\omega$ reproduces [Peshier '04]
3. our ansatz:
   $$s^{BW}(T) = \int dM\, s_{qp}(T,M) \cdot BW(m,M,\Gamma)$$

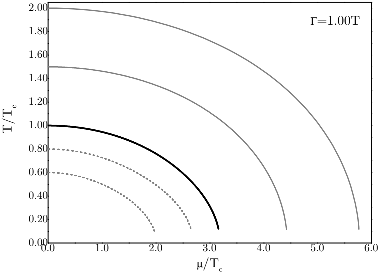
[Blaschke, Bugaev '03] [Andronic, Braun-M., Stachel '06] [Biro, Levai, Van, Zimanyi '06]

Results

- large widths remove crossings

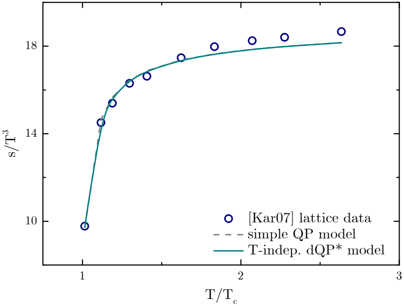






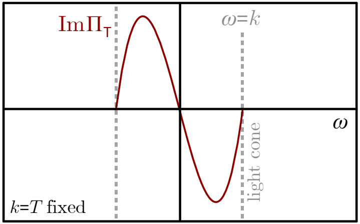
- use [Karsch '07] lattice data for $N_f = 2 + 1$ (hep-ph/0701210):

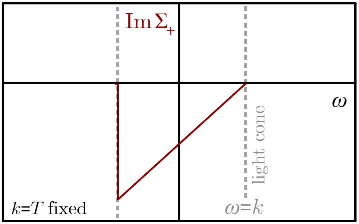
  $\rightarrow$ $\Gamma = 0.00038T$
  $T_s = \text{-}0.8234\, T_c$
  $\lambda = 8.601$



Damping contributions

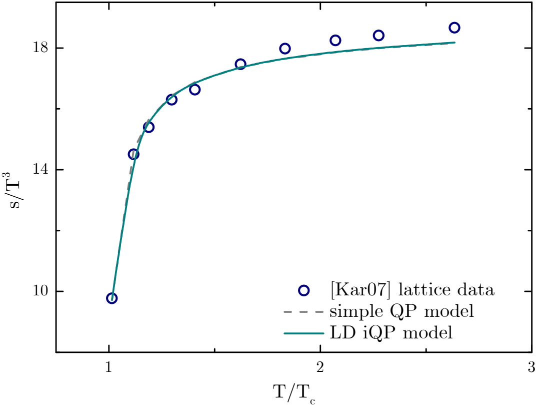
- consider exact expression $\tilde{s} = \int d\omega \int dk\, \sigma(\omega, k) \cdot F(\mathrm{Im}\Pi(\omega, k))$
- use HTL self-energies





$\longrightarrow$ complex below the light-cone only
$\longrightarrow$ "Landau damping"

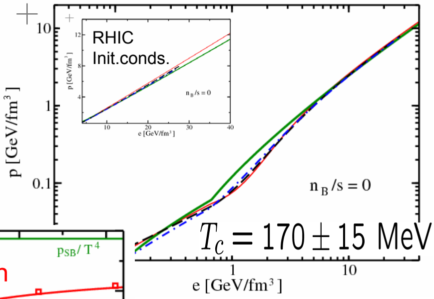
Results for $N_f = 2 + 1$



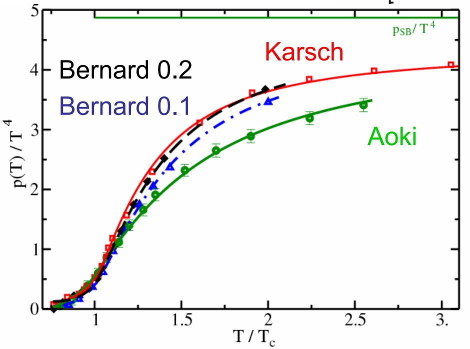
- characteristics ugly $\longrightarrow$ not all degrees of freedom incorporated
  $\longrightarrow$ future: evaluation of full HTL expression $\longleftrightarrow$ [Romatschke '04]

2+1 EOS ready?

$\mu = 0$



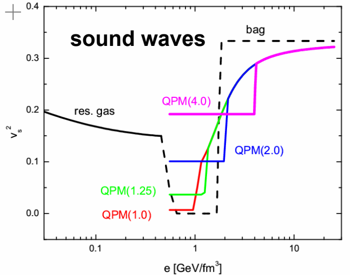
$T_c = 170 \pm 15$ MeV

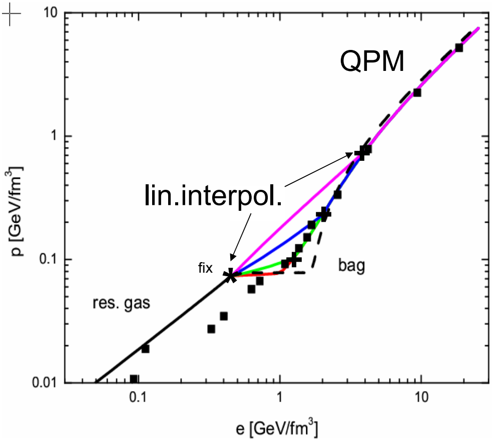


[M. Bluhm]

A family of EOS's $\mu_B \ll T$

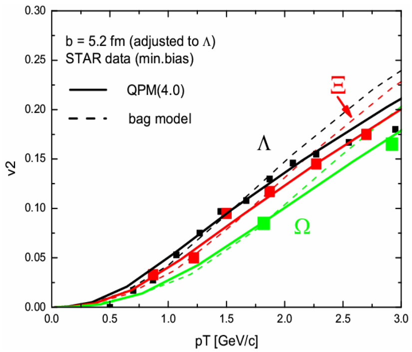
- interpolate between hadron gas and QPM description





Elliptic flow from relativistic hydrodynamics

- calculate elliptic flow using relativistic hydro code
- compare with experimental data
  - low p_T: bag model description a little more accurate
  - high p_T: bag model fails, QPM in good agreement



Summary

- derived a handy expression for 2-loop QCD entropy within the CJT formalism
- damping effects and quasi-particle widths included in both
  - an physical ansatz which can be well justified and
  - a systematic approach using the HTL self-energies
- simple QPM takes into account all relevant degrees of freedom
- EOS $(T,\ \mu > 0)$ in agreement with experimental data