Ekman boundary layers in a fluid filled precessing cylinder

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Astro MHD 29th September 2020

HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

Outline

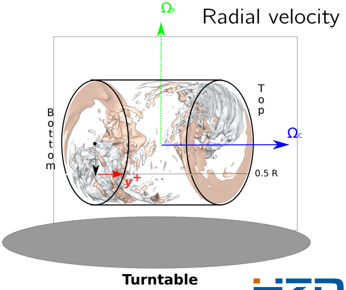
1. Scope of the study
2. Ekman layer theory
3. Application to precessing cylinder endwall and results from simulation
4. Extrapolation and connection with former results

Research questions and problems

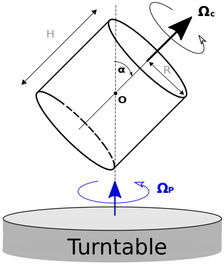
- What is the behaviour of the boundary layer on the cylnder endcaps
- How it evolves in the phase space of Ek (cylinder rotations) and Po (precession force strenght) numbers? $\rightarrow$ Instabilities, turbulence...
- Does the endwall boundary layers interact/adapt with the bulk flow?

$\Downarrow$

Problem: finding reference scales for the study of boundary layer (thickness, reference velocity). The precession driven flows are intrinsecally 3D



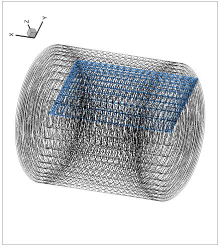
Math formulation and tools



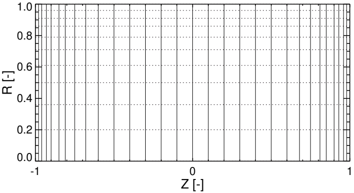
$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\boldsymbol{\nabla} p - 2\,Po\hat{\boldsymbol{k}} \times \boldsymbol{u} + Ek\nabla^2 \boldsymbol{u}$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$Po = \frac{\Omega_p}{\Omega_c} \quad Ek = \frac{\nu}{\Omega_c R^2} \quad \Gamma = \frac{H}{R}$$



$\Downarrow$ Numerical



Solving through DNS code SEMTEX http://users.monash.edu.au/~bburn/semtex.html

General theory of Ekman layer

- Ekman boundary layer: type of boundary layer developed in rotating systems, characterized by the balance between Coriolis and viscous forces
- Study methods: math (stability analysis), DNS, lab experiment, atmospheric measurements. Linear laminar theory: scaling with $\sqrt{Ek}$

Caldwell, Van Atta, J. Fluid Mech. (1970), 44

Main parameter to classify the Ekman layer:

$$Re_\delta = \frac{U_{ref}\delta_{Ek}}{\nu}$$

- U_{ref} is the reference velocity (geostrophic) at the BL edge
- δ_{Ek} the thickness $\approx \sqrt{Ek} = \sqrt{\nu/(\Omega L^2)}$

		Theory		Experiment		
Type of instability	Quantity	Faller & Kaylor	Lilly	Faller, Faller & Kaylor	Tatro *et al.*	Present work
Type II (class A)	Critical Reynolds number	55	55	< 70	56·3 + 116·8 *Ro*	56·7
	Wavelength, in Ekman depths	24	21	22 to 33	27·8 ± 2·0	(Frequency 61% of that predicted by Lilly)
	ϵ†	− 15°	− 20°	+ 5° to − 20°	0 to − 8°	
	Velocity divided by geostrophic	0·50	0·57	—	0·16	
Type I (class B)	Critical Reynolds number	118	110	125 ± 5	124·5 + 7·32*Ro*	—
	Wavelength, in Ekman depths	11	11·9	10·9	11·8	—
	ϵ†	10°–12°	8°	+ 14·5 ± 2·0°	+ 14·8° ± 0·8°	—
	Velocity divided by geostrophic	0·33(11°)	0·094	0·023(14·5°)	0·034	—

ϵ† is the angle between wavefront and geostrophic wind.

TABLE 1. Summary of Ekman layer instabilities

Fully turbulent Ekman layer for $Re_\delta > 150$

HZDR

Idea for precession problem (weak regime)

First step: decomposing and filtering the DNS.

$$A_{mnk} = \int_V \boldsymbol{u}_{DNS} \cdot \boldsymbol{u}^*_{mnk} dV \,/ N_{mnk}$$

$$\boldsymbol{u}_{DNS} = \underbrace{\overbrace{\sum_{k=1}^{K} A_{00k}\, \boldsymbol{u}_{00k}\,(r)}^{axisymmetric} + \overbrace{\sum_{m=1}^{M}\sum_{k=1}^{K} \frac{1}{2}\left[A_{m0k}\, \boldsymbol{u}_{m0k}\,(r,\varphi)) + c.c\right]}^{non-axisymmetric}}_{Geostrophic}$$

$$+ \underbrace{\sum_{n=1}^{N}\sum_{k=1}^{K} \frac{1}{2}\left[A_{0nk}\, \boldsymbol{u}_{0nk}\,(z,r) + c.c\right]}_{axisymmetric \quad oscillation}$$

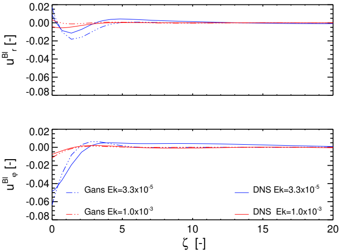
$$+ \underbrace{\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{2K} \frac{1}{2}\left[A_{mnk}\, \boldsymbol{u}_{mnk}\,(z,r,\varphi) + c.c\right]}_{inertial \quad waves} + \boldsymbol{u}^{BI}$$

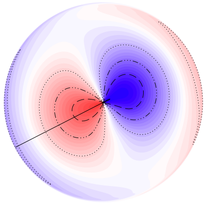
$\Rightarrow$

(m,n,k)	$\lvert A_{mnk} \rvert$
$(1,1,1)$	6.97×10^{-2}
$(0,0,2)$	2.02×10^{-3}
$(0,2,1)$	1.79×10^{-3}
$(2,0,1)$	1.42×10^{-3}
$(1,3,2)$	9.83×10^{-4}
$(1,1,2)$	6.58×10^{-4}
$(1,1,1)^-$	6.26×10^{-4}
$(1,5,1)$	5.89×10^{-4}

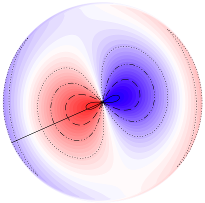
$Ek = 3.3 \times 10^{-5}$ $\quad Po = 10^{-3}$

$\downarrow$





DNS



Linear analytical theory

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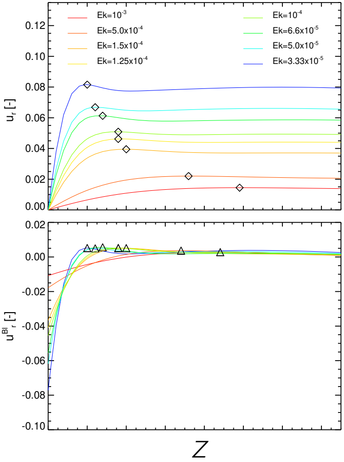
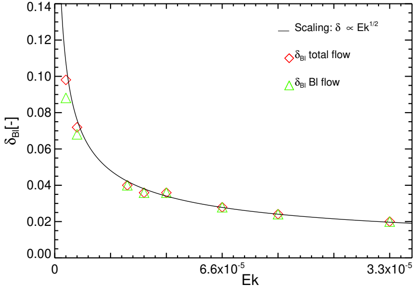
Thickness in weak precession

Precession driven flows in cylinder present Ekman layers on the endwalls. (Meunier, Gans, Zhang, Kong...)

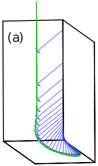


Checking the scale $Po/\sqrt{Ek} \ll 1$



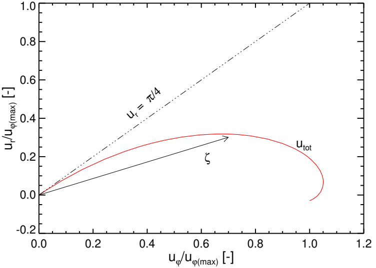
Particular features of precessing Ekman layer

Precession Ekman layer has the scaling of pure rotating flows but...
there are different behaviours: Ekman Spiral.



Spalart et al. Phys. of Fluids (2008), **20**, *101507*

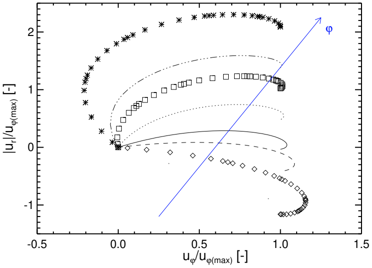
Spiral at $\varphi \approx 0$ for $Po/\sqrt{Ek} \ll 1$



$m = 1$

$\Rightarrow$

Poincaré force gives an azimuthal distribution.



The φ impacts on the axial vorticity of the flow (bulk and BL).

HZDR

General study (up to Po)

Kong, et al., Geo. e Astro. Fluid Dynamics (2014)

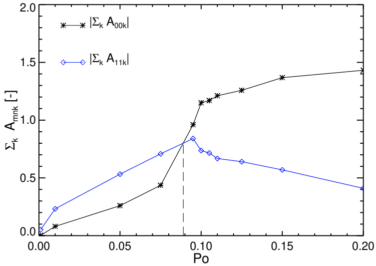
$$\boldsymbol{u}_{DNS} = \underbrace{\overbrace{\sum_{k=1}^{K} A_{00k}\, \boldsymbol{u}_{00k}(r)}^{axisymmetric} + \overbrace{\sum_{m=1}^{M}\sum_{k=1}^{K} \frac{1}{2}\left[A_{m0k}\, \boldsymbol{u}_{m0k}(r,\varphi)) + c.c\right]}^{non-axisymmetric}}_{Geostrophic}$$

$$+ \underbrace{\sum_{n=1}^{N}\sum_{k=1}^{K} \frac{1}{2}\left[A_{0nk}\, \boldsymbol{u}_{0nk}(z,r) + c.c\right]}_{axisymmetric\quad oscillation}$$

$$+ \underbrace{\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{2K} \frac{1}{2}\left[A_{mnk}\, \boldsymbol{u}_{mnk}(z,r,\varphi) + c.c\right]}_{inertial\quad waves} + \boldsymbol{u}^{BI}$$

$$\Longrightarrow$$

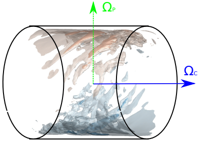
$Re = 10000,\ \alpha = 90°$



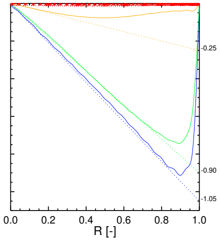
Emergence of an axisymmetric-geostrophic current for large Po.
Amplitude A_{00} with azimuthal and axial wave umber $(m,n) = (0,0)$.

$$A_{mnk} = \int_V \boldsymbol{u}_{DNS} \cdot \boldsymbol{u}^*_{mnk}\, dV \,/ N_{mnk}$$

$u_z \to 0$ in the bulk for large Po



$$U_g(r)\hat{\boldsymbol{\varphi}} = \sum_k A_{00k}\, \boldsymbol{u}_{00k}(r)$$

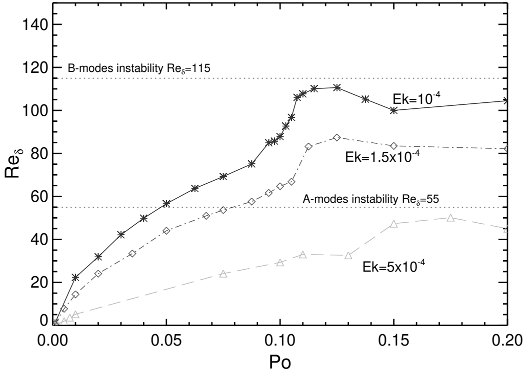


$$\Omega_g = \frac{dU_g(r)}{dr}\Big|_{r=0}$$

Characterization of Ekman layer

Definition of Re_δ in precessing cylinder:

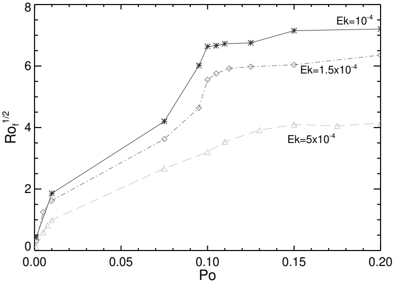
$$Re_\delta = \frac{(\Omega_g R^2)\sqrt{Ek}}{\nu}$$

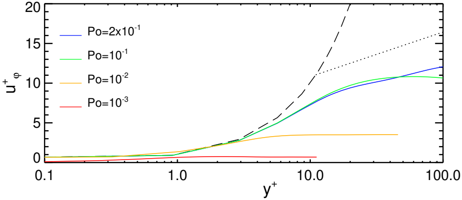


- Profiles depend by $Ek = \nu/(R^2\Omega_c)$
- Peaks around $Po \approx 0.125$ for small Ek
- no cross of B-mode instability and no fully turbulent boundary layer (on the endwalls)

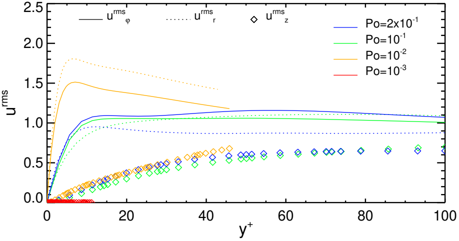
Other quantities

$$y^+ = \frac{z u_\tau}{\nu} \qquad u_\tau = \sqrt{\partial u_\varphi / \partial z|_{wall}}$$





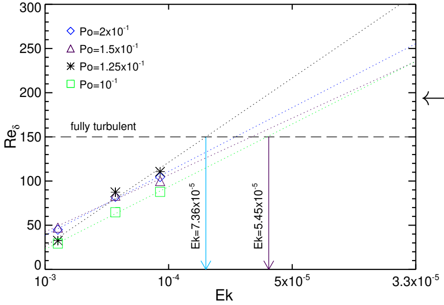
$$Ro_f = \frac{u_\tau^2}{f'\nu} \qquad f' = 2\Omega_c + U_g/r + dU_g/dr$$



$$u^{rms} = \frac{\sqrt{\langle u'_j u'_j \rangle}}{u_\tau}$$

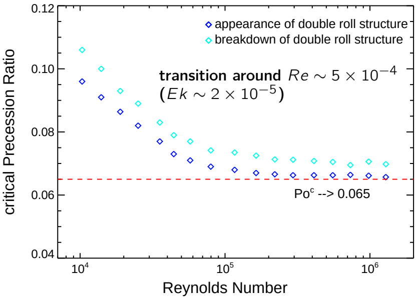
- NO turbulent phenomena.
- Flow follow the viscous sub-layer but not achievement of log-law
- fluctuations strenght small

Connection with previous experiments



← Extrapolation for turbulent Ekman layer

Relation with experiment



- critical precession ratio for transition to turbulence in dependence of Re.
- UDV measurements: up to $Re \approx 2 \times 10^6$ show **asymptotic behaviour** for $Re \geqslant 10^5$. $Po \rightarrow 0.065$
- width of instable region seems to be largely **independent** of Re

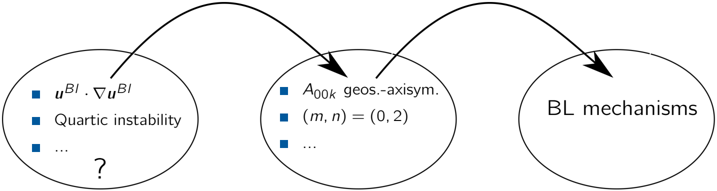
Discussions

Summary

- characterization of the bulk flow structure through A_{mnk}
- Geostrophic-axisymmetric flow dominance for strong precession
- characterization of Ekman (endwalls) layer in precessing cylinder $f(Ek, Po)$
- Appearence of instabilities but no fully turbulent Ekman layer (in the range of our simultaions)
- Reasonable prediction of turbulence onset (consistency with asymptotic profile shown by experiments)

Limitations of this study:

- results valid for large precession angle α and not extreme Ek.
- turbulent study requires finer mash (e.g 10 grid points $0 < y^+ < 10$).
- no insights to decouple bulk mechanism and Ekman mechanism: **<u>what causes what</u>**?



Possible future study

Experiments on the Ekman layer (PIV ??).

Theoretical connection with bulk flows phenomena

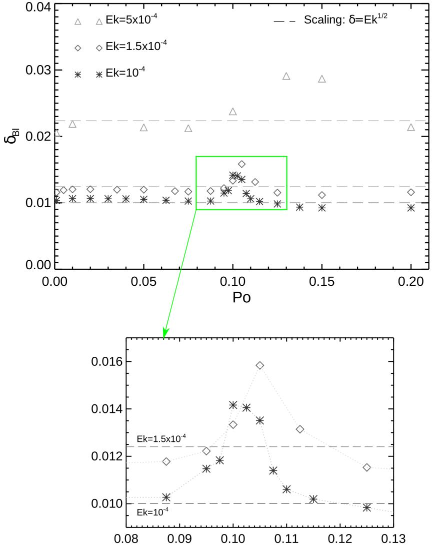
Deeper analalysis on the connection of $(m, n) = (0, 2)$ with the Ekman suction (average thickness could increase around $Po \approx 0.10$)

Ekman (endwall) layers phenomena in MHD context

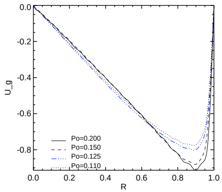
The end

Thank you for your attention

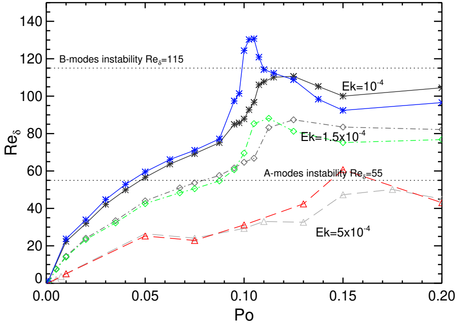
Appendix



δ_{BI}
Po
Ek=5x10^{-4}
Ek=1.5x10^{-4}
Ek=10^{-4}
Scaling: δ=Ek$^{1/2}$



U_g
R
Po=0.200
Po=0.150
Po=0.125
Po=0.110



Re$_\delta$
Po
B-modes instability Re$_\delta$=115
A-modes instability Re$_\delta$=55
Ek=10^{-4}
Ek=1.5x10^{-4}
Ek=5x10^{-4}