

# NUCLEAR LEVEL DENSITY PREDICTIONS

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# STATISTICAL CALCULATIONS OF NEUTRON CAPTURE RADIATION\*

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## Abstract

STATISTICAL CALCULATIONS OF NEUTRON CAPTURE RADIATION. The capture gamma-ray spectrum, the multiplicity, the population of the levels (isomer ratio) and the line density is calculated with a computer program using formulas for the level density and transition probability. With parameters for the level density and for the transition probability from other experiments good agreement is achieved in many cases. The number of detected lines in a spectrometer is expected to increase with about the square root of the sensitivity of a spectrometer.

## 1. INTRODUCTION

Measurement of thermal neutron capture radiation is used in most cases to develop the lower part of the level scheme of the final nucleus. But more information can be gained from neutron capture radiation. The numerous lines from the capture state to the intermediate levels and from the intermediate levels to the discrete low levels depend on the level density and transition probability. I would like to discuss the difficulties and some results of calculations which relate the level density and the transition probability to the whole measured capture radiation.

There are several thousand levels between the compound state and the ground state in heavier nuclei. Since most of these levels, their spins and parities or even the respective transition probabilities are not known, statistical assumptions on level density, spin distribution and transition probability must be used to calculate the neutron capture spectrum. It is obvious that the formulas for level density and transition probability, which are also not so well known, are a rough simplification. All special relations between levels are neglected, such as exist, for instance, in rotational bands. The K quantum number is not used. Anomalies in the capture spectrum cannot be explained by statistical calculations such as the 'gold bump' at 5 to 6 MeV, discussed by Bartholomew [1], or the strong transitions from the capture state in  $^{114}\text{Cd}$  [2] and  $^{165}\text{Dy}$  [3] to levels near 3 MeV. But it should be possible to get some information on the usefulness of a statistical model.

The publications on a statistical model for neutron capture radiation discuss two questions: (1) isomer ratios and (2) calculation of the neutron capture gamma spectrum, especially of the unresolved part above 2 MeV.

Huizenga and Vandenberg [4, 5] describe a simple model to calculate isomer ratios. This model has been used and improved by many authors [6-9]. Pönitz [10] suggests a cascade model for the calculation of isomer ratios. Several publications apply this or similar models [10-15].

\* Part of a Habilitationsschrift, Technische Hochschule, Munich, 1968.

# **Fifth International Symposium on Capture Gamma-Ray Spectroscopy and Related Topics**

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# **Capture Gamma-Ray Spectroscopy and Related Topics-1984**

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Nucleus	$I_i - I_f$	Level	Trans-Energy	X-Value	$\times 10^{-3}$	Ref.
$^{174}\text{Yb}$	$2_{-1}^{+2}$	1849	1771	.12(3) <sup>a</sup>		
	$0_5^{+0}$	1895	1895	.10(3)		
	$2_{-1}^{+2}$	1956	1878	.09(3)		
	$0_2^{+0}$	1478	1478	<.051		20,37
	$0_3^{+0}$	1884	1884	<.175		
	$0_2^{+0}$	1199	1199	.28(3)	38	
	$2_{-2}^{+2}$	1277	1184	.34(3) <sup>a</sup>		
	$4_{-4}^{+2}$	1450	1144	.17(1) <sup>a</sup>		
	$6_{-6}^{+2}$	1731	1099	.16(2) <sup>a</sup>		
	$0_3^{+0}$	1434	1434	>.15		
$^{178}\text{Hf}$	$2_{-2}^{+2}$	1496	1403	.44(3) <sup>a</sup>		
	$0_4^{+0}$	1444	1444	.59(4)		
	$2_{-2}^{+2}$	1562	1468	.23(2) <sup>a</sup>		
	$0_5^{+0}$	1772	1772	>.2		
	$2_{-2}^{+2}$	1818	1725	.53(4) <sup>a</sup>		
	$4_{-4}^{+2}$	1956	1650	.37(4) <sup>a</sup>		
	$0_2^{+0}$	1087	1087	.0006	22	
	$0_3^{+0}$	1478	1478	.0072		
	$0_4^{+0}$	1704	1704	1.70		
	$0_5^{+0}$	1765	1765	0.05		
$^{188}\text{Os}$	$0_6^{+0}$	1825	1825	.15		
	$0_7^{+0}$	1966	1966	>16		
	$0_2^{+0}$	1135	1135	<.005	22	
	$0_3^{+0}$	1403	1403	.092		
	$0_4^{+0}$	1823	1823	<.03		
	$0_5^{+0}$	1919	1919	.06		
	$0_2^{+0}$	985	985	1.5(2)	26	
	$0_2^{+0}$	985	985	1.5(2)	26	

a - Assumes pure E0 + E2

b - Assumes 2<sup>+</sup> level

## STRUCTURE AND STATISTICAL ASPECTS OF EXTENSIVE LEVEL SCHEMES FROM (n, $\gamma$ ) AND TRANSFER REACTIONS

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### ABSTRACT

The very precise spectrometers for (n, $\gamma$ ) and (n,e) reactions at the ILL, Grenoble, and the high resolution Q3D spectrograph at the Munich Tandem Accelerator yield very detailed information on nuclear transitions and excitations. These results allow the establishment of level schemes which are rather complete in a given spin and energy region and which contain frequently more than 70 levels with spin and parity information. Recently the nuclei  $^{20}\text{F}$ ,  $^{24}\text{Na}$ ,  $^{28}\text{Al}$ ,  $^{36}\text{Cl}$ ,  $^{40}\text{K}$ ,  $^{41}\text{K}$ ,  $^{42}\text{K}$ ,  $^{114}\text{Cd}$ ,  $^{134}\text{Cs}$ ,  $^{155}\text{Sm}$ ,  $^{154}\text{Eu}$ ,  $^{155}\text{Gd}$ ,  $^{161}\text{Dy}$ ,  $^{163}\text{Dy}$  and several actinide nuclei have been investigated. The limits of this method and the possibilities to identify nuclear structures will be discussed. In particular the mixing of single particle and vibrational excitations in deformed nuclei will be treated. A statistical analysis of these extensive level schemes gives information on the level density, on the strength of gamma transitions and on gamma multiplicities of levels. A systematic survey of gamma multiplicities of low spin levels shows that these multiplicities increase only slowly after the first 20 levels while the spread seems to decrease. Level density parameters for the constant temperature and Bethe formulae are determined. These formulae reproduce nicely the experimental level densities.

The distribution of the E1 and M1 strength of primary transitions is compared with theoretical predictions. The energy dependence of dipole transitions in the Cl and K isotopes agrees better with an  $E^5$  proportionality than with  $E^3$  observed for heavier nuclei. Non-statistical distribution of M1 strength in the sd shell nuclei was seen and might be explained by nuclear structure effects.

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## Simple models for the Nuclear Level Densities

### Back-shifted Fermi Gas (BSFG)

$$\rho_{BSFG} = \frac{e^{2\sqrt{a(E-E_1)}}}{12\sqrt{2}a^{1/4}(E-E_1)^{5/4}} \quad a, E_1 : \text{parameters}$$

### Constant Temperature (CT)

$$\rho_{CT} = \frac{1}{T} e^{(E-E_0)/T} \quad T, E_0 : \text{parameters}$$

Both models describe well the level density at least up to the neutron binding energy

## *Our approach:*

- 1) Determine ***empirically*** the two parameters for both BSFG and CT models, by fitting the low energy known levels and the mean level spacing at the neutron binding energy, for 310 nuclei between  $^{18}\text{F}$  and  $^{251}\text{Cf}$
- 2) **Find correlations** between the empirical parameters and various nuclear structure observables (masses, pairing energies, etc.).
- 3) **Deduce simple formulas** that describe the empirical parameters of *all* 310 nuclei, and can be used to extrapolate to other nuclei.

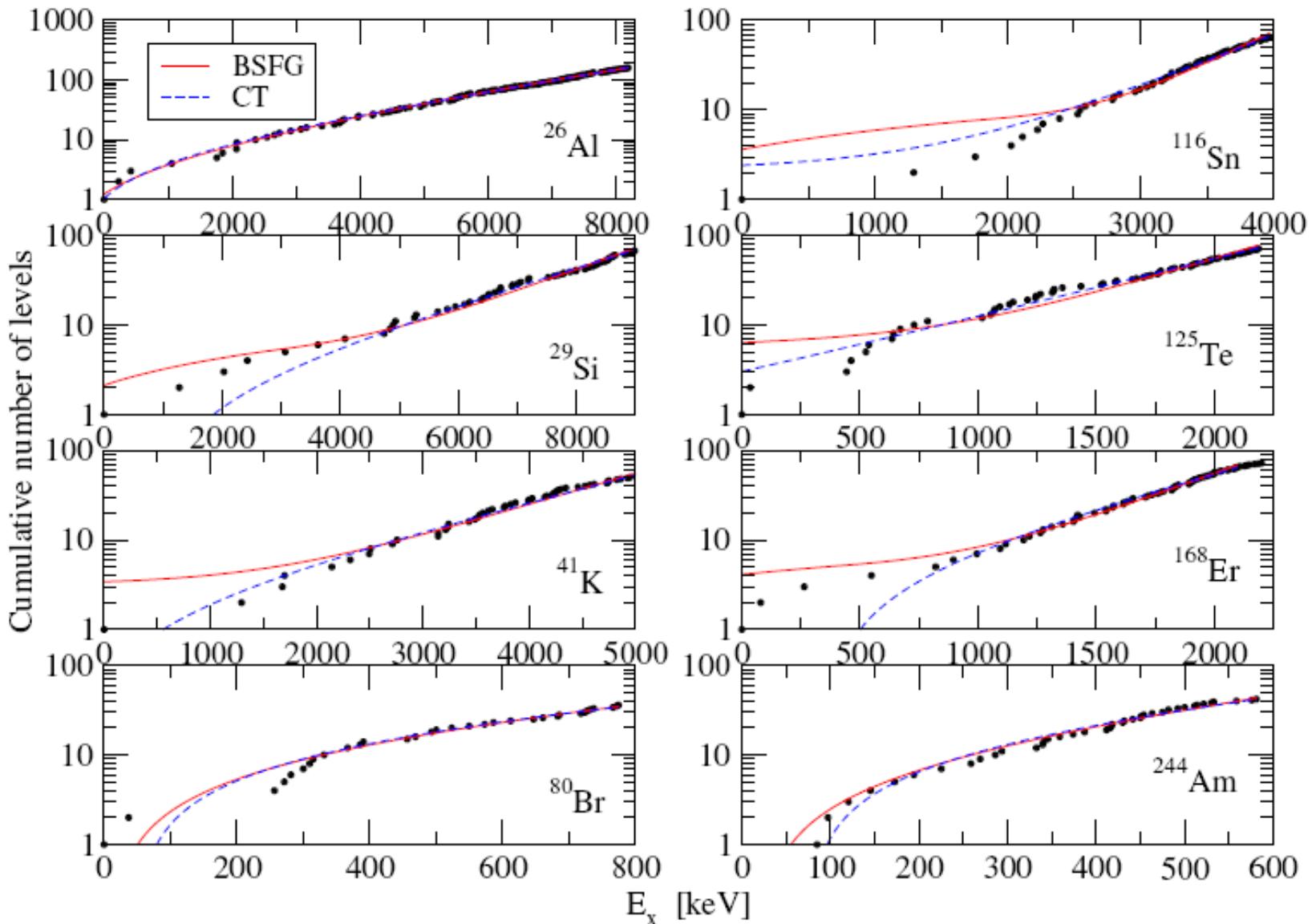
Phys. Rev. C72(2005)044311

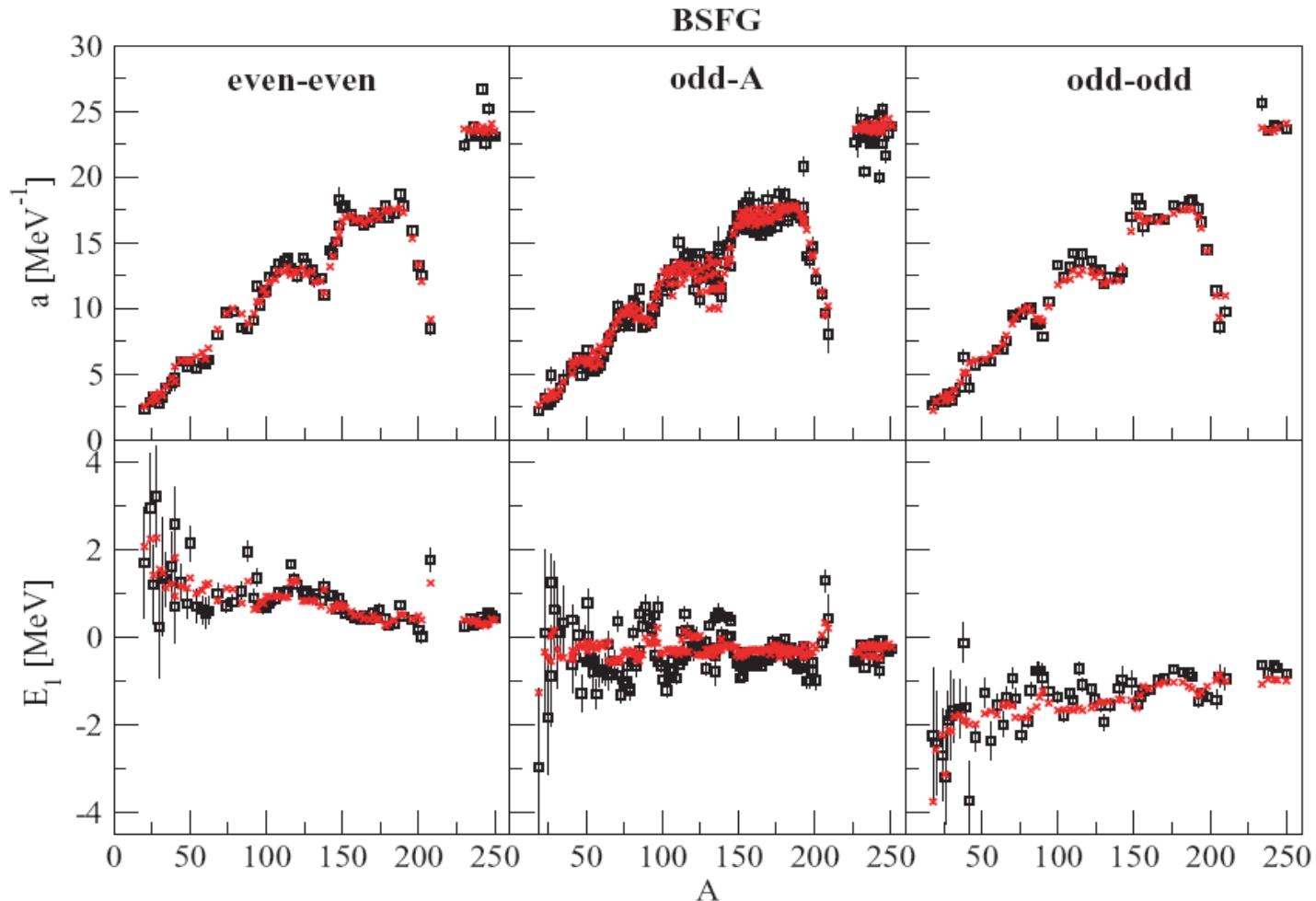
Phys. Rev. C80(2009)054310

IoP Conf. Ser. 338(2012)012028

# Experimental Cumulative Number of Levels N(E)

Resonance density is included in the fit

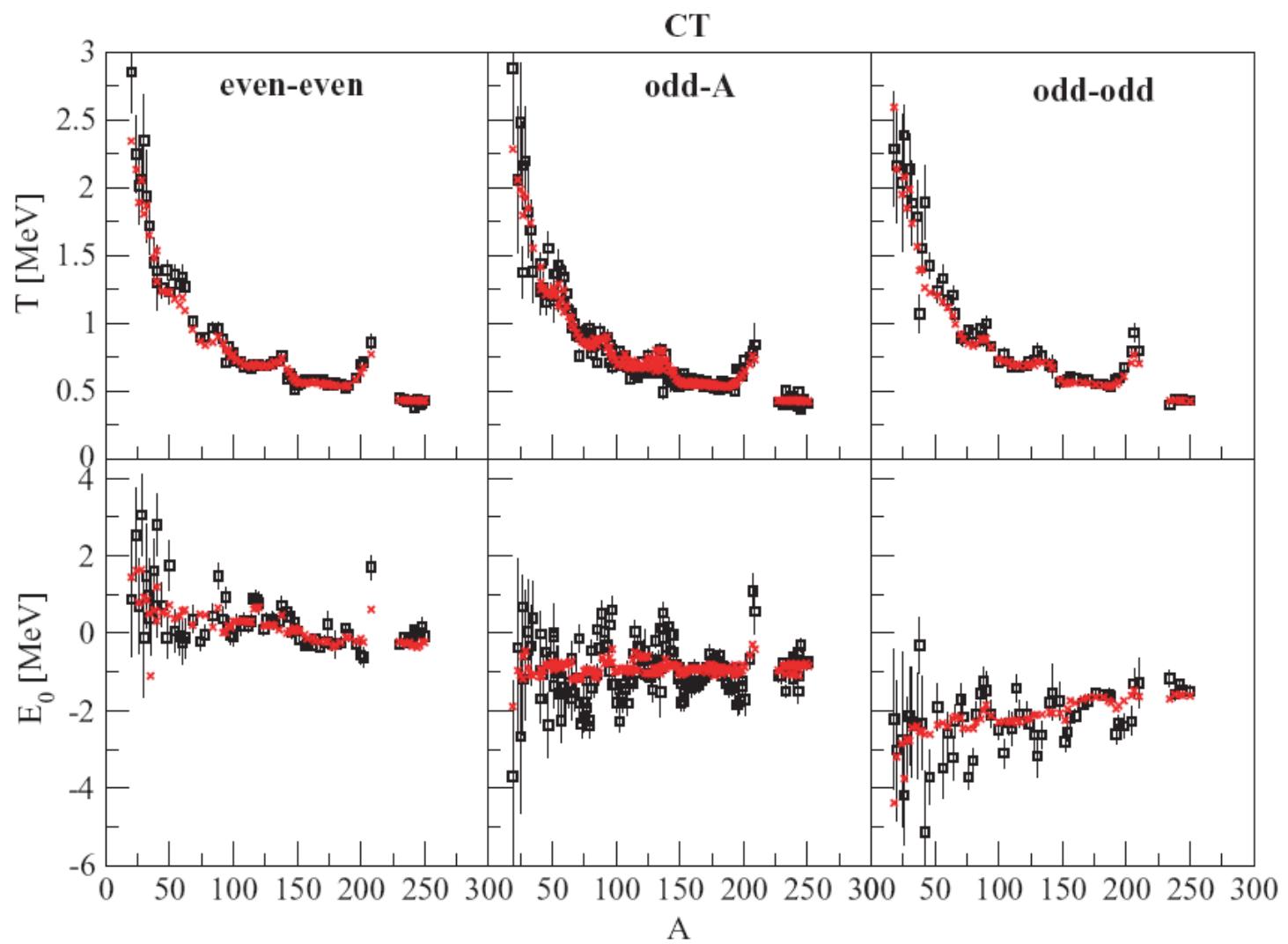




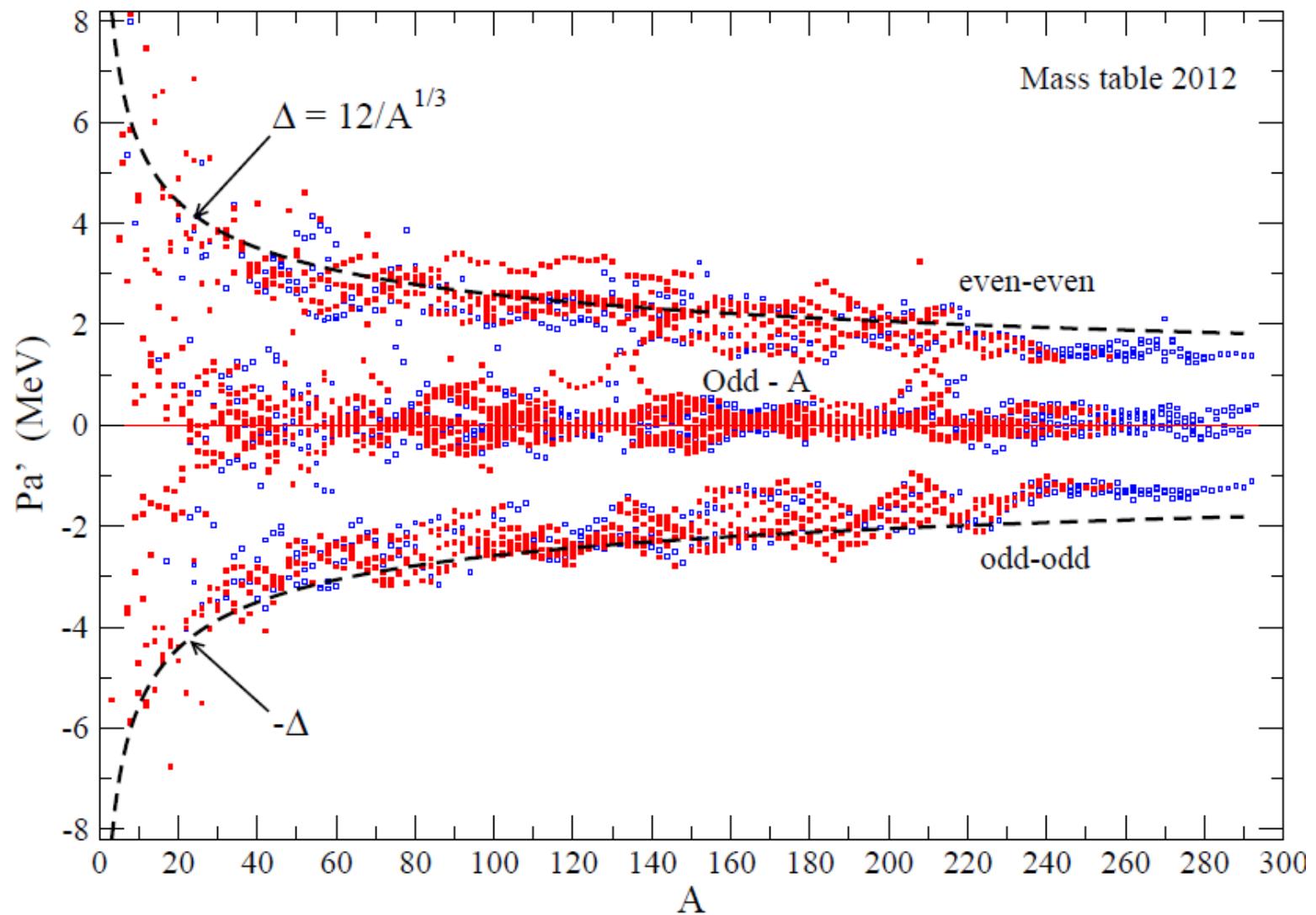
$$\mathbf{a = (0.199 + 0.0096 S') A^{0.869}; \quad E_1 = -0.381 + 0.5 Pa'}$$

$\mathbf{S = M_{exp} - M_{weiz}}$  : *shell correction*;  $\mathbf{S' = S + 0.5 Pa'}$  ;  $\mathbf{Pa'}$  : *deuteron pairing*

All from mass table

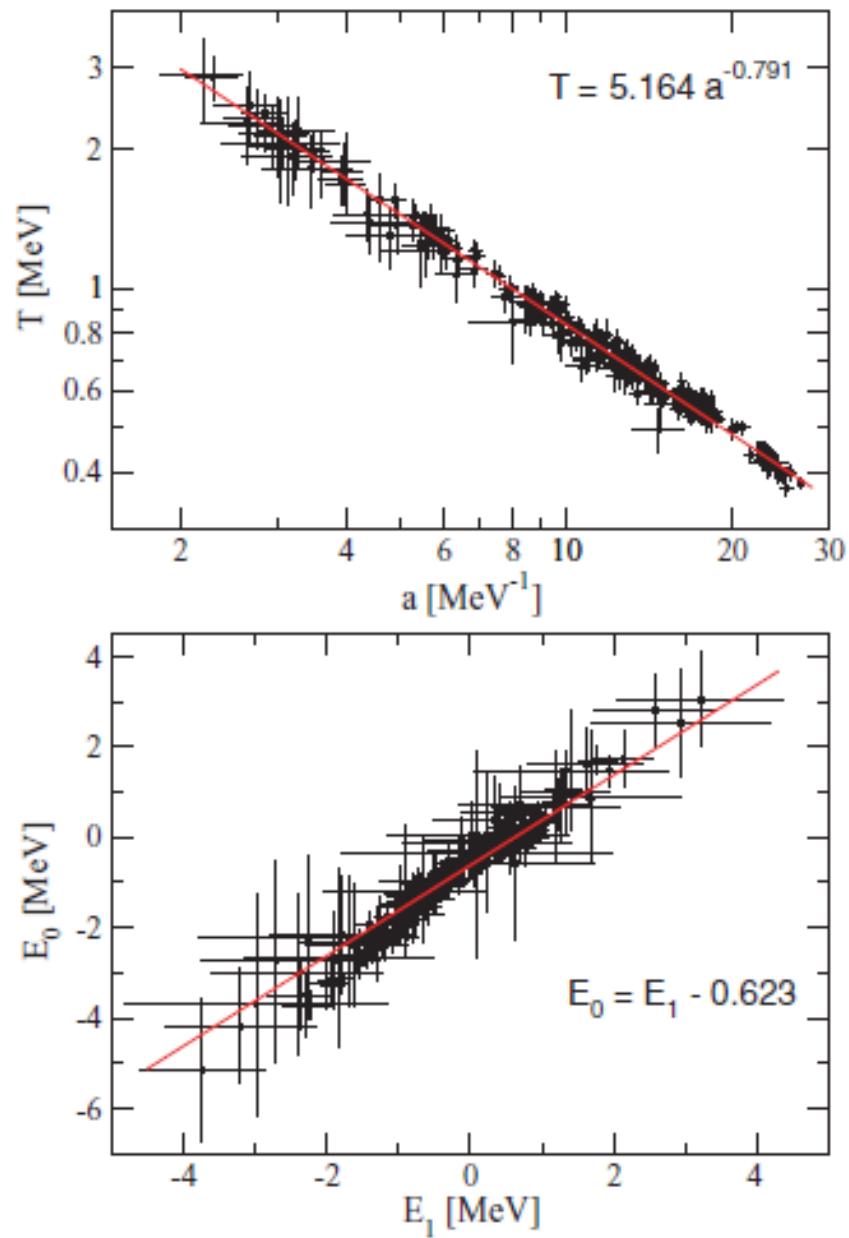


$$T = A^{-2/3} / (0.0597 + 0.00198 S'); \quad E_0 = -1.004 + 0.5 Pa'$$

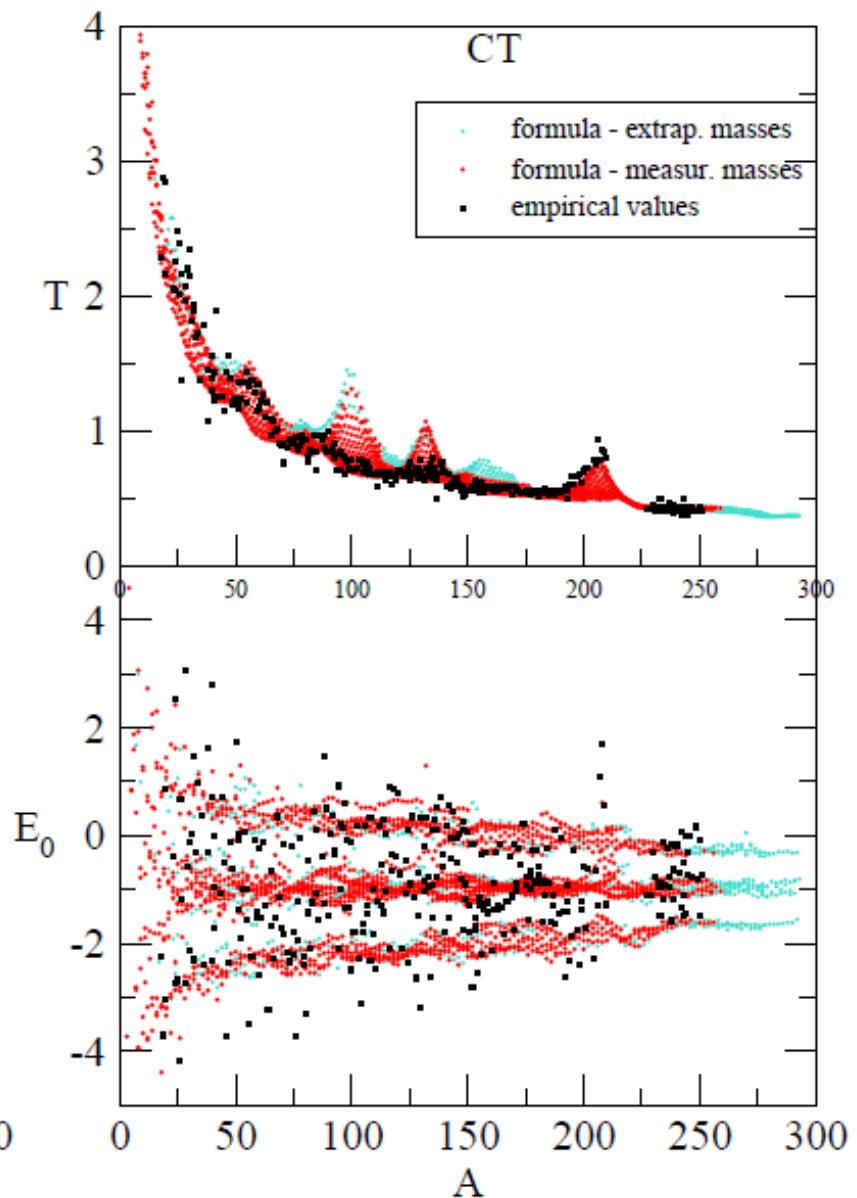
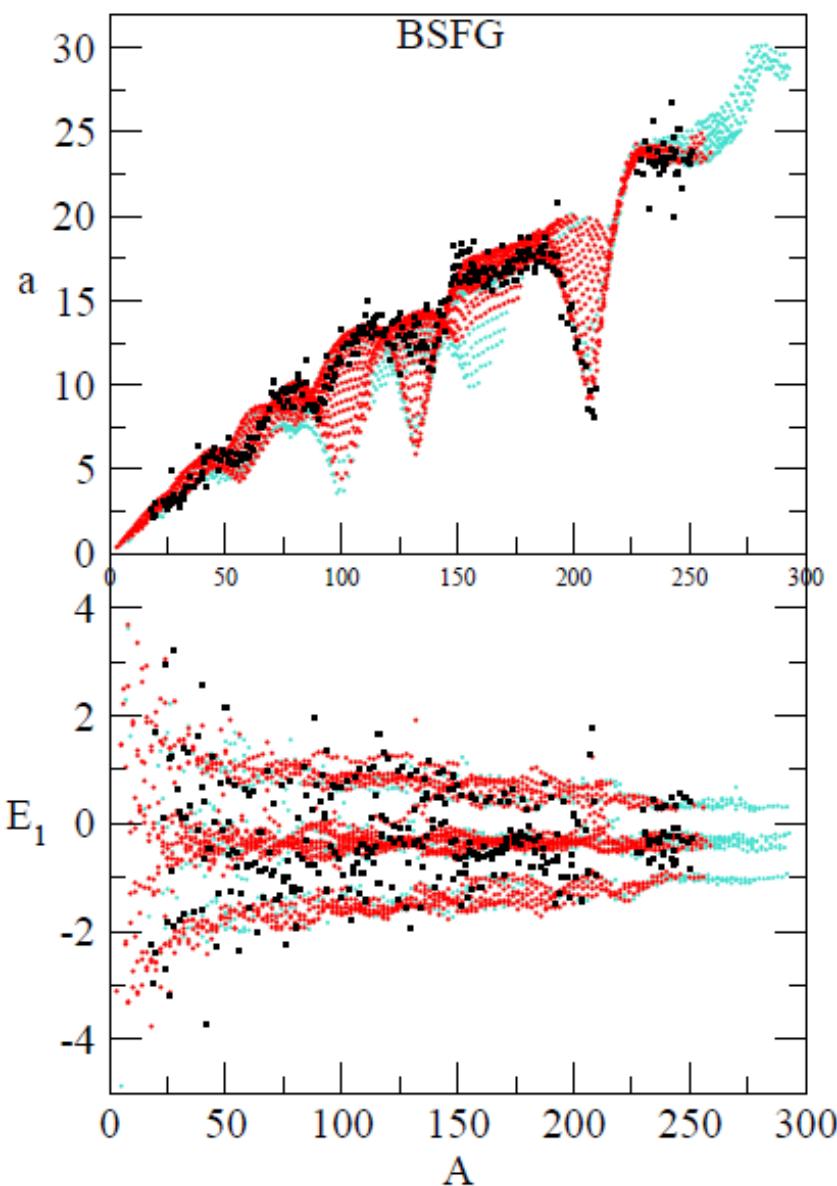


Deuteron pairing:  $\text{Pa}' = [\text{M.E.}(A+2,Z+1) + \text{M.E.}(A-2,Z-1) - 2 \text{ M.E.}(A,Z)]/2$

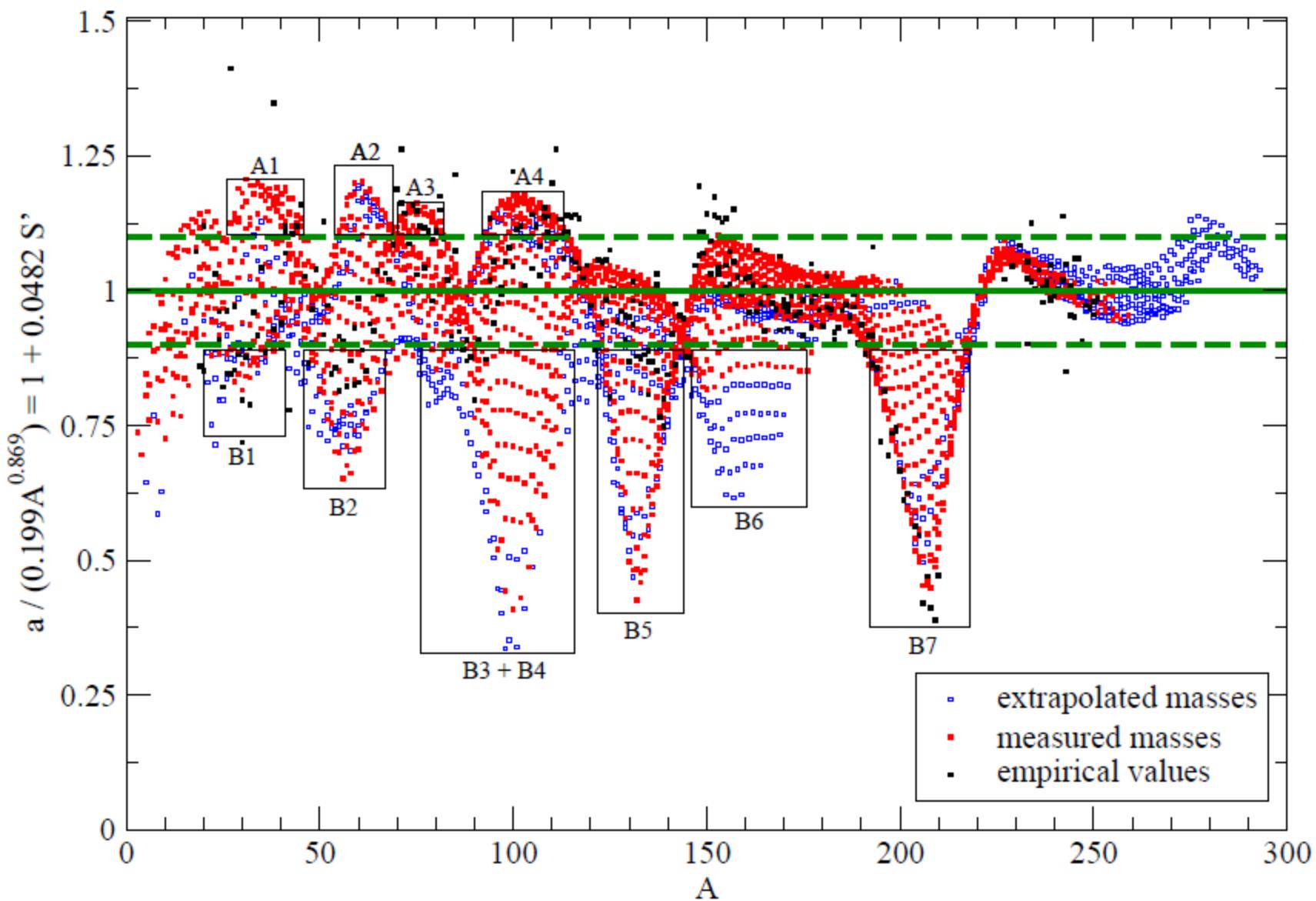
## Correlations between the parameters of the BSFG and CT models



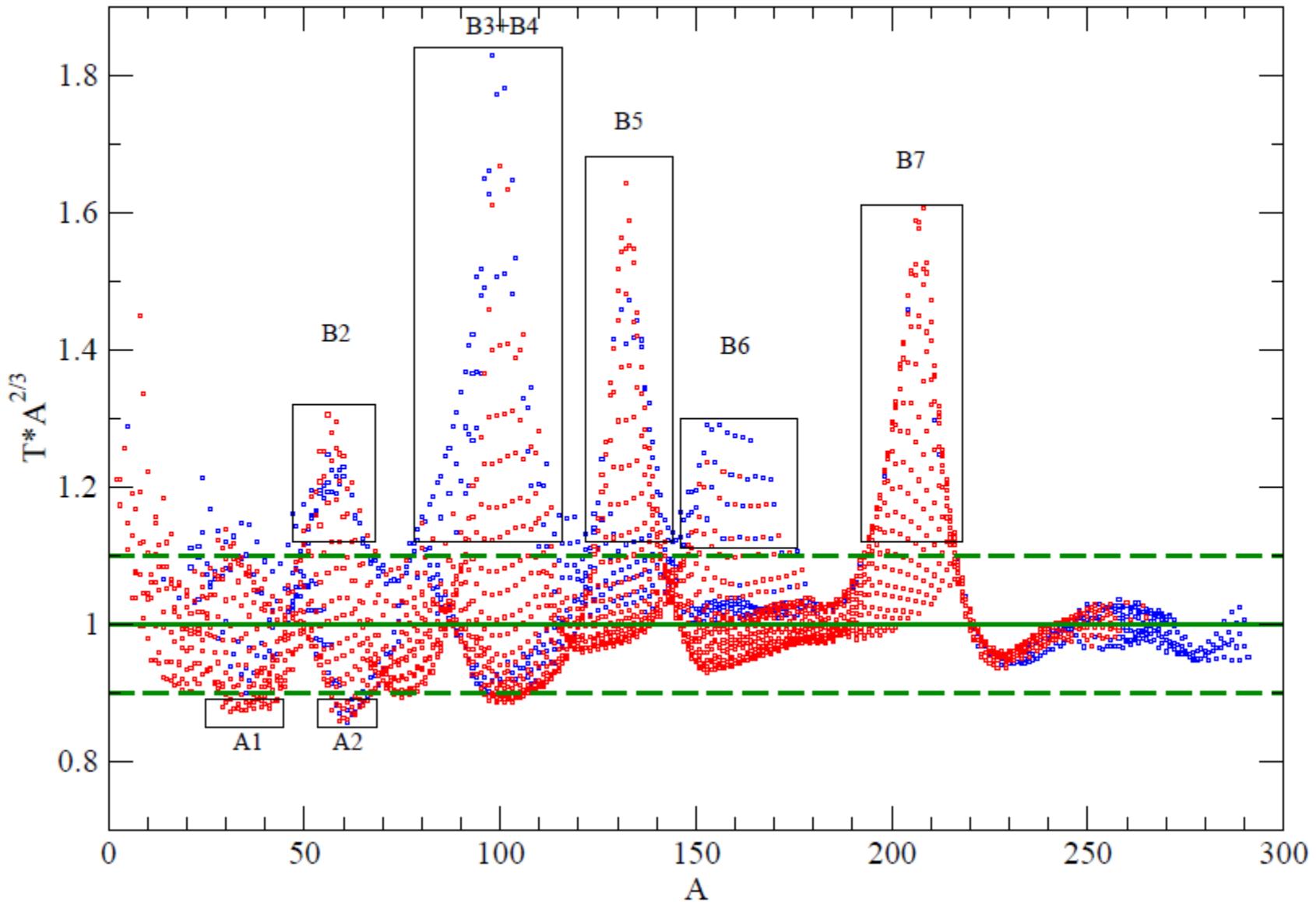
## Level density parameters predicted with mass table 2012



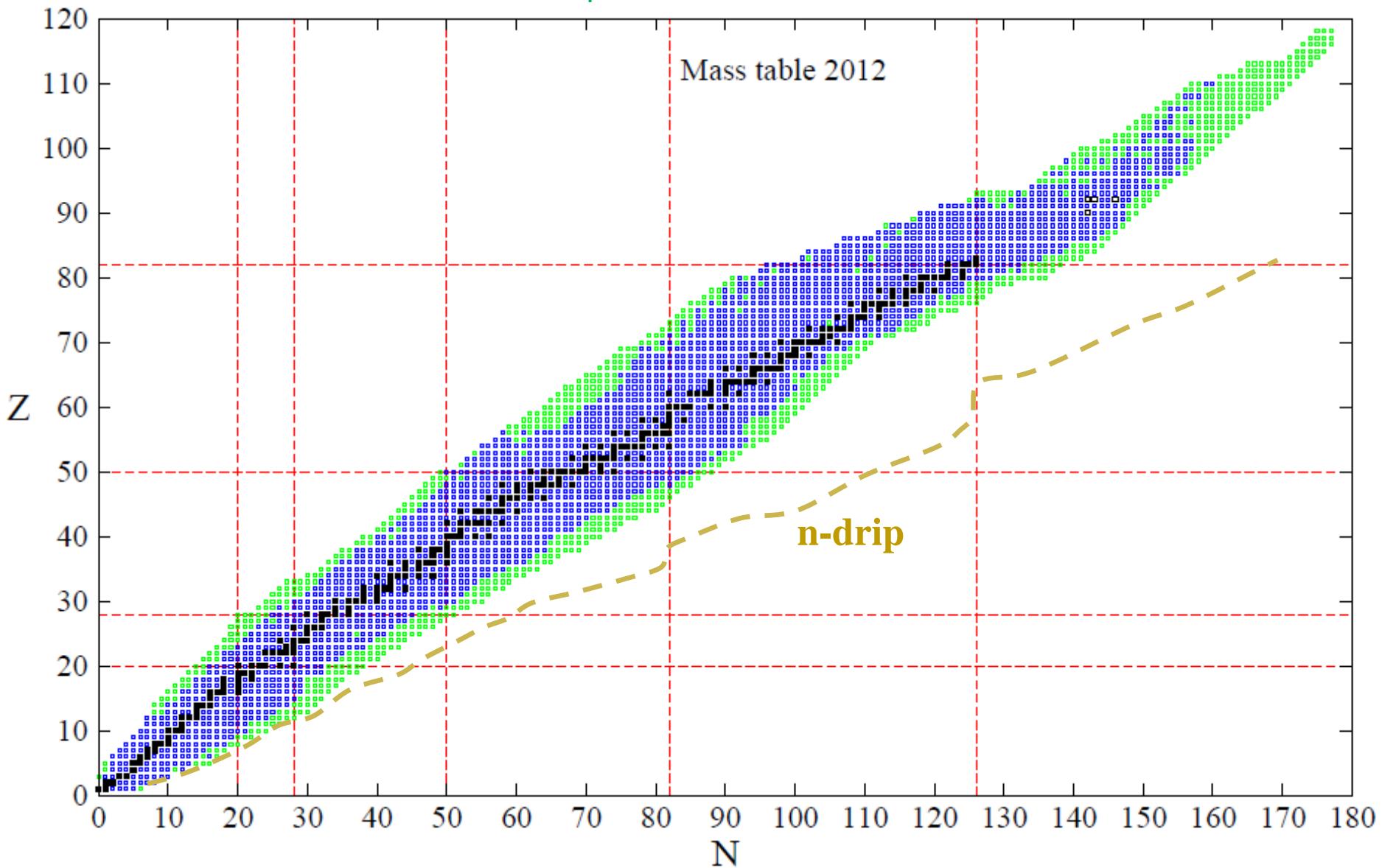
'Normalized' a-parameter (BSFG) predicted from the mass table 2012



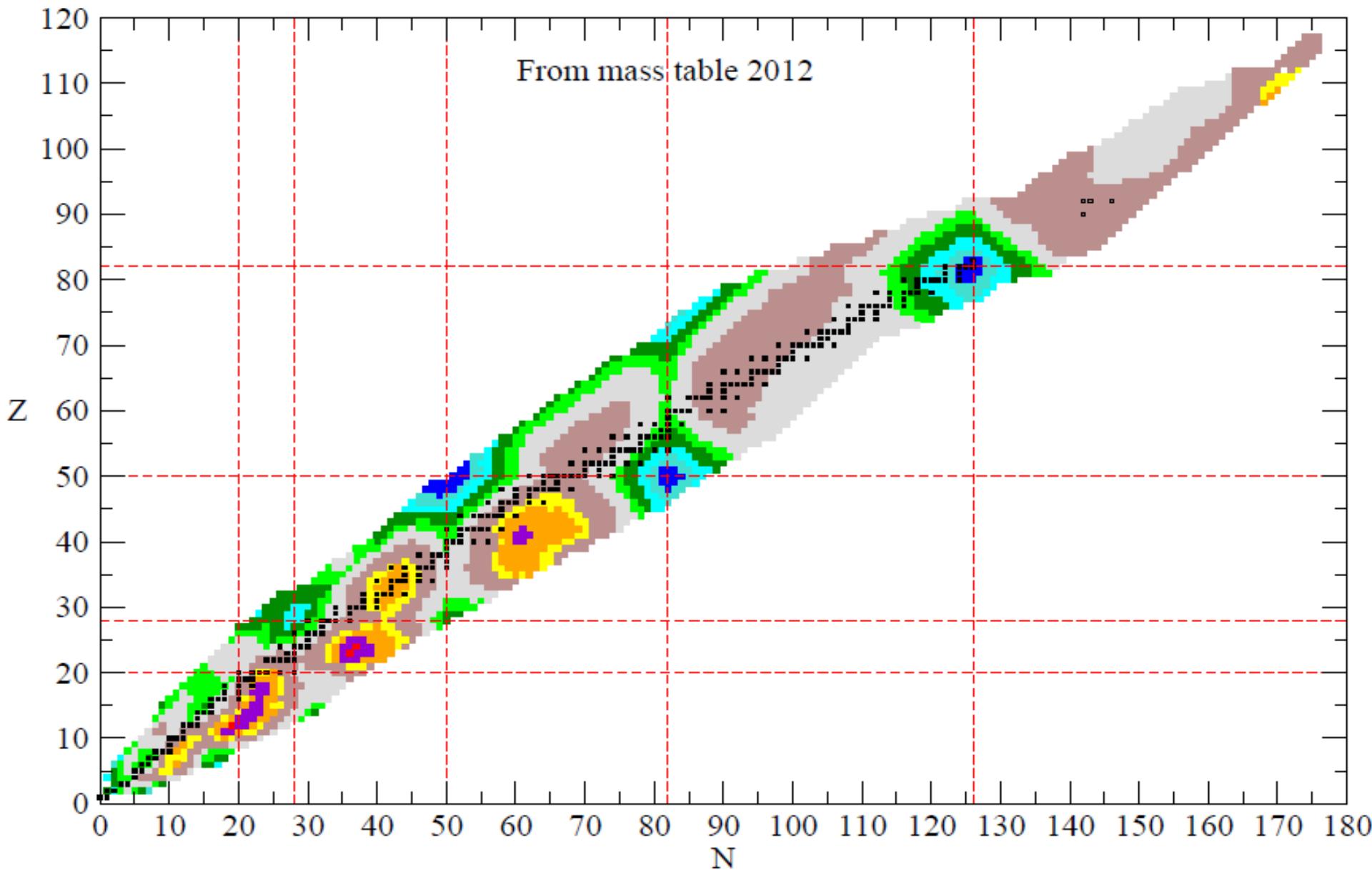
'Normalized' T-parameter (CT) predicted from the mass table 2012



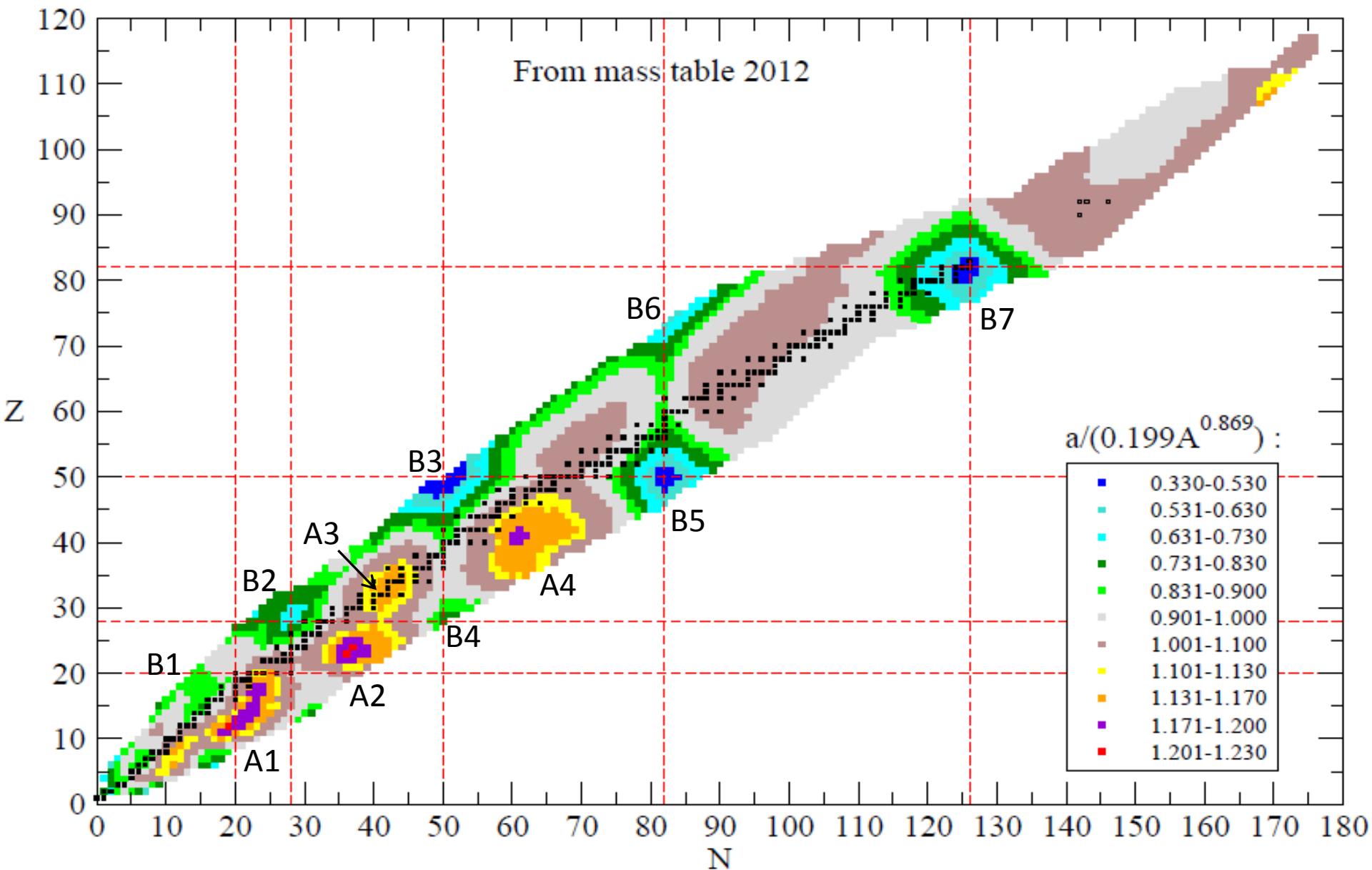
# Measured and extrapolated masses in 2012 mass table

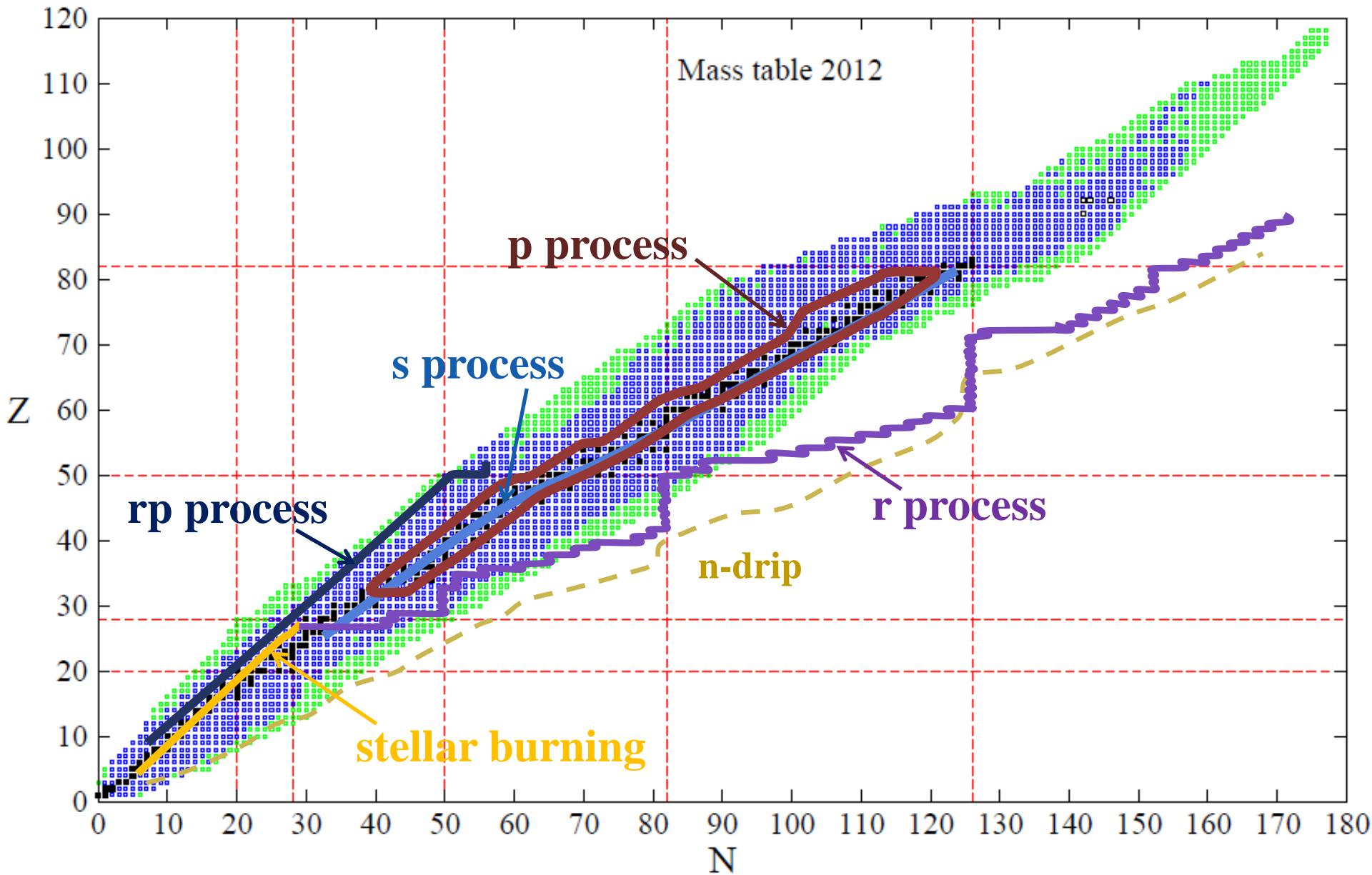


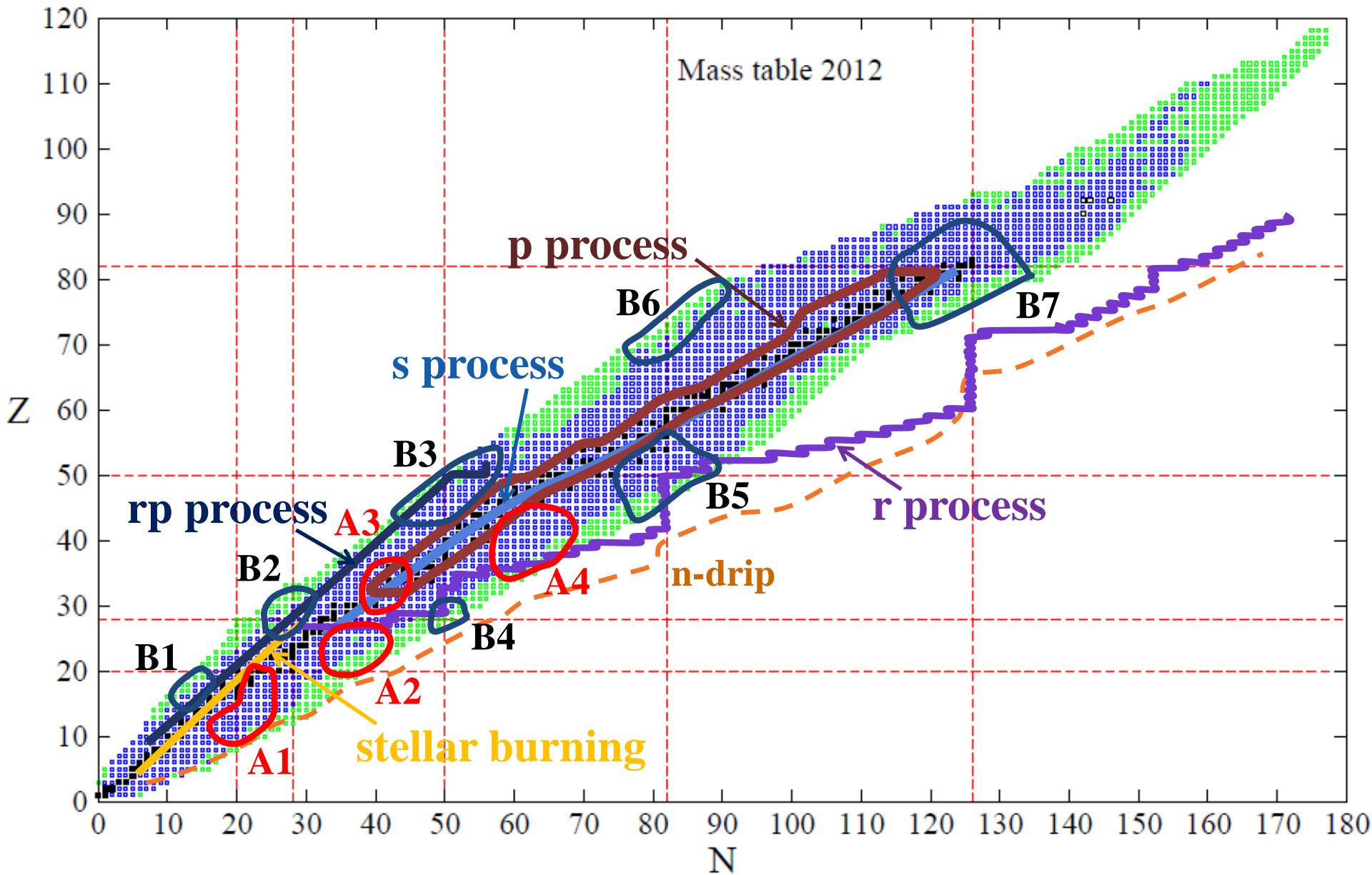
# 'Normalized' a-parameter predicted from 2012 mass table



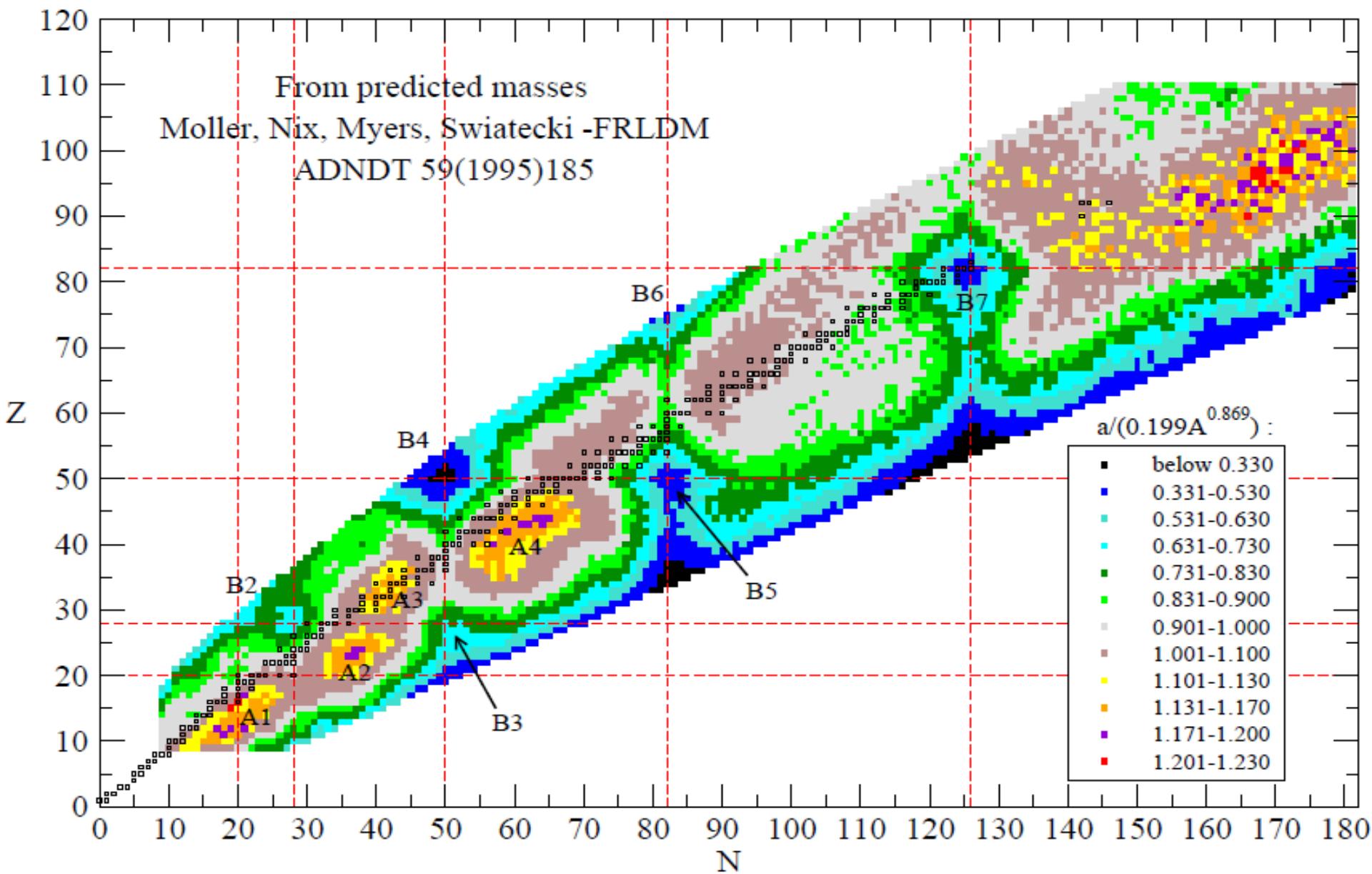
# 'Normalized' a-parameter predicted from mass table 2012

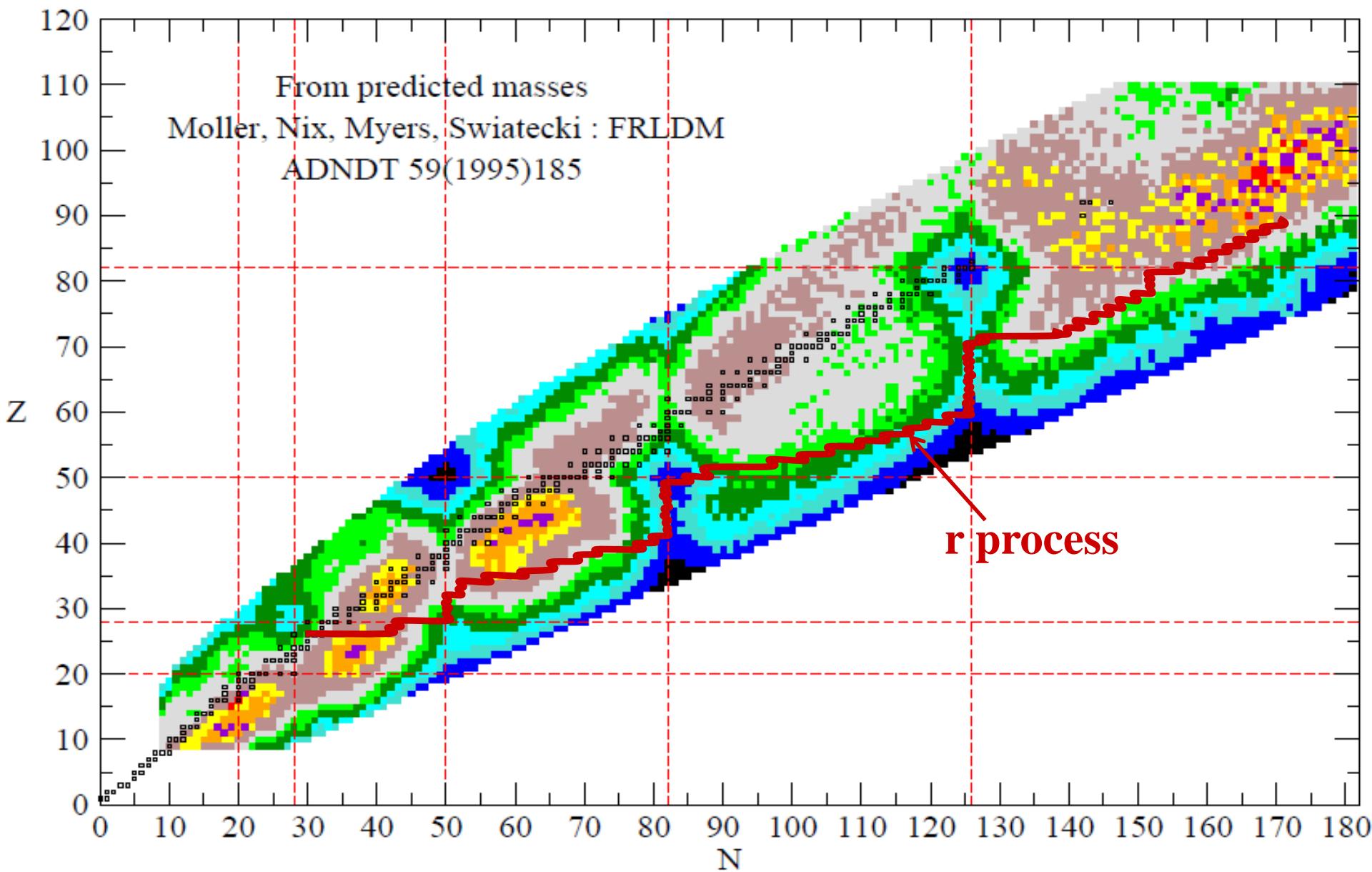




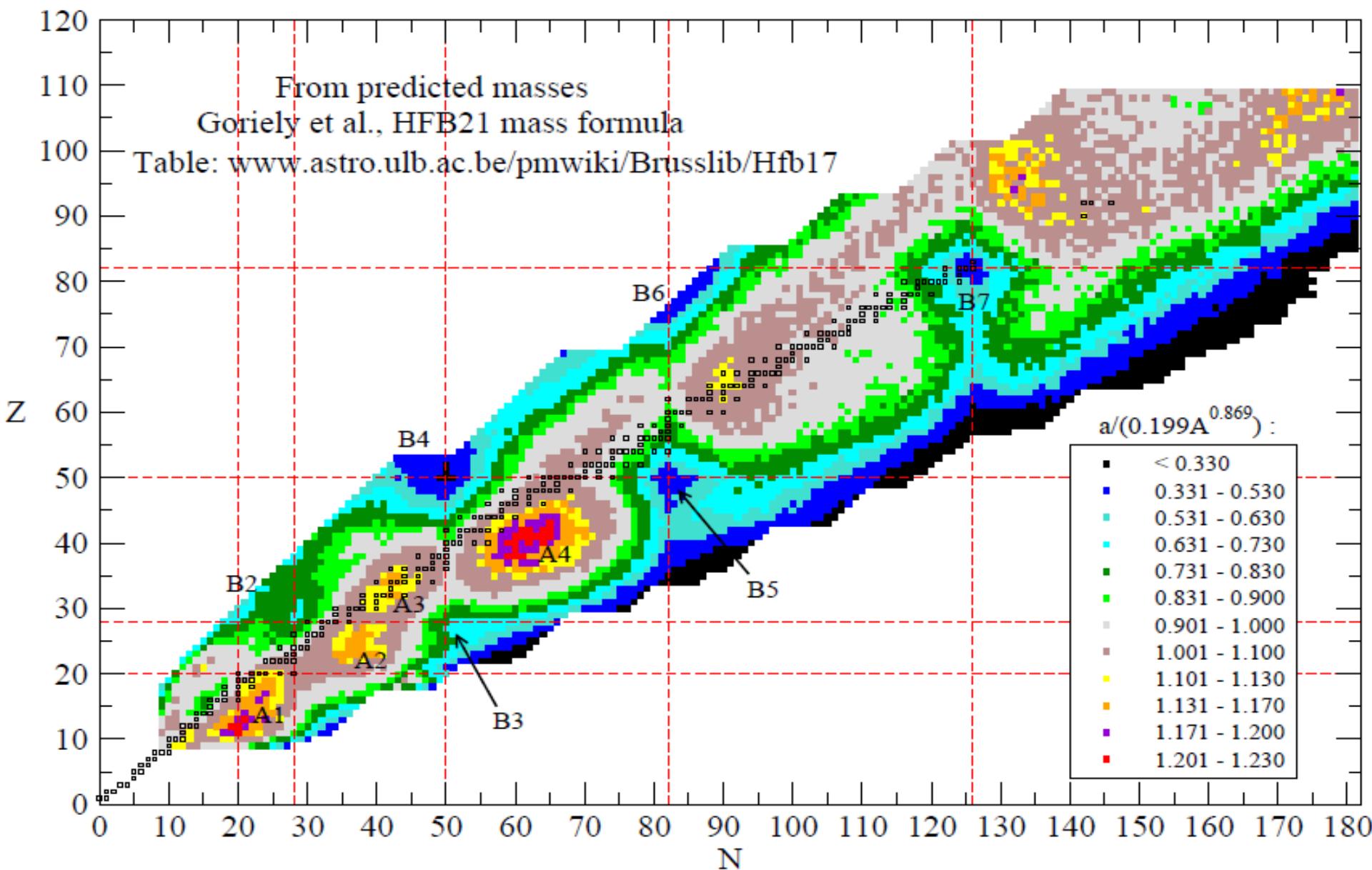


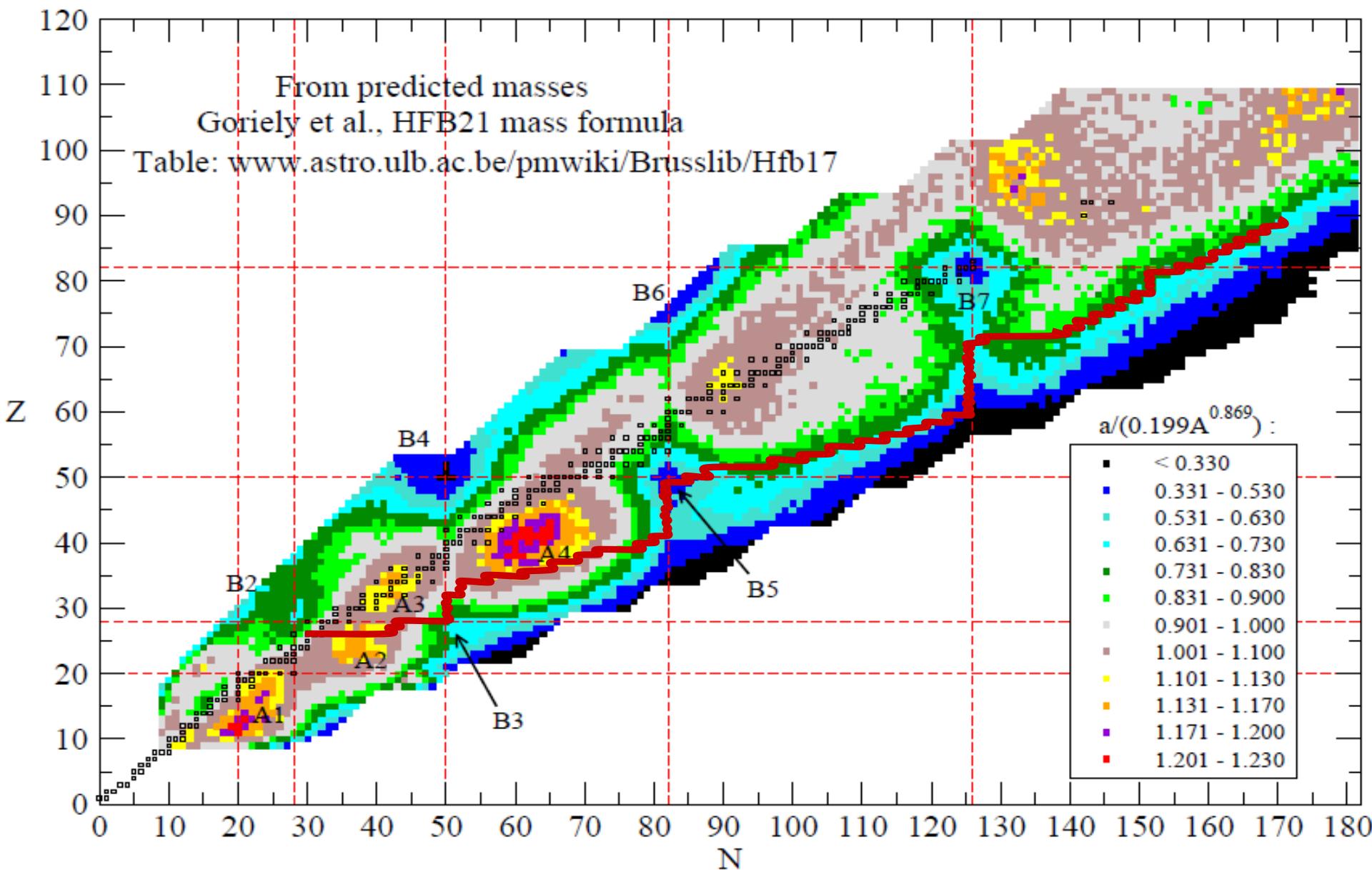
# 'Normalized' a-parameter predicted from Moller-Nix table

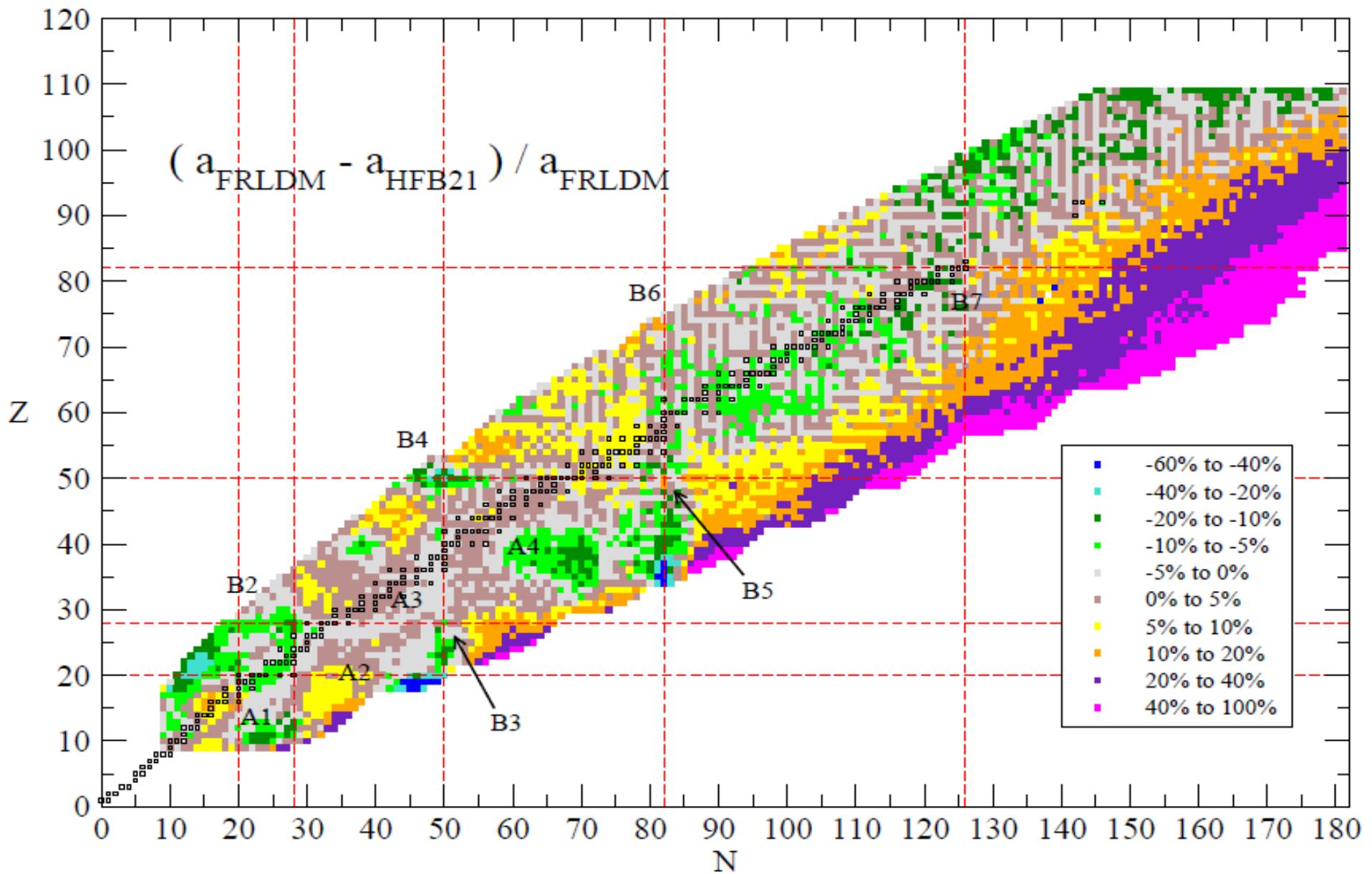


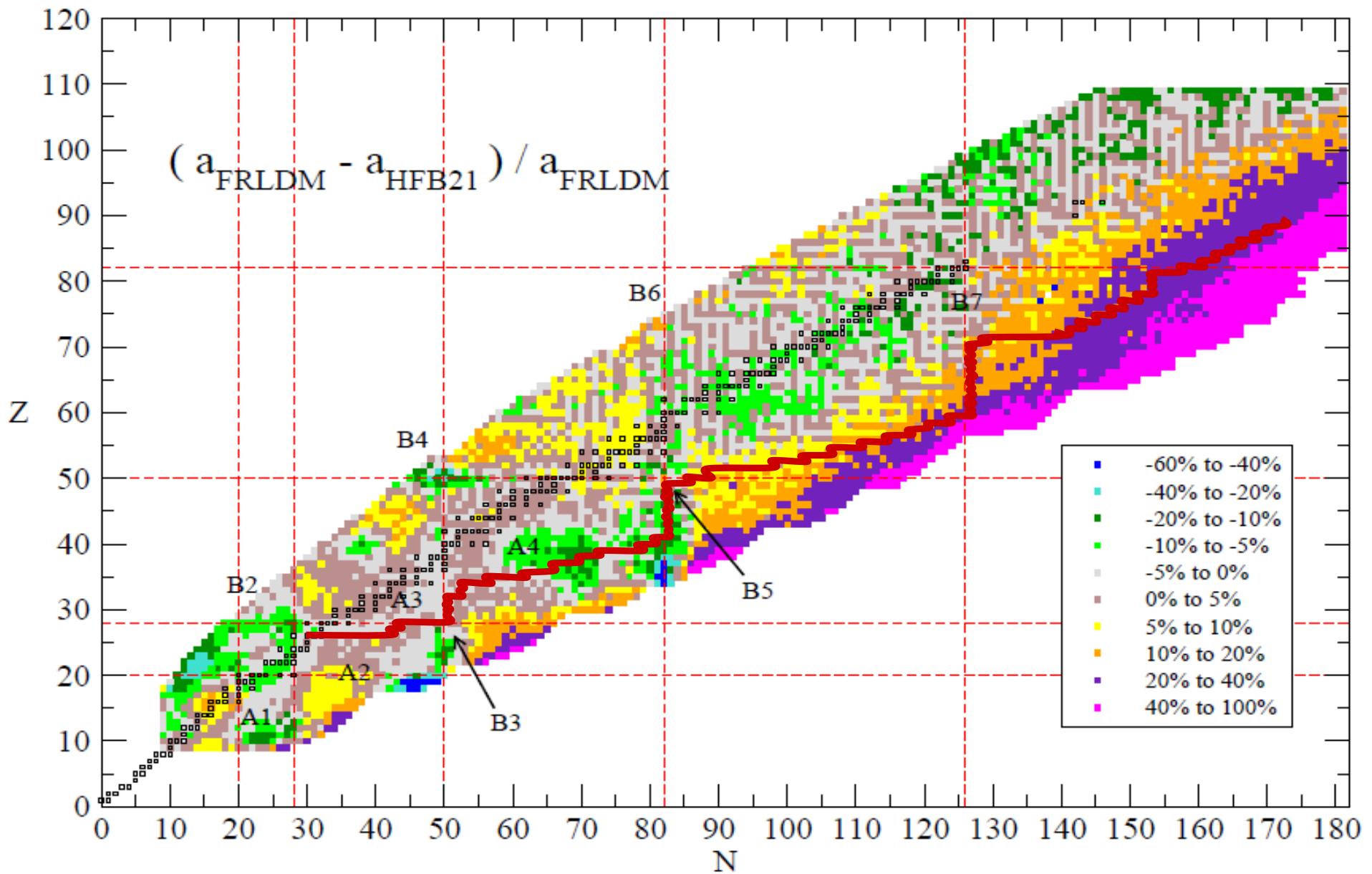


# 'Normalized' a-parameter predicted from HFB21









## **STATISTICS**

Nr. of nuclei with experimentally determined level densities: 310

### **Mass table 2012**

Nr. of nuclei with tabulated mass :	3348
with measured masses :	2433
with extrapolated masses :	915
Nr. of nuclei with normalized-a within $1 \pm 0.1$ :	2031
below 0.9 :	1037
above 1.1 :	994

### **Moller-Nix mass table**

Nr. of nuclei from mass table ( $8 < Z < 111$ , $N < 182$ ) :	6400
Nr. of nuclei with normalized-a within $1 \pm 0.1$ :	2980

### **HFB21 mass table**

Nr. of nuclei from mass table ( $8 < Z < 110$ , $N < 182$ ) :	6091
Nr. of nuclei with normalized-a within $1 \pm 0.1$ :	2892