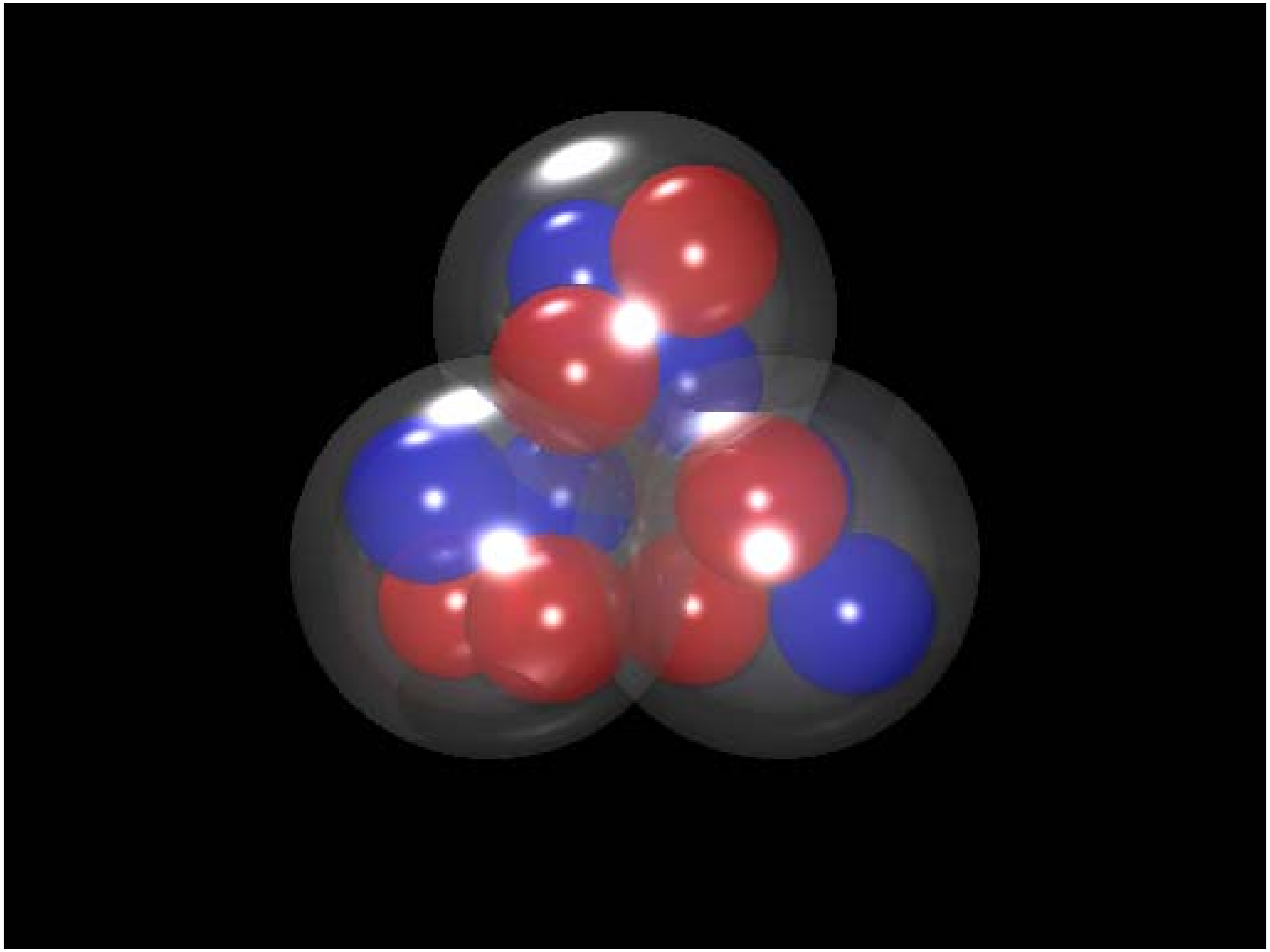

Structure of Rotational Bands in Alpha-Cluster Nuclei

- Introduction
- Structure of ^{12}C: Hoyle band
- Alpha-cluster model (Wheeler, Brink, Robson, …)
- Algebraic Cluster Model
- Summary and conclusions

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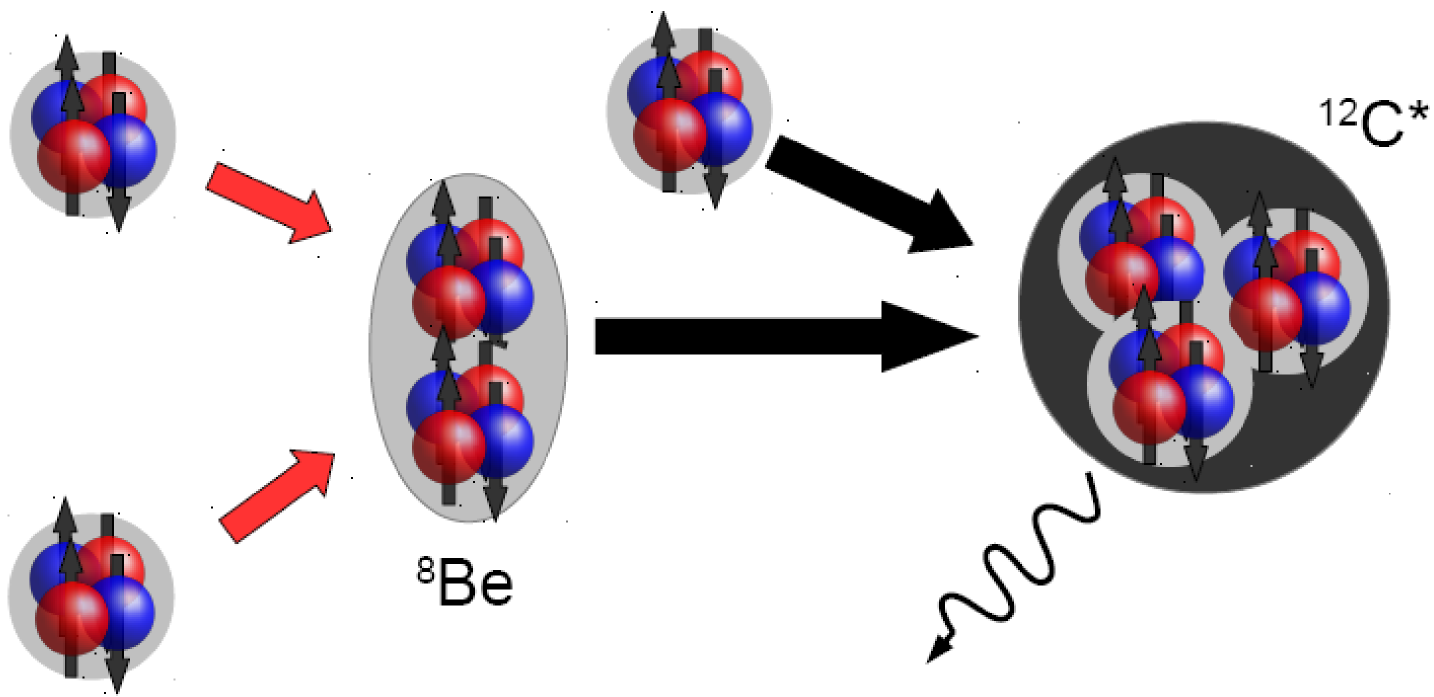
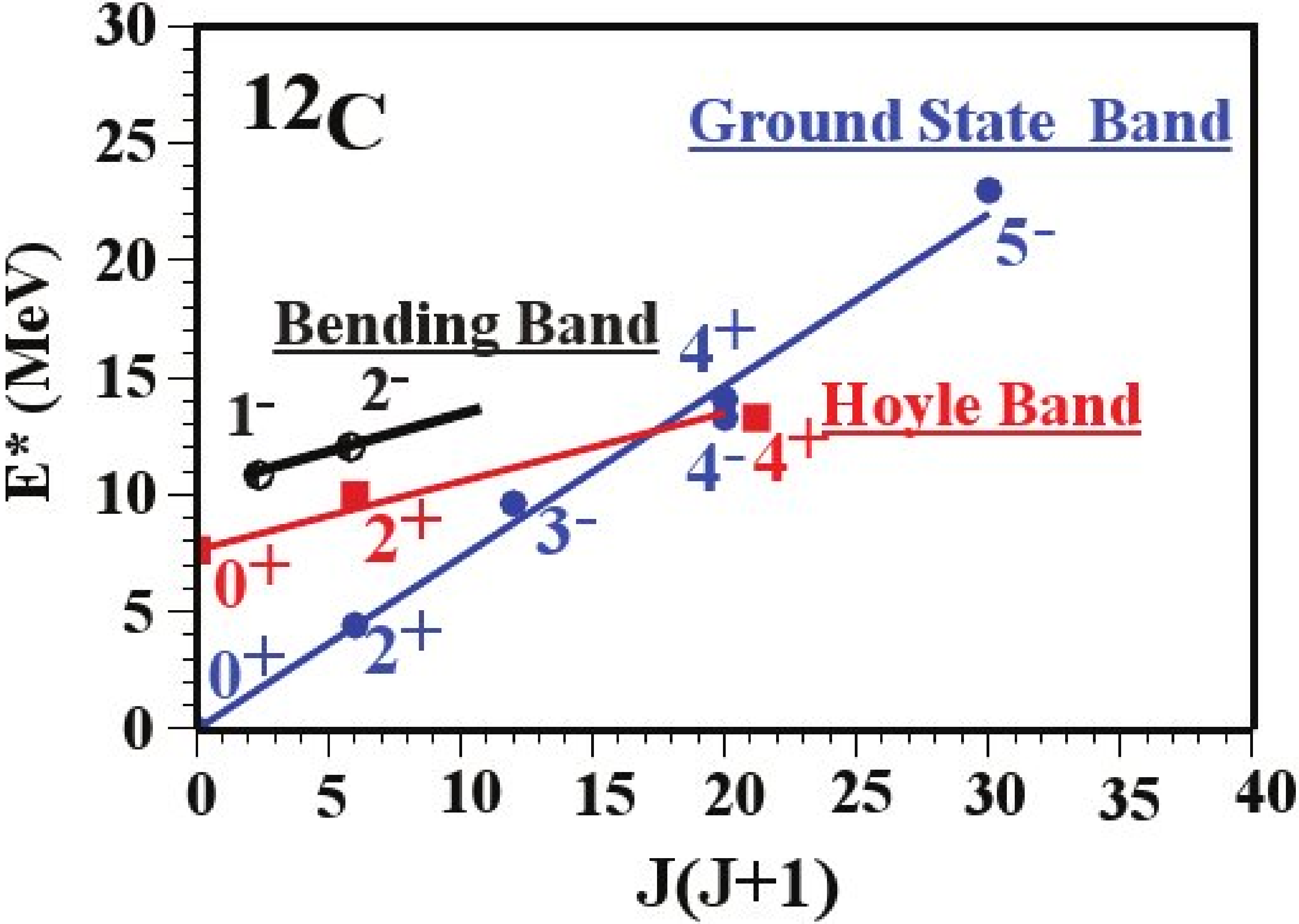
Triple-Alpha Process

In 1953, Fred Hoyle predicted the existence of a state near 7.68 MeV [a].



[a] F. Hoyle, D. N. F. Dunbar, W. A. Wenzel, and W. Whaling, Phys. Rev. 92, 1095 (1953).

Tzany Kokalova, INPC 2013, Florence



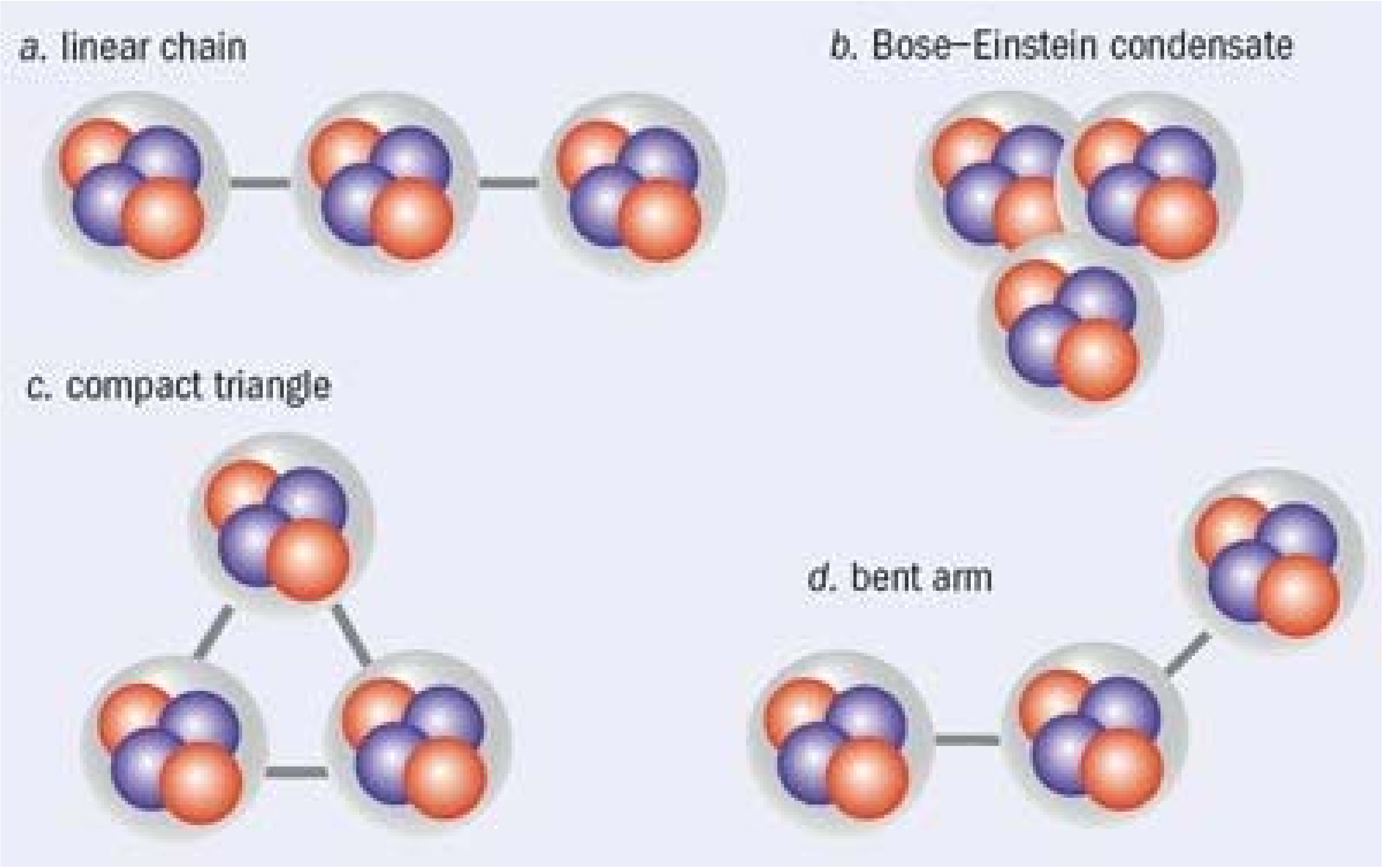
^{12}C
Ground State Band
Bending Band
Hoyle Band
1^-
2^-
0^+
2^+
3^-
4^+
4^-
5^-
E* (MeV)
J(J+1)
0
5
10
15
20
25
30
35
40

Recent Work

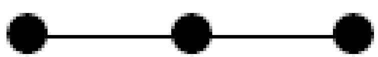
- Alpha-cluster model (Wheeler 1937, Brink 1966, Robson, 1978)
- AMD (Kanada-Enyo, PTP, 2007)
- FMD model (Chernykh et al, PRL, 2007)
- BEC-like cluster model (Funaki et al, PRC, 2009)
- Ab initio no-core shell model (Roth et al, PRL, 2011)
- Lattice EFT (Epelbaum et al, PRL, 2011, 2012)
- No-core symplectic model (Dreyfuss et al, PLB, 2013)
- Algebraic Cluster Model (2002, 2014)
- and many others

- Review: Freer & Fynbo, PPNP 78, 1 (2014)

Geometric Configurations

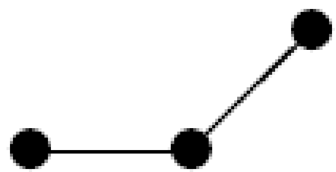


Alpha-Cluster Configurations



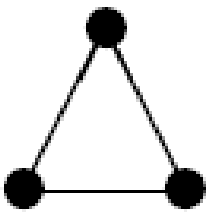
$\mathcal{D}_{\infty h}$

Linear



$\mathcal{C}_{2v}$

Bent

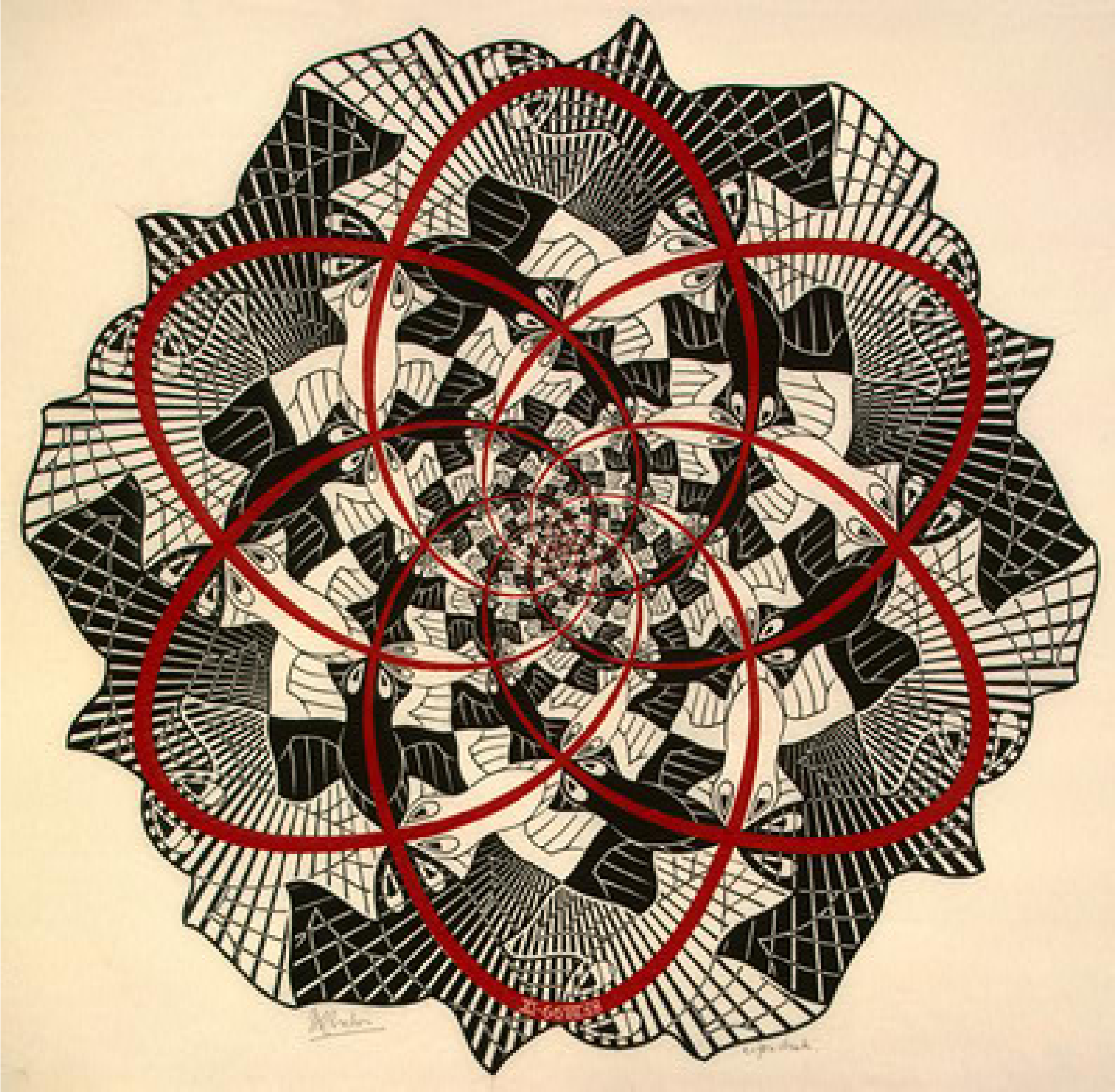


$\mathcal{D}_{3h}$

Triangular

Examine rotational structure!

Morinaga, PR 101, 254 (1956)
Epelbaum et al, PRL 109, 252501 (2012)
Wheeler (1937), Robson (1978)
Bijker & Iachello, AP 298, 334 (2002)



Algebraic Cluster Model (ACM)

- For k dof introduce a SGA of U(k+1)
- 2-body systems: U(4) model
- 3-body systems: U(7) model
- 4-body systems: U(10) model
- Applications: hadrons, molecules, alpha-cluster nuclei

ACM for Three-Body Systems

Two relative Jacobi vectors

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \ , \qquad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

Building blocks $\quad b_\rho^\dagger \ , \ b_\lambda^\dagger \ , \ s^\dagger$

Total number of bosons $\quad N = n_s + n_\rho + n_\lambda$

Identical Clusters

Explicit construction of harmonic oscillator states with good permutation symmetry

Kramer & Moshinsky, Nucl. Phys. 82, 241 (1966)

In ACM, wave functions are generated numerically, the permutation symmetry S_3 is determined by the interchange P(12) and the cyclic permutation P(123)

Wave Functions

- Labeled by $\left|[N],\alpha,L_t^P\right\rangle$
- Total number of bosons: N
- Angular momentum L and parity P
- Permutation symmetry: t=S(ymmetric) for alpha-cluster nuclei

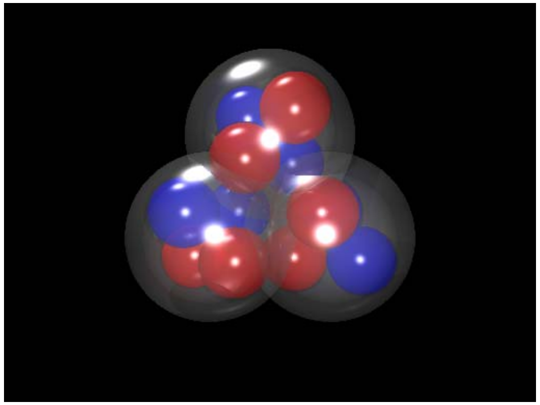
Bijker & Iachello, PRC 61, 067305 (2000)
Bijker & Iachello, AP 298, 334 (2002)

Three-body Clusters: Oblate Top

$$
\begin{aligned}
H &= \xi_1 \,(R^2\, s^\dagger s^\dagger - b_\rho^\dagger \cdot b_\rho^\dagger - b_\lambda^\dagger \cdot b_\lambda^\dagger)\,(\mathrm{h.c.}) \\
&\quad + \xi_2 \left[(b_\rho^\dagger \cdot b_\rho^\dagger - b_\lambda^\dagger \cdot b_\lambda^\dagger)\,(\mathrm{h.c.}) + 4 (b_\rho^\dagger \cdot b_\lambda^\dagger)(\mathrm{h.c.}) \right] \\
&\quad + \kappa_1\, \vec{L} \cdot \vec{L} + \kappa_2\, (b_\rho^\dagger \cdot \tilde{b}_\lambda - b_\lambda^\dagger \cdot \tilde{b}_\rho)\,(\mathrm{h.c.})
\end{aligned}
$$

$R^2 = 0$	:	anharmonic oscillator
$R^2 = 1,\ \xi_1 > 0,\ \xi_2 = 0$	:	deformed oscillator
$R^2 \neq 0,\ \xi_1,\ \xi_2 > 0$	:	oblate top

Equilibrium shape:
equilateral triangle



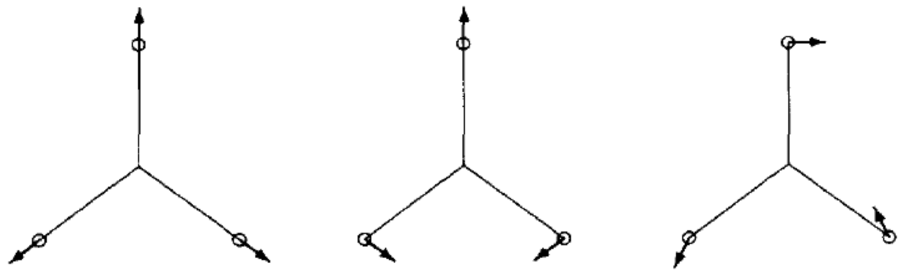
Bijker & Iachello, Ann. Phys. 298, 334 (2002)

Rotations and Vibrations

- Excitations of an oblate top

$$E \approx \omega_1(\nu_1 + \frac{1}{2}) + \omega_2(\nu_2 + 1) + \kappa_1 L(L+1) + \kappa_2(K \mp 2\ell_2)^2$$

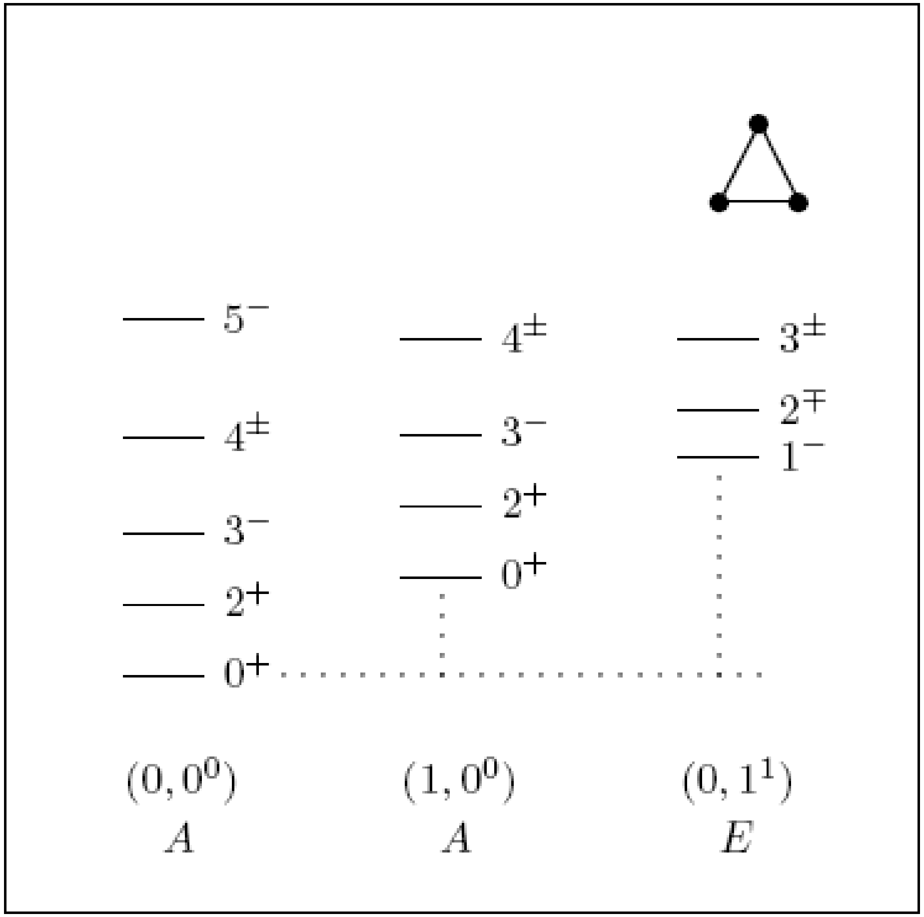
- Frequencies $\omega_1 = 4NR^2\xi_1 \ , \quad \omega_2 = \frac{4NR^2}{1+R^2}\xi_2$



$\nu_1(A_1)$ $\nu_2^{l_2}(E)$

Breathing vibration Bending vibration

Energy Spectrum



Ground state and Hoyle band (breathing vibration)

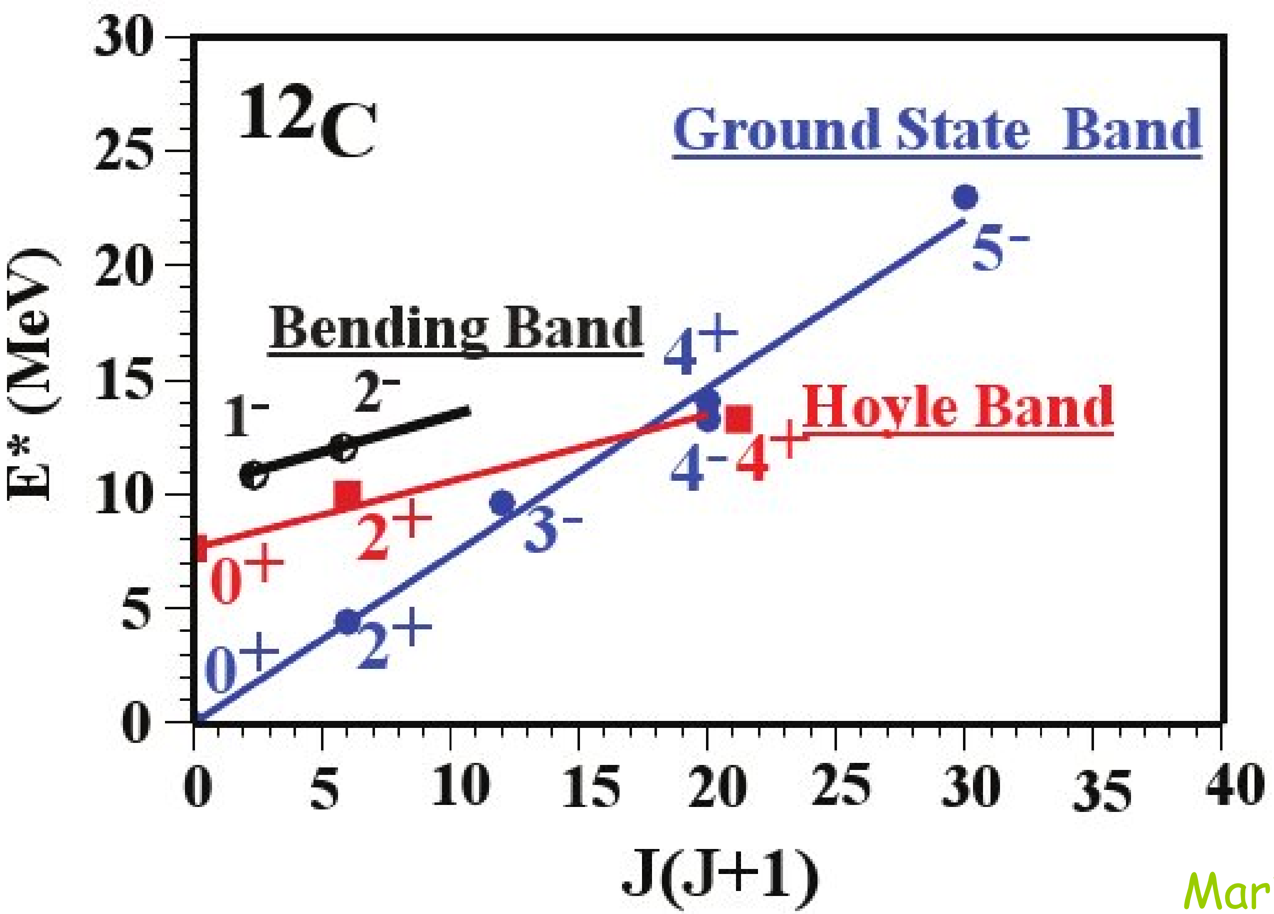
$$L^P = 0^+, 2^+, 3^-, 4^\pm, 5^-, \ldots$$

Bending vibration

$$L^P = 1^-, 2^\pm, 3^\pm, \ldots$$

Fingerprint of triangular shape with D_{3h} symmetry

Rotational Bands



Marín-Lámbarri et al,
PRL 113, 012502 (2014)

Experimental Studies

gs	3^-	Kokalova et al, PRC 87, 057307 (2013)
gs	4^-	Freer et al, PRC 76, 034320 (2007)
		Kirsebom et al, PRC 81, 064313 (2010)
gs	5^-	Marín-Lámbarri et al, PRL 113, 012502 (2014)
Hoyle	2^+	Itoh et al, PRC 84, 054308 (2011)
		Freer et al, PRC 86, 034320 (2012)
		Zimmerman et al, PRL 110, 152502 (2013)
Hoyle	4^+	Freer et al, PRC 83, 034314 (2011)
Hoyle	$3^-, 4^-$	Some evidence for negative parity strengths between 11 and 14 MeV
		Freer et al, PRC 76, 034320 (2007)

Energy Formula

$$
\begin{aligned}
E = E_0 \; &+ \; \omega_1(\nu_1 + \frac{1}{2})\left(1 - \frac{\nu_1 + 1/2}{N}\right) \\
&+ \; \omega_2(\nu_2 + 1)\left(1 - \frac{\nu_2 + 1}{N + 1/2}\right) \\
&+ \; \kappa_1 \, L(L+1) + \kappa_2 \, (K \mp 2\ell_2)^2 \\
&+ \; \left[\lambda_1 \, (\nu_1 + \frac{1}{2}) + \lambda_2 \, (\nu_2 + 1)\right] L(L+1) \, .
\end{aligned}
$$

Vibrations

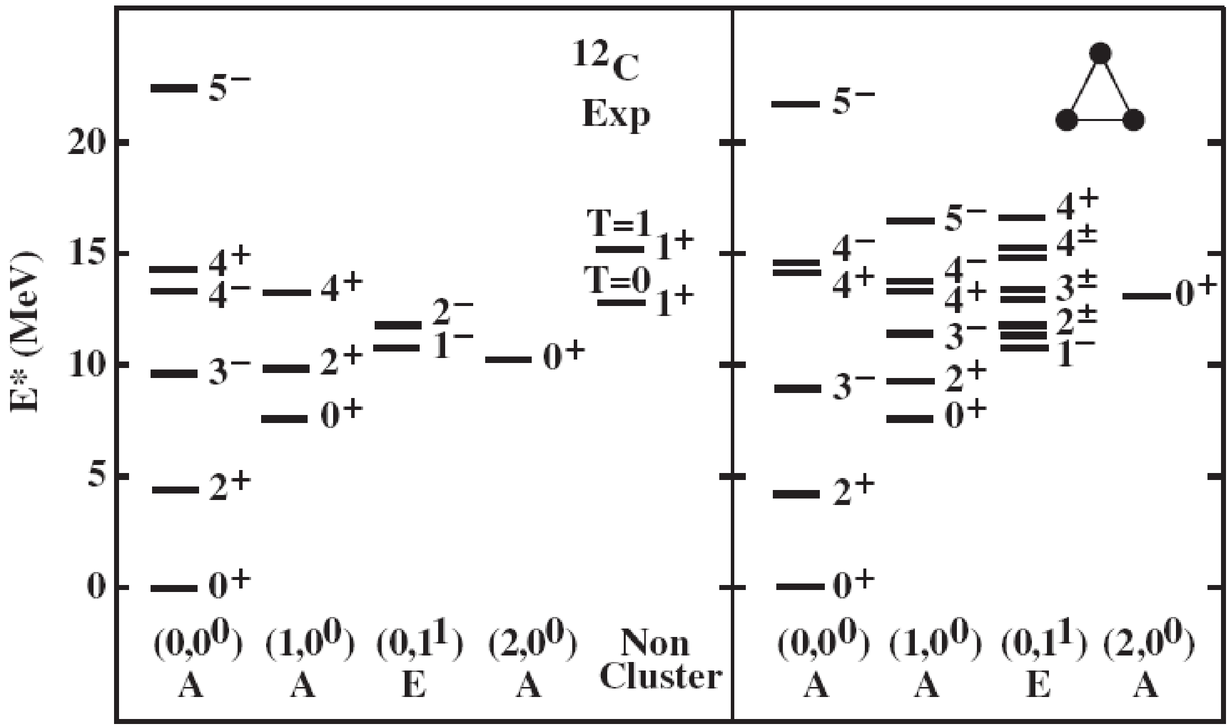
Rotations

Vibration-rotation couplings

Marín-Lámbarri et al,
PRL 113, 012502 (2014)

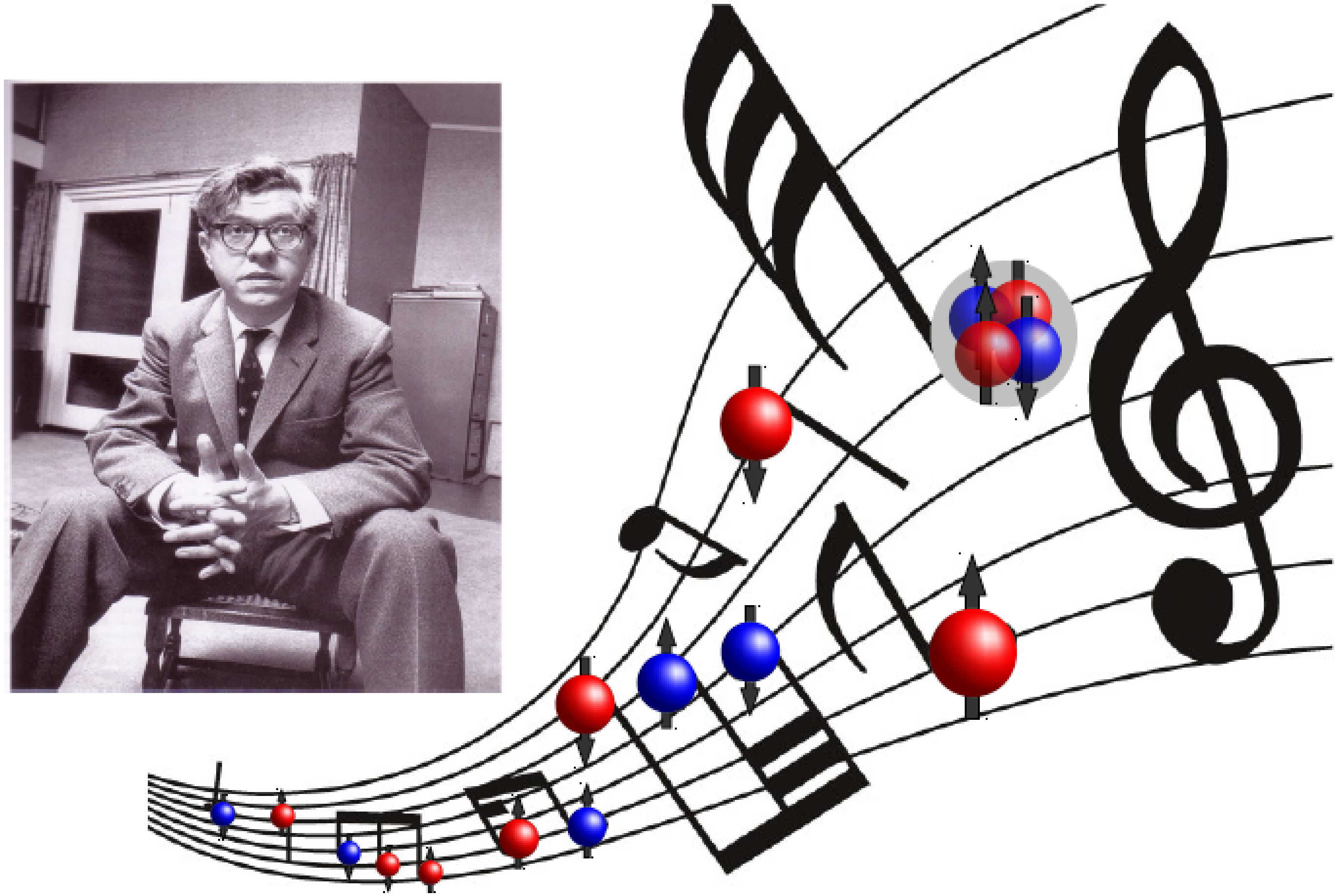
ACM for ^{12}C

rotational structure, the electromagnetic transition



Marín-Lámbarri, Bijker, Freer, Gai, Kokalova, Parker, Wheldon, PRL 113, 012502 (2014)

Who plays in the Hoyle band?



Tzany Kokalova, INPC 2013, Florence

Electric Transitions

Form factor

$$F(0^+ \to L^P) = a_L\, j_L(q\beta)$$

$$a_L^2 = \frac{2L+1}{3}\left[1+2P_L(-\frac{1}{2})\right]$$

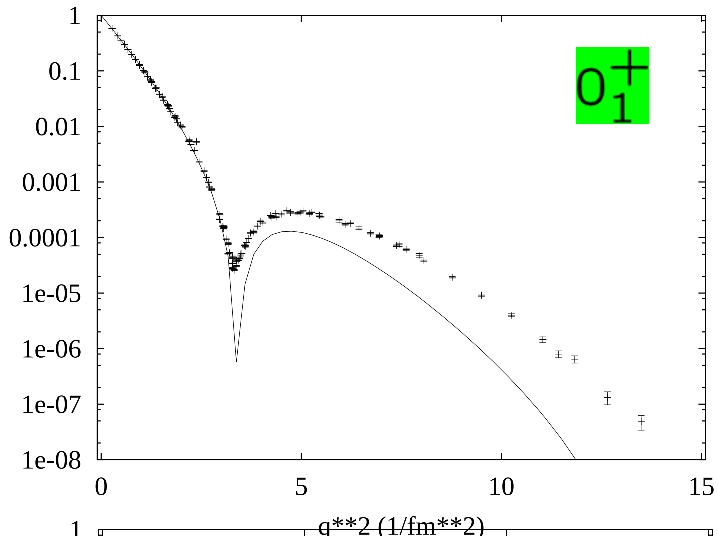
Long-wavelenght limit

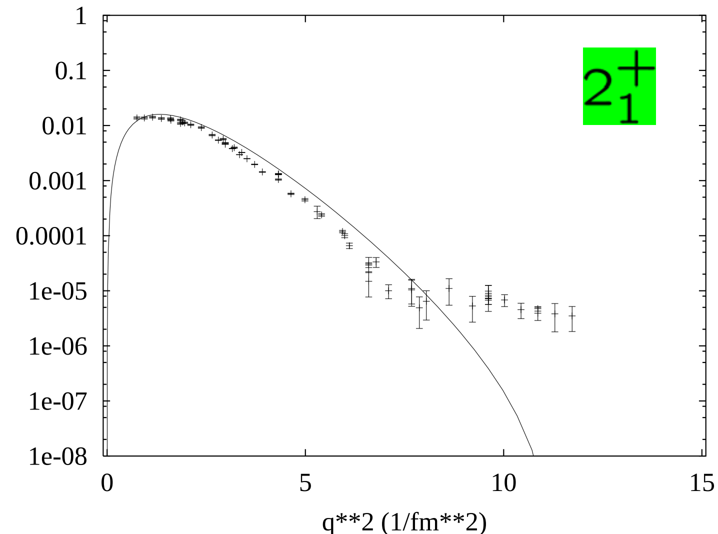
$$B(EL;0^+ \to L^P) = \left(\frac{Ze}{3}\right)^2 \frac{2L+1}{4\pi}\left[3+6P_L(-\frac{1}{2})\right]\beta^{2L}$$

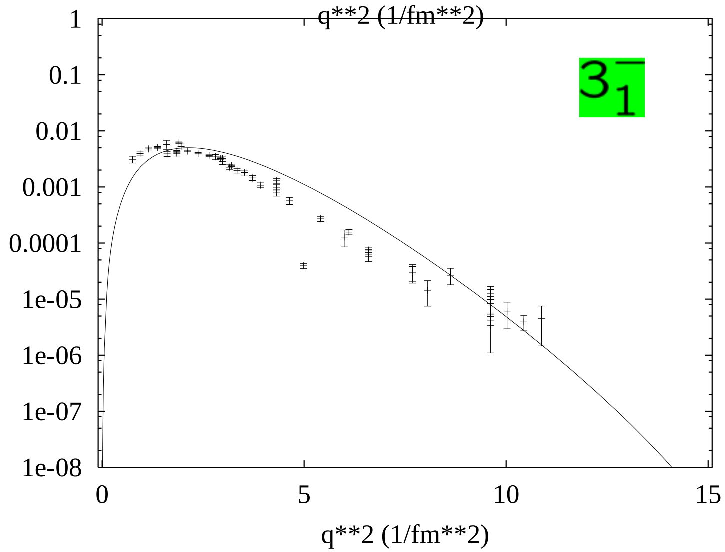
	Th.	Exp.	
$B(E2;2_1^+ \to 0_1^+)$	7.1	7.6 ± 0.4	$e^2\mathrm{fm}^4$
$B(E3;3_1^- \to 0_1^+)$	45	103 ± 17	$e^2\mathrm{fm}^6$
$B(E4;4_1^+ \to 0_1^+)$	48		$e^2\mathrm{fm}^8$
$B(E2;0_2^+ \to 2_1^+)$	0.10	13.1 ± 1.8	$e^2\mathrm{fm}^4$
$M(E0;0_2^+ \to 0_1^+)$	0.14	5.5 ± 0.2	fm^2
$\langle r^2\rangle^{1/2}$	2.468	2.468 ± 0.12	fm

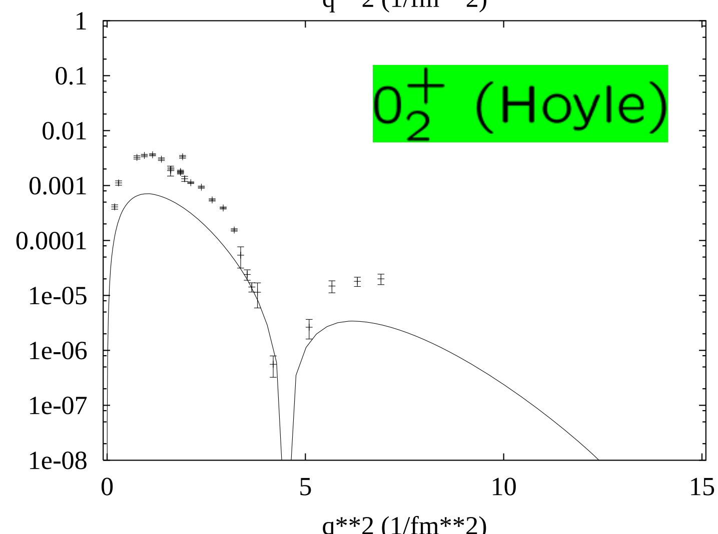
$$\begin{aligned}
B(EL;0_1^+ \to 1_1^-) &= 0 \\
B(EL;0_1^+ \to 2_1^+) &= (Ze)^2\,\frac{5}{4\pi}\,\frac{1}{4}\,\beta^4 \\
B(EL;0_1^+ \to 3_1^-) &= (Ze)^2\,\frac{7}{4\pi}\,\frac{5}{8}\,\beta^6 \\
B(EL;0_1^+ \to 4_1^+) &= (Ze)^2\,\frac{9}{4\pi}\,\frac{9}{64}\,\beta^8 \\
B(EL;0_1^+ \to 4_1^-) &= 0 \\
B(EL;0_1^+ \to 5_1^-) &= (Ze)^2\,\frac{11}{4\pi}\,\frac{35}{128}\,\beta^{10}
\end{aligned}$$

Form Factors









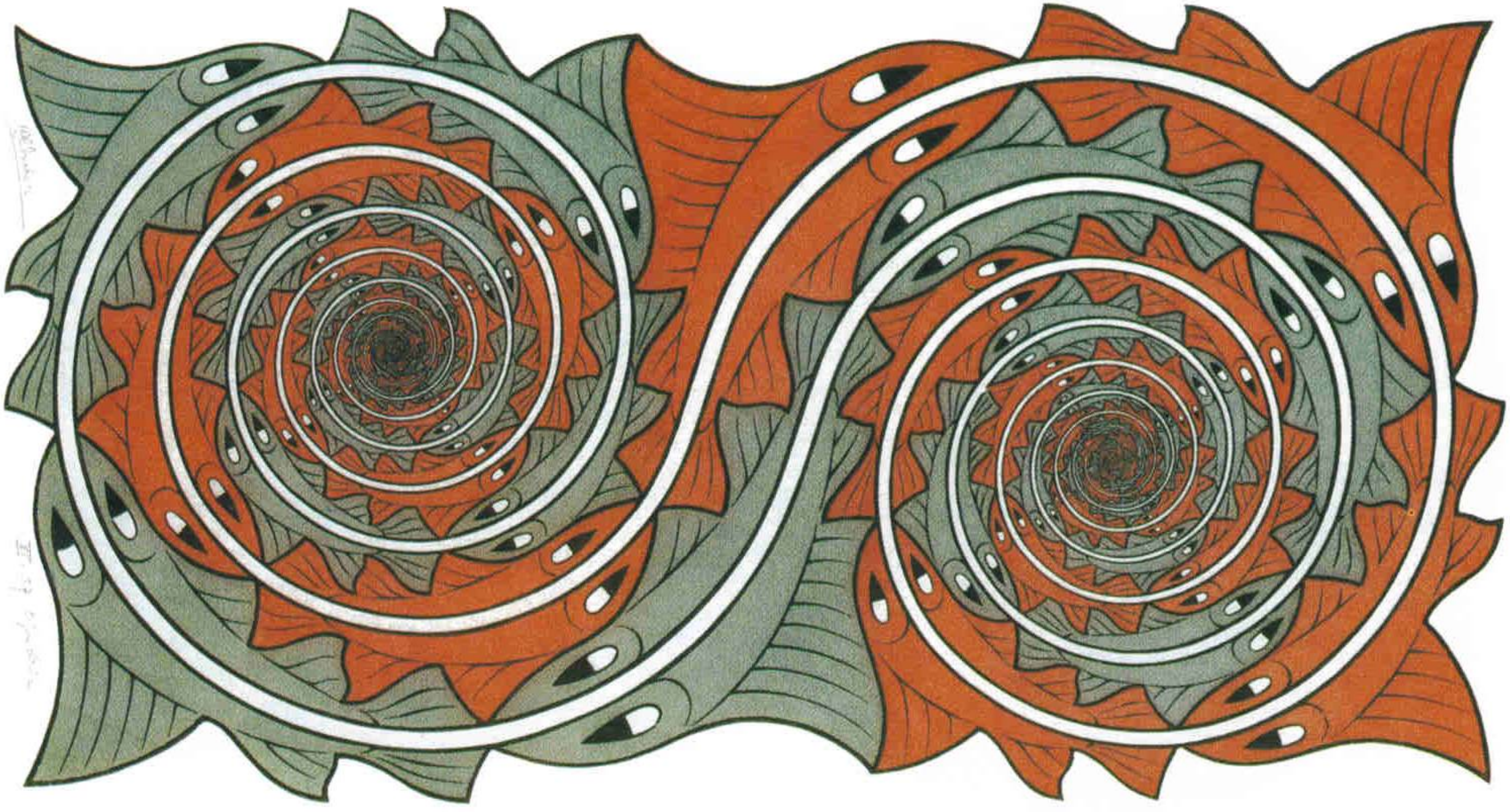
Estimate of Hoyle Radius

- Moments of inertia and radii of ground state (i=gs) and Hoyle band (i=H)

$$\frac{1}{2\mathcal{I}_i} = \frac{1}{Am\beta_i^2(1+2/\alpha\beta_i^2)}$$
$$\left\langle r^2 \right\rangle_i^{1/2} = \sqrt{\beta_i^2 + 3/2\alpha}$$

- Radii

$$\left\langle r^2 \right\rangle_{\mathrm{gs}}^{1/2} = 2.47 \text{ fm}$$
$$\left\langle r^2 \right\rangle_{\mathrm{H}}^{1/2} = 3.45 \text{ fm}$$



Summary and Conclusions

- Algebraic Cluster Model
- Oblate top with triangular symmetry for ^{12}C
- Rotational bands fingerprints of geometric configuration of alpha clusters
- Ground state band: triangular
- Hoyle band: bent-arm, triangular?
- Search for negative parity states 3^-, 4^- (Gai et al)
- Spherical top with tetrahedral symmetry for ^{16}O (Bijker & Iachello, PRL 112, 152501, 2014)

Signals of Alpha Clusters

- Structure of Rotational Bands (Marín-Lámbarri et al, PRL 113, 012502, 2014)
- Giant Dipole Resonances (He et al, PRL 113, 032506, 2014)
- Relativistic Nuclear Collisions (Broniowski et al, PRL 112, 112501, 2014)