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Disentangling the nuclear shape coexistence in even-even Hg isotopes using the interacting boson model

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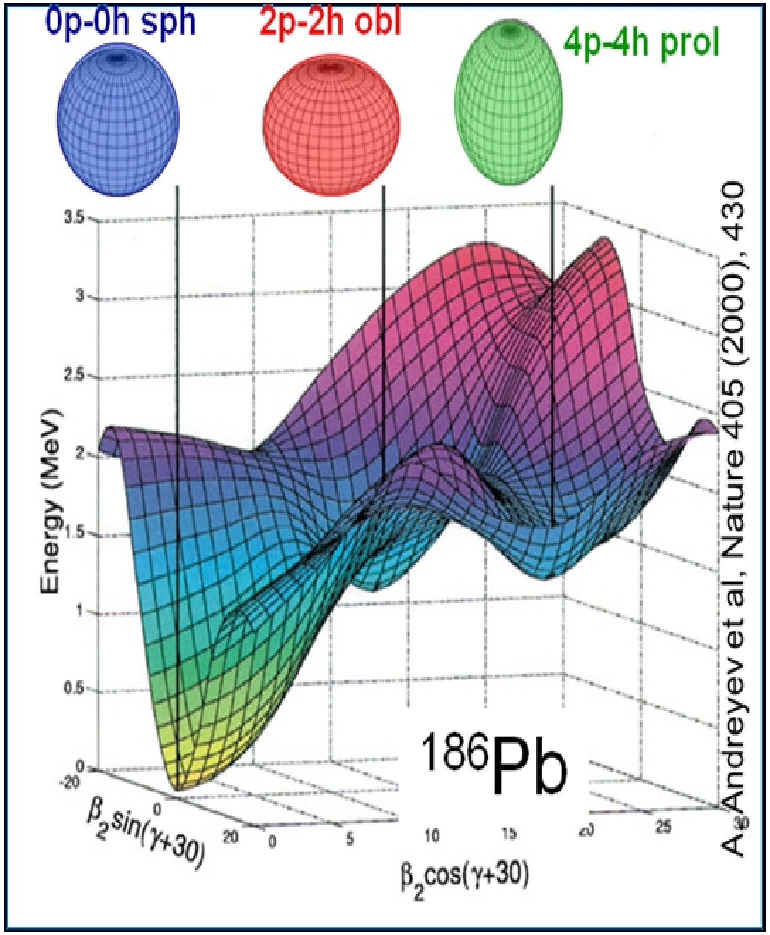
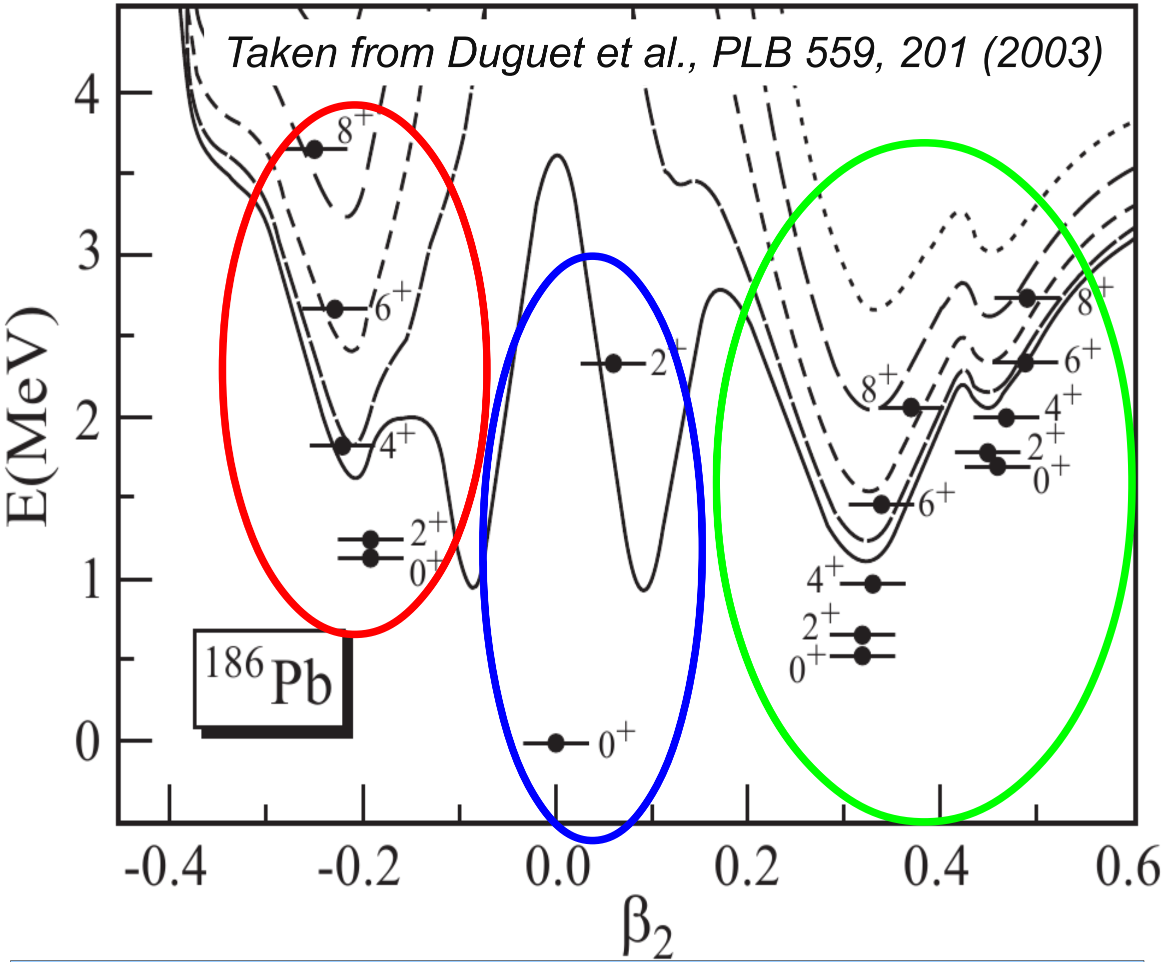
Mean Field

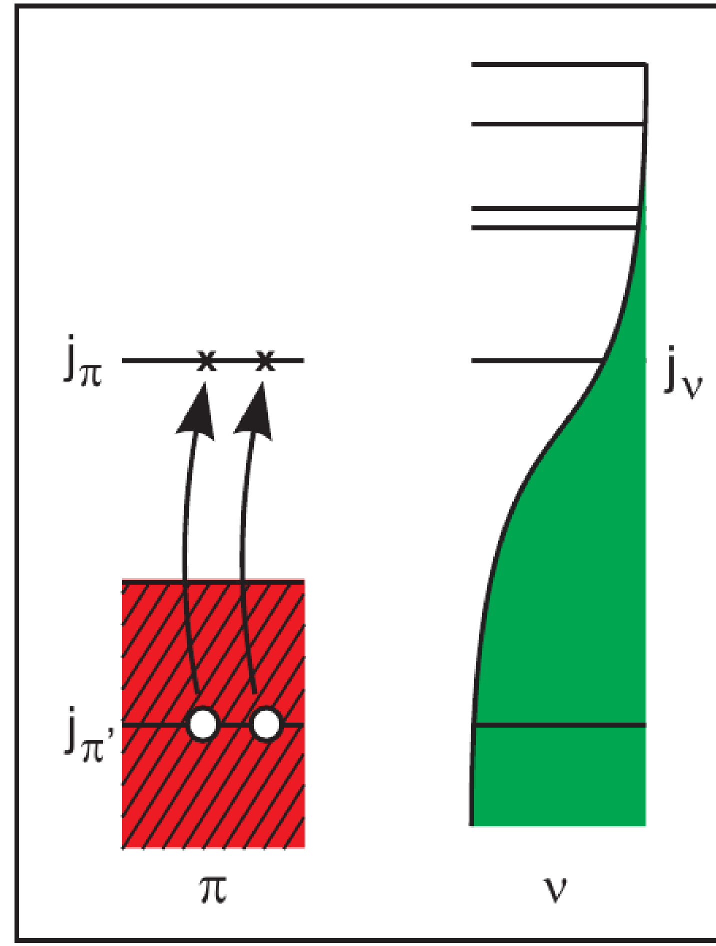


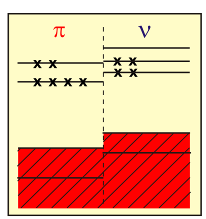


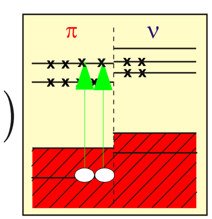
The angular momentum projected mean field plus the Generator Coordinate Method generates different bands with very different deformation.

Shell Model

- For nuclei near to closed shells, either for neutrons or for protons, it can be energetically favorable to have excitations of 2p-2h, 4p-4h … crossing the energy gap.
- The np-nh excitations have a lower excitation energy than expected due to the correlation energy: pairing and deformed correlations.
- Restricted to light and medium-heavy nuclei, at present.

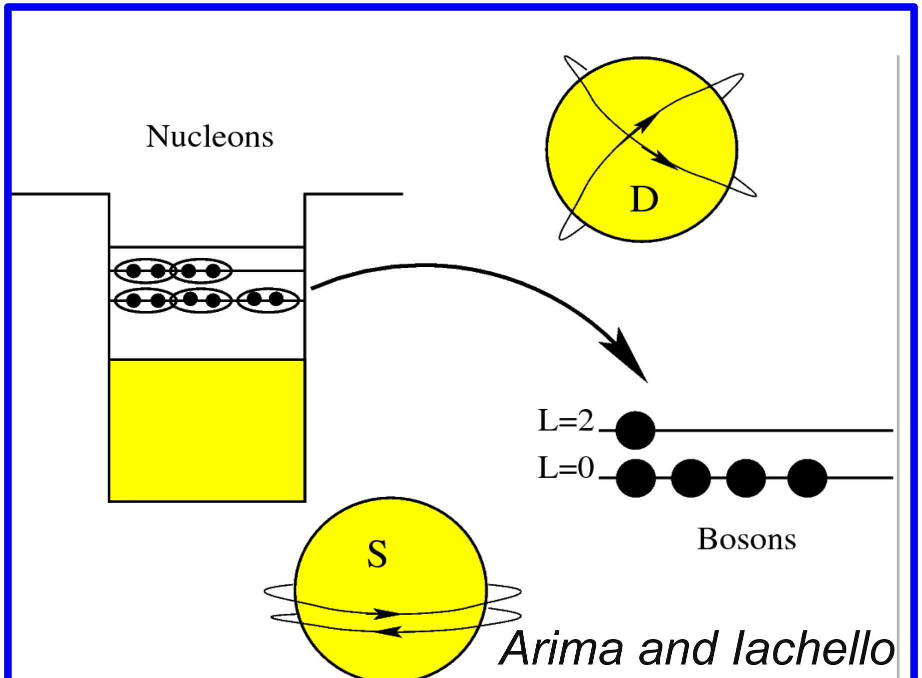


$\phi(J,M) = a(J,M)$ 

$+\ b(J,M)$ 

In heavy nuclei the huge model space imposes some kind of truncation: symmetry dictated truncation.

Interacting Boson Model (IBM)



Nucleons couple preferably in pairs with angular momentum either equal to 0 (S) or equal to 2 (D). Those pairs are then described by means of bosons: s and d.

$$s^\dagger, d_m^\dagger (m = 0, \pm 1, \pm 2)$$

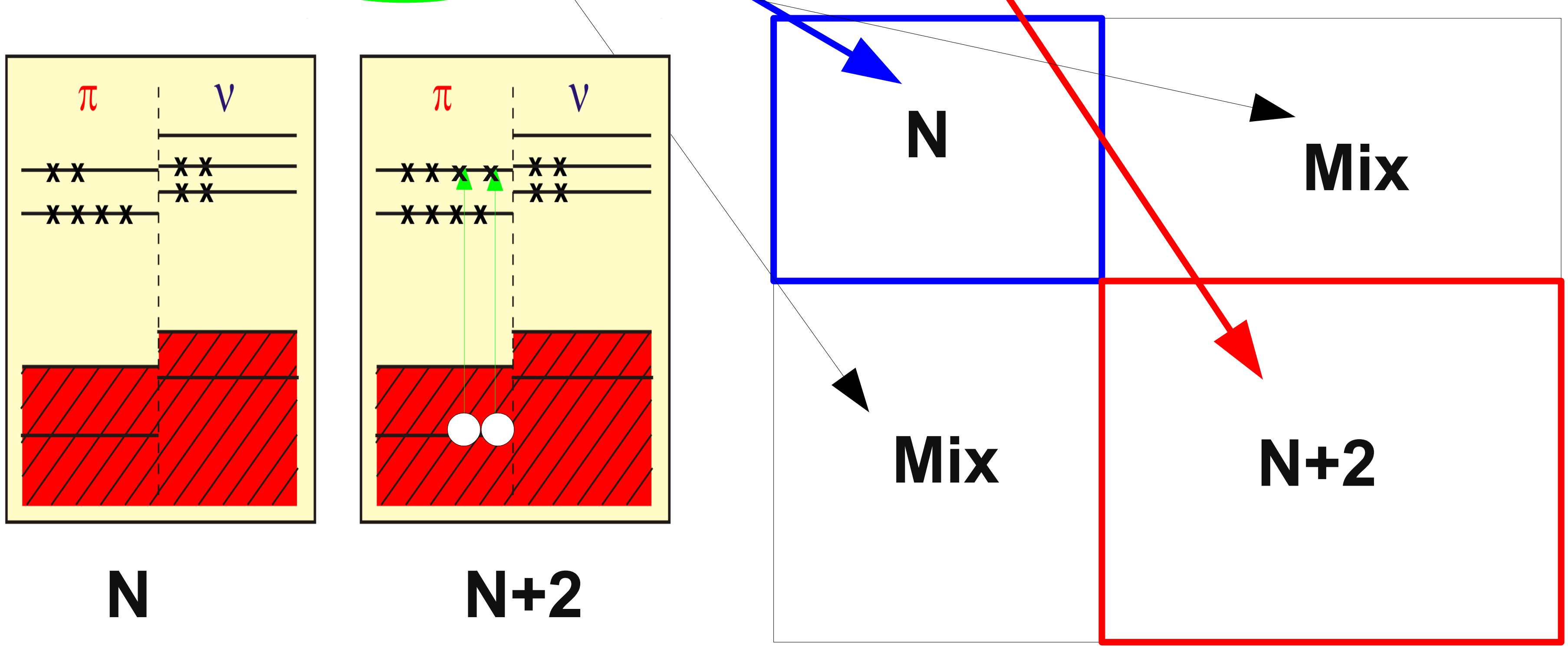
$$s, d_m (m = 0, \pm 1, \pm 2)$$

$$\hat{H}_{ECQF} = \epsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa ' \hat{L} \cdot \hat{L}$$

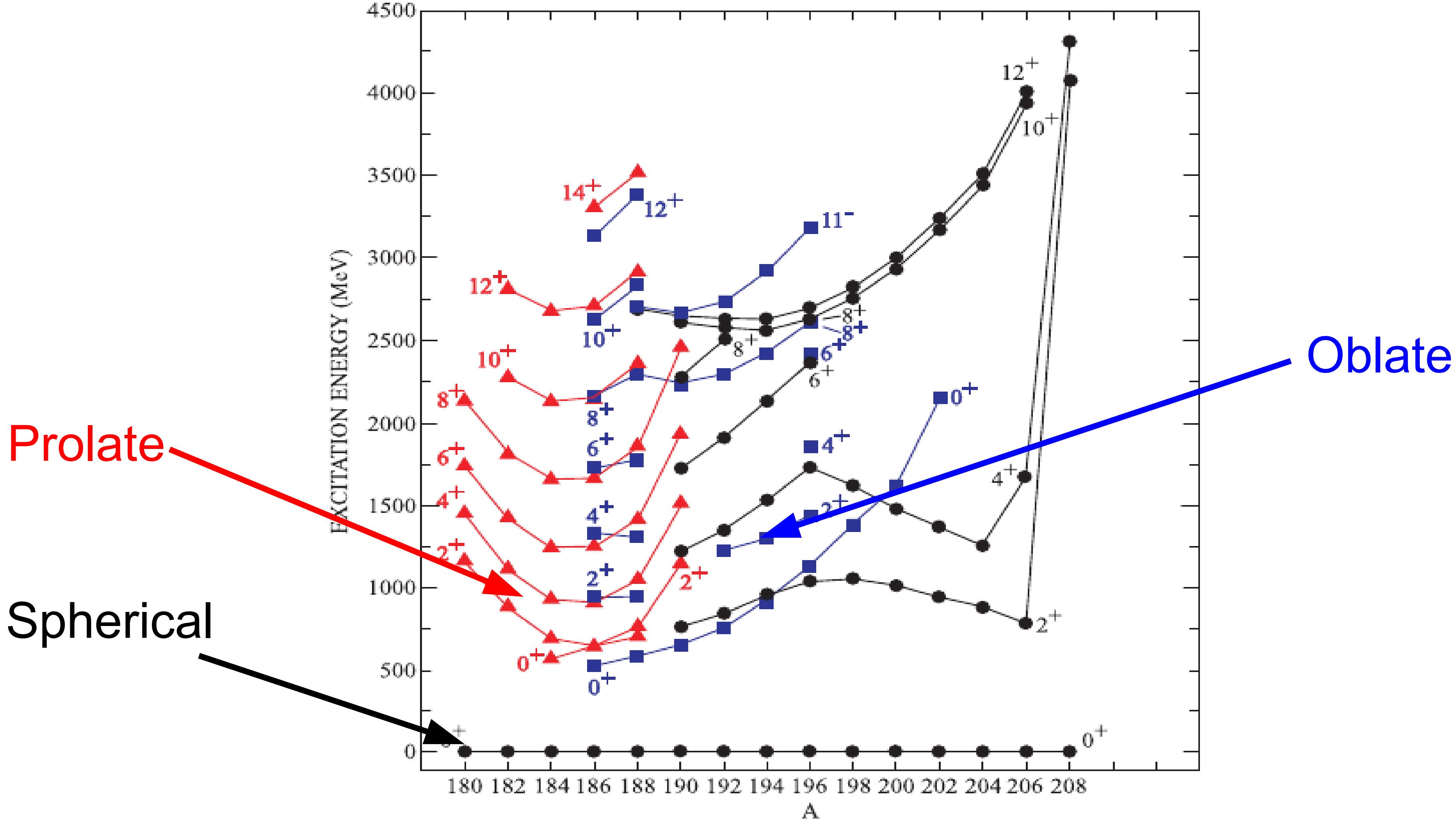
Interacting Boson Model

(configuration mixing)

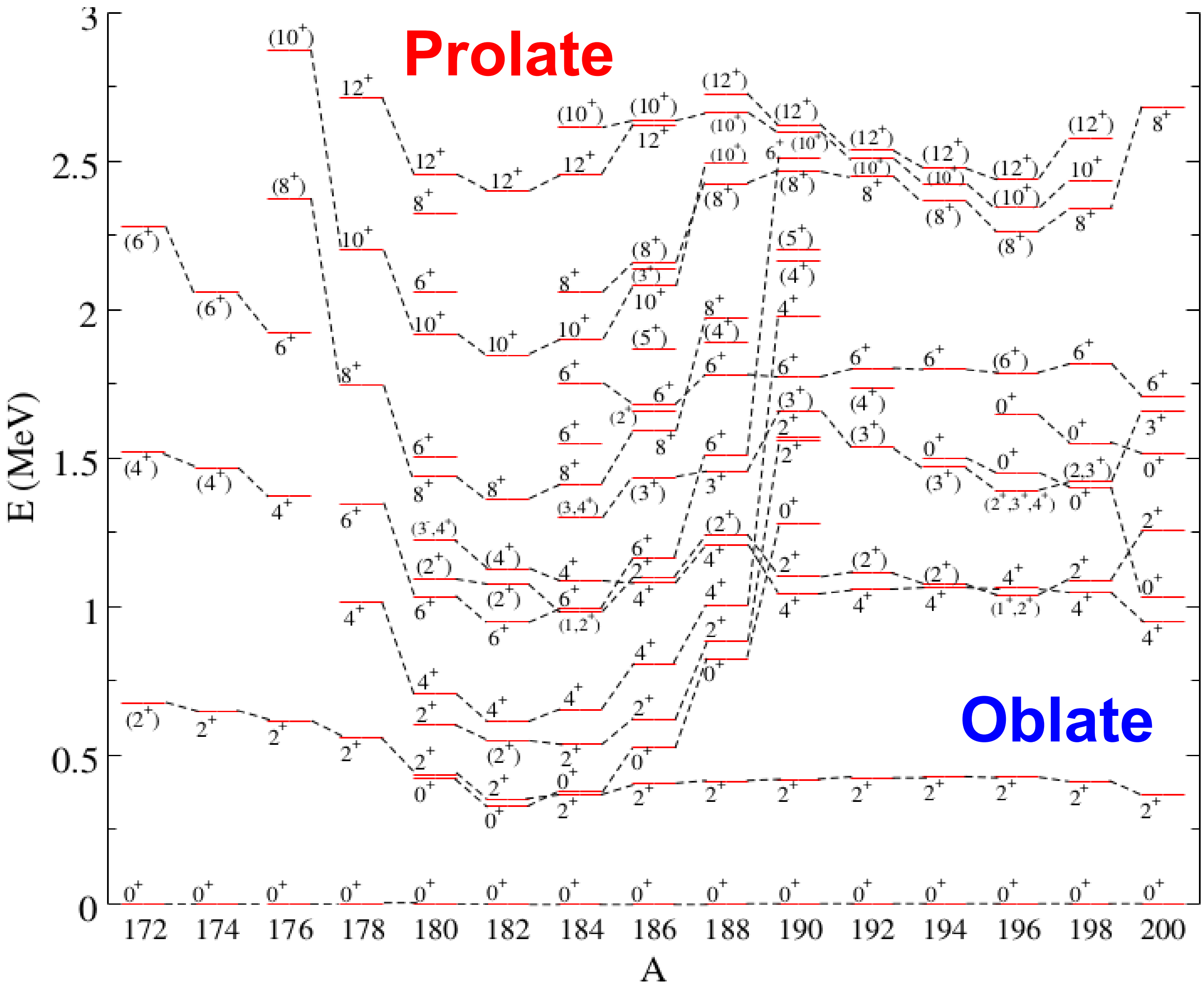
$$\hat{H} = \hat{P}_N^\dagger \hat{H}_{\text{ECQF}}^N \hat{P}_N + \hat{P}_{N+2}^\dagger \left(\hat{H}_{\text{ECQF}}^{N+2} + \Delta^{N+2}\right) \hat{P}_{N+2} + \hat{V}_{\text{mix}}^{N,N+2},$$



Pb isotopes

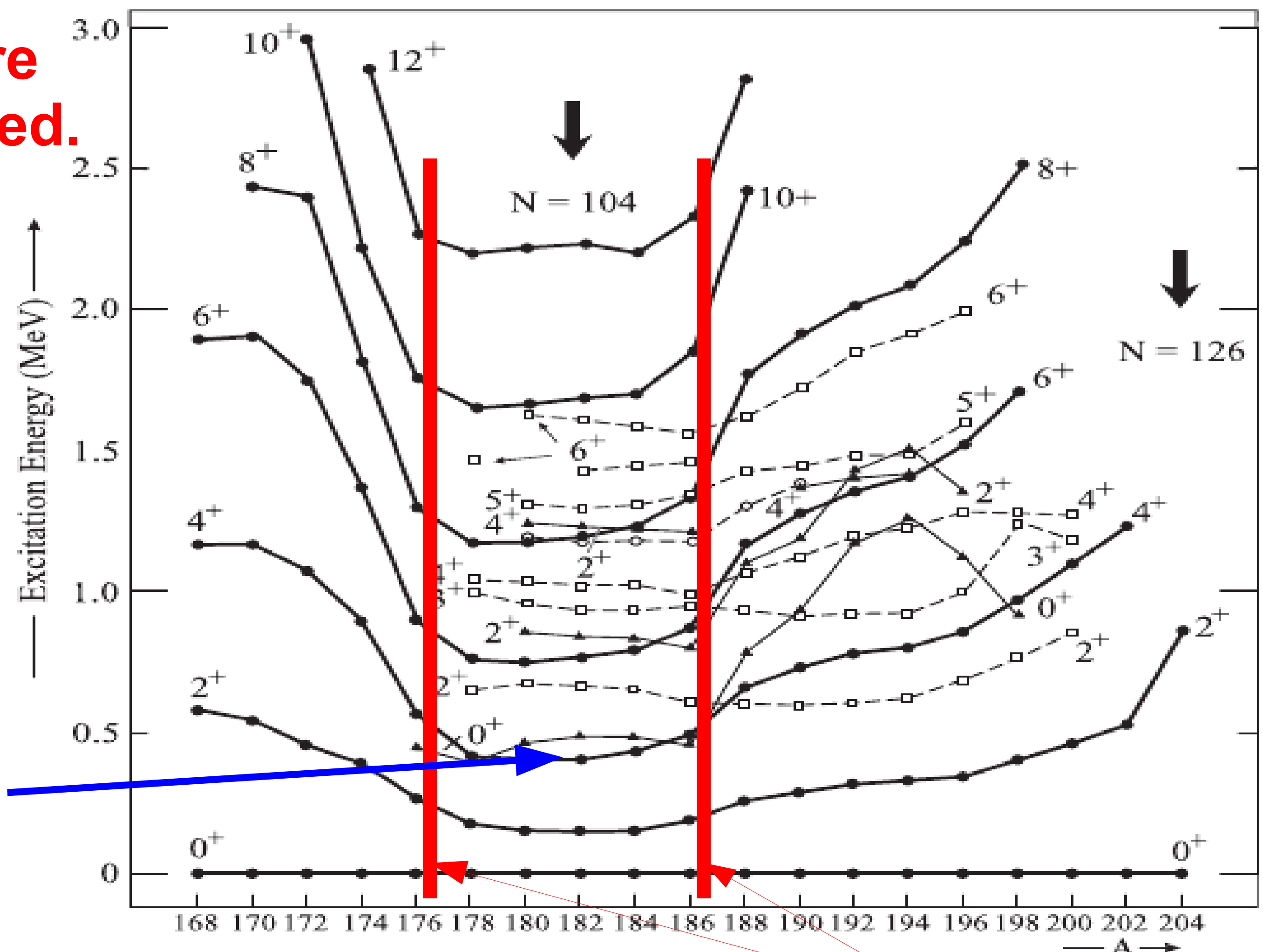


Hg isotopes


Prolate
Oblate
E (MeV)
A
0
0.5
1
1.5
2
2.5
3
172
174
176
178
180
182
184
186
188
190
192
194
196
198
200

Pt isotopes

The two families are not clearly separated. Strong mixing!!



Flat behavior

Sudden drop

JEGR and K. Heyde, NPA 825, 39 (2009),
JEGR, V. Hellemans, and K. Heyde, PRC 84, 014331 (2011).

How to fix the parameters for Hg

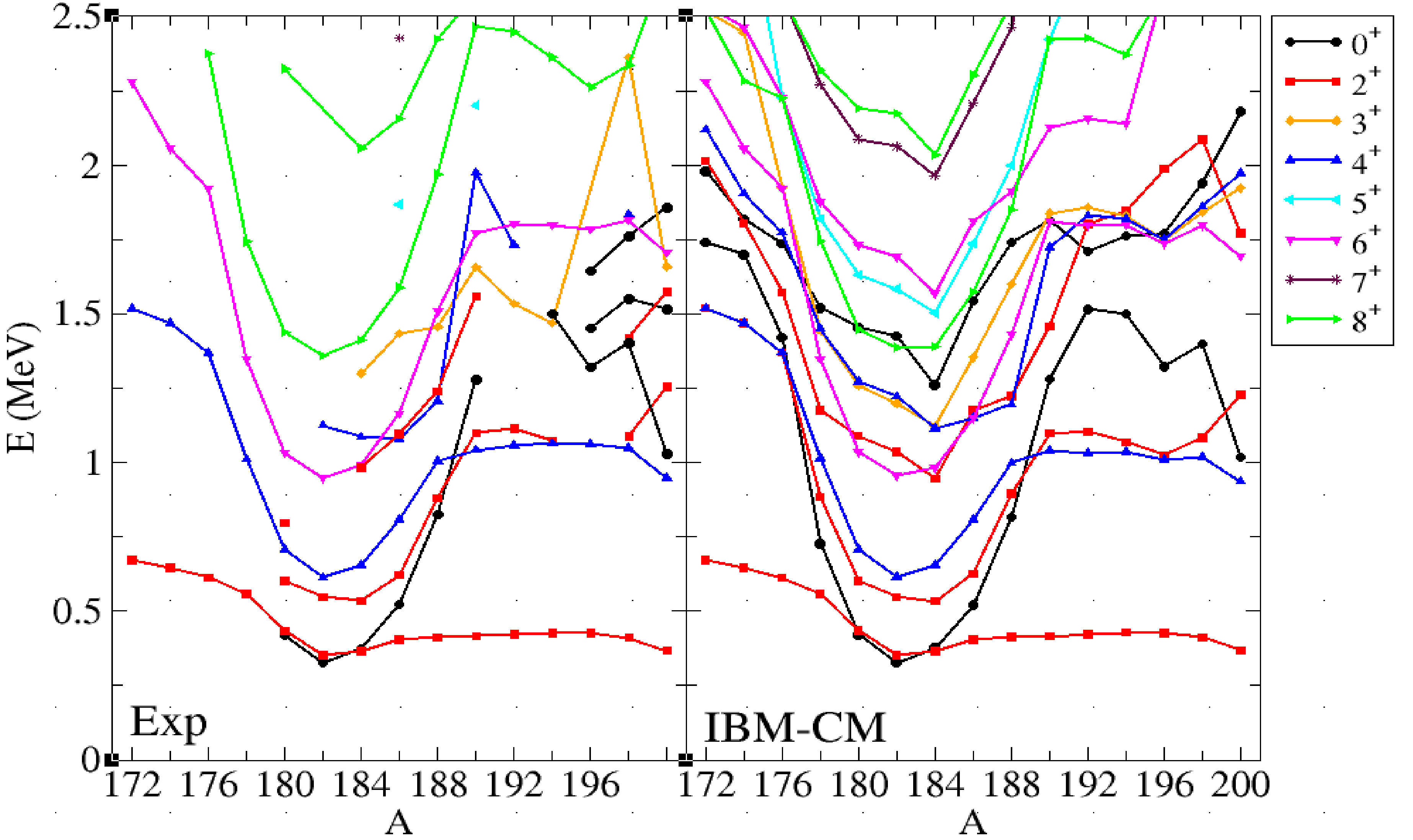
Least squares fit to the experimental data, including excitation energies and absolute B(E2) transitions.

$$\chi^2 = \frac{1}{N_{data} - N_{par}} \sum_{i=1}^{N_{data}} \frac{(X_i(data) - X_i(IBM))^2}{\sigma_i^2}$$

Error (keV)	States
$\sigma = 0.1$	2_1^+
$\sigma = 1$	$4_1^+, 0_2^+, 2_2^+$
$\sigma = 10$	$2_3^+, 3_1^+, 4_2^+, 6_1^+, 8_1^+$
$\sigma = 100$	$2_4^+, 3_1^+, 4_3^+, 6_2^+$

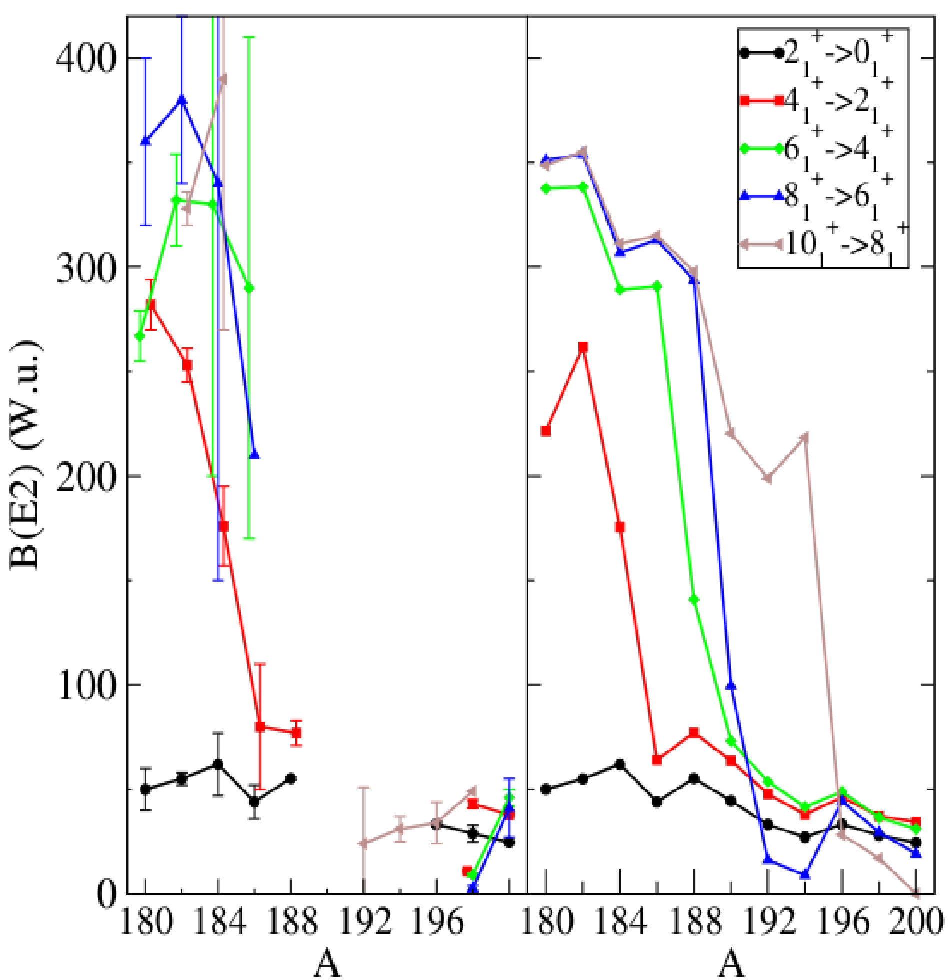
+ all the known B(E2) transitions

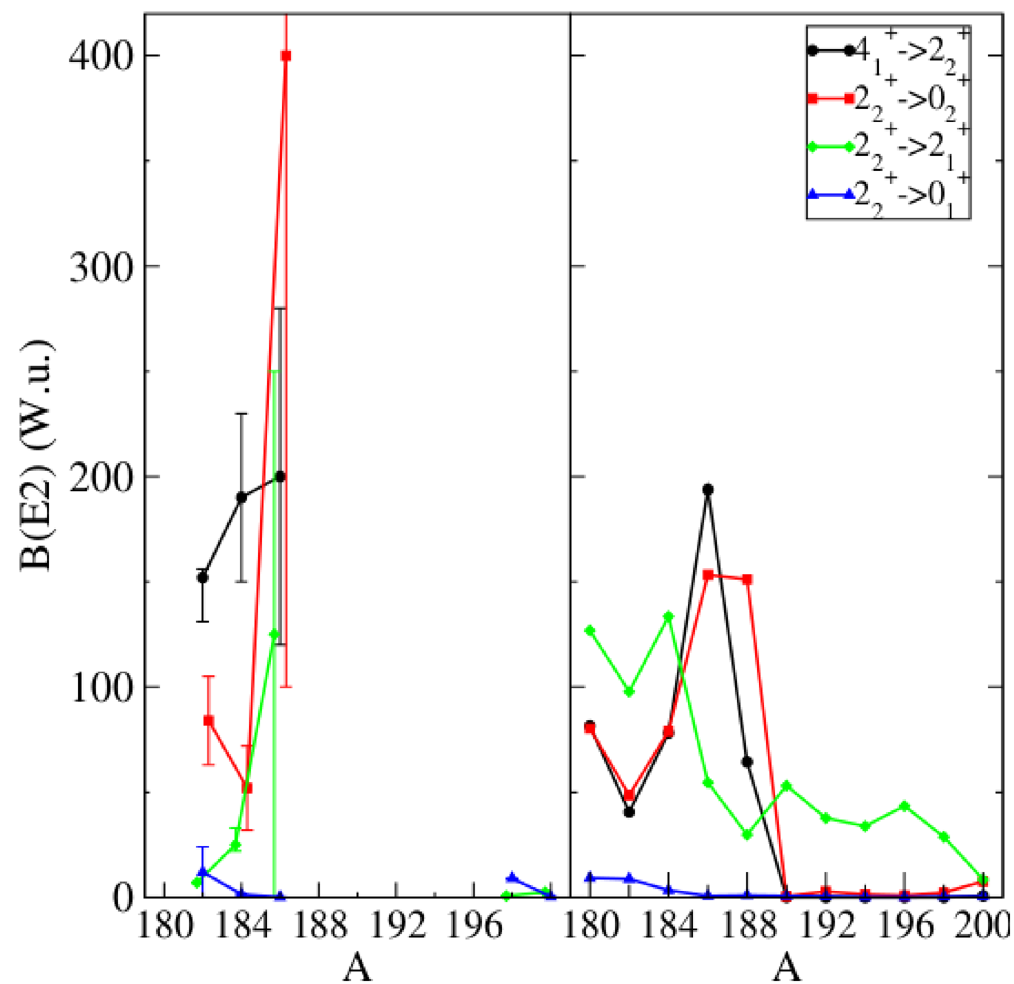
IBM configuration mixing



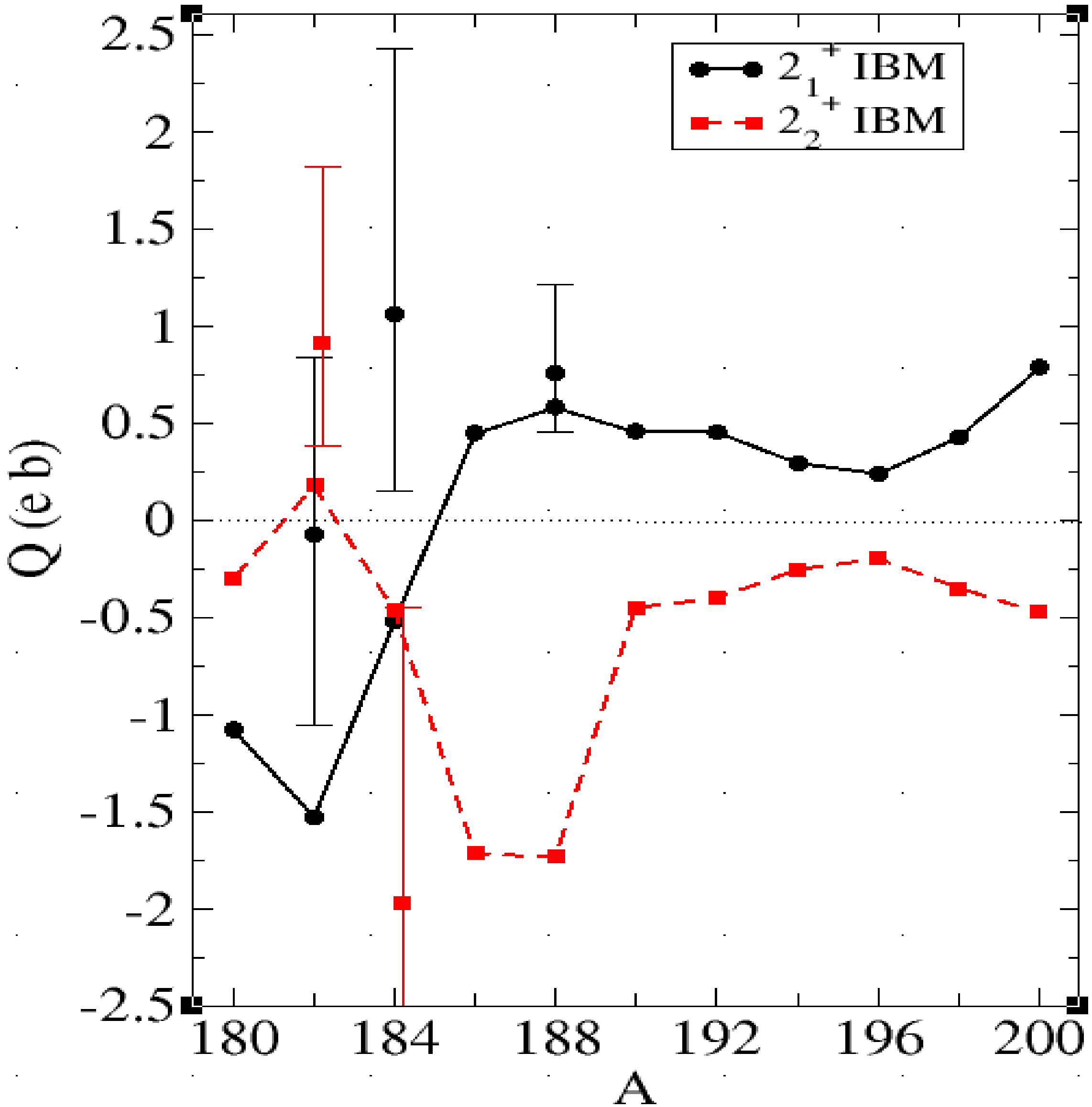
JEGR and K. Heyde, PRC 89, 014306 (2014).

B(E2) transition rates

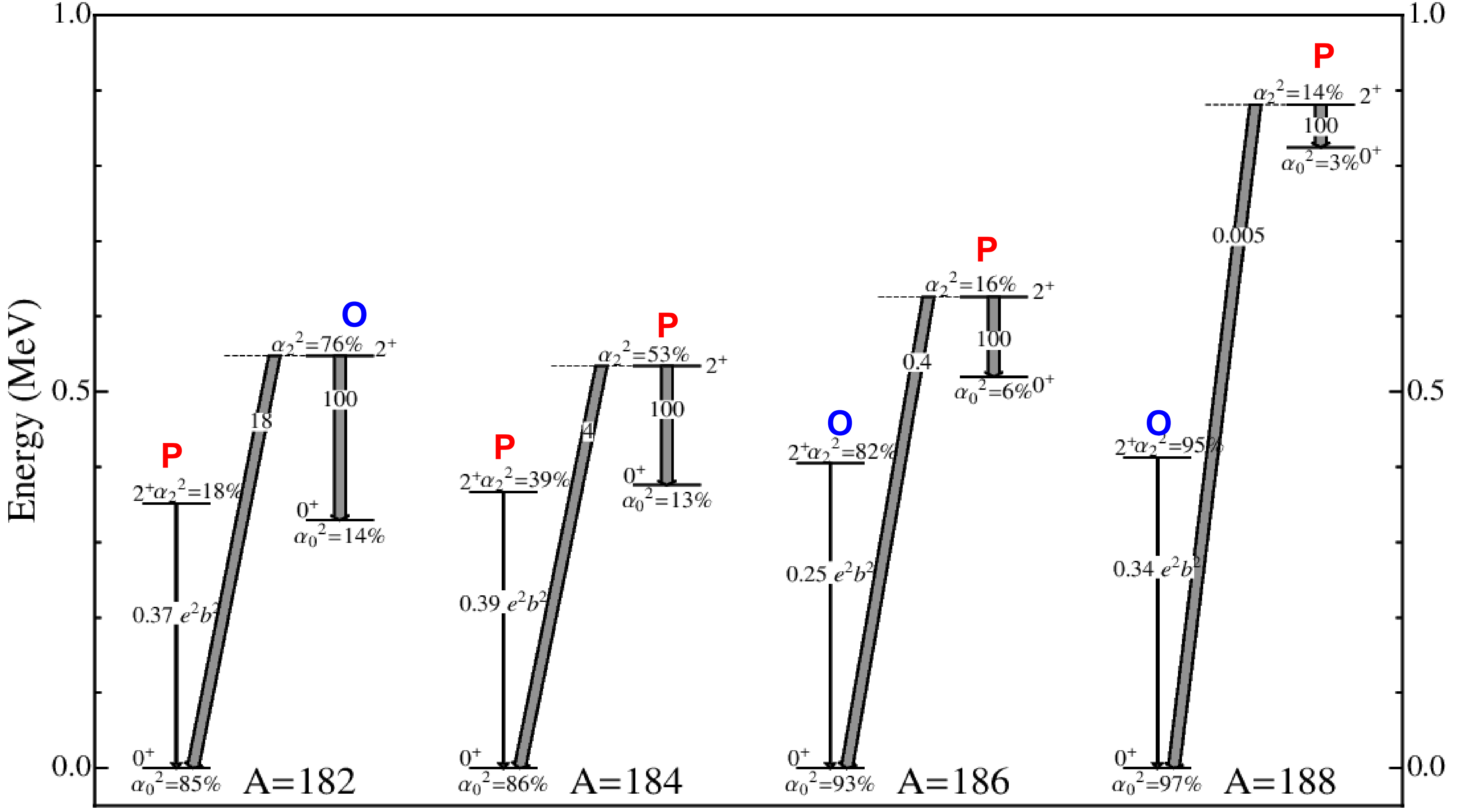




Quadrupole moments



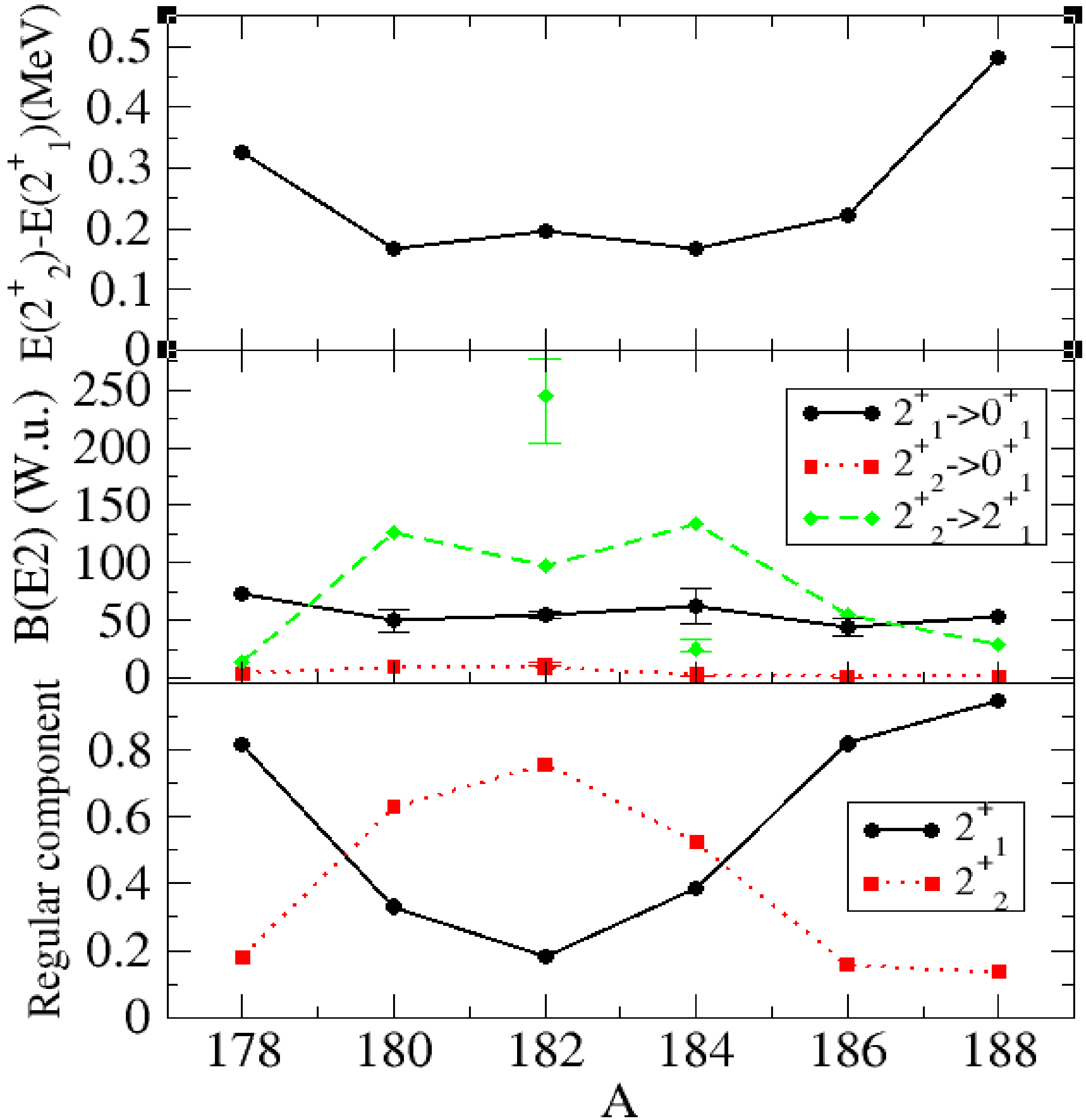
Mixture of states



K. Wrzosek-Lipska, Session 17 (Wednesday)

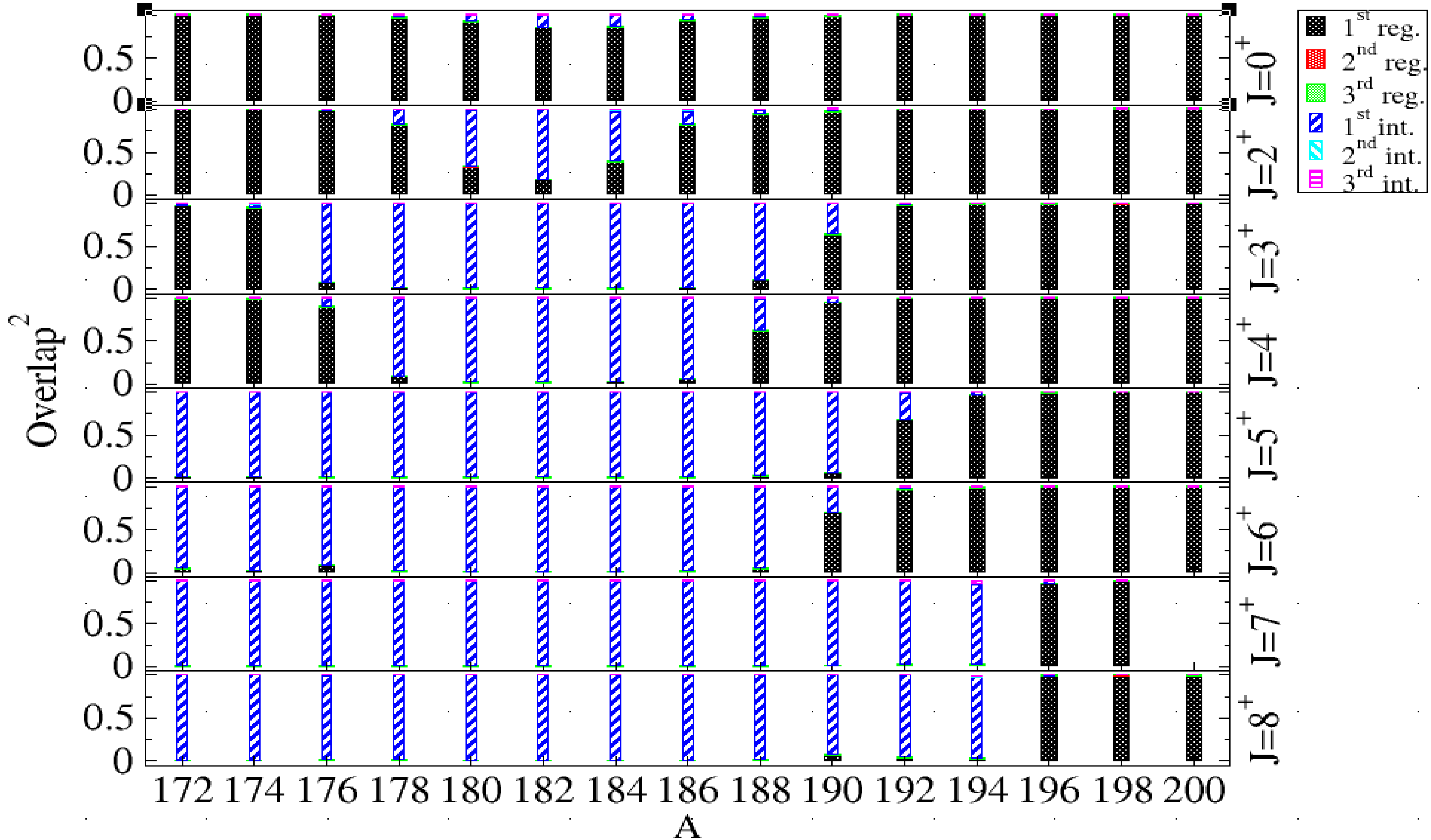
N. Bree, K. Wrzosek-Lipska, et al., Phys. Rev. Lett. 112, 162701 (2014).

More on mixture of 2^+ states

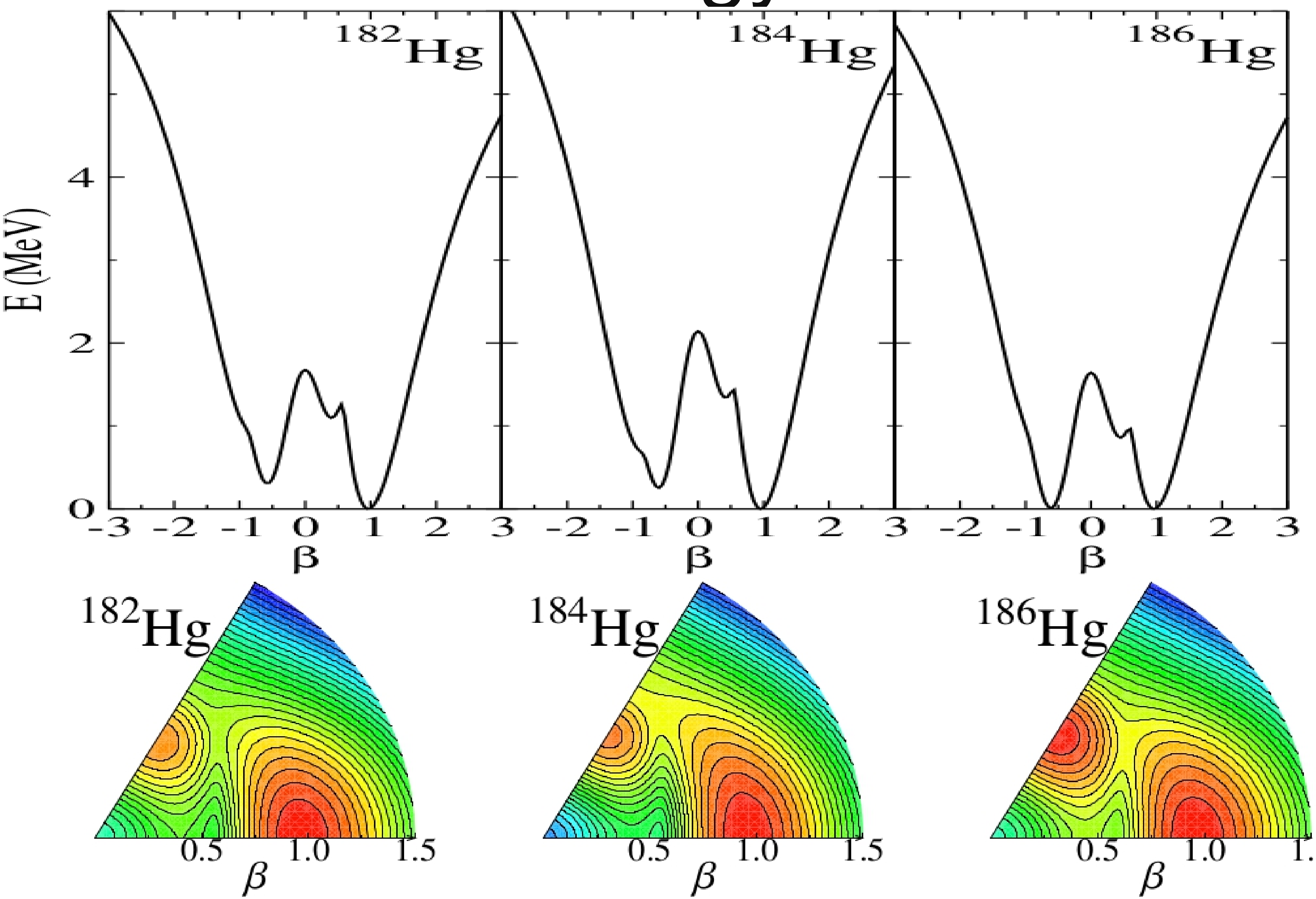


Decomposition in an intermediate basis

First state



IBM-CM energy surfaces



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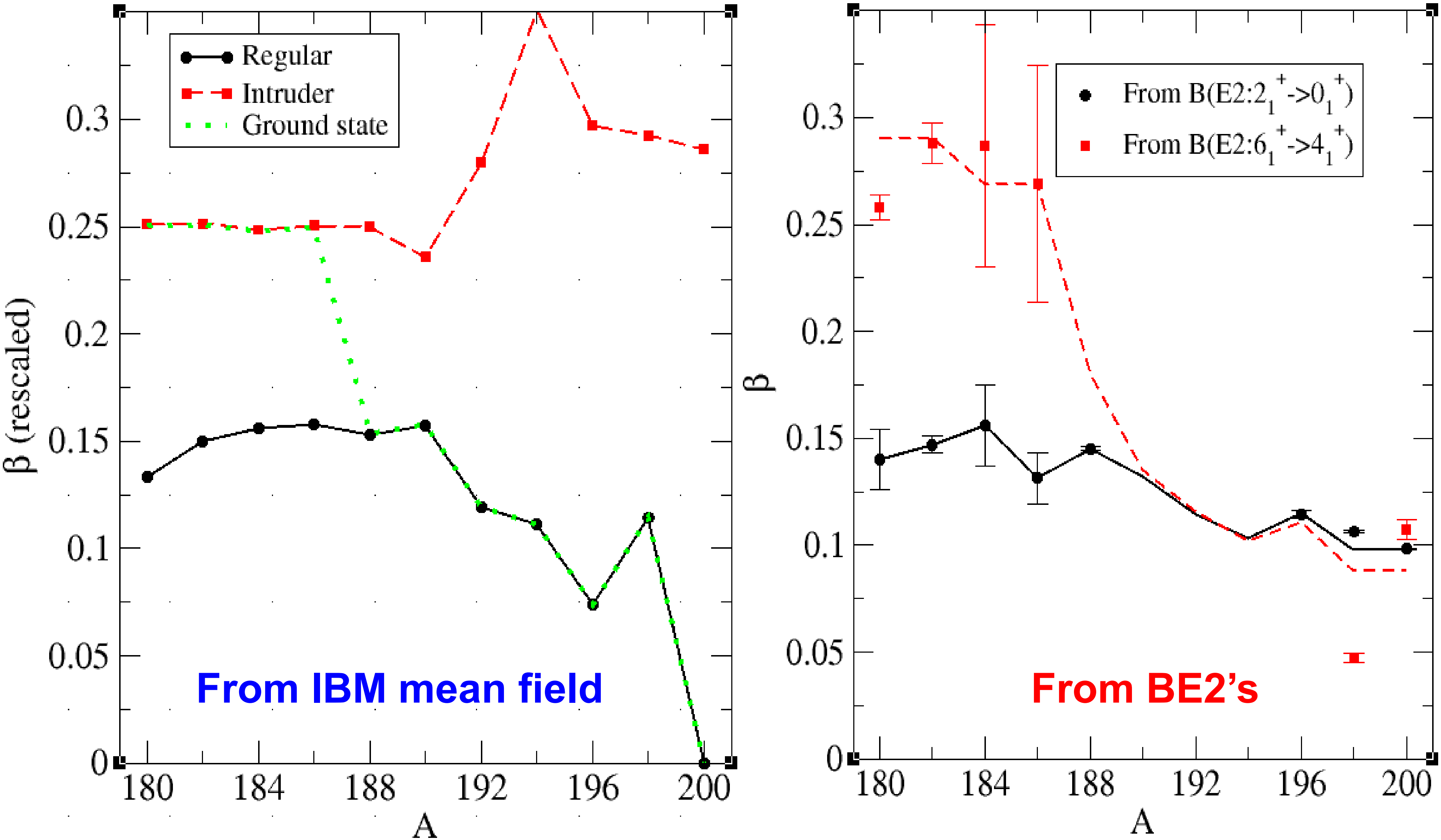
Deformation value for the ground state



Quadrupole invariants

Definitions:

$$q_{2,i} = \sqrt{5}\langle 0_i^+||[\hat{Q}\times\hat{Q}]^{(0)}|0_i^+\rangle,$$

$$q_{3,i} = -\sqrt{\frac{35}{2}}\langle 0_i^+[\hat{Q}\times\hat{Q}\times\hat{Q}]^{(0)}|0_i^+\rangle$$

$$q_{2,i} = \sum_r \langle 0_i^+||\hat{Q}||2_r^+\rangle\langle 2_r^+||\hat{Q}||0_i^+\rangle,$$

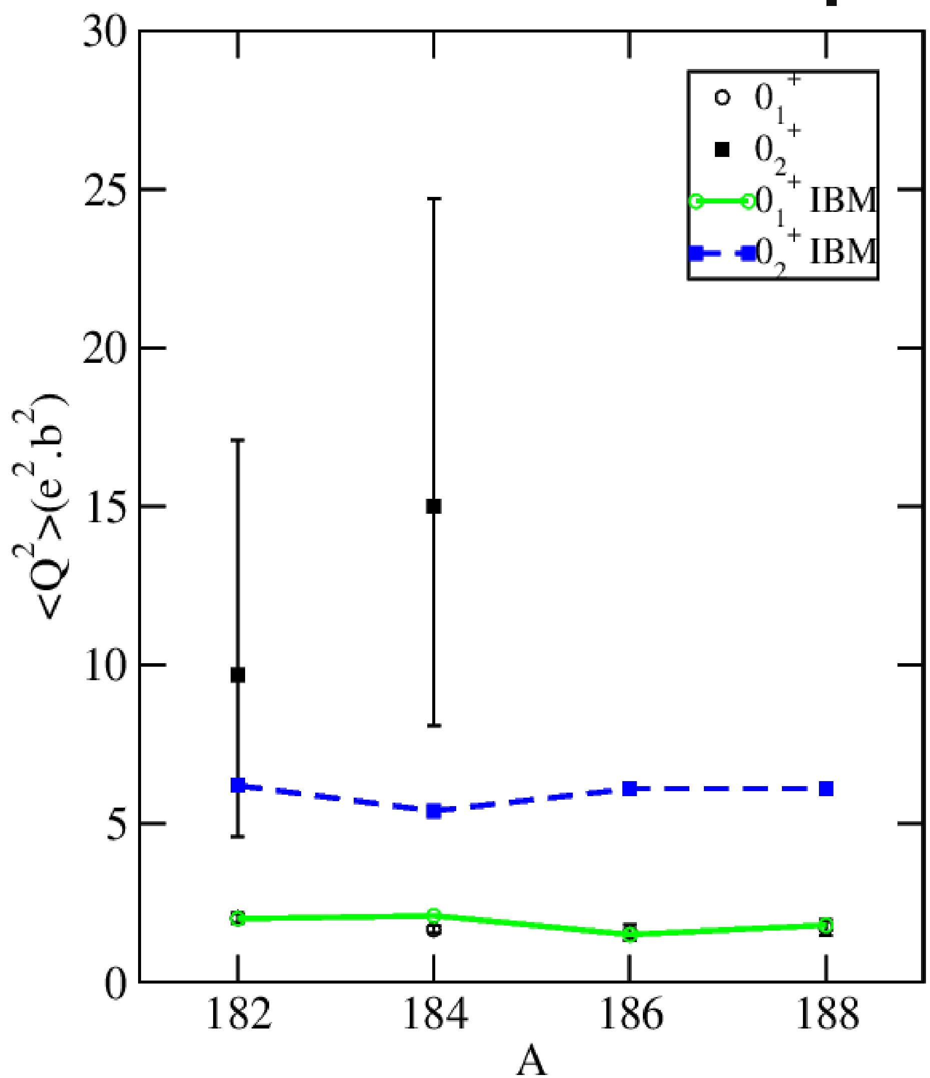
$$q_{3,i} = -\sqrt{\frac{7}{10}}\sum_{r,s}\langle 0_i^+||\hat{Q}||2_r^+\rangle\langle 2_r^+||\hat{Q}||2_s^+\rangle\langle 2_s^+||\hat{Q}||0_i^+\rangle$$

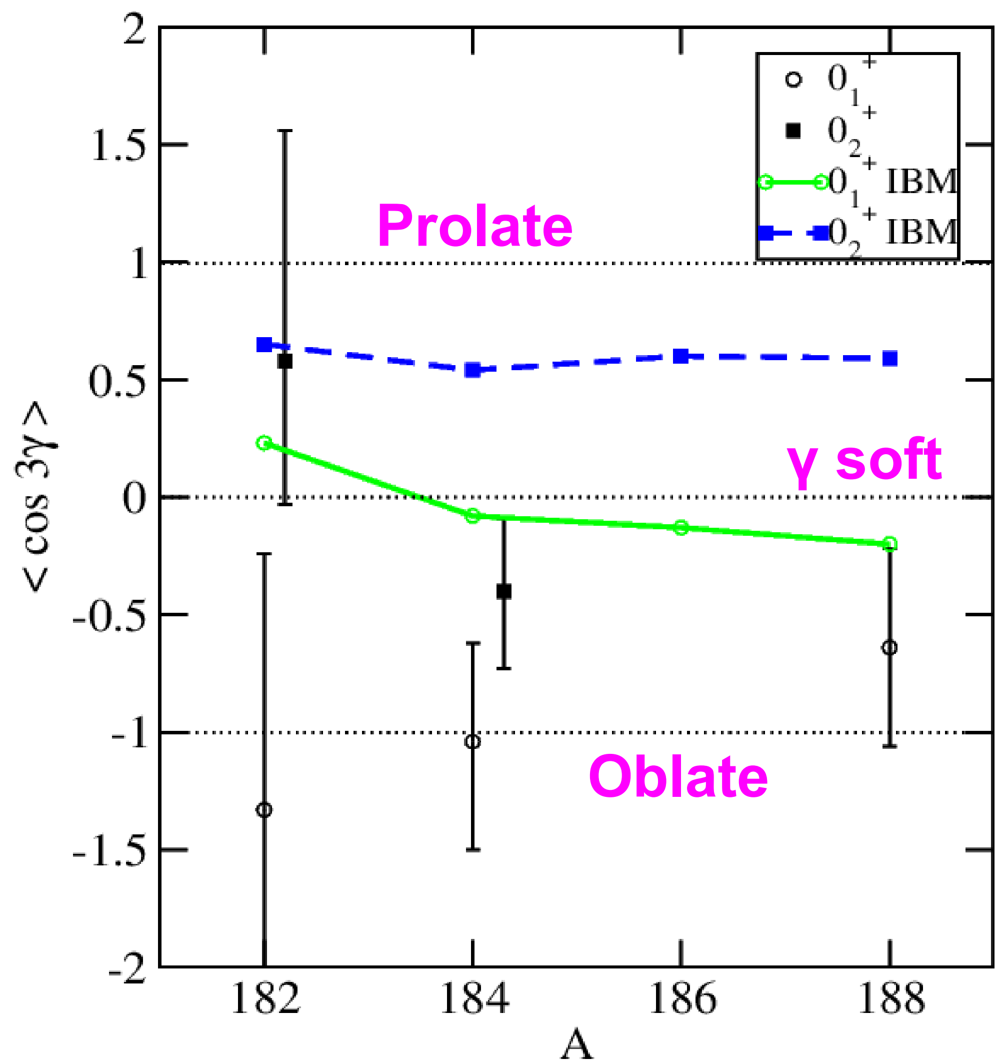
Geometric interpretation

$$q = \sqrt{\langle q^2\rangle},$$

$$\gamma = \frac{60}{\pi}\arccos\frac{q_3}{q_2^{3/2}}$$

Quadrupole invariants





K. Wrzosek-Lipska, Session 17 (Wednesday)

N. Bree, K. Wrzosek-Lipska, et al., Phys. Rev. Lett. 112, 162701 (2014).

Summary and conclusions

- We have given a detailed description of even-even Hg isotopes using the interacting boson model including configuration mixing: excitation energies, BE2's, quadrupole moments, deformation, radii, ...

- IBM provides a description compatible with sophisticated mean-field calculations and at the same time gives a very precise description of spectroscopic properties.

- In Hg isotopes the effect of the coexistence is shown in the 2^+ states instead than in the 0^+ (as for Pt), anyhow configuration mixing is somehow *concealed.*

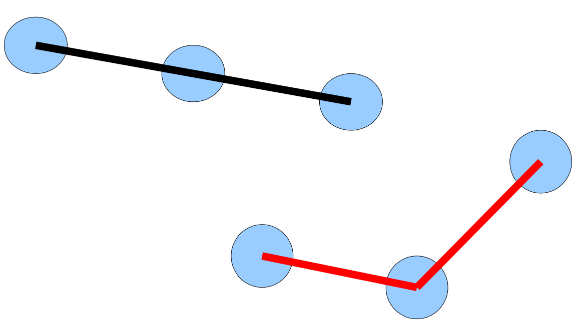
- In Pb nuclei three configurations coexist, spherical, oblate and prolate, two in the case of Hg, oblate and prolate, in both cases with a *weak mixing*, while in the case of Pt the *strong mixing* between both configurations hides the presence of two configurations.

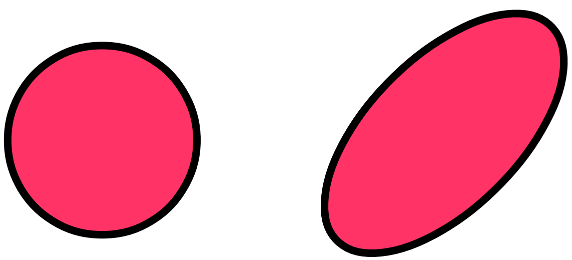
Thank you

What is shape coexistence?

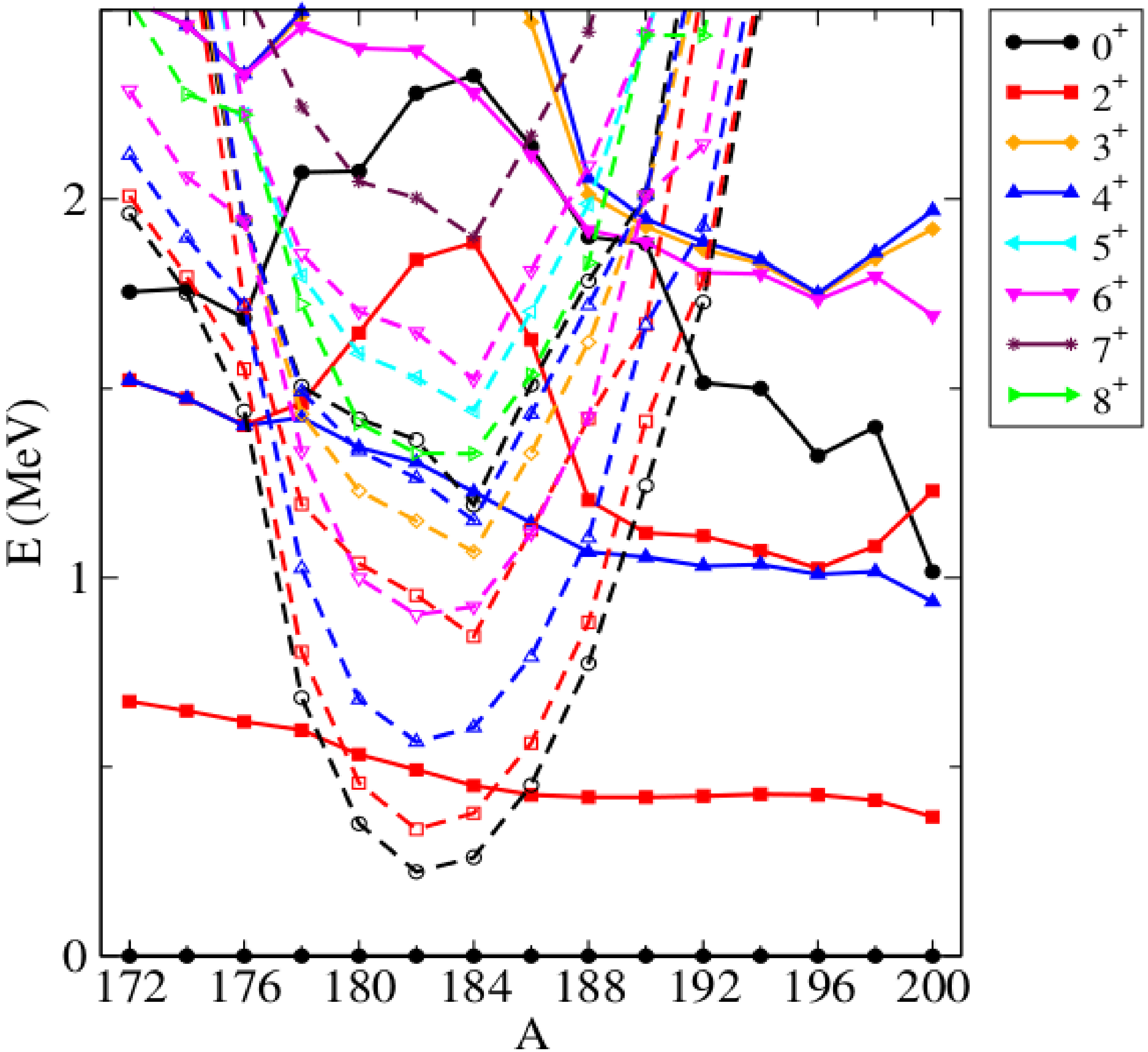
It appears in quantum systems where eigenstates with very different shapes coexist.

Therefore, it is implicit the existence of a geometric interpretation.





Unperturbed energies



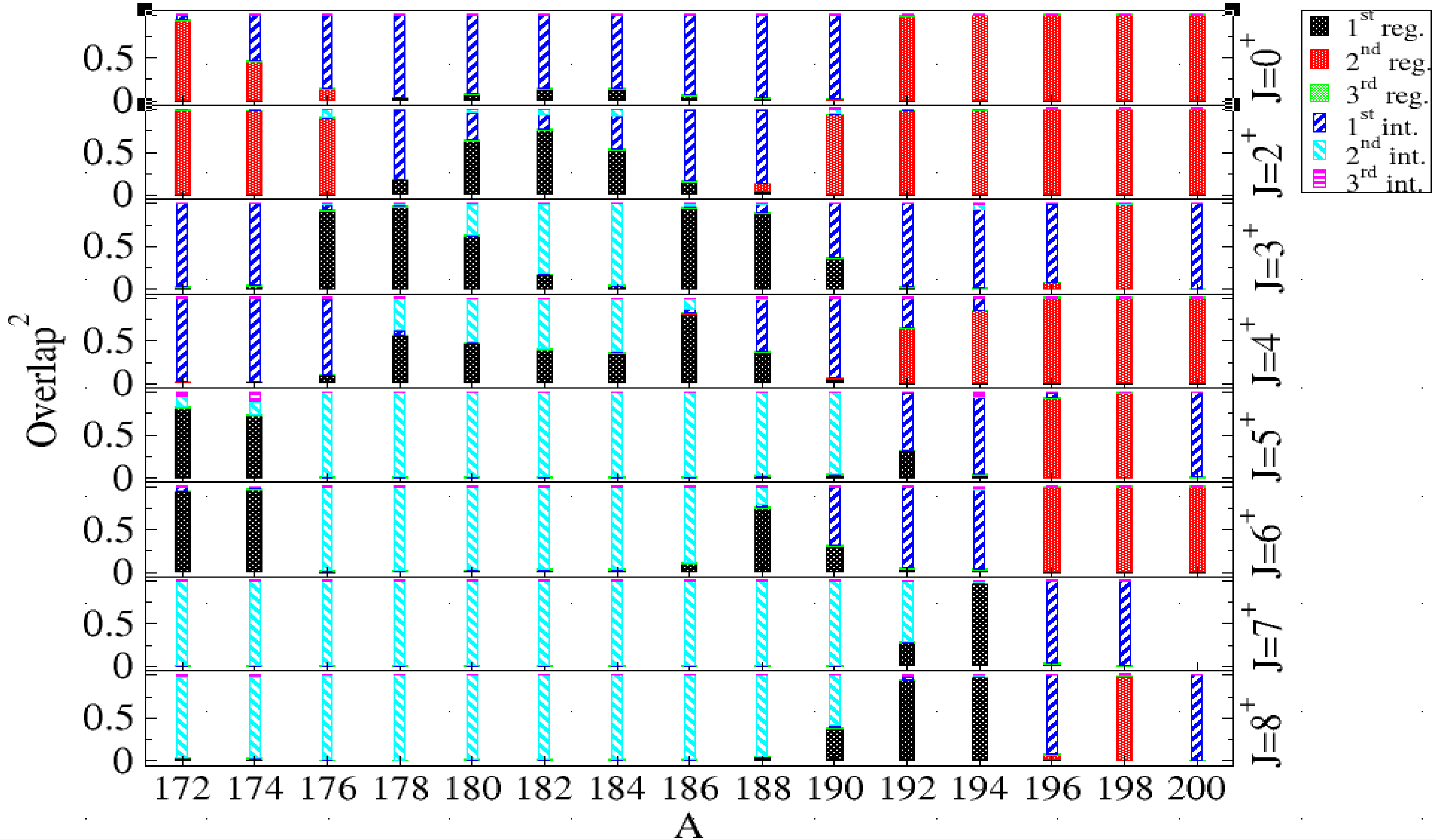
The parameters

TABLE II: Hamiltonian and $\hat{T}(E2)$ parameters resulting from the present study. All quantities have the dimension of energy (given in units of keV), except χ_{N+2} which is dimensionless and e_N and e_{N+2} which are given in units $\sqrt{\text{W.u.}}$ The remaining parameters of the Hamiltonian, *i.e.*, χ_N, ε_{N+2}, κ'_N, and κ'_{N+2} are equal to zero, except $\Delta^{N+2} = 3480$ keV and $w_0^{N,N+2} = w_2^{N,N+2} = 20$ keV.

Nucleus	ε_N	κ_N	χ_N	κ_{N+2}	χ_{N+2}	e_N	e_{N+2}
^{172}Hg	845.0	-41.38	0.01	-20.70	-1.29	-	-
^{174}Hg	888.6	-40.21	0.02	-19.63	1.25	-	-
^{176}Hg	906.4	-34.99	0.02	-27.99	0.01	-	-
^{178}Hg	1032.4	-50.27	0.15	-37.56	0.13	-	-
^{180}Hg	1152.1	-54.39	0.36	-38.72	-0.19	1.38	2.41
^{182}Hg	1253.4	-58.46	0.39	-39.91	-0.17	1.11	2.24
^{184}Hg	1321.9	-58.12	0.41	-38.74	-0.11	1.14	1.94
^{186}Hg	1097.6	-56.95	0.36	-39.57	-0.16	1.07	2.11
^{188}Hg	839.4	-53.17	0.20	-38.61	-0.17	1.42	2.13
^{190}Hg	703.3	-57.59	0.13	-42.57	0.01	1.42*	2.13*
^{192}Hg	697.3	-42.57	0.25	-26.55	-0.60	1.42*	2.13*
^{194}Hg	615.8	-44.49	0.19	-21.34	-1.32	1.42*	2.13*
^{196}Hg	545.9	-39.79	0.16	-18.00	-0.85	1.81	2.72*
^{198}Hg	449.2	-54.08	0.31	-18.00	-0.85	1.83	-
^{200}Hg	499.3	-45.73	1.07	-18.00	-0.85	1.97	-

Decomposition in an intermediate basis

Second state



Quadrupole invariants

Isotope	State	$\langle q^2 \rangle$ (e^2b^2)		$\langle \cos 3\gamma \rangle$		γ (deg)	
		Theo.	Exp.	Theo.	Exp.	Theo.	Exp.
182	0_1^+	2.00	$2.02(^{+16}_{-15})$	0.23	$-1.33(^{+109}_{-87})$	25.5	-
	0_2^+	6.19	$9.7(^{+74}_{-51})$	0.65	$0.58(^{+98}_{-61})$	16.6	-
184	0_1^+	2.08	1.66(12)	-0.08	$-1.04(^{+42}_{-46})$	31.5	-
	0_2^+	5.40	$15.0(^{+97}_{-69})$	0.54	$-0.40(^{+31}_{-33})$	19.2	-
186	0_1^+	1.45	$1.56(^{+23}_{-25})$	-0.13	-	32.6	-
	0_2^+	6.07	-	0.60	-	17.7	-
188	0_1^+	1.80	1.72(26)	-0.20	$-0.64(42)$	33.8	-
	0_2^+	6.06	-	0.59	-	18.0	-
196	0_1^+	1.06	-	-0.14	-	32.7	-
	0_2^+	0.68	-	-0.41	-	38.2	-
198	0_1^+	0.89	-	-0.29	-	35.6	-
	0_2^+	0.52	-	-0.57	-	41.7	-
200	0_1^+	0.80	-	-0.79	-	47.3	-
	0_2^+	0.58	-	-0.87	-	50.3	-

Quadrupole invariants: step by step

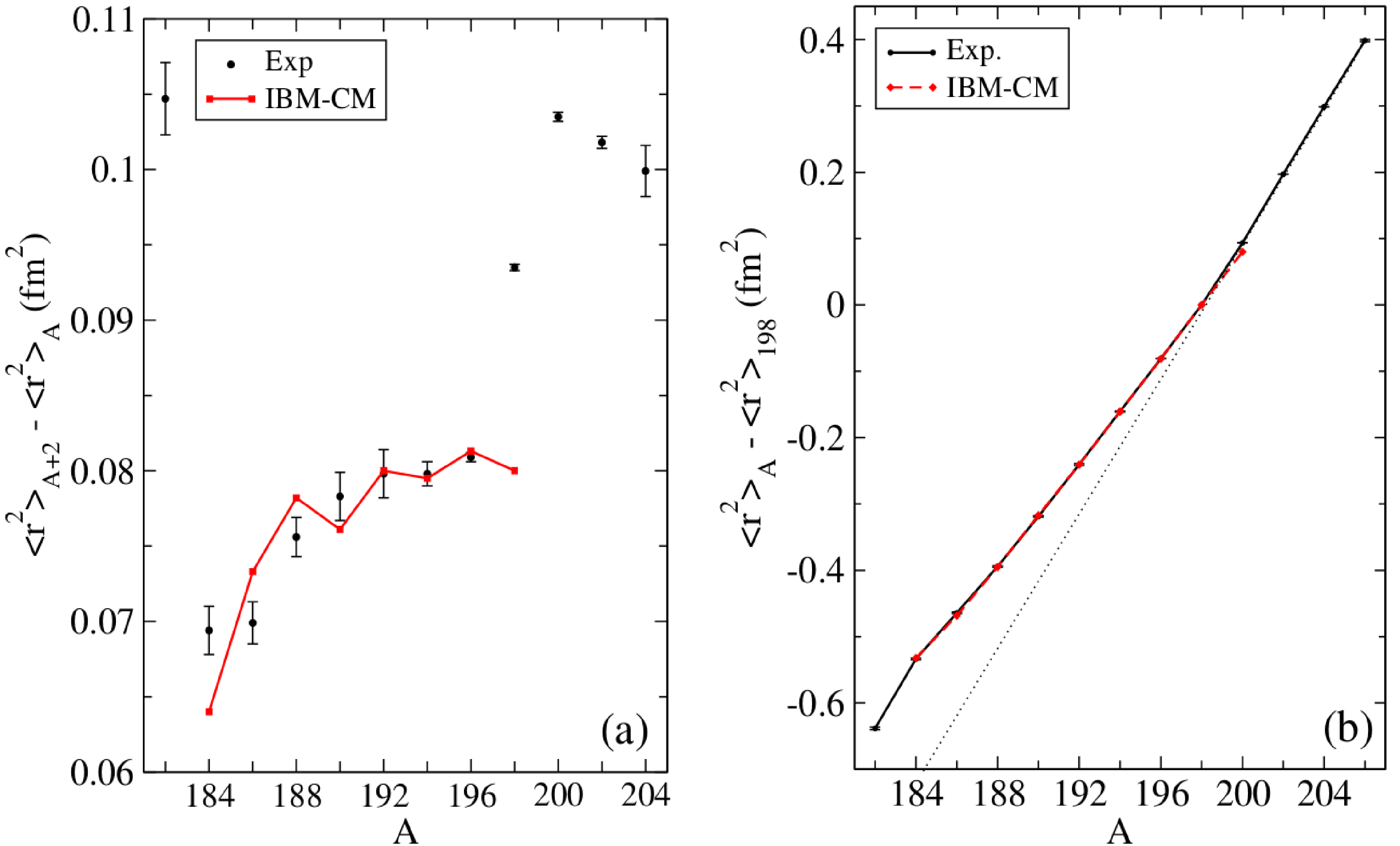
	$\langle q^2\rangle$ (e^{2}b^2)					
i	1	2	3	4	5	Exact
0_1^+	1.93	2.03	2.03	2.04	2.06	2.08
0_2^+	2.25	4.71	5.33	5.36	5.39	5.40

	$\langle \cos 3\gamma\rangle$					
i	1	2	3	4	5	Exact
0_1^+	0.41	-0.13	-0.15	-0.15	-0.08	-0.08
0_2^+	0.38	1.03	0.51	0.52	0.54	0.54

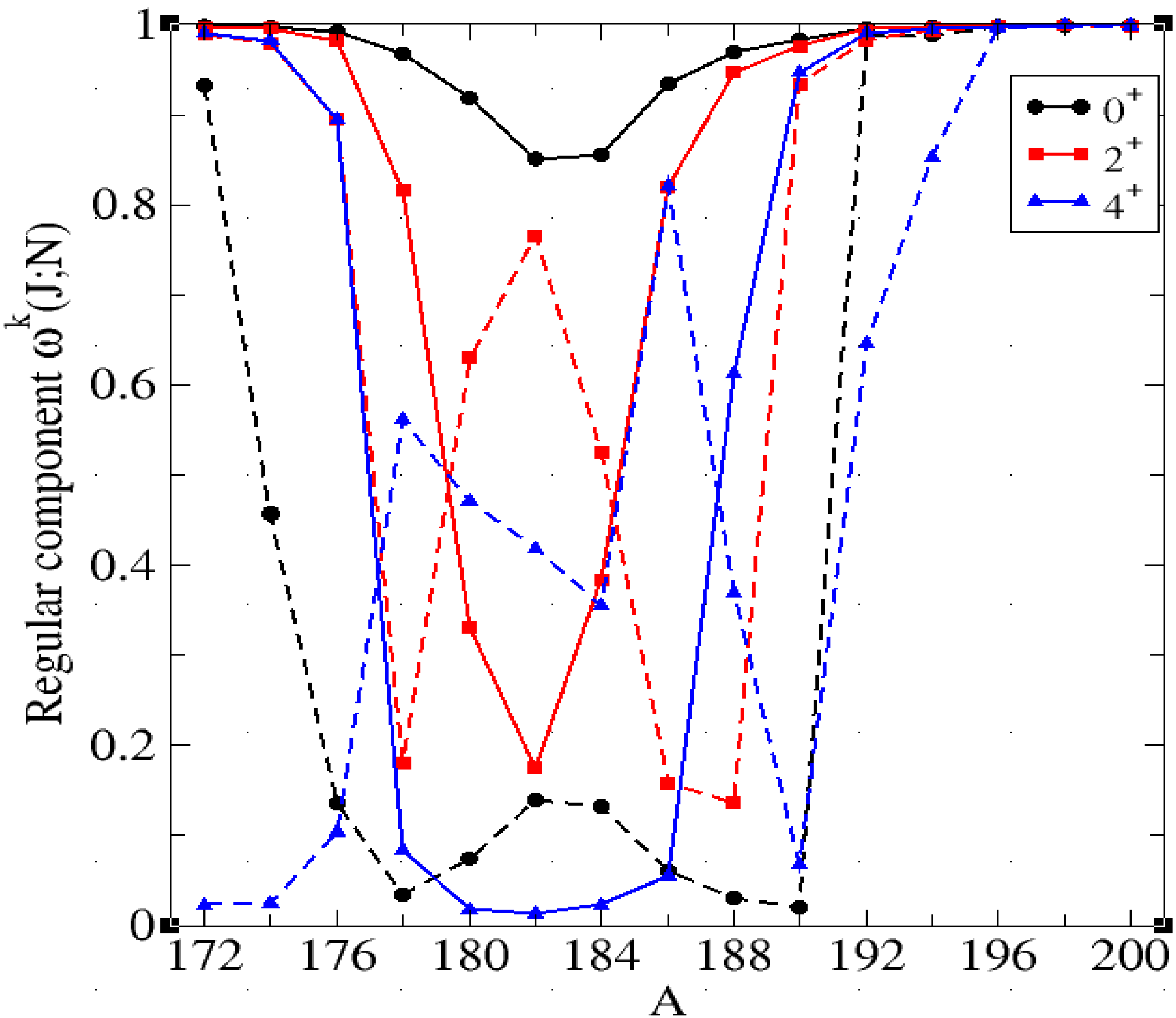
<q^2> and <cos 3γ> for 0^+_1 and 0^+_2 as a function of the number of 2^+ states included in the sum: ^{184}Hg.

More new experimental matrix elements are welcome!!

Isotopic shift



Decomposition of the yrast state wave function



Decomposition of the yrast state wave function resulting from the mixing