Triaxiality of heavy nuclei as essential feature to predict radiative capture

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Radiative capture of fast neutrons

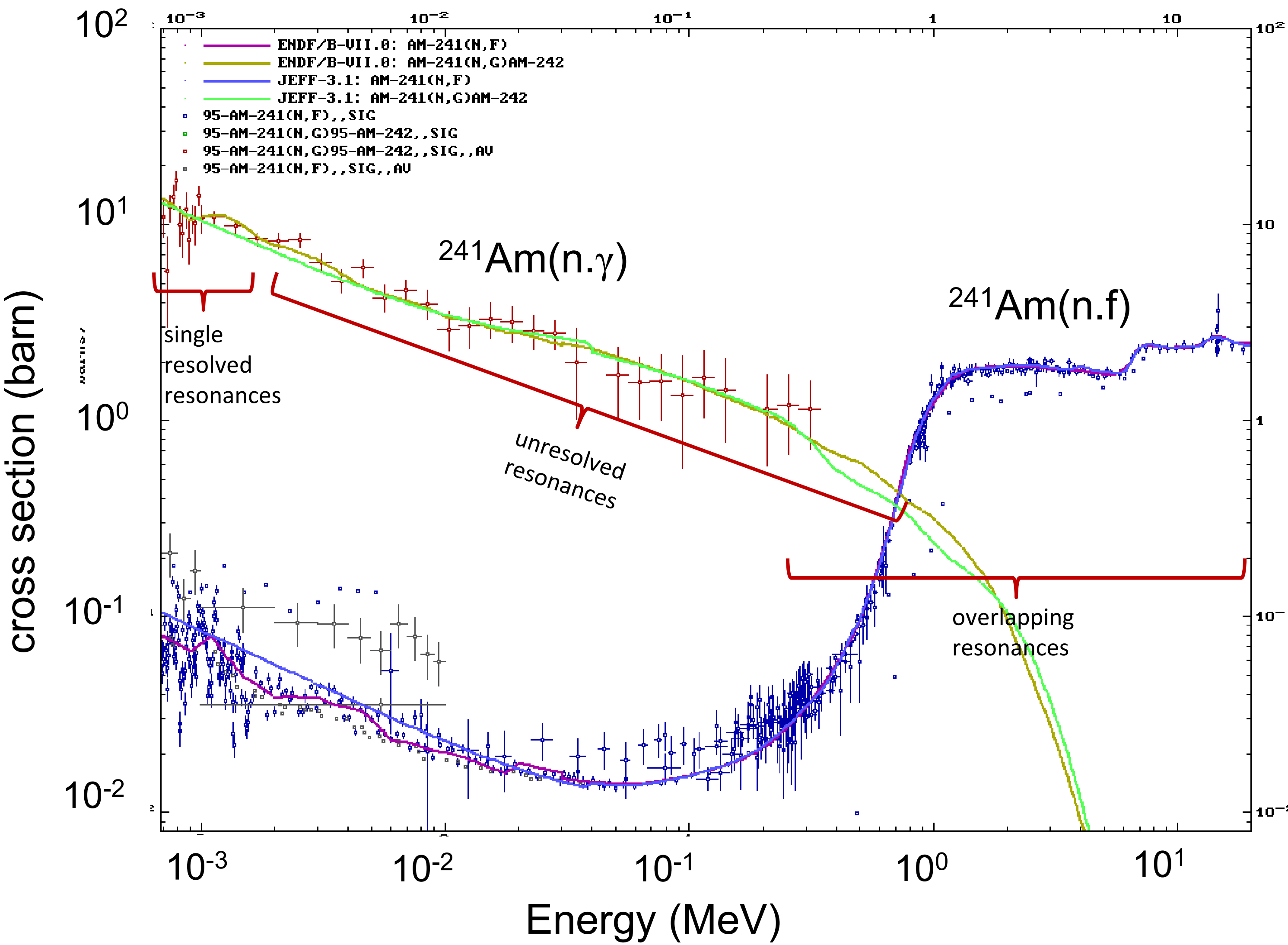
Level densities and nuclear shape

Electric dipole and other radiative strength

Predictions for capture cross sections /Maxwellian averages

Triaxiality as general property of many heavy nuclei

*Neutron induced **fission** in competition to **radiative neutron capture***



***better measurements** needed*
*as well as **model calculations***

Salvatores. NEA report No. 6090 (2006)

Radiative capture (ℓ_n) ***,*** *averaged over resonances* ***r***
and summed over final bound states **b,** *reached by γ-decay of multipolarity λ=1*

$$\langle\sigma(\mathrm{n},\gamma)\rangle_r \cong 2\pi^2\, ƛ_n^2 \cdot \sum_{\ell_n}(2\ell_n+1)\cdot\langle\frac{\Gamma_n\cdot\bar{\Gamma}_\gamma}{\Gamma_n+\bar{\Gamma}_\gamma}\rangle_r\cdot\rho(E_r,Jr); \qquad \bar{\Gamma}_\gamma=\sum_{J_b} g\,\frac{f_1(E_\gamma)\,E_\gamma^3}{\rho(E_r,J_r)}$$

$E_n > 3$ **keV**, $\bar{\Gamma}_\gamma \ll \Gamma_n$, $E_\gamma = E_r - E_b$; ***Axel-Brink hypothesis***

$$\Rightarrow\ \langle\frac{\Gamma_n\cdot\bar{\Gamma}_\gamma}{\Gamma_n+\bar{\Gamma}_\gamma}\rangle_r \cong \langle\bar{\Gamma}_\gamma(E_\gamma,J_b\leftrightarrow J_r)\rangle_r = \sum_{J_b} g'\int_0^{E_r}\frac{f_1(E_\gamma)\,E_\gamma^3}{\rho(E_r,J_r)}\,\rho(E_b,J_b)\,\mathrm{d}E_\gamma$$

s-capture by I=0 target, $\ell_n=0$, $J_r = ½^+$

$$\Rightarrow\ \langle\sigma(\mathrm{n},\gamma)\rangle_r \cong 2\pi^2\, ƛ_n^2\cdot\sum_{J_b} g'\int_0^{E_r} f_1(E_\gamma)\,E_\gamma^3\cdot\rho(E_b,J_b)\,\mathrm{d}E_\gamma$$

cross section is proportional to ***photon strength*** $f_\lambda(E_\gamma)$ *and to* ***level density*** $\rho(E_b, J_b)$,
both *are influenced by nuclear symmetry, incl. non-axial deformation (i.e.* ***triaxiality****)*

J.H. Gibbons et al., Phys. Rev. 122 (1961) 182

G. A. Bartholomew et al., Adv. Nucl. Phys. 7 (1972) 229

Phase transition** (pt) **from BCS regime to Fermi gas [Gilbert & Cameron, 1965]

=> *the intrinsic quasiparticle state density $\omega(E_x)$ at excitation energy E_x is well approximated by*

$$\omega_{\mathrm{qp}}(E_{\mathrm{x}}) = \frac{1}{T}\exp\left(\frac{E_x - E_0}{T}\right) \quad \text{for } E < E_{\mathrm{pt}} \qquad \text{and} \qquad \omega_{\mathrm{qp}}(E_{\mathrm{x}}) = \frac{\exp\left(2\sqrt{\mathrm{a}\cdot(E_{\mathrm{x}} - E_{bs})}\right)}{\sqrt{\frac{144}{\pi}\mathrm{a}^{1/2}(E_{\mathrm{x}} - E_{bs})^{5/2}}} \quad \text{for } E \geq E_{\mathrm{pt}}$$

$$t_{\mathrm{pt}} = \Delta_{\mathrm{o}}\cdot \mathrm{e}^{\mathrm{C}}/\pi = 0.567\cdot\Delta_0\,; \quad E_{\mathrm{pt}} = \mathrm{a}\cdot t_{\mathrm{pt}}^{\,2} + E_{\mathrm{con}} - \delta E$$

*In infinite **nuclear matter** (nm) the **parameter a** is related to the nucleon's Fermi energy ε_F and it determines the energy $\boldsymbol{E_{con}}$ of the pairing induced **condensation; δ**a and **δ**E correct for finite nuclei :*

$$\mathrm{a}_{\mathrm{nm}} = \frac{\pi^2\cdot \mathrm{A}}{4\,\varepsilon_{\mathrm{F}}}\;;\quad \mathrm{a} = \mathrm{a}_{\mathrm{nm}} + \delta\mathrm{a}\;;\quad \delta\mathrm{a} = \alpha\cdot \mathrm{A}^{2/3} \quad \textit{and} \quad E_{\mathrm{con}} = \frac{3\,\mathrm{a}_{\mathrm{nm}}}{2\,\pi^2}\,\Delta_0^2; \quad \textit{backshift: } E_{\mathrm{bs}} = E_{\mathrm{con}} - \delta E$$

*This gives the intrinsic **quasi-particle state density** in a finite nucleus,*
*but **<u>not</u>** yet the number of levels of **well defined spin.***

The underlying symmetry has to defined before:

1. ***spherical*** ⇒ ***only q-p states***
2. ***axial*** ⇒ ***q-p states and axial rot-vib***
3. ***non-axial = triaxial*** ⇒ ***q-p states and arbitrary rotations***
4. ***no reflection symmetry*** ⇒ ***q-p states, arbitrary rotations, octupole deformation***

Phase transition** (pt) **from BCS regime to Fermi gas [Gilbert & Cameron, 1965]

=> the intrinsic quasiparticle state density $\omega(E_x)$ at excitation energy E_x is well approximated by

$$\omega_{qp}(E_x) = \frac{1}{T}\exp\left(\frac{E_x - E_0}{T}\right) \quad \text{for } E < E_{pt} \qquad \text{and} \qquad \omega_{qp}(E_x) = \frac{\exp\left(2\sqrt{a\cdot(E_x - E_{bs})}\right)}{\sqrt{\frac{144}{\pi}\, a^{1/2}(E_x - E_{bs})^{5/2}}} \quad \text{for } E \geq E_{pt}$$

$$t_{pt} = \Delta_o \cdot e^C/\pi = 0.567\cdot\Delta_0\,;\; E_{pt} = a\cdot t_{pt}^2 + E_{con} - \delta E$$

*In infinite **nuclear matter** (nm) the **parameter** **a** is related to the nucleon's Fermi energy ε_F and it determines the energy **E_{con}** of the pairing induced **condensation; δ**a and **δ**E correct for finite nuclei :*

$$a_{nm} = \frac{\pi^2\cdot A}{4\,\varepsilon_F}\,;\;\; a = a_{nm} + \delta a\,;\;\; \delta a = \alpha\cdot A^{2/3} \quad \textit{and} \quad E_{con} = \frac{3\,a_{nm}}{2\,\pi^2}\,\Delta_0^2; \quad \textit{backshift: } E_{bs} = E_{con} - \delta E$$

***Collectivity with respect to 3 axes** increases the level density by pulling quasiparticle modes down into collective (e.g. rotational) bands built on top of each intrinsic state*

$$\rho\,(E_x, J, \pi) \cong \sum_{\tau}^{2J+1} \exp\left(-\frac{\sum_i E_i(J,\tau)}{t}\right)\cdot\omega_{qp}(E_x) \cong \frac{\sqrt{8\pi}\,\sigma_1\sigma_2\sigma_3}{2\cdot 4\cdot\sqrt{8\pi}\,\sigma^3}\cdot(2J+1)\cdot e^{-\frac{(J+1/2)^2}{2\sigma^2}}\cdot\omega_{qp}(E_x) \xrightarrow{\text{small } J} \frac{2J+1}{2\cdot 4}\,\omega_{qp}(E_x)$$

Summation *"over the different rotational levels in a given band having the same value of J"*, labeled by τ;
reduction by 4 accounts for $\mathcal{R}$-symmetry [Bjørnholm, Bohr, Mottelson; Rochester conf. 1974]

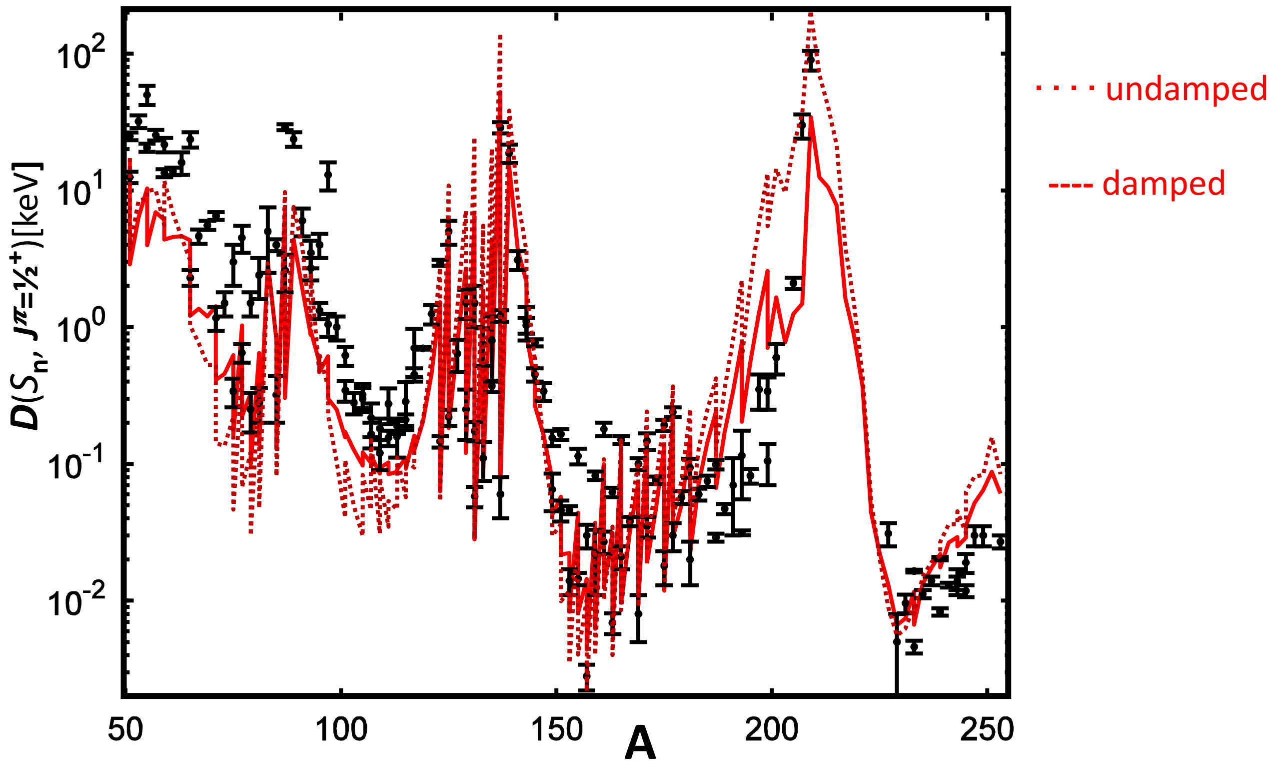
*The **'old'** redistribution of quasi-particle states into levels of distinct spin **implicitly assumes spherical symmetry** of the nucleus **at $E_x = S_n$** and neglects the nuclear modes which differ from quasi-particle excitations* [Vigdor & Karwowski, 1982]**:**

$$\rho_{sph}(E_x, J, \pi) \approx \frac{2J+1}{2\cdot\sqrt{8\pi}\cdot\sigma^3}\cdot e^{-\frac{(J+1/2)^2}{2\sigma^2}}\cdot\omega_{qp}(E_x) \xrightarrow{\text{small } J} \frac{2J+1}{2\cdot\sqrt{8\pi}\cdot\sigma^3}\,\omega_{qp}(E_x) \quad \text{with } \sigma = \sqrt{\frac{\mathfrak{I}\cdot t}{\hbar^2}}$$

***Average spacings** of **s-wave resonances** at* $\mathbf{S_n}$, $I_R=½^+$ *is well reproduced*

*when allowing **triaxiality** in 146 nuclei with* **50<A<254**

with nearly no sensitivity to β & γ



Damping of shell effect** (à la Kataria & Kapoor) **has some influence, it needs no new parameter.

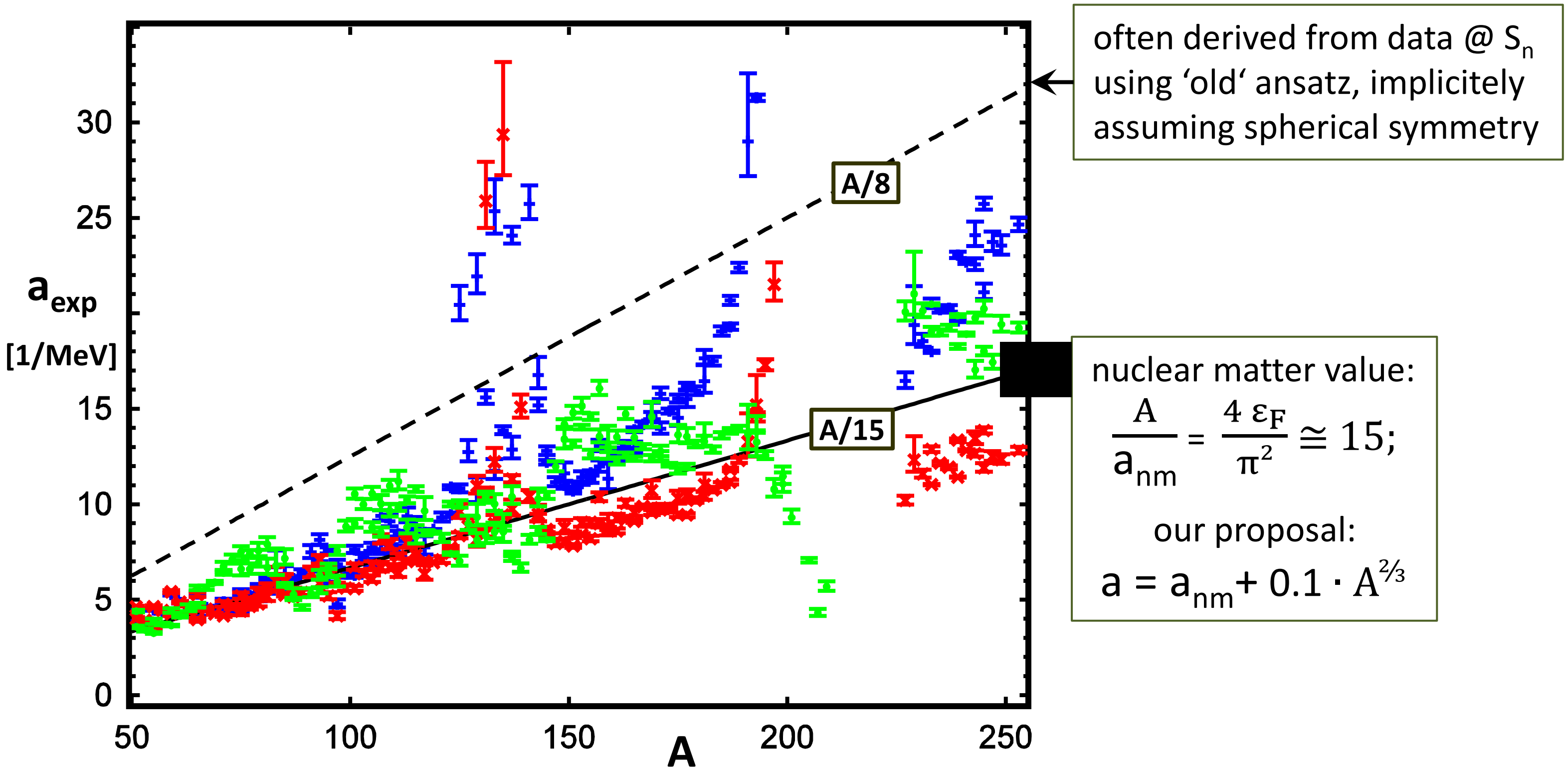
The only quantity fitted to obtain this absolute scale agreement is the surface term **α = 0.1** !

***And: ℛ -symmetry** is assumed, as well established for heavy nuclei (away from scission saddle point),*

*whereas usually **spherical** or **axial symmetry** are assumed **ad hoc**.*

Level density parameter **a** *can be related to observed s-resonance distances at S_n;*

*data can be converted approximately without surface term – **not** assuming shape symmetry.*



***without** shell correction*

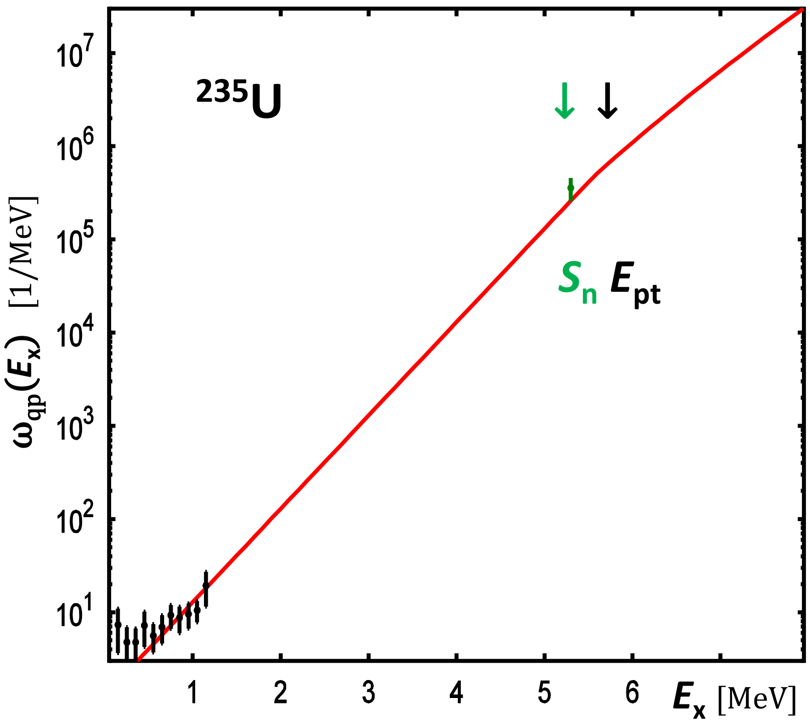
The data were converted** with shell correction from **LDM

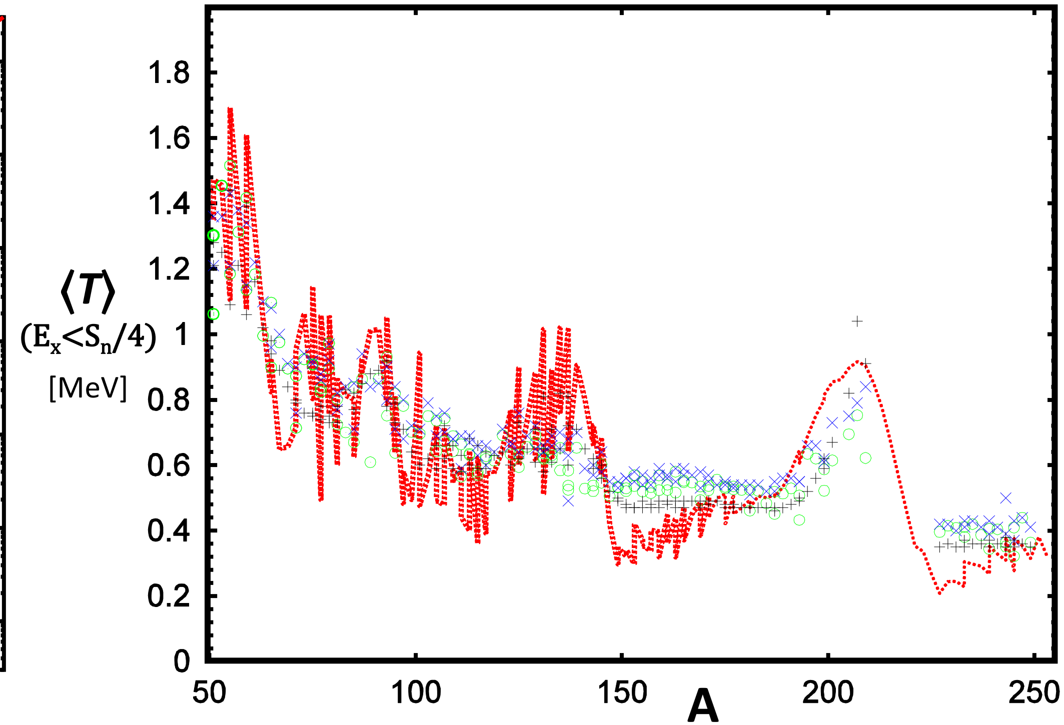
with shell correction from old LDM

Note: **a** has strong influence on E_x-dependence of $\rho \approx \exp(2\sqrt{a \cdot (E_x - E_{bs})}\,)$;
it should thus **not** be adjusted to A-dependence alone.
We stay close to nuclear matter value.

Data: //www-nds.iaea.org/RIPL-3/
Myers and Swiatecki, Ark. Fizik 36 (1967) 343
Myers and Swiatecki, Nucl. Phys. A 81 (1966) 1

*The **level density** formalism compares well to bound states and s-wave resonances – and the temperatures derived from them:*





Inverting the formula

$$\frac{1}{D(E,J)} = \rho(E,J) \cong \frac{2J+1}{4} \cdot \omega_{qp}(E) \cdot e^{-\frac{(J+\frac{1}{2})^2}{2\sigma^2}}$$

allows to extract ω_{qp} from the observed average level distances D(E,J) for all J;

when σ is taken from systematics.

from D_{exp}(bound levels, various J^π)

from D_{exp}(S_n, ½$^+$) .

*The 'nuclear' temperatures $T = \frac{\rho}{\partial\rho/\partial E}$ as derived **locally** from level data – with **significant scatter** – by:*

Koning et al., NPA 810 (08) 13 +

v.Egidy et al., PRC 80 (09) 054310 x

Belgya et al., RIPL-2/3 o

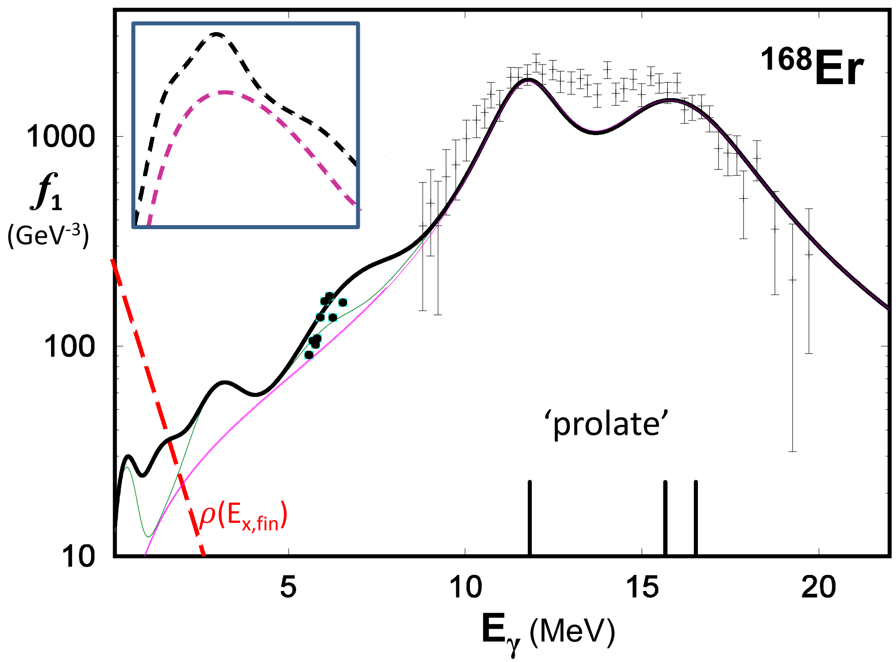
*agree quite well to our **prediction: ------, based on one global parameter only and loss of symmetry.***

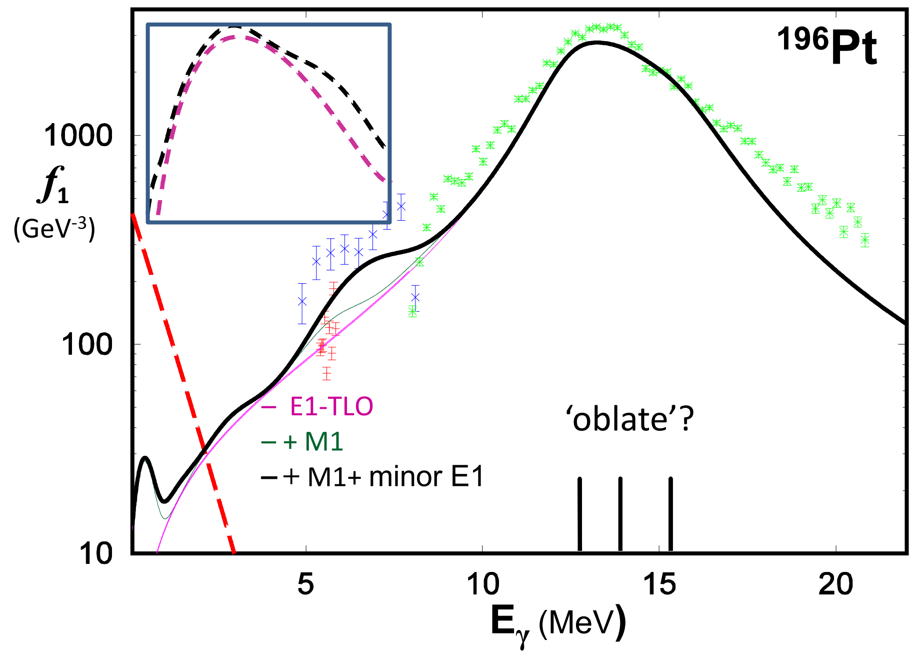
Belgya, Capote et al.,(2012) www-nds.iaea.org/RIPL-3/levels/levels/

Overlap between final level density $\rho(E_x)$ and photon width $\Gamma(E_\gamma)$ peaks at ≈ 3 MeV;

it determines 1st photon yield and sensitivity of radiative capture cross sections to $\rho(Ex)$ and $f(E\gamma)$.

*Additional **'minor'** strength near 3-5 MeV (scissors M1, pygmy E1, $(2^+\otimes3^-)_{1-}$) leads to some enhancement.*





Triple Lorentzian E1-PSF* (TLO) *causes ≲ 80% of yield;

minor components non-negligible ► need of new experimental investigations.

*Good description of **dipole strength** data in **IVGDR** and (n,γ)-data **in the tail***

using axis ratios from HFB and widths $\Gamma_k \propto E_k^{1.6}$.

Gurevich et al., NPA,351(81) 257

Mughabghab & Dunford, PLB 487(00)155

Massarczyk et al., PRC **87**(13) 044306

Goryachev & Zalesnyi, Yad. Fiz. **27** (78) 1479

ERINDA

HZDR

Dipole strength function f_1

***triple** Lorentzian* (**TLO**) *works well:*

for even and odd nuclei:

absolute scale,

two global parameters.

But: *data below S_n, (obtained differently),*

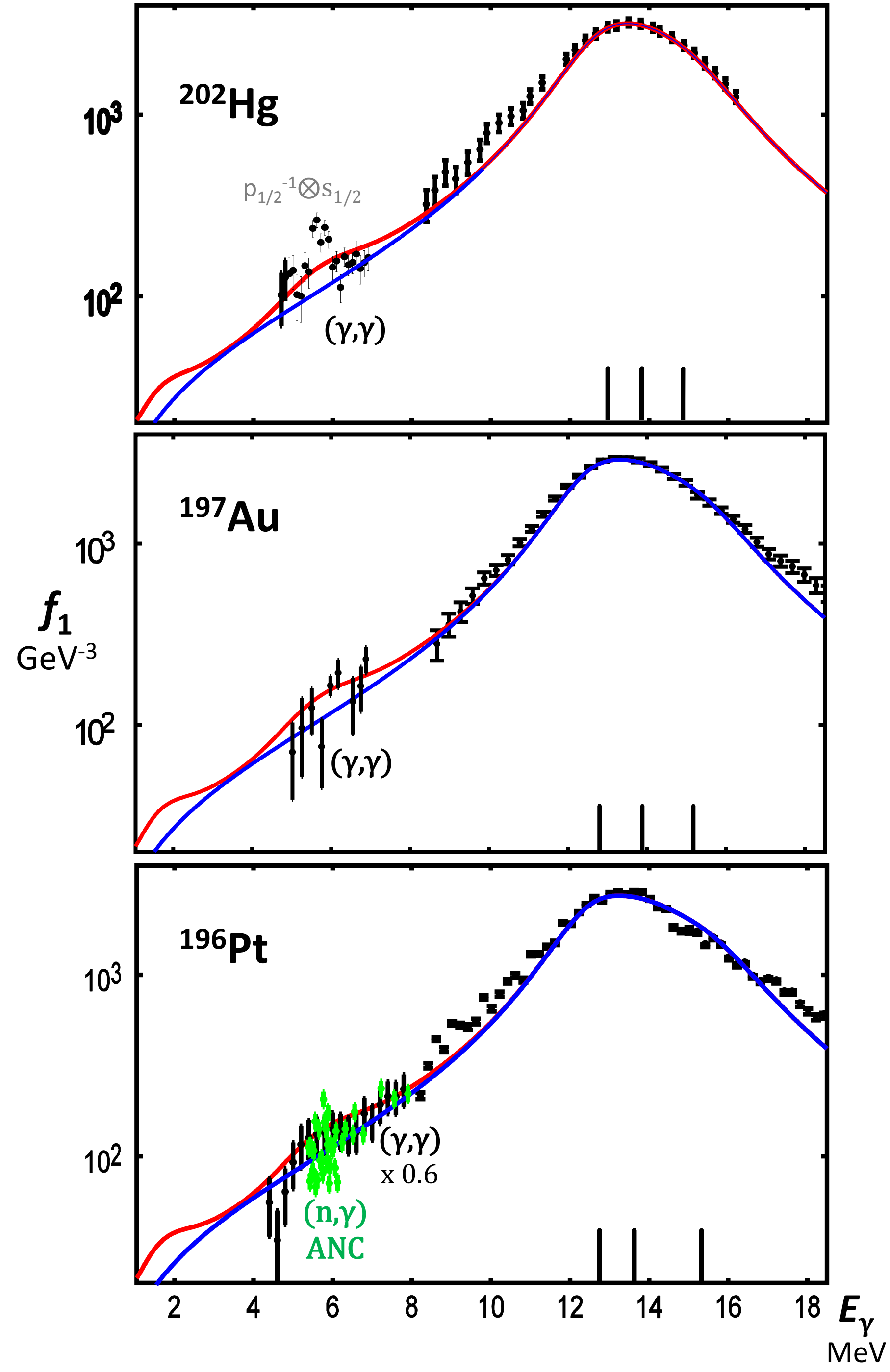
have large uncertainty and

exceed **TLO** → *minor components:*

pigmy-E1, *scissors*-M1, $(3^-\otimes 2^+)1^-$.

Important: *data below 4 MeV are missing,*

needed for radiative capture!



R.M. Laszewski and P. Axel, Phys. Rev. C 19 (1979) 342
G. A. Bartholomew, CGS Studsvik, (1969)
R. Massarczyk et al., Phys. Rev. C 87 (2013) 044306
S.F. Mughabghab, C.L. Dunford, Physics Letters B 487(2000)155

Various collective modes contribute to the photon strength in radiative capture:

E1: ***IVGDR***, *fit by* ***TLO*** *with* ***sum rule (TRK)*** *and global* ***spreading width*** $\Gamma \propto E_{GDR}^{1.6}$: $\int f dE \approx 10\ GeV^{-2}$

isoscalar(IS) **E1** *strength in* ***'pygmy'*** *resonance at* $E_{py} \approx 0.5 \cdot E_{GDR} \approx 6\ MeV$: $\int f dE \lesssim 0.06\ GeV^{-2}$

vibration-coupling : $(2^+ \times 3^-)1^-$ @ $E_{sum} \approx 3\ MeV$; $I \propto B(E2) \cdot B(E3)$: $\int f dE \lesssim 0.04\ GeV^{-2}$

M1: ***orbital*** (scissors) ***mode*** @ $\approx 3\ MeV$; $I_{sc} \approx Z^2 \cdot \beta^2$: $\int f dE \lesssim 0.3\ GeV^{-2}$

$\approx 3\ MeV$

isoscalar and isovector components of spin-flip mode @ $\approx 7\ MeV$: $\int f dE \lesssim 0.1\ GeV^{-2}$

'zero pole' originating from a recoupling of nucleon spins within equal configurations: $\int f dE \lesssim 0.01 GeV^{-2}$

E2: *quadrupole vibrations* @ $\approx 1\text{-}2\ MeV$ *contribute* $\int f dE < 10^{-2}$, *the GQR* @ $\gtrsim 9\ MeV$ $\int f dE < 0.2\ GeV^{-2}$.

The parameters of these minor contributions to strength are approximated based on intensive experimental studies at e-beams, which determine transition strength $f_\lambda(0 \rightarrow E_{coll})$ *from ground.*

Axel-Brink hypothesis *predicts same strength on top of any quasi-particle state* E_{qp}, *causing collectively enhanced decay transitions* $f_\lambda(E_x \rightarrow E_{qp}) = f_\lambda(0 \rightarrow E_\gamma = E_x - E_{coll})$.

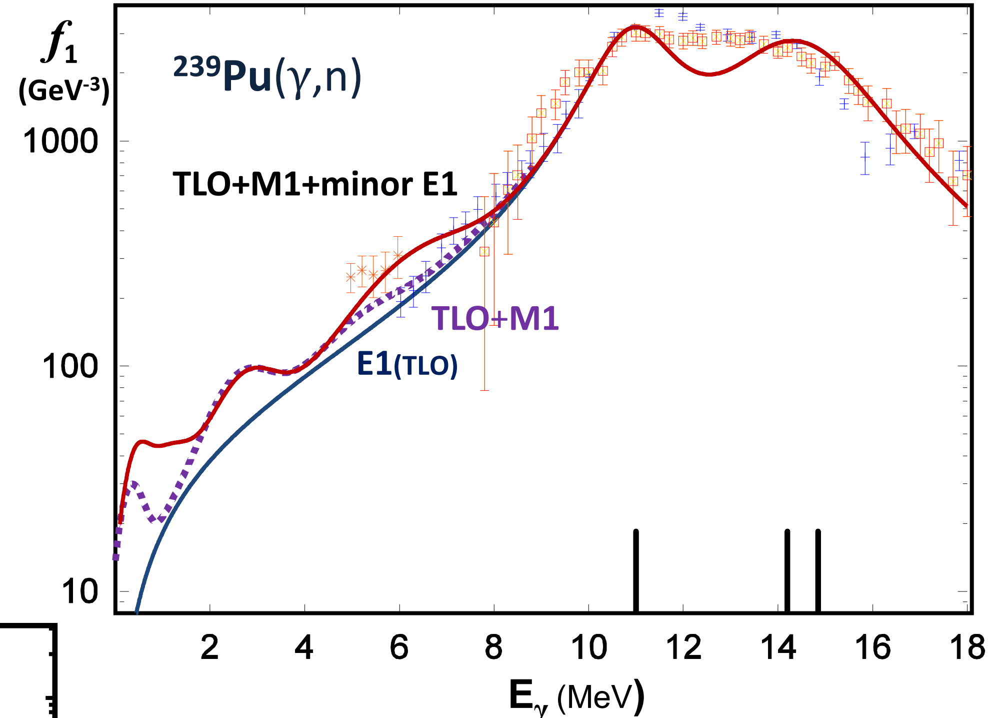
Respective structures may appear in CN-reaction spectra (BNL, LASL, Oslo, Ohio ..) and they contribute to ***radiative capture*** *of n and p – especially for* $E_\gamma \approx 3\ MeV$.

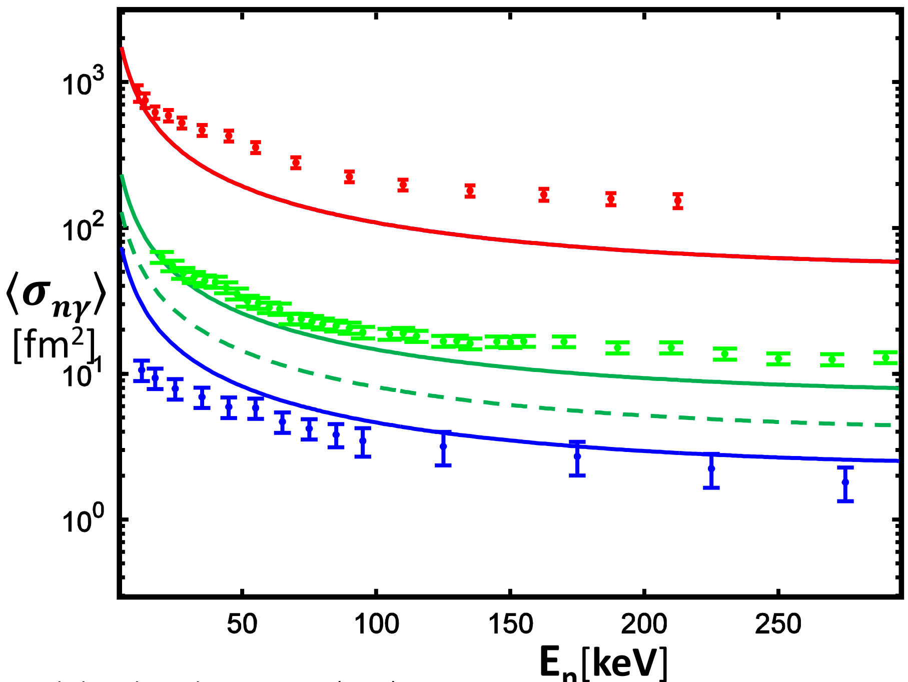
Poelhekken et al., PLB 278 (92) 423
Pysmenetska et al., Phys. Rev. C 73 (2006) 017302
von Garrel et al., Phys. Rev. C 73 (2006) 054315
Kneissl et al., J. Phys. G 32 (2006) R217
Andrejtscheff et al., Phys. Lett. B 506 (2001) 239

Heyde et al., Rev,Mod.Phys 82 (2010) 2365
Enders et al., Phys. Rev. C 71 (2005) 014306
Richter, Prog. Part. Nucl. Phys. 34 (1995) 261
Schwengner et al., Phys. Rev. L 111 (2013) 232504

The ***photon strength*** *and* ***level density parametrizations*** *presented here also work well for* ***actinides:***

Calculation agrees well to data without any new parameter,
indicating a possible use for transmutation applications;
other channels ⇒ overshoot.





^{232}Th(n,γ) × 10

^{238}U(n,γ) + minor
- - - - TLO-E1 only

^{240}Pu(n,γ) ÷ 10

Photon strength other than GDR-tail (TLO)
=> *isovector* E1
has ≲ 50 % influence on radiative capture
mainly orbital M1, isE1, (E3⊗E2)1$^-$.

K. Wisshak et al., Nucl.Sc.E.137,183 (2001)
W. Poenitz, Nucl.Sc.E.57,300 (1975)
L.W.Weston,J.H.Todd, Nucl.Sc.E.63,143 (1977)

+ G.M. Gurevich et al., Nucl. Phys. A 351, 257 (1981)
⌶ B.L. Berman et al., Phys. Rev. C 34 (1986) 2201
x Y. Birenbaum, et al., Phys. Rev. C 36 (1987)1293

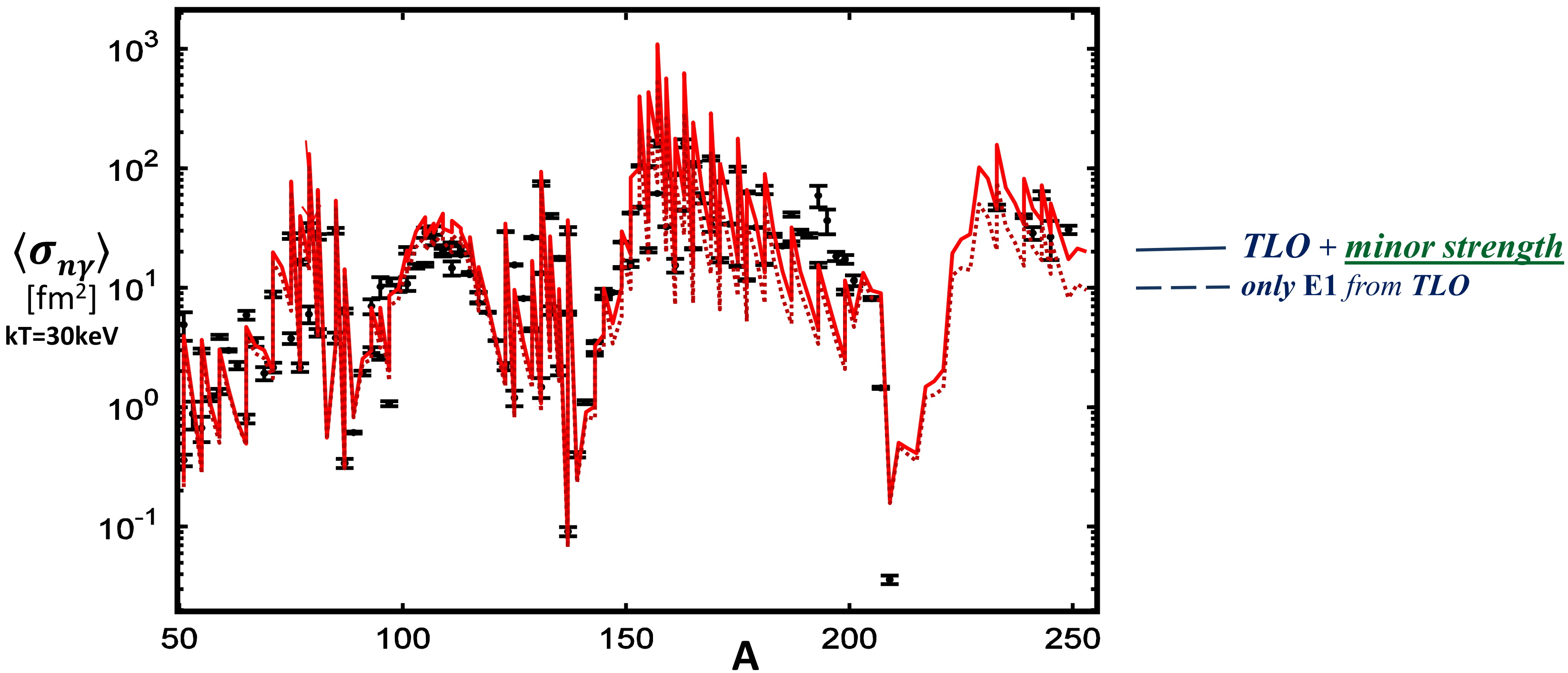
Maxwellian averages from simultaneous global predictions for

*average **level distances** at S_n and **photon widths** for **radiative neutron capture** (unresolved resonance region)*

*=> **test of** the **TLO-photon strength $f_1(E_\gamma)$** and the **level density parameterization.***

Maxwellian averages are a good measure for keV neutrons

$$\langle\sigma\rangle_{kT} = \frac{2}{\sqrt{\pi}} \frac{\int_0^\infty \sigma(E_n)\, E_n\, e^{-E_n/kT}\, dE_n}{\int_0^\infty E_n\, e^{-E_n/kT}\, dE_n} \qquad \Phi = dN/dE_n \sim \sqrt{E_n} \cdot e^{-E_n/kT}$$



***good agreement to Maxwellian averages for >100 nuclei** with predominant s-capture.*

Global predictions are possible, as ⟨σ⟩ depend significantly only on a –

and also on $f_1(E_\gamma)$, on the nuclear symmetry, and the choice of shell correction δW_o;

Dillmann et al., PRC81 (10) 015801; Pritychenko et al., At.D and Nucl.DT 96 (2010) 645; AIP Conf. Proc. **819**, 123; www.kadonis.org www.nndc.bnl.gov/astro

Myers & Swiatecki, Ark. Fyzik. 36 (67) 343;

Recommendations:

1. ***Consider the presence of <u>triaxiality</u>*** *at higher E_x* ***for***
   a. *the extrapolation of dipole strength from* IVGDR *data*
   b. *the projection of ω_{qp} to $\rho(E_x)$ out of the intrinsic into the 'lab' frame*
2. ***Make sure to use FG level density formulae with $a \cong a_{nm}$,***

   otherwise the dependence of ρ on E_x will be too steep and σ_{CN} too big.

   There is a good chance that 'your' Hauser-Feshbach code needs a change.

Acknowledgements:

Discussions with H. Feldmeier, K.-H. Schmidt, R. Schwengner and H. Wolter

are gratefully acknowledged.

*For **level density** and **photon strength** in heavy nuclei **triaxiality** plays an important role – an issue rarely adressed in **model calculations,** (e.g. Hartree-Fock-Bogolyubov).*

*Probabely the disregard of **triaxiality** is related to numerical problems (of theorists), resulting from performing a 3D-**angular momentum projection** before the variation.*

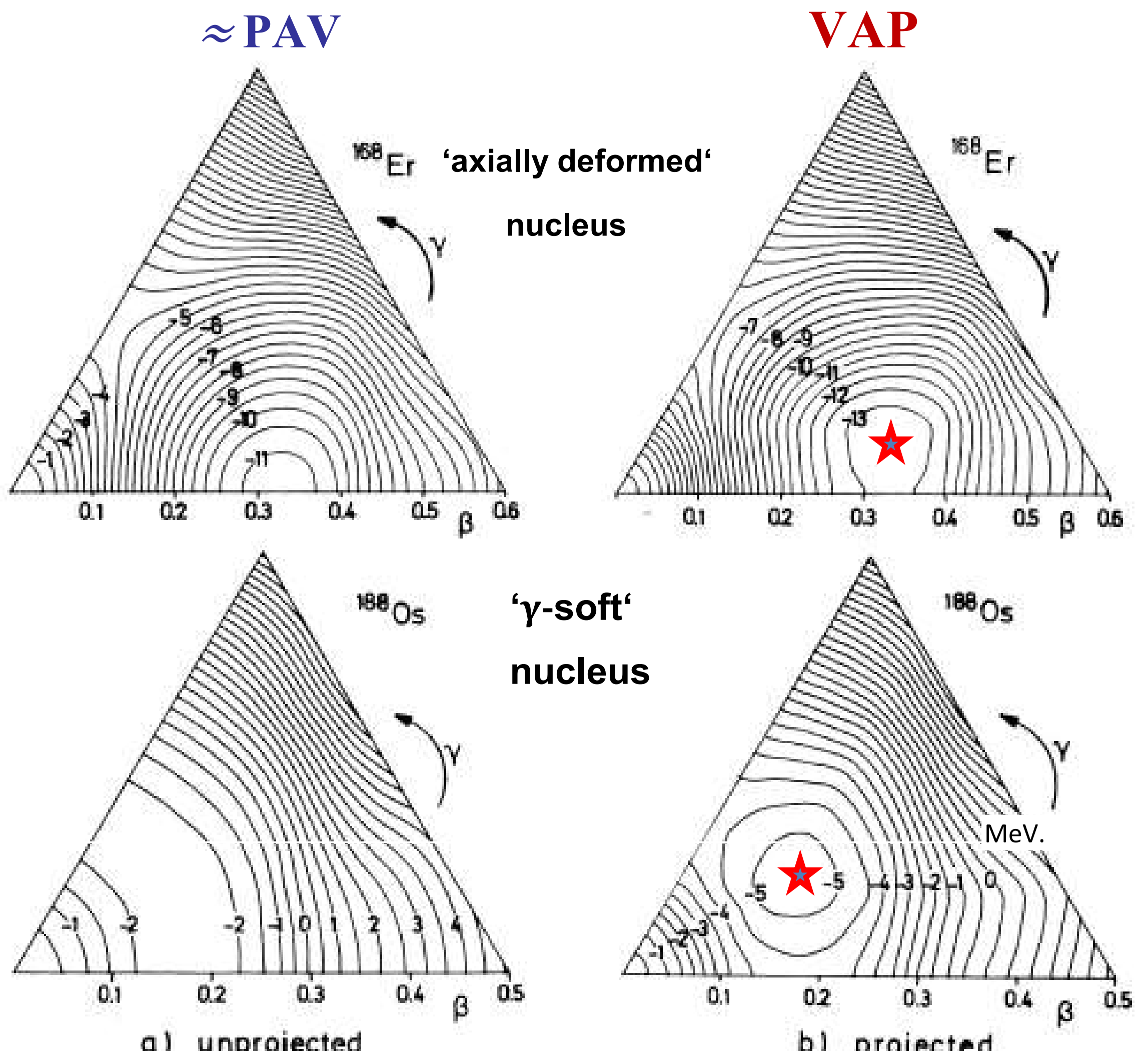


FIG. 1. Energy surface in the β-γ plane for the nuclei ^{168}Er and ^{188}Os (a) without angular momentum projection and (b) with exact three-dimensional angular momentum projection. The units on the equipotential lines are megaelectron-

Hayashi, Hara, Ring , PRL 53 (1984) 337