Recent development of projected shell model based on many-body techniques

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Nuclear structure models

- Shell-model diagonalization method
  - Based on quantum mechanical principles
  - Growing computer power helps extending applications
  - A single configuration contains no physics
  - Huge basis dimension required, severe limit in applications
- Mean-field approximations
  - Applicable to any size of systems
  - Fruitful physics around minima of energy surfaces
  - No configuration mixing, results depending on quality of mean-field
  - States with broken symmetry, cannot study transitions
- Algebraic models
  - Based on symmetries, simple and elegant
  - Serve as important guidance for complicated calculations

Deformed basis vs spherical basis

- Most nuclei are deformed. To describe a deformed nucleus, a spherical shell model needs a huge configuration space, thus has no obvious advantage.
- J.P. Elliott was the first to take the advantage of a deformed many-body basis and developed the SU(3) shell model.
- For heavy nuclei, the original Elliott SU(3) scheme is no longer valid;
  - one may use generalized SU(3) schemes if symmetries exist.
  - or more generally, one can start from a deformed basis and apply angular-momentum-projection technique.

A method related to mean-field, shell model, algebraic models

- Angular-momentum projection method based on mean-field solutions
  - Start from intrinsic bases (e.g. solutions of deformed mean-field) and select most relevant configurations
  - Use angular momentum projection technique to transform them to laboratory basis (many-body technique)
  - Diagonalize Hamiltonian in the projected basis (configuration mixing, a shell-model concept)
  - Numerical results can be discussed using algebraic models

The Projected Shell Model:

- K. Hara, Y. Sun, *Int. J. Mod. Phys.* E 4 (1995) 637

The procedure

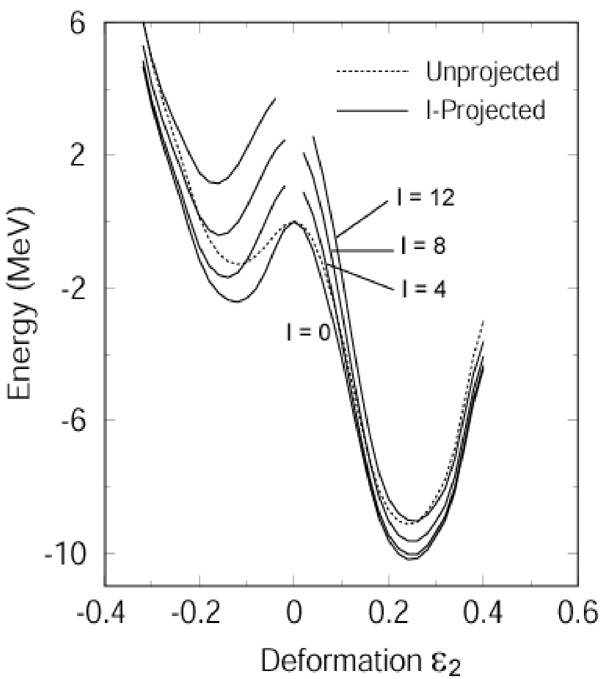
- Take a set of quasiparticle states at a fixed deformation (e.g. solutions of HF, HFB or HF + BCS)
- Select configurations (qp vacuum + multi-qp states near the Fermi level)
- Project them onto good angular momentum (if necessary, also parity, particle number) to form a basis in laboratory frame
- Diagonalize a two-body Hamiltonian in the projected basis

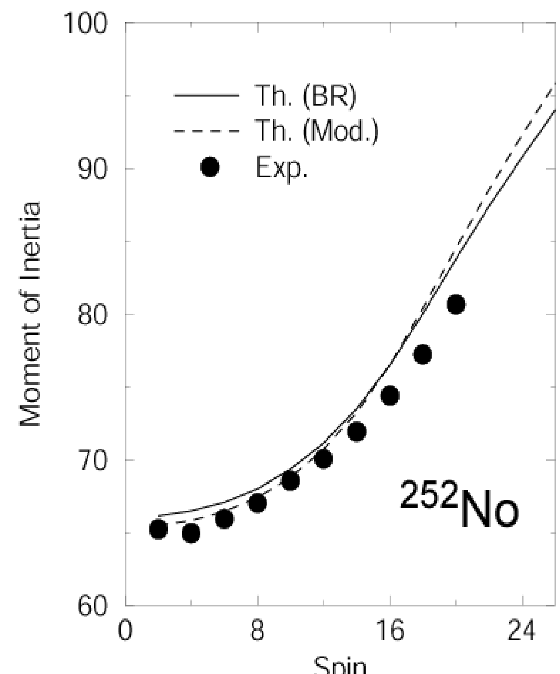
- This model has worked well for spectrum description for nuclei with stable deformation (and super-deformation or superheavy nuclei)

K. Hara, Y. Sun, *Int. J. Mod. Phys.* E 4 (1995) 637

Example of a good axially-deformed rotor

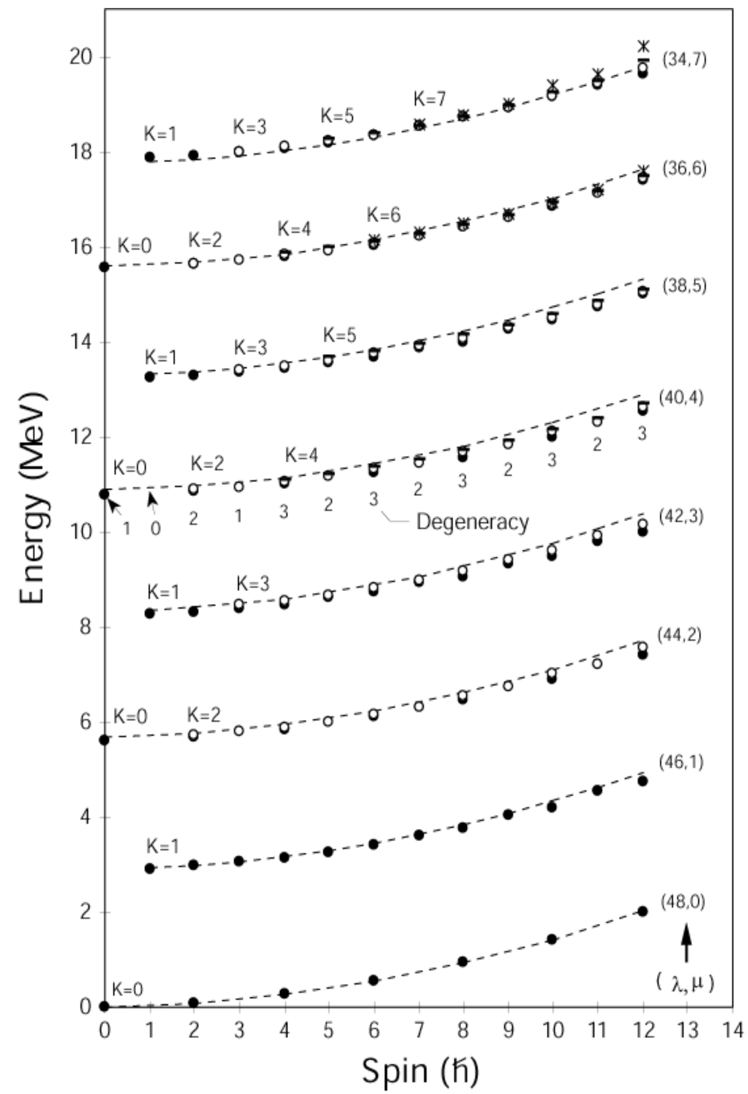
- Angular-momentum-projected energy calculation shows a deep prolate minimum
  - A very good rotor with axially-deformed shape
  - Quasi-particle excitations based on the same deformed potential





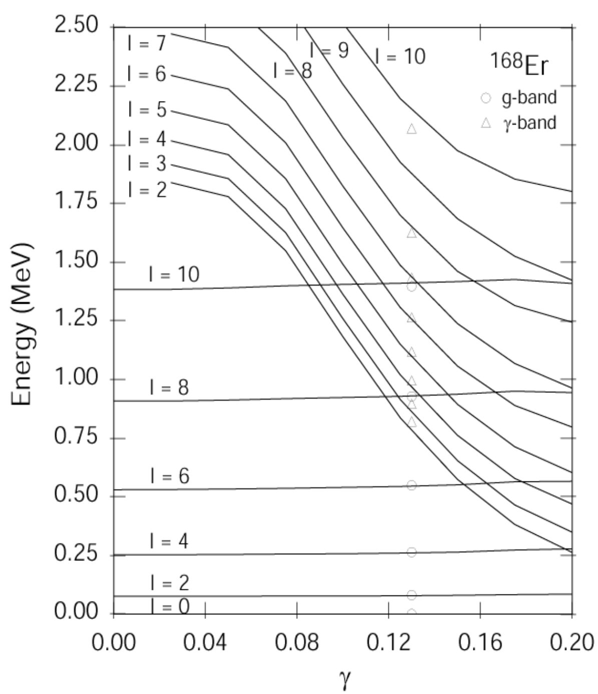
Emergence of SU(3) symmetry

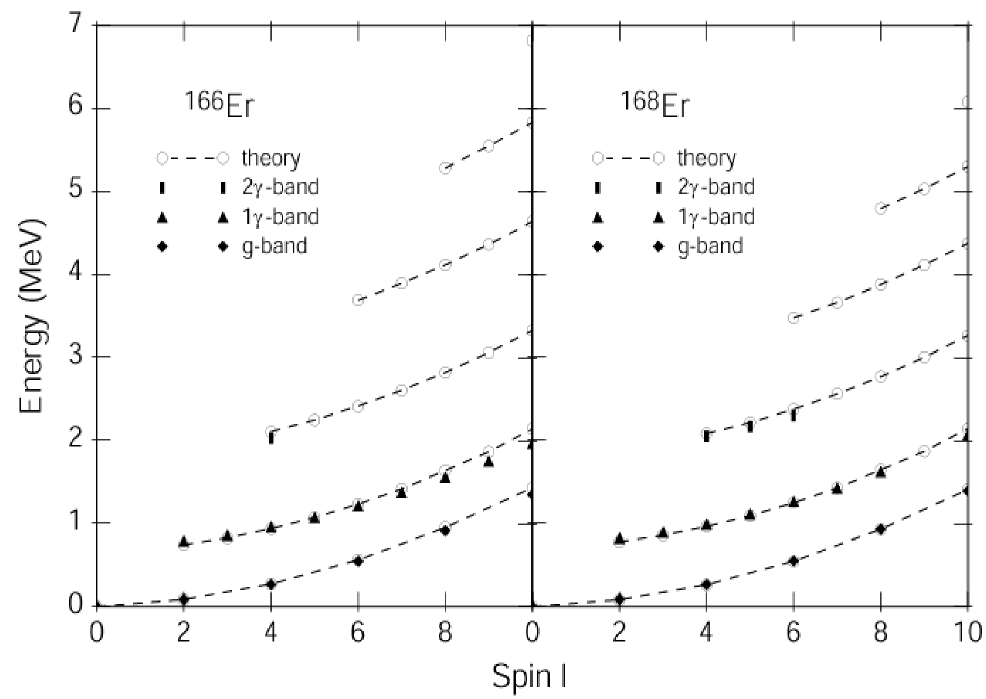
- Nearly perfect SU(3) symmetry emerges from a.-m.-projection
  - Project on separate BCS vacuum of $|\phi_\nu\rangle$ and $|\phi_\pi\rangle$, then couple the projected states $|I_\sigma\rangle = N^I \hat{P}^I |\phi_\sigma\rangle$ to form the basis $|(I_\nu \otimes I_\pi) I\rangle$
  - Diagonalize the Hamiltonian in the coupled basis
  - Multi-phonon scissors mode is predicted
  - Y. Sun *et al*., *PRL* 80 (1998) 672; *NPA* 703 (2002) 130



γ-vibrational states

- γ-vibration states cannot be obtained when axial symmetry in the basis states is assumed
- Need 3-dimensional angular-momentum projection performed on a triaxially deformed basis





Y. Sun et al. *Phys. Rev.* C61 (2000) 064323

γ-vibrations

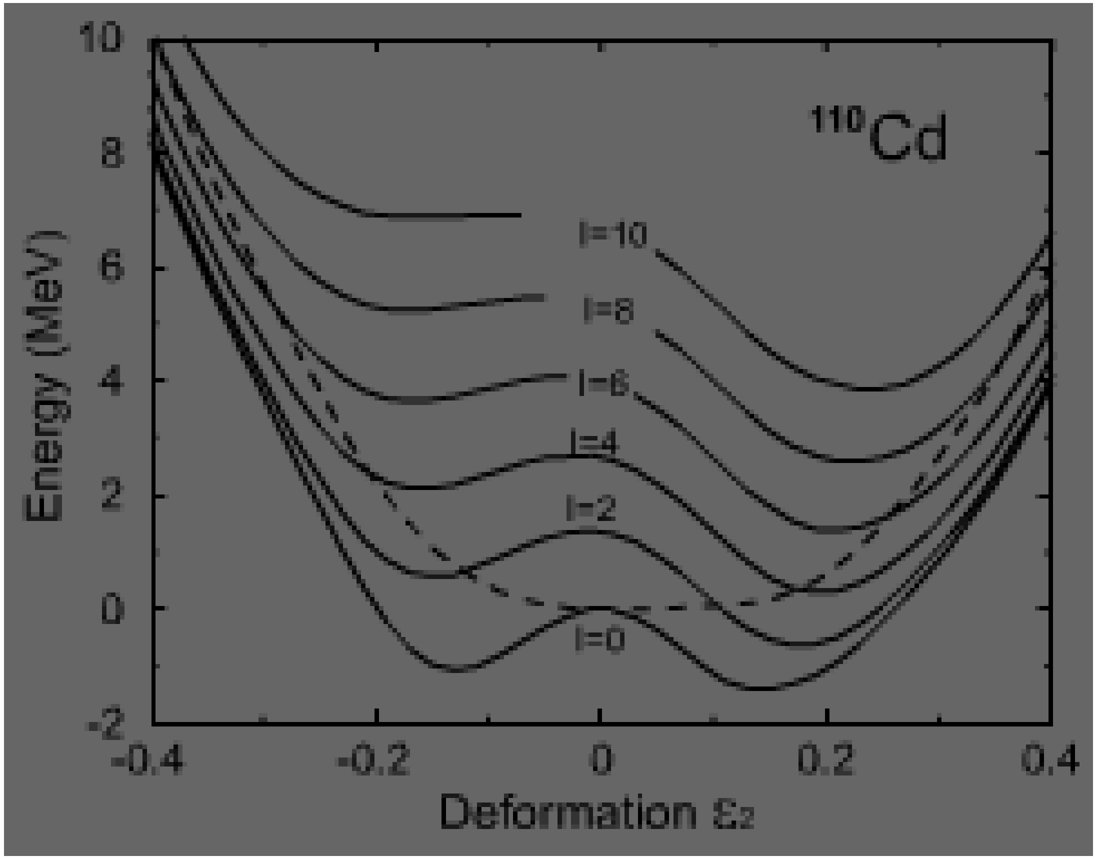
- Calculated transition rates confirm the multi-phonon structure

Table 1. Comparison of all known experimental in-band and inter-band $B(E2)$ values (associated errors in parenthesis) and calculated ones in W.u. for ^{168}Er. $K = 4^+$ lifetimes from ref. [2], $K = 0^+$, and $K = 2^+$ lifetimes and $B(E2)$ values from ref. [8] and all the references therein.

$(I, K)_i \rightarrow (I, K)_f$	$B(E2)_{exp}$ (W.u.)	$B(E2)_{TPSM}$ (W.u.)
$(2,0)_i \rightarrow (0,0)_f$	207 (6)	228.6
$(4,0)_i \rightarrow (2,0)_f$	318 (12)	326.9
$(6,0)_i \rightarrow (4,0)_f$	$440^{(a)}$ (30)	361.2
$(8,0)_i \rightarrow (6,0)_f$	350 (20)	380.0
$(10,0)_i \rightarrow (8,0)_f$	302 (21)	393.0
$(2,2)_i \rightarrow (0,0)_f$	4.80 (17)	2.7
$(2,2)_i \rightarrow (2,0)_f$	8.5 (4)	4.5
$(2,2)_i \rightarrow (4,0)_f$	0.62 (4)	0.3
$(3,2)_i \rightarrow (2,0)_f$	> 0.19	4.9
$(3,2)_i \rightarrow (4,0)_f$	> 0.13	2.7
$(4,2)_i \rightarrow (2,0)_f$	1.7 (4)	1.3
$(4,2)_i \rightarrow (4,0)_f$	8.7 (18)	5.5
$(4,2)_i \rightarrow (6,0)_f$	1.13 (25)	0.7
$(5,2)_i \rightarrow (4,0)_f$		3.9
$(5,2)_i \rightarrow (6,0)_f$		3.7
$(6,2)_i \rightarrow (4,0)_f$	0.78 (19)	0.8
$(6,2)_i \rightarrow (6,0)_f$	6.4 (16)	5.7
$(6,2)_i \rightarrow (8,0)_f$	2.4 (7)	1.1
$(7,2)_i \rightarrow (6,0)_f$		3.3
$(7,2)_i \rightarrow (8,0)_f$		4.4
$(8,2)_i \rightarrow (6,0)_f$	1.3 (6)	0.5
$(8,2)_i \rightarrow (8,0)_f$	1.8 (8)	5.7
$(8,2)_i \rightarrow (10,0)_f$	120 (50)	1.4
$(4,4)_i \rightarrow (2,2)_f$	3.4 (19)	11.9
$(4,4)_i \rightarrow (3,2)_f$	2.2 (13)	7.1
$(4,4)_i \rightarrow (4,2)_f$	$1.7^{(b)}$ (9)	2.7
$(4,4)_i \rightarrow (5,2)_f$	$0.7^{(b)}$ (3)	0.6
$(4,4)_i \rightarrow (6,2)_f$	2.0 (13)	0.1
$(5,4)_i \rightarrow (3,2)_f$	5 (5)	7.7
$(5,4)_i \rightarrow (4,2)_f$	4 (3)	8.6
$(5,4)_i \rightarrow (5,2)_f$	1.8 (15)	4.6
$(5,4)_i \rightarrow (6,2)_f$	0.8 (7)	1.3
$(5,4)_i \rightarrow (7,2)_f$	7 (6)	0.2

P. Boutachkov et al. *Eur. Phys. J.* A15 (2002) 455

Example of softness – no definite shapes

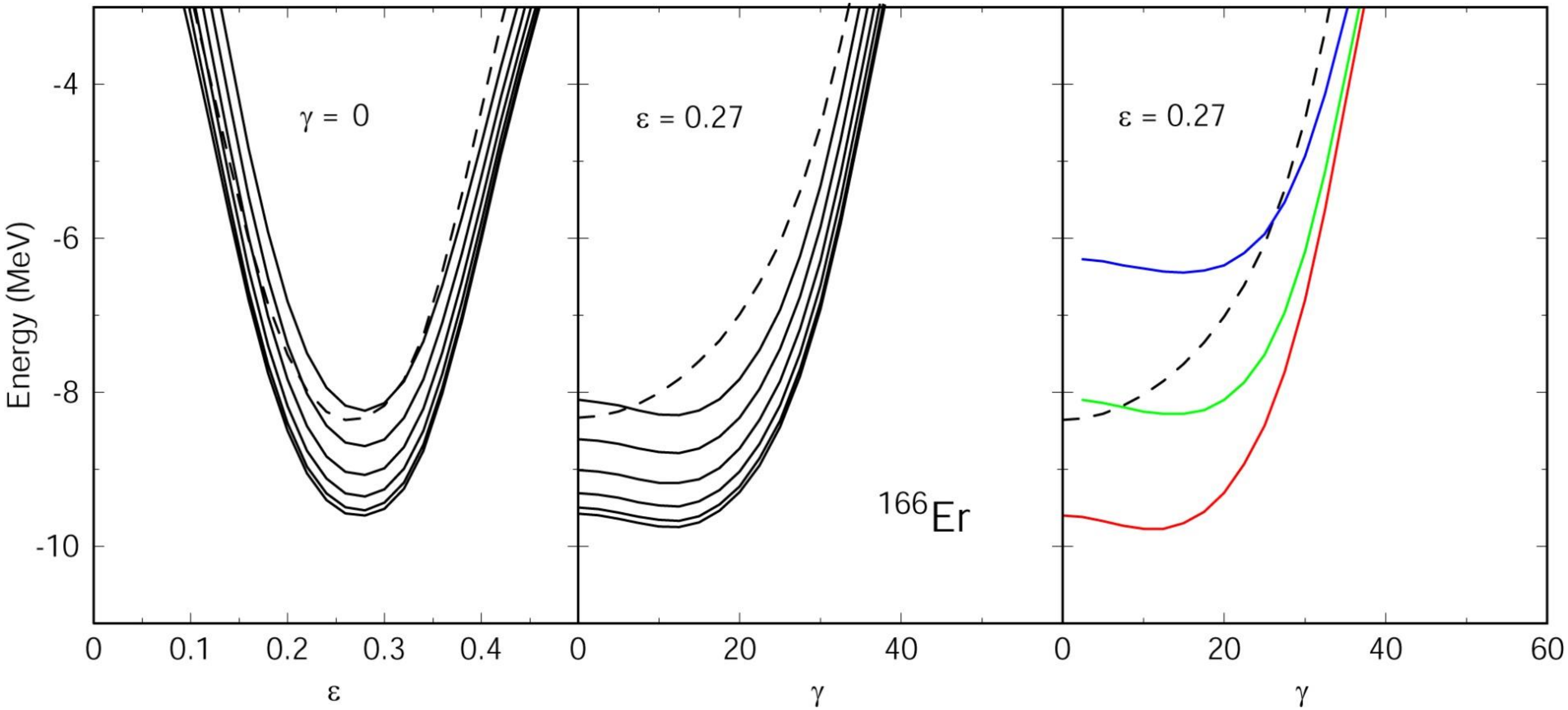


Mean-field calculation shows a spherical shape.

Projected calculation shows shallow minima separated by a low energy barrier.

Shapes may be developed with rotation.

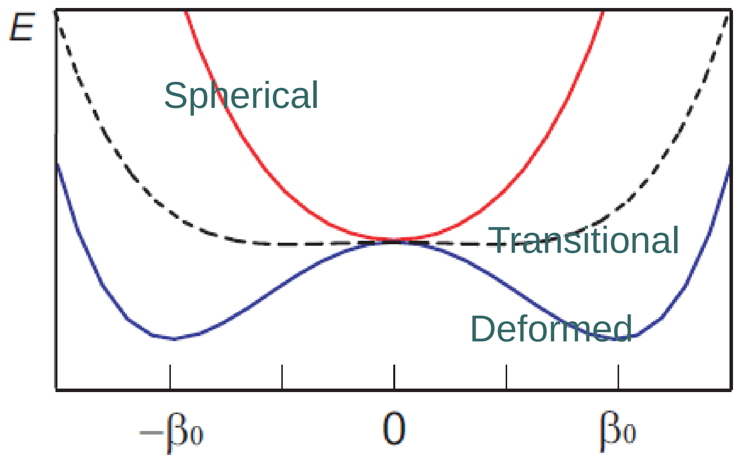
γ-softness in well-deformed nuclei



Angular-momentum-projected energy surfaces as functions of ε and γ

Description of a system with soft potential surfaces

- A spherical nucleus described by spherical shell model.
- A deformed nucleus described by deformed shell model.
- Transitional ones are *difficult*. A better wavefunction is a superposition of many states of deformation parameter β.



Schematic energy potential for spherical (red), transitional (dashed), and deformed (blue) nuclei.

$$\left|\Psi^I\right\rangle = \int f^I(\beta)\left|\Phi^I(\beta)\right\rangle d\beta$$

$$\left|\Phi^I(\beta)\right\rangle = \hat{P}^I\left|\phi(\beta)\right\rangle \qquad \{\beta\} = \{\beta_1, \beta_2, \beta_3, \ldots\}$$

Generate Coordinate Method (GCM)

- GCM starts with a general ansatz for a trail wave function

$$|\Psi\rangle = \int da\, f(a)\, |\Phi(a)\rangle$$

with $\{a\} = a_1, a_2, \ldots, a_i$ being generate coordinates

- $f(a)$ is a weight function, determined by solving the Hill-Wheeler Equation

$$\mathscr{H} f = E \mathscr{N} f$$

with the overlap functions

$$\begin{aligned} \mathscr{H}(a, a') &= \left\langle \Phi(a) \middle| \hat{H} \middle| \Phi(a') \right\rangle, \\ \mathscr{N}(a, a') &= \langle \Phi(a) | \Phi(a') \rangle \end{aligned}$$

Projected Generate Coordinate Method (PGCM)

- Choosing generate coordinate as ε_2, an improved wave function

$$\left|\Psi^{I,N}\right\rangle = \int d\varepsilon_2 f^{I,N}(\varepsilon_2)\left|\Phi^{I,N}(\varepsilon_2)\right\rangle$$

$$\left|\Phi^{I,N}(\varepsilon_2)\right\rangle = \hat{P}^I \hat{P}^N \left|\Phi_0(\varepsilon_2).\right\rangle$$

- Hamiltonian

$$\hat{H} = \hat{H}_0 - \frac{\chi}{2}\sum_\mu \hat{Q}_\mu^+ \hat{Q}_\mu - G_M \hat{P}^+ \hat{P} - G_Q \sum_\mu \hat{P}_\mu^+ \hat{P}_\mu$$

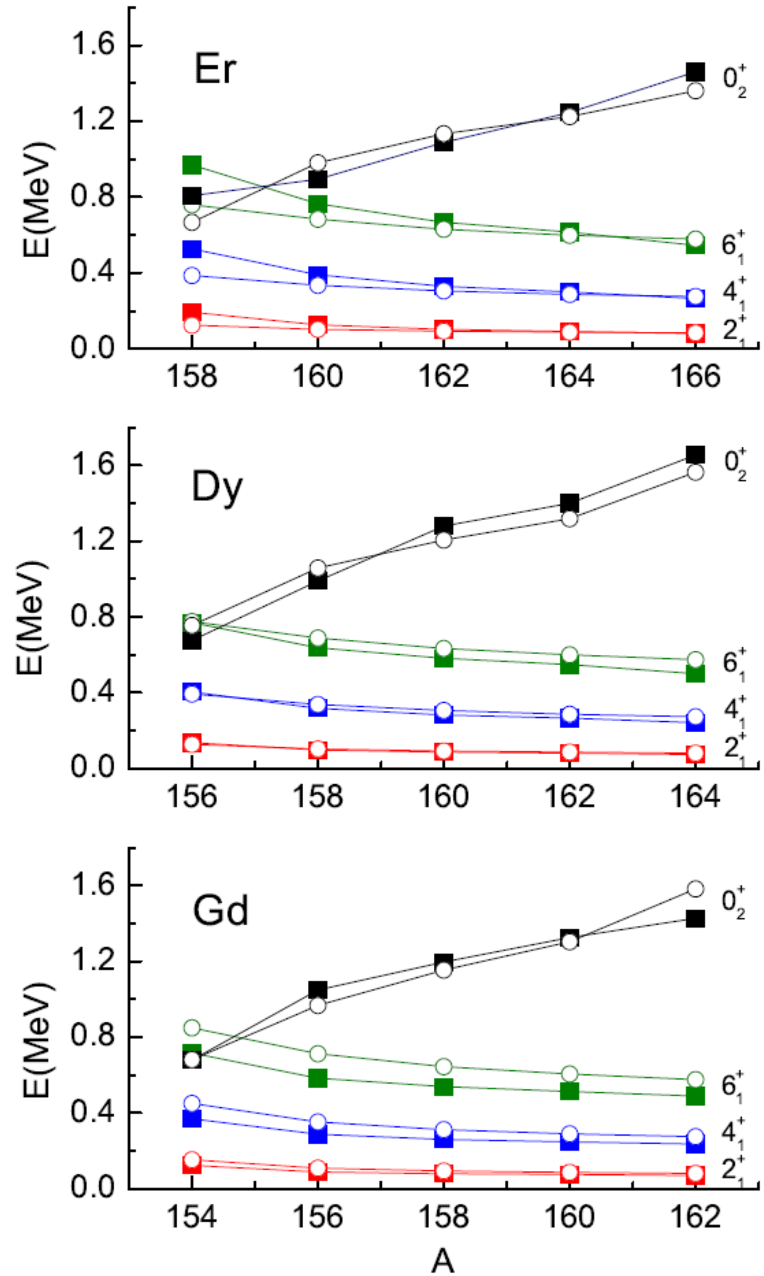
with a fixed set of parameters (fixed χ, G_M, and G_Q) is diagonalized for a chain of isotopes.

F.-Q. Chen, Y. Sun, P. Ring, *Phys. Rev.* C88 (2013) 014315

Energy levels

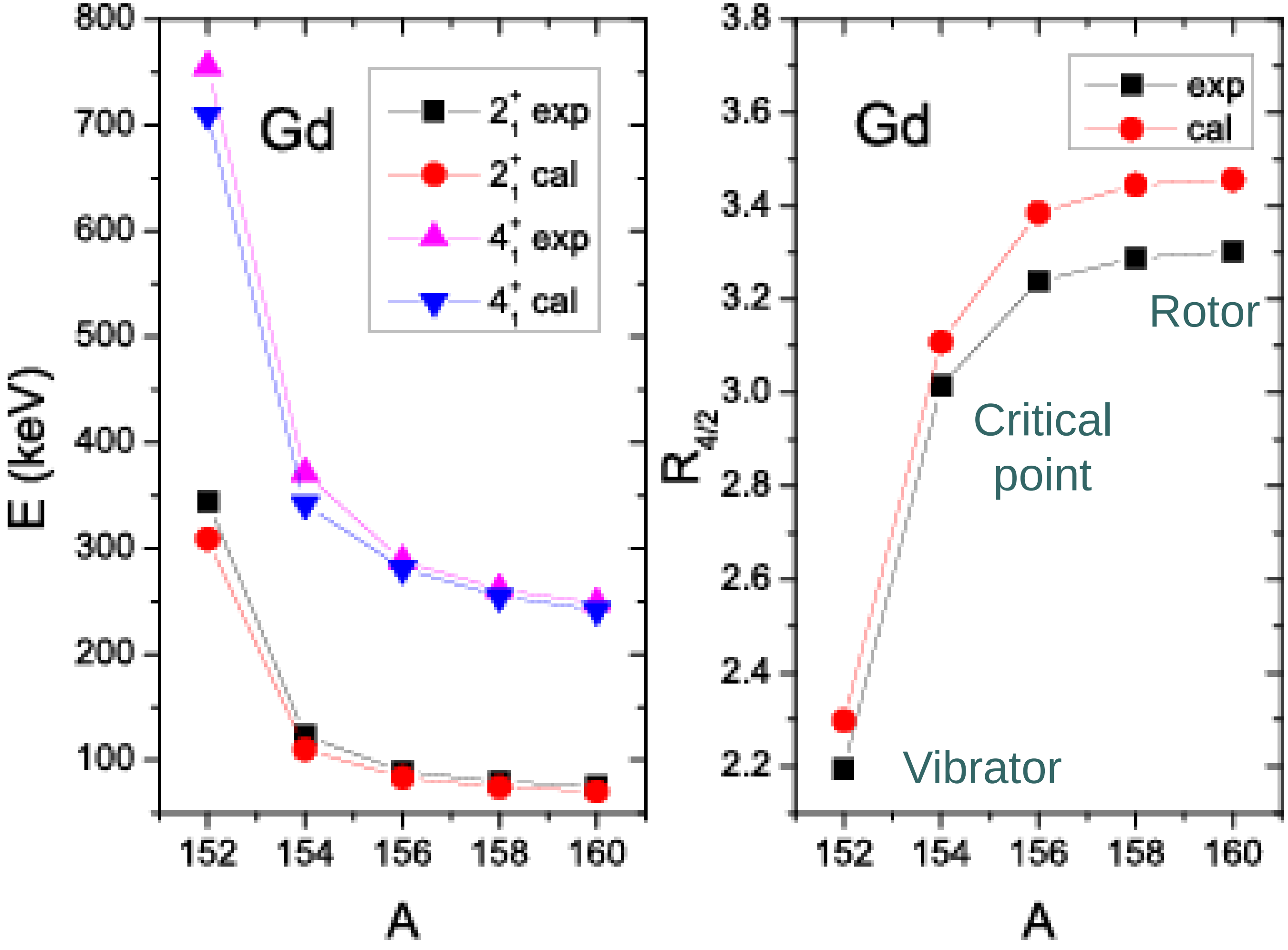
- Comparison of energy levels of 2_1^+, 4_1^+, and 6_1^+ of ground band and excited 0_2^+ state
  - Exp data (filled squares)
  - Calculations (open circles)

for isotopes from N=90 (transitional) to N=98 (well-deformed) nuclei



N=90
N=98

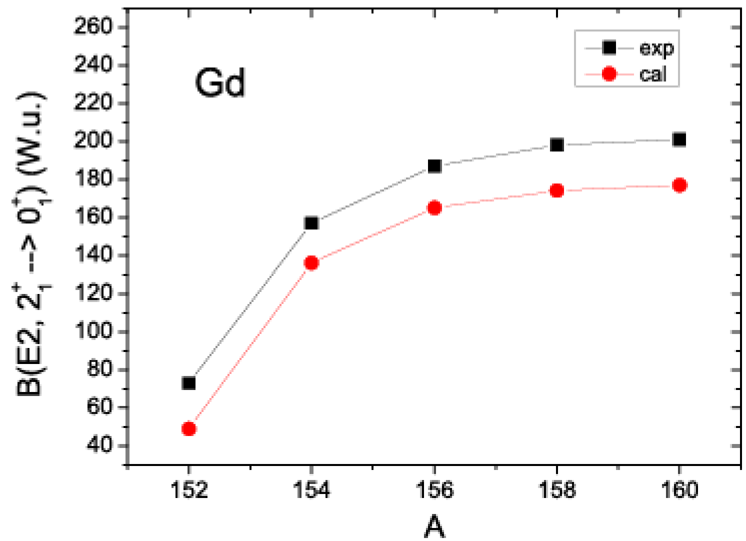
Spherical-deformed shape phase transition

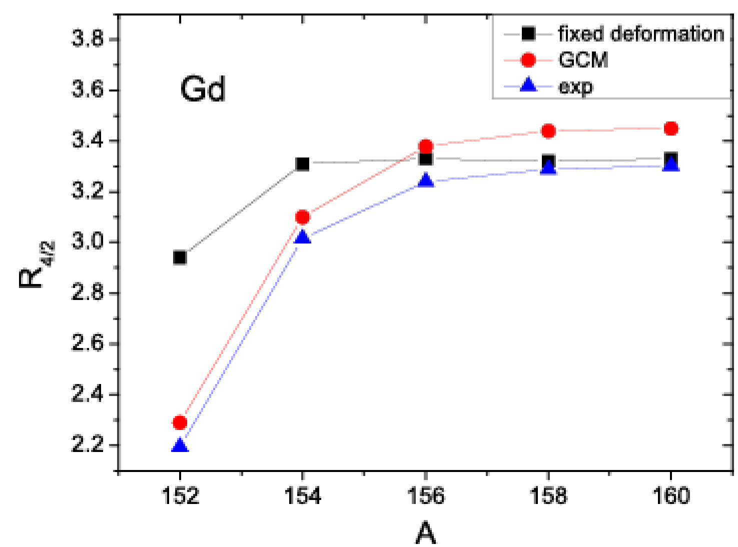


Spherical-deformed shape phase transition

- Drastic changes in electric quadrupole transition B(E2, $2^+ \rightarrow 0^+$) from vibrator ^{152}Gd (N=88), to critical point ^{154}Gd (N=90), to rotor $^{156\text{-}160}$Gd (N>90).

- Black squares show if use only one fixed deformation ε_2 in the calculation, transitional feature cannot be reproduced.





Distribution function

- The Hill-Wheeller Equation diagonalizes the Hamiltonian in a non-orthogonal basis, and therefore, $f(\varepsilon_2)$ is not a proper quantity to analyze the GSM wave function.

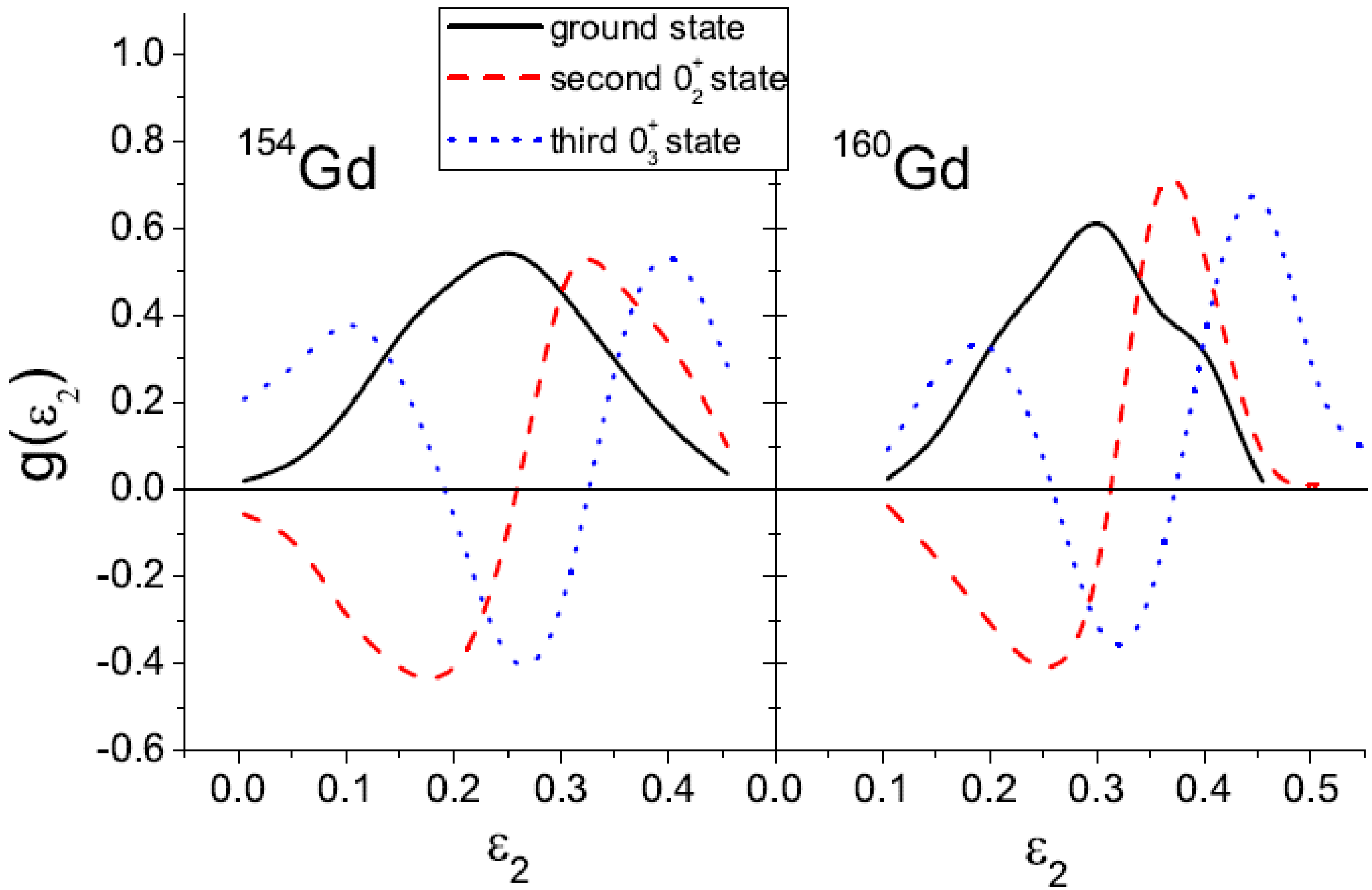
- Transformation of $f(\varepsilon_2)$ to an orthogonal basis gives

$$g(\varepsilon_2) = \int \mathcal{N}^{1/2}(\varepsilon_2, \varepsilon_2')f(\varepsilon_2')d\varepsilon_2'$$

which can be used to present the distribution of the GCM wave functions.

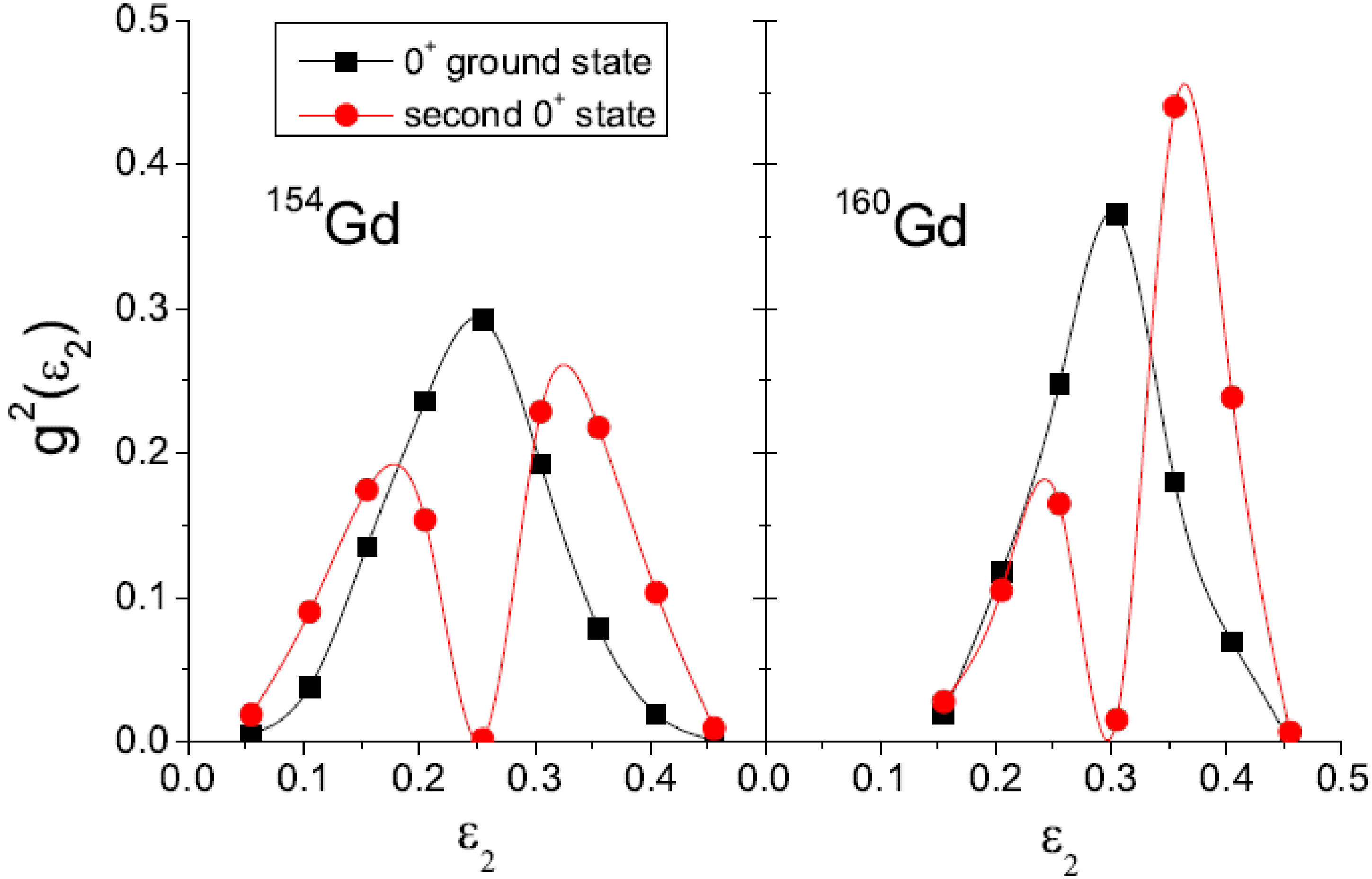
- $g^2(\varepsilon_2)$ represent the probability function.

Distribution function of deformation



Calculated distribution function of deformation for the first three 0^+ states in ^{154}Gd and ^{160}Gd

Probability function of deformation



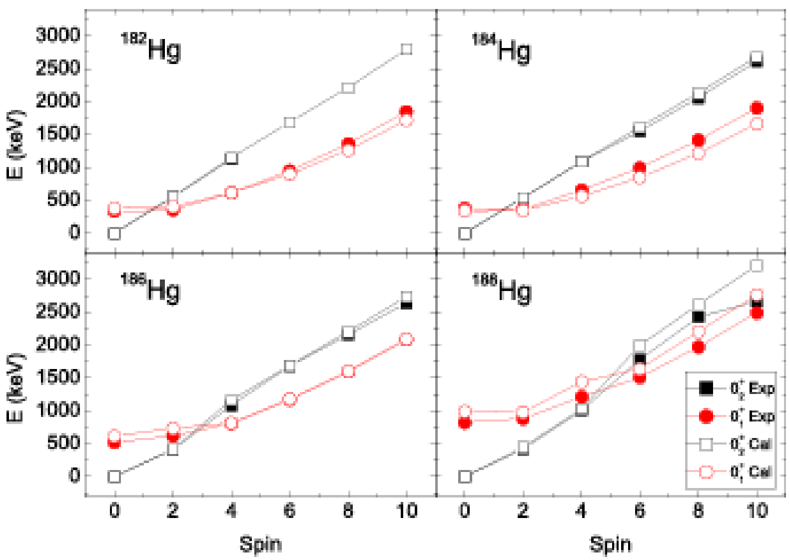
Calculated probability function of deformation for ground state 0_1^+ and excited 0_2^+ state in ^{154}Gd and ^{160}Gd.

Probability function of deformation

- Peak of the Gaussian defines deformation
  - ^{160}Gd being more deformed than ^{154}Gd
- The distribution is wider for ^{154}Gd
  - reflecting the softness of this nucleus
- The distribution for 0_2^+ is much more fragmented
  - reflecting a vibrational nature of these states
- For 0_1^+ , system stays mainly at system's deformation with the largest probability
- For 0_2^+ , system shows two peaks having different heights lying separately at both sides of the equilibrium
  - indicating an anharmonic oscillation
  - prefering to have a larger probability in the site of larger deformation

Hg isotopes

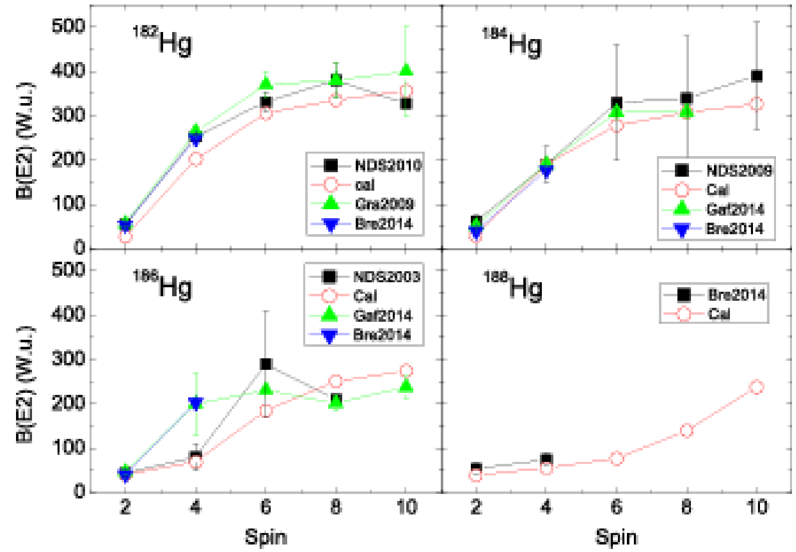
Energy levels for two 0^+ bands



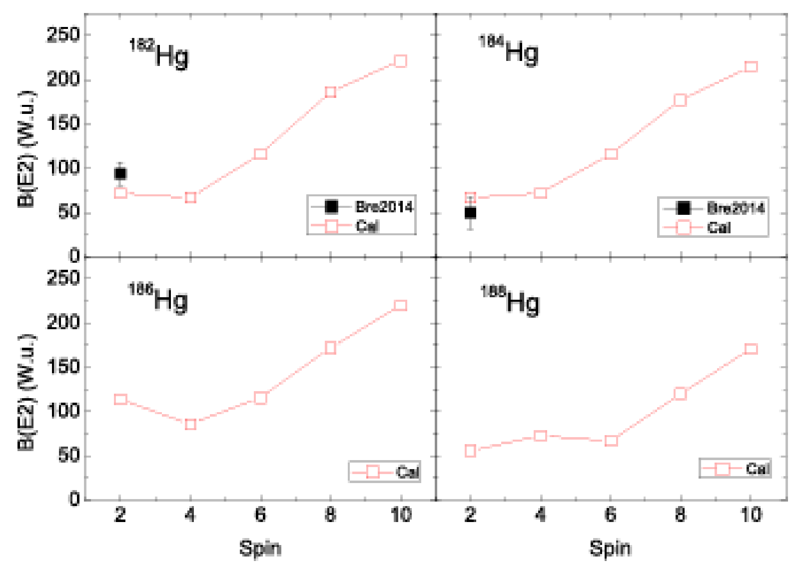
Main features can not be described when superposition is taken only for prolate deformation.

Need superposition for both prolate and oblate deformations.

B(E2) for first 0^+ band

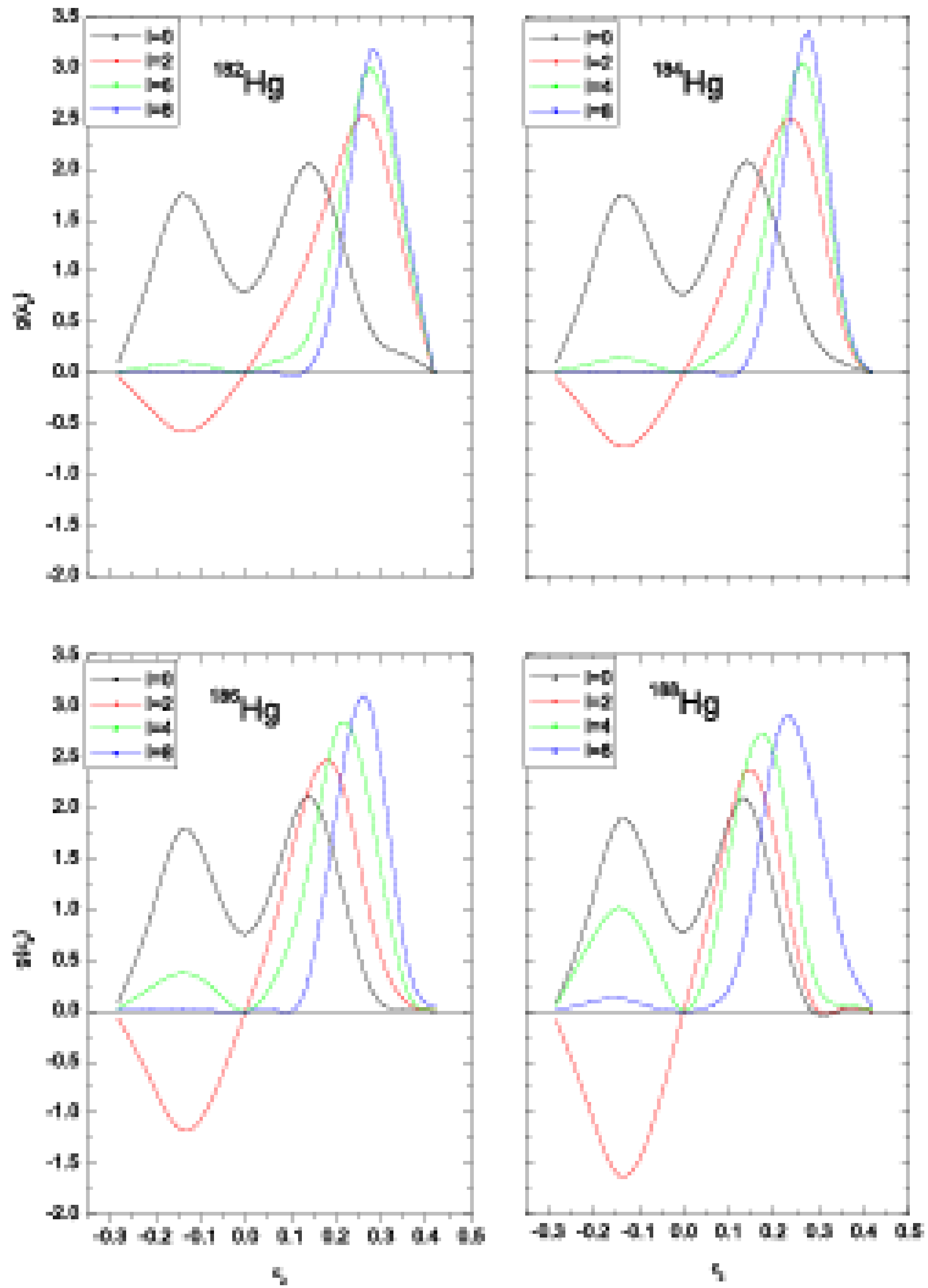


B(E2) for second 0^+ band



Hg isotopes

Distribution function for the first 0^+ band

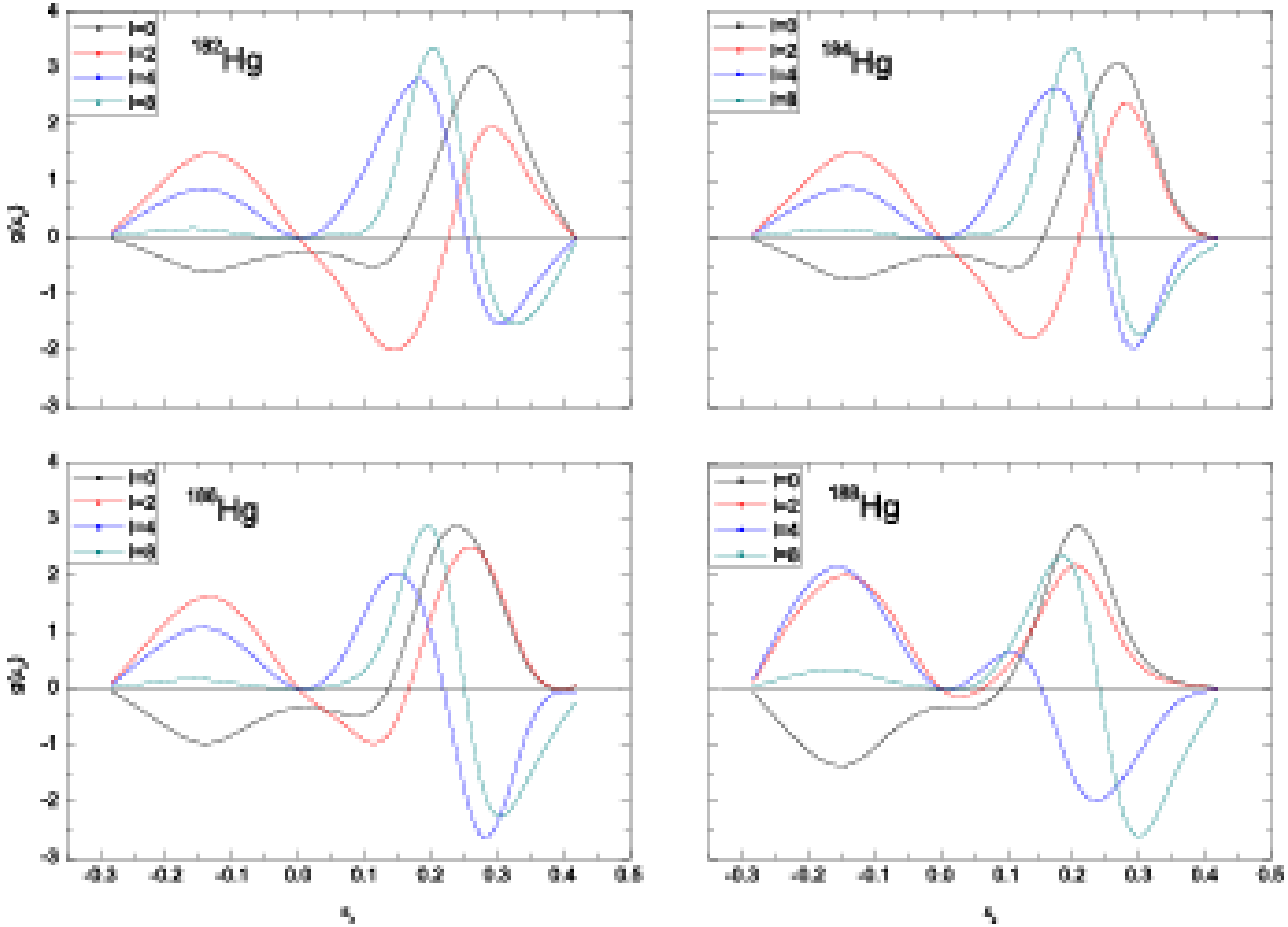


I=0: nearly spherical, two peaks distributed around zero deformation

I=2: has one node, but distributed more on prolate side

I=4 or higher: mainly peaked on the prolate side

Distribution function for the second 0^+ band



I=0: nearly prolately deformed

I=2: has two nodes, but developed to co-existing shapes at ^{188}Hg

I=4 or higher: shape developed rapidly. Finally mainly peaked on the prolate side with one node

a.-m.-projected multi-quasi-particle states based on a fixed deformation

- Even-even nuclei:

$$\left\{\hat{P}^{I}_{MK}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\nu}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\pi}\alpha^{+}_{\pi}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\pi}\alpha^{+}_{\pi}|0\rangle, \ldots\right\}$$

- Odd-odd nuclei:

$$\left\{\hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\pi}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\pi}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\pi}\alpha^{+}_{\pi}\alpha^{+}_{\pi}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\pi}\alpha^{+}_{\pi}\alpha^{+}_{\pi}|0\rangle, \ldots\right\}$$

- Odd-neutron nuclei:

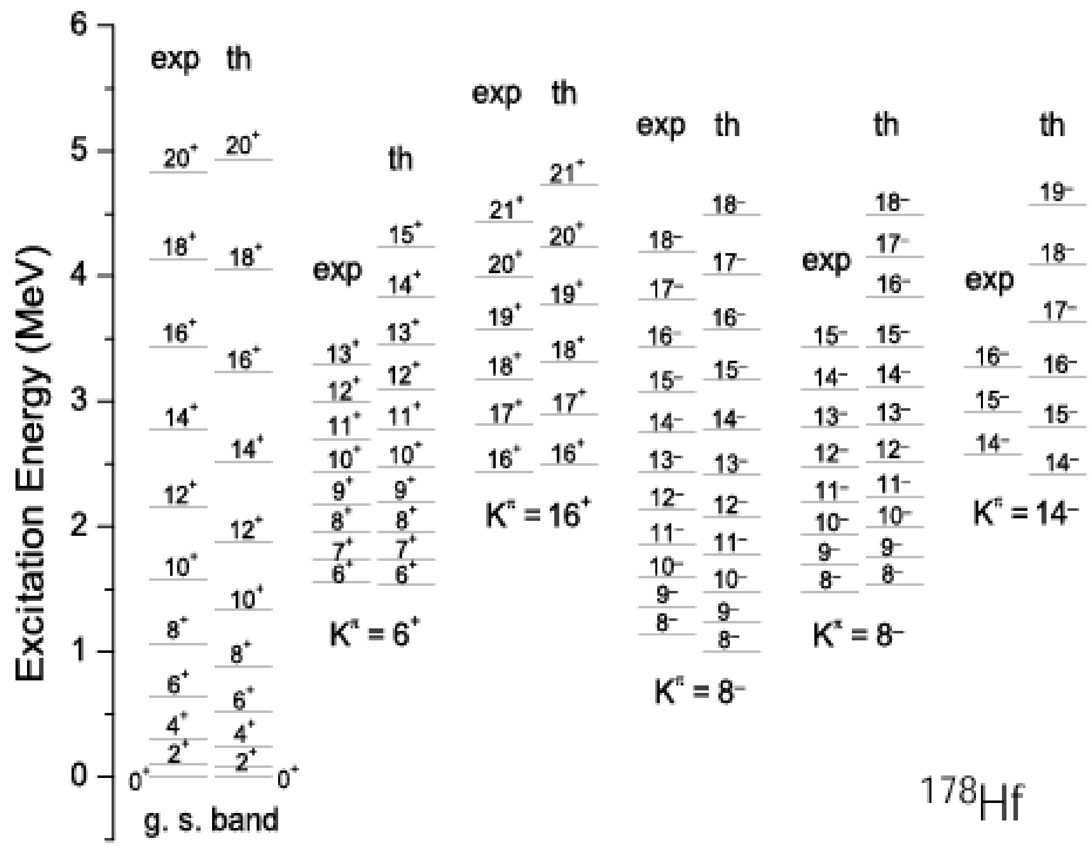
$$\left\{\hat{P}^{I}_{MK}\alpha^{+}_{\nu}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\pi}\alpha^{+}_{\pi}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\pi}\alpha^{+}_{\pi}|0\rangle, \ldots\right\}$$

- Odd-proton nuclei:

$$\left\{\hat{P}^{I}_{MK}\alpha^{+}_{\pi}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\pi}|0\rangle, \hat{P}^{I}_{MK}\alpha^{+}_{\nu}\alpha^{+}_{\nu}\alpha^{+}_{\pi}\alpha^{+}_{\pi}\alpha^{+}_{\pi}|0\rangle, \ldots\right\}$$

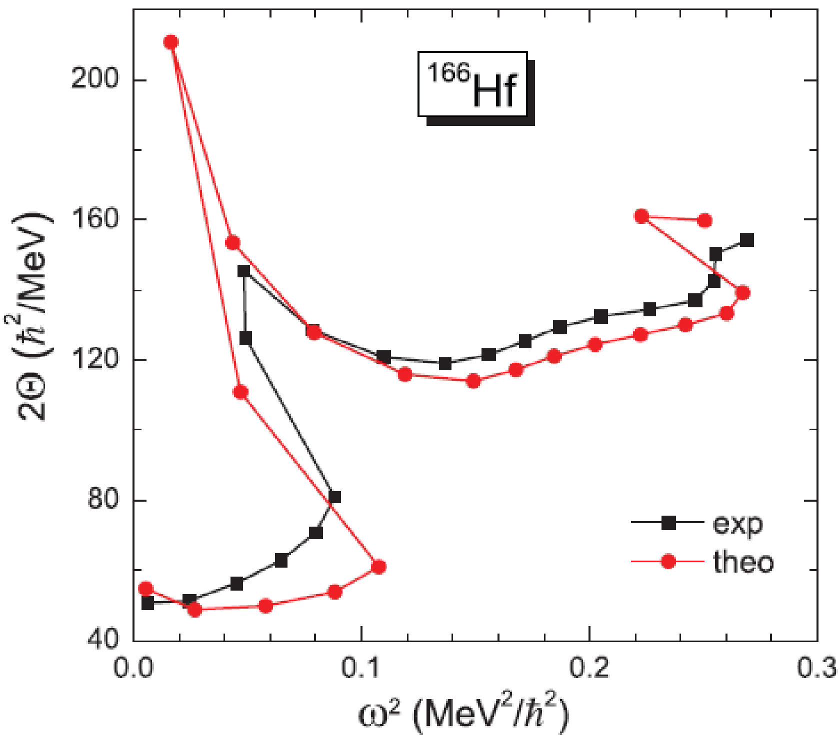
Multi-quasiparticle excitations

- 0-, 2-, 4-qp states of ^{178}Hf
- Data:
  - S.M. Mullins *et al*, *Phys. Lett.* B 393 (1997) 279
- Theory:
  - Y. Sun *et al*, *Phys. Lett.* B 589 (2004) 83

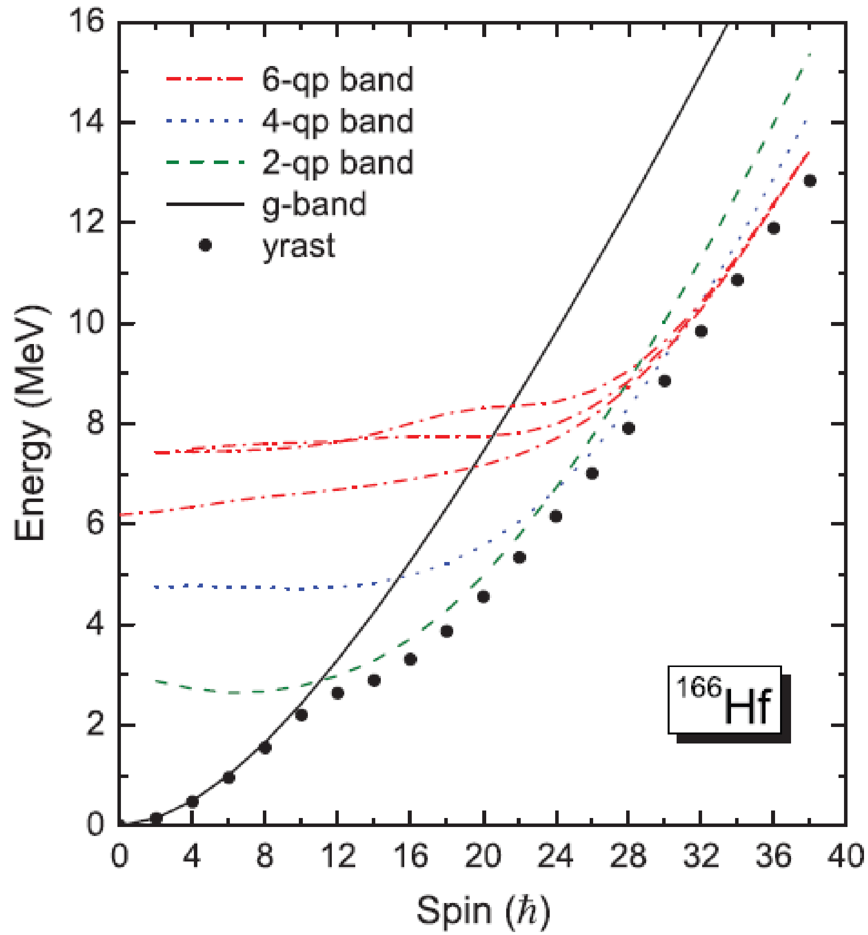


Multi-quasiparticle computation using the Pfaffian algorithm

- Calculation of projected matrix elements usually uses the generalized Wick theorem
- A matrix element having n (n’) qp creation or annihilation operators respectively on the left- (right-) sides of the rotation operator contains $(n + n - 1)!!$ terms in the expression – a problem of combinatorial complexity
- Use of the Pfaffian algorithm:
  - L.M. Robledo, Phys. Rev. C 79 (2009) 021302(R).
  - L.M. Robledo, Phys. Rev. C 84 (2011) 014307.
  - T. Mizusaki, M. Oi, Phys. Lett. B 715 (2012) 219.
  - M. Oi, T. Mizusaki, Phys. Lett. B 707 (2012) 305.
  - T. Mizusaki, M. Oi, F.-Q. Chen, Y. Sun, Phys. Lett. B 725 (2013) 175



A third band-crossing is described.

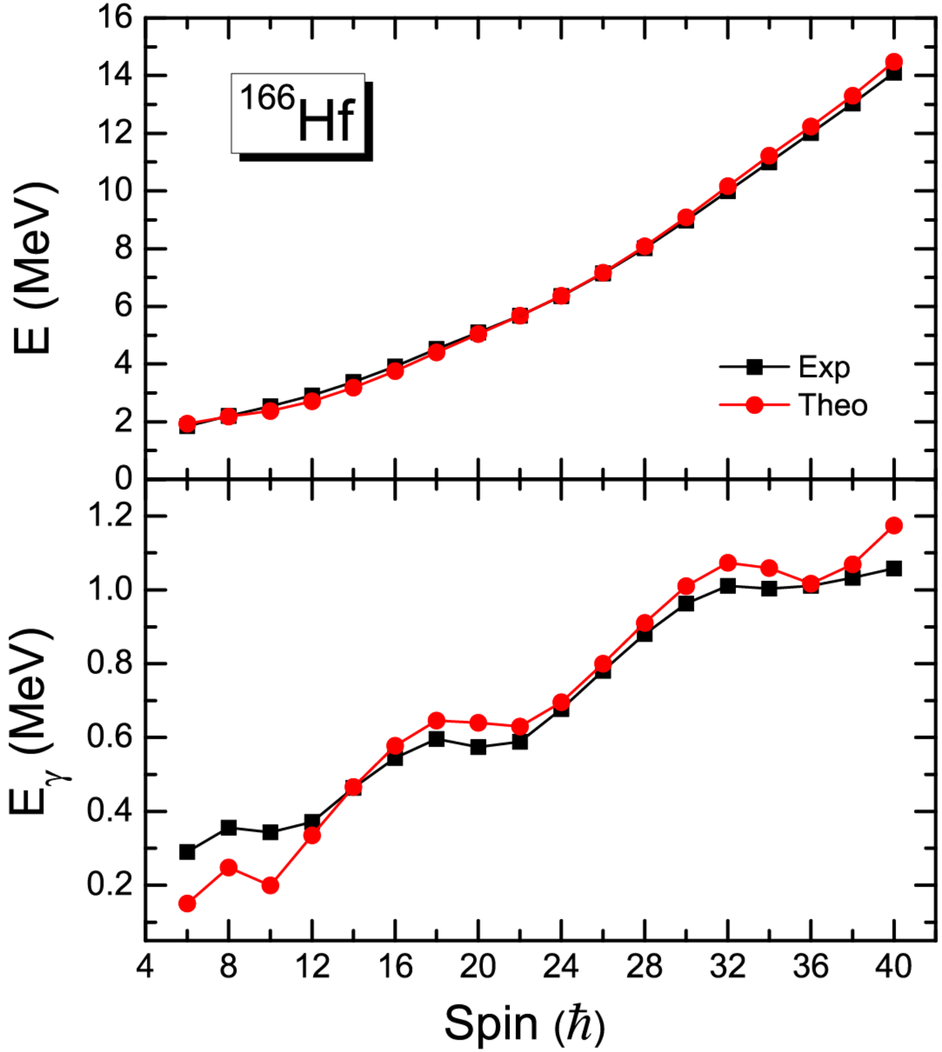


$$\{|\Phi\rangle,\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}|\Phi\rangle,\ a^\dagger_{\pi_i}a^\dagger_{\pi_j}|\Phi\rangle,\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}|\Phi\rangle,$$
$$\times\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}|\Phi\rangle,\ a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}|\Phi\rangle,$$
$$\times\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}a^\dagger_{\nu_m}a^\dagger_{\nu_n}|\Phi\rangle,\ a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}a^\dagger_{\pi_m}a^\dagger_{\pi_n}|\Phi\rangle,$$
$$\times\ a^\dagger_{\pi_i}a^\dagger_{\pi_j}a^\dagger_{\nu_k}a^\dagger_{\nu_l}a^\dagger_{\nu_m}a^\dagger_{\nu_n}|\Phi\rangle,\ a^\dagger_{\nu_i}a^\dagger_{\nu_j}a^\dagger_{\pi_k}a^\dagger_{\pi_l}a^\dagger_{\pi_m}a^\dagger_{\pi_n}|\Phi\rangle\}$$

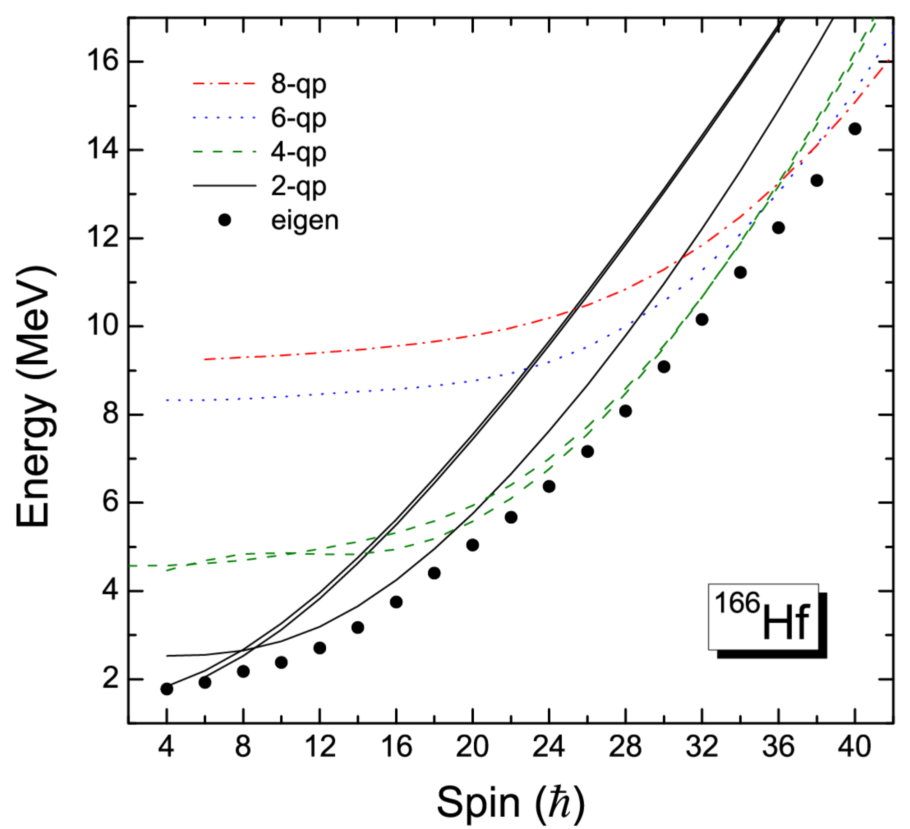
Extension of configuration space to 6-qps.

L.-J. Wang et al. *Phys. Rev.* C90 (2014) 011303(R)

Example for very high-spin states



Calculation including 8-qps based on a fixed deformation



Summary

- New development in the Projected Shell Model:
  - We improved the PSM wave function by superimposing (angular-momentum and particle-number) projected states with different deformation ε_2
  - The method can be applied to problems of soft nuclei, shape co-existence, phase transition, etc.
  - excited 0^+ states can be described together with the ground state in an equal footing
- High order multi-quasiparticle states using the Phaffian algorithm
  - To overcome the problem in the classical Wick's theorem for matrix-element calculation
  - Computer code can be developed when large number of quasiparticle excitations are included.

Collaborators

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