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New type of helical magnetorotational instability in rotating flows with positive shear

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Outline

1.Overview of helical and azimuthal magnetorotational instabilities (**HMRI** and **AMRI**) at radially increasing angular velocity, i.e., positive shear – the role of the **upper Liu limit**

2.Motivation

-- Can HMRI and AMRI still survive at astrophysically relevant, smaller positive shear, breaking the Liu limit/constraint

3. Revealing a new type of double-diffusive HMRI at positive shear

-- local WKB analysis

-- 1D global linear stability analysis:
Power-law (constant Rossby number) rotation profile
MHD Taylor-Couette (TC) flow

4. **Summary and implications**

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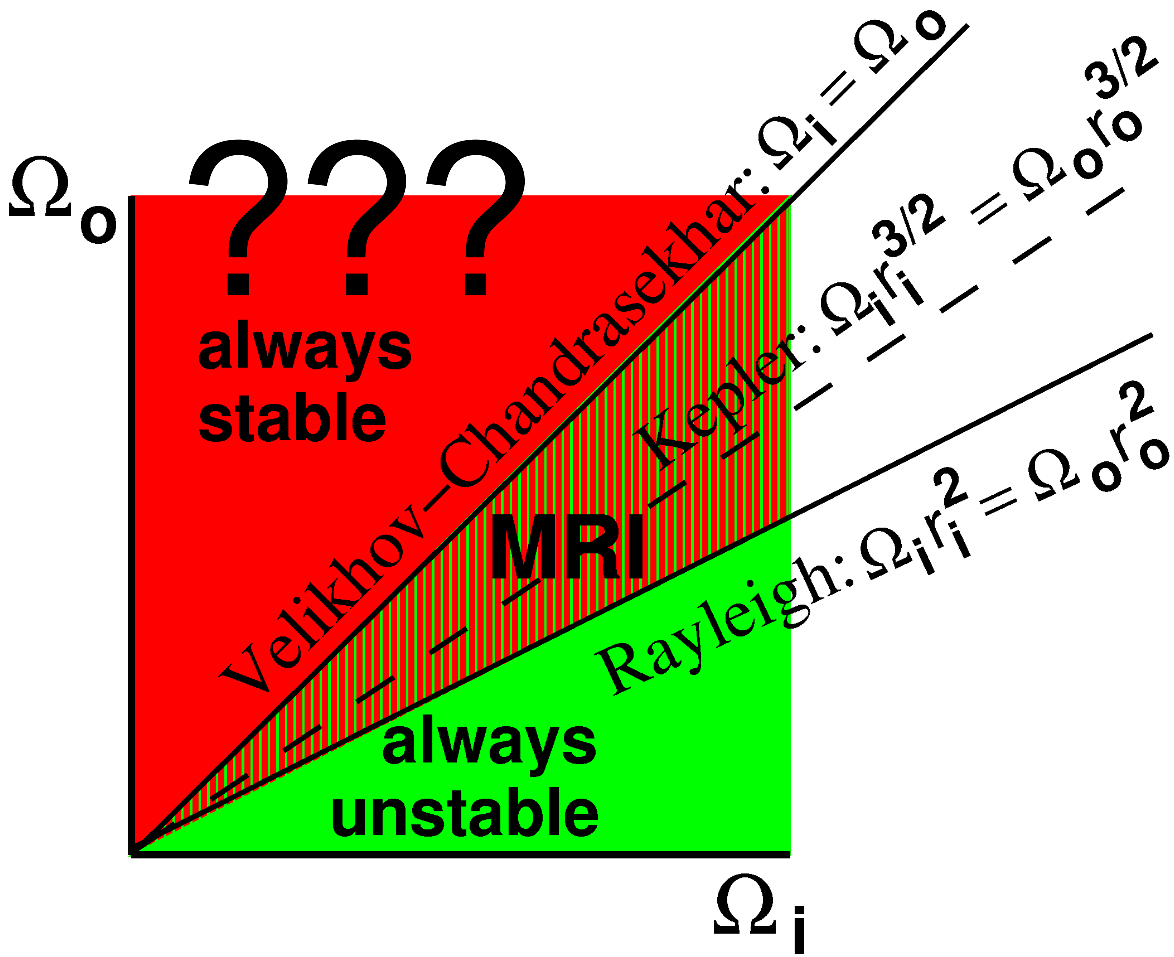
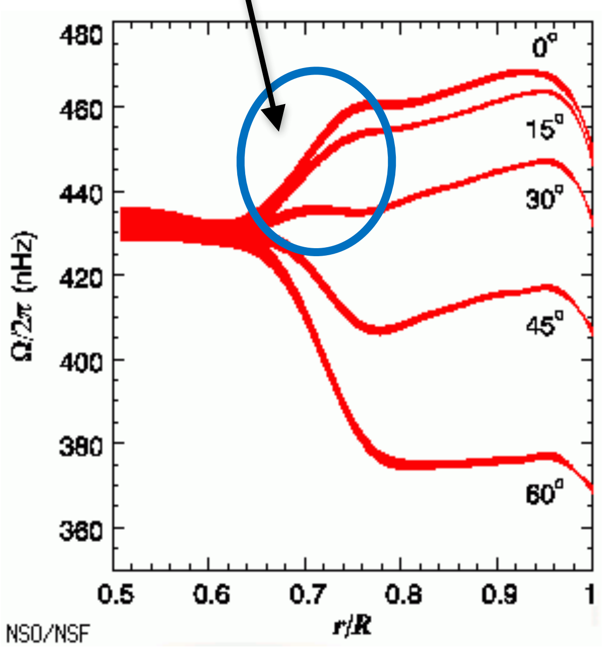
Basic problem: Are rotational flows with positive shear always stable?



Possible relevance for the near-equatorial parts of the solar tachocline



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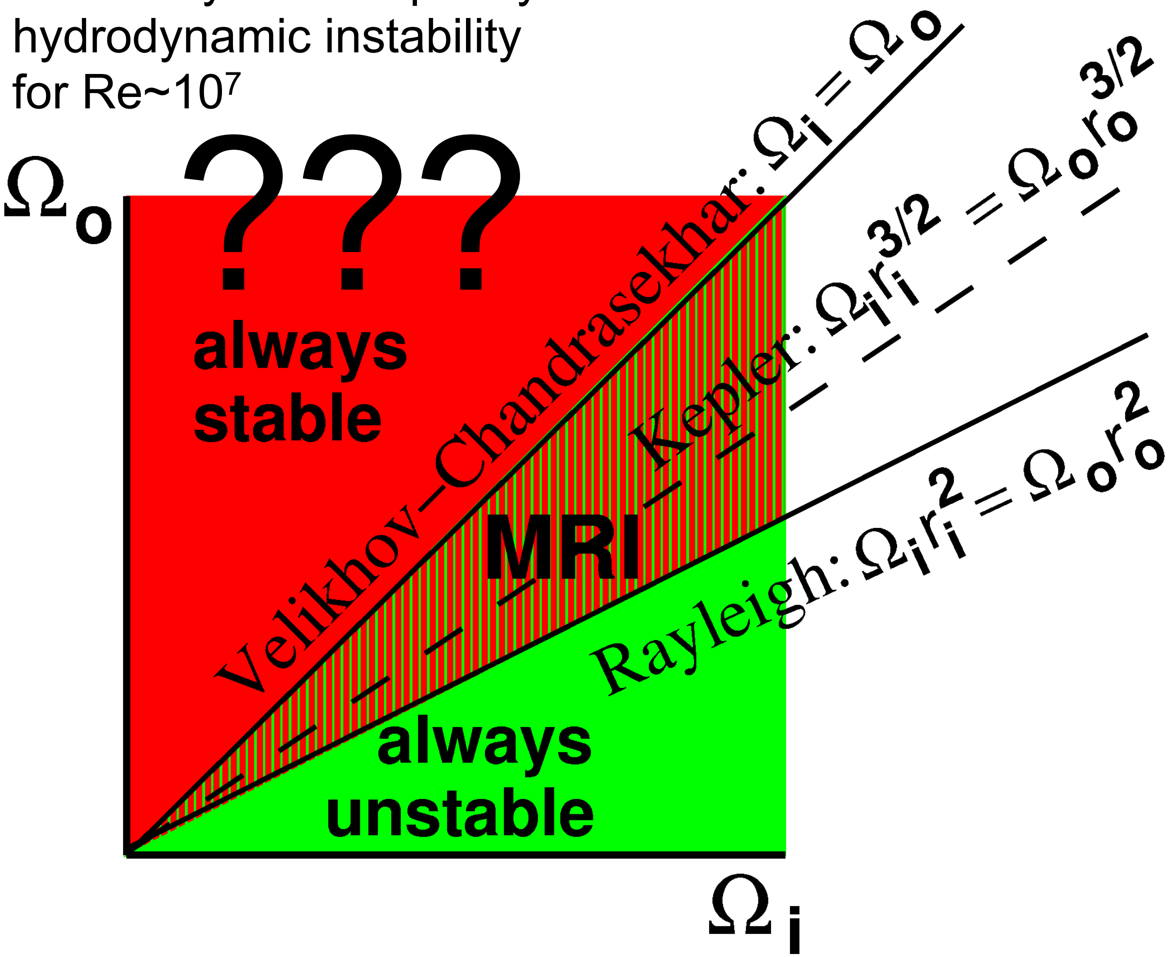
Deguchi, Phys. Rev. E 95 (2017), 021102

Non-axisymmetric purely hydrodynamic instability for Re~10^7



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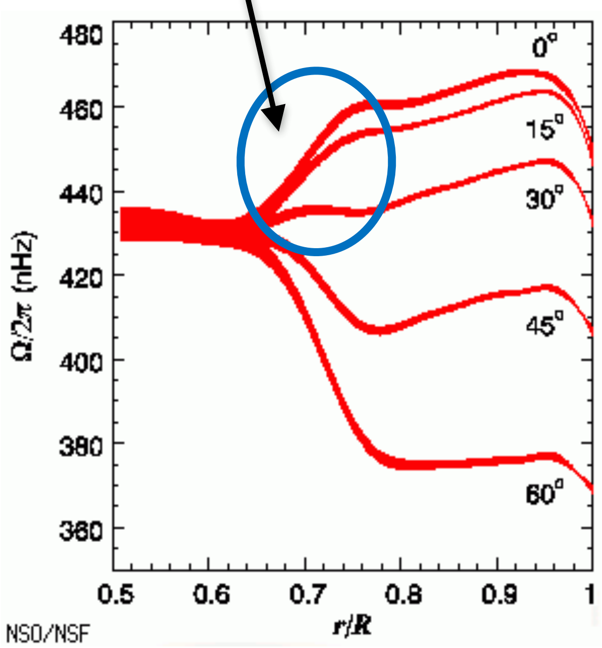

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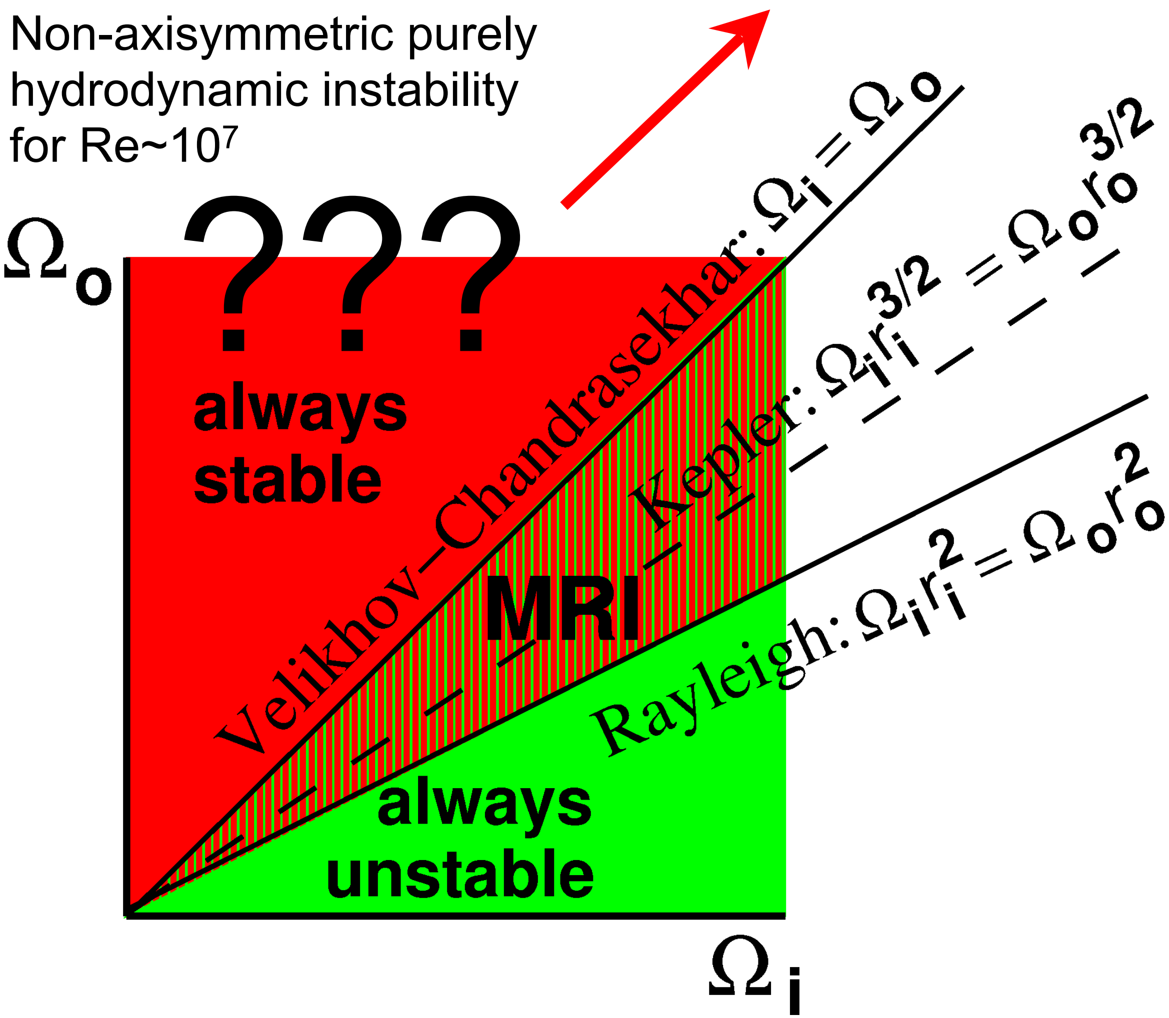
HERE: Prospects for **magnetic** destabilization



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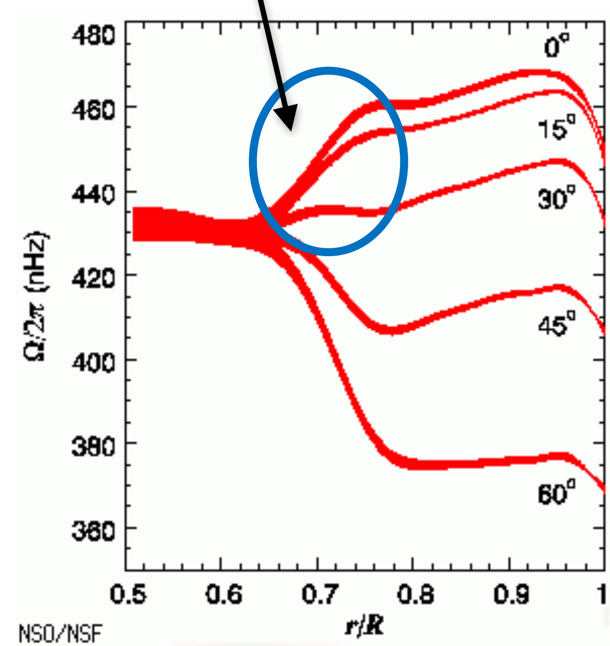
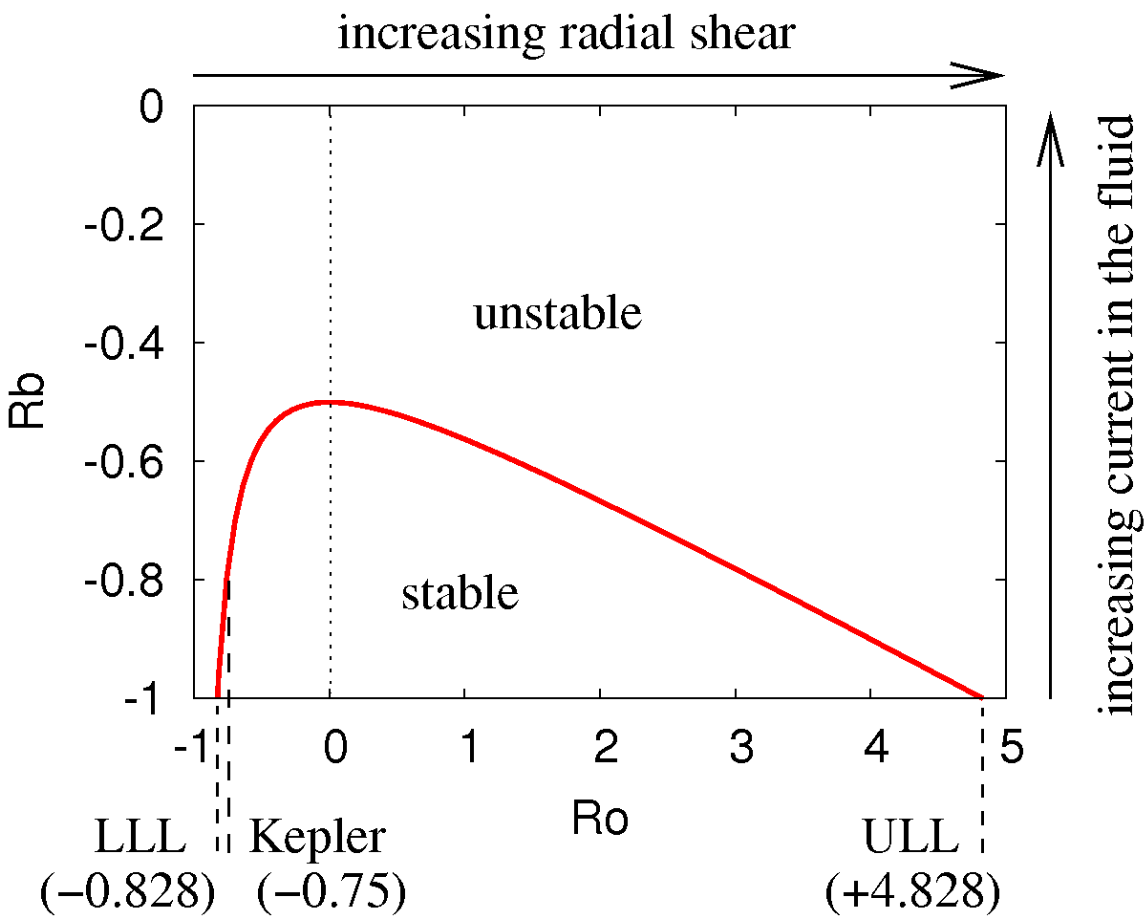


Can magnetic fields destabilize rotational flows with positive shear? HMRI and AMRI



Lower Liu Limit

Upper Liu Limit

$$\mathrm{Ro} = \frac{r}{2\Omega}\frac{\partial \Omega}{\partial r}$$

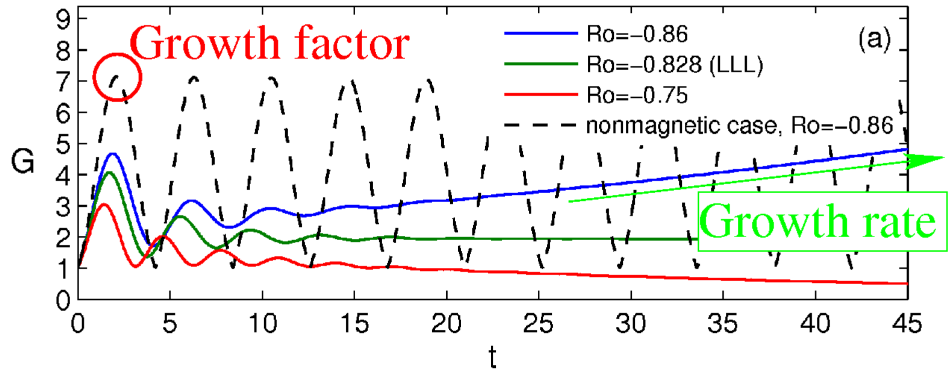
$$\mathrm{Rb} = \frac{r}{2\omega_{A\phi}}\frac{\partial \omega_{A\phi}}{\partial r}$$

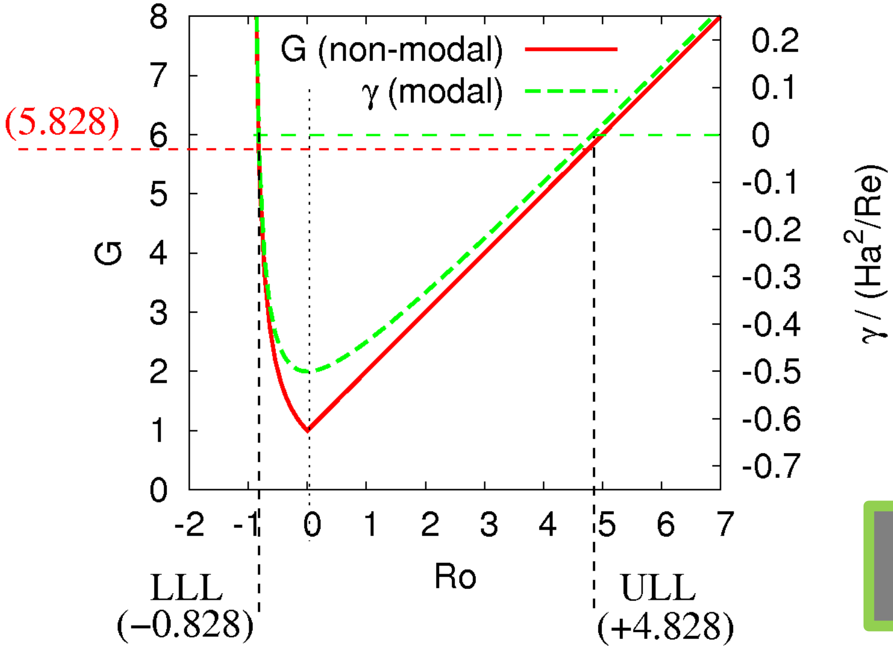
$$\boxed{Rb = -\frac{1}{8}\frac{(Ro+2)^2}{Ro+1}}$$

Kirillov and Stefani, Phys. Rev. Lett. 111 (2013), 061103; JFM 760 (2014), 591

Liu, Goodman, Herron, Ji, Phys. Rev. E 74 (2006), 056302

Link between non-modal growth and dissipation-induced instabilities





Any physical reason for the lower and upper Liu limits (LLL and ULL) of the shear for the emergence of HMRI?

YES!

Analytical link between **non-modal growth factor G** of purely hydro-dynamic flows with **modal growth rate** γ of dissipation-induced HMRI

$$\boxed{G_m = (1 + Ro)^{sgn(Ro)}}$$

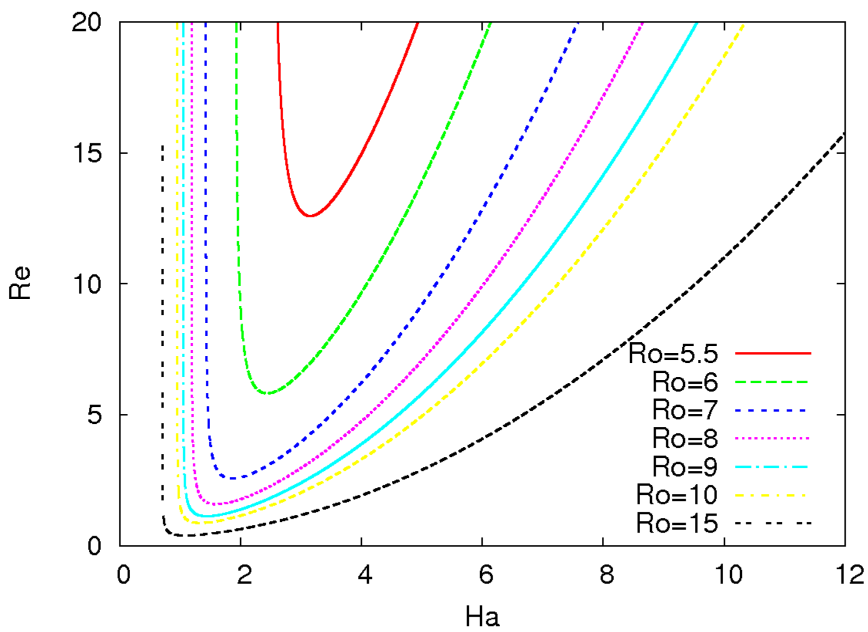
$$\gamma = \frac{Ha^2}{Re}\left[\frac{(Ro+2)^2}{8(1+Ro)} - 1\right] = \frac{Ha^2}{Re}\left[\frac{(G_m+1)^2}{8G_m} - 1\right]$$

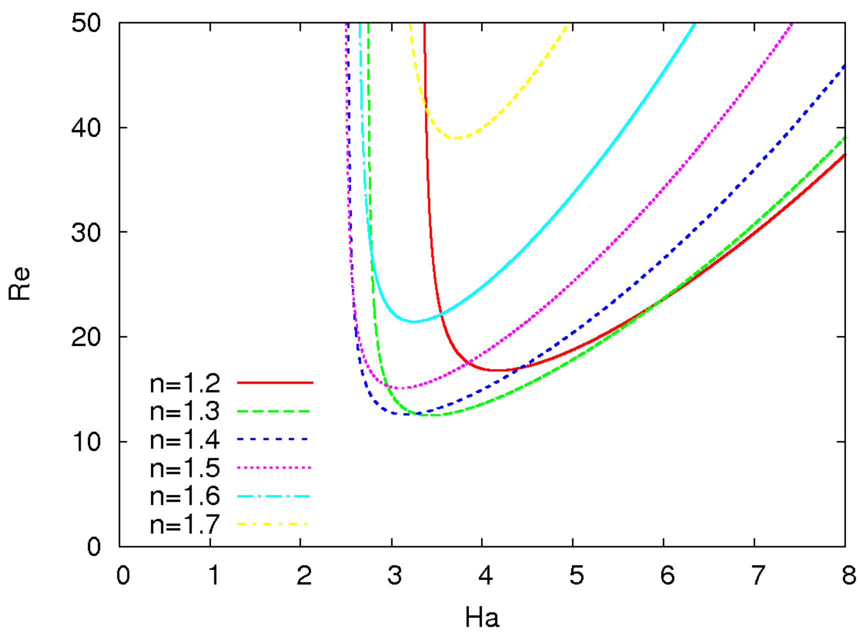
Mamatsashvili and Stefani, PRE 94, 051203 (R) (2016)

AMRI at positive shear -„Super-AMRI“ -operates only at $Ro>Ro_{ULL}$ =4.828

Stability curve as a solution of :

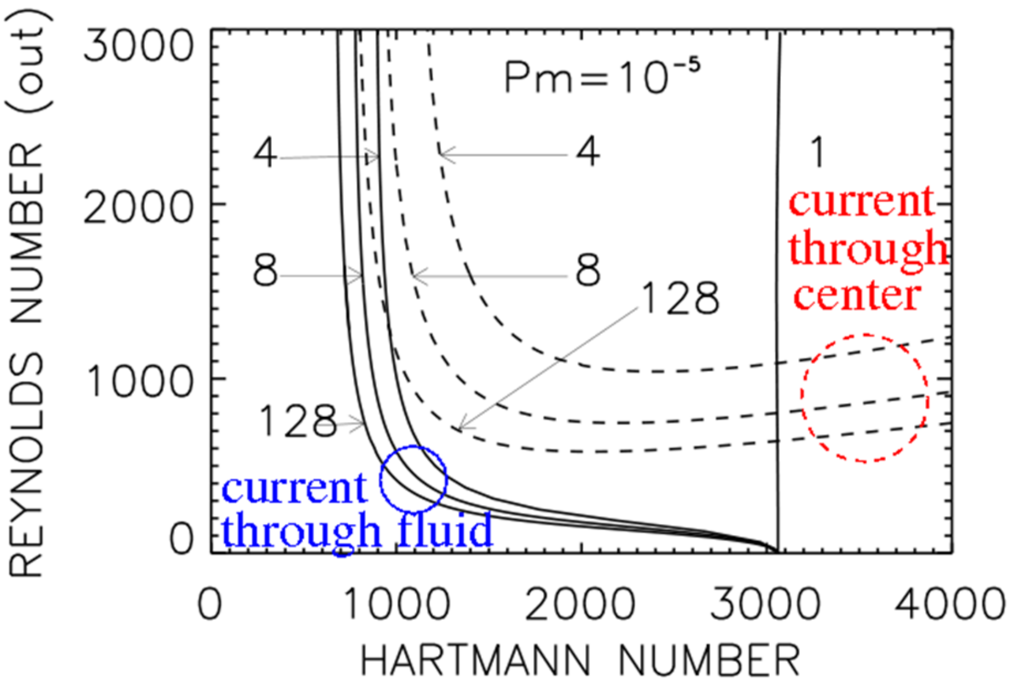
$$Re^2=\frac{1}{4}\frac{[(1+Ha^2n^2)^2-4\,Ha^2Rb(1+Ha^2n^2)-4Ha^4n^2][1+Ha^2(n^2-2Rb)]^2}{Ha^4Ro^2n^2-\left[\left(1+Ha^2(n^2-2Rb)\right)^2-4Ha^4\,n^2\right][Ro+1]}$$





Stefani and Kirillov, Phys. Rev. E 92 (2015), 051001

Super-AMRI also found in 1D global stability analysis



(dashed lines are the stability boundaries of Super-AMRI at different $\mu=\Omega_{out}/\Omega_{in}=4, 8, 128>1$)

Rüdiger et al., Phys. Fluids 28 (2016), 014105

Correct translation of the unstable range at high Ro>Ro_{ULL} in the local analysis into the ratio of outer and inner cylinders‘ angular frequencies for super rotation $\mu=\Omega_{out}/\Omega_{in}$ > 1 in the global Taylor-Couette flow case is crucial!

Taylor-Couette (TC) -Experiment with a small gap is needed (at least r_{in}/r_{out}~0.8),
Minimum central current ~ 30 kA

These types of Super-HMRI/AMRI are a bit frustrating because...

...the upper Liu limit value $Ro>Ro_{OLL}$ =4.828 is quite a large positive shear compared to those found in astrophysical objects or in usual lab experiments.
(for example, the positive shear in the equatorial parts of the solar tachocline is only about Ro~0.7)

→ Open questions:

- Can any sort of HMRI and/or AMRI still survive at astrophysically relevant, smaller positive shear?
- Can such an instability, if present, still operate at wider gap width in TC flows (e.g., for $\boldsymbol{r_{in}/r_{out}\sim 0.5}$) corresponding to the **PROMISE** and **DRESDYN** MRI/TI experiments

Surprisingly, the answer is YES!

We revealed a novel type of double-diffusive axisymmetric HMRI at arbitrary positive shear - so-called Type 2 Super-HMRI

Mamatsashvili et al., Phys. Rev. Fluids, 4, (2019), 103905

Equilibrium: Taylor-Couette flow with

Inner and outer radii: $r_{in}, r_{out}, \ \eta = r_{in} / r_{out}$

Stationary inner, $\Omega_{in} = 0$, and rotating outer, Ω_{out} cylinders

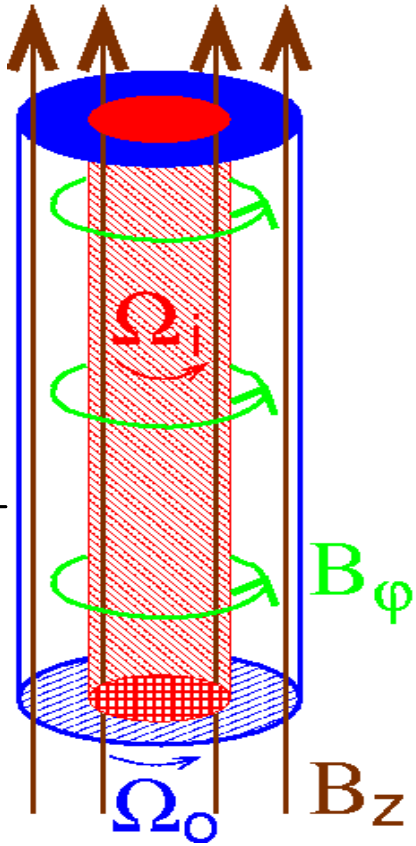
Angular velocity profile: $\Omega(r) = \dfrac{\Omega_{out}}{1-\eta^2}\left(1 - \dfrac{r_{in}^2}{r^2}\right)$

Current-free helical magnetic field: $\mathbf{B} = (0, B_\varphi(r), B_z), \quad B_\varphi = \beta B_z \dfrac{r_{out}}{r}$

As distinct from most other studies on HMRI, this setup has a radially increasing angular frequency, $d\Omega / dr > 0$,

1. Hydrodynamically stable: $Ro = (r / 2\Omega) d\Omega / dr > 0,$

2. Stable against standard MRI (at $\beta = 0$)



First method: WKB analysis

Axisymmetric (*m=0*) perturbations $\propto \exp(\gamma t + ik_r r + ik_z z)$

γ is the (complex) eigenvalue, k_r and k_z are the radial and axial wavenumbers

Dispersion relation – 4-th order polynomial

$$\boxed{\gamma^4 + a_1\gamma^3 + a_2\gamma^2 + (a_3 + \mathrm{i}b_3)\gamma + a_4 + \mathrm{i}b_4 = 0,}$$

with the coefficients:

Kirillov and Stefani, JFM 760 (2014), 591

$$a_1 = 2k^2 Re^{-1}(1 + Pm^{-1}),$$

$$a_2 = 2(k_z^2 + 2\alpha^2\beta^2)Ha^2 Re^{-2} Pm^{-1} + 4\alpha^2(1 + Ro) + k^4 Re^{-2}\left(1 + 4Pm^{-1} + Pm^{-2}\right)$$

$$a_3 = 8(1 + Ro)\alpha^2 k^2 Re^{-1} Pm^{-1} + 2k^2 Re^{-3} Pm^{-1}\left(1 + Pm^{-1}\right)[k^4 + (k_z^2 + 2\alpha^2\beta^2)Ha^2]$$

$$a_4 = 4\alpha^2 k^4 Pm^{-2}\left[(1 + Ro)Re^{-2} + \beta^2 Ha^2 Re^{-4}\right] + 4\alpha^2 k_z^2 Ro Ha^2 Re^{-2} Pm^{-1} + Re^{-4} Pm^{-2}\left(k_z^2 Ha^2 + k^4\right)^2$$

$$\boxed{b_3 = -8\alpha^2\beta k_z Ha^2 Re^{-2} Pm^{-1}}$$

$$b_4 = 4k_z^3\beta\left[Ro\left(1 - Pm^{-1}\right) - 2Pm^{-1}\right] Ha^2 Re^{-3} Pm^{-1}$$

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The parameters in detail

Total wavenumber: $k = (k_r^2 + k_z^2)^{1/2}, \quad \alpha = \dfrac{k_z}{k}$

Reynolds number: $\mathrm{Re} = \dfrac{\Omega_{out} r_{out}^2}{\nu}$ (ν - viscosity)

Magnetic Reynolds number: $\mathrm{R}m = \dfrac{\Omega_{out} r_{out}^2}{\eta}$ (η - resistivity)

Magnetic Prandtl number: $Pm = \dfrac{\nu}{\eta} = \dfrac{Rm}{\mathrm{Re}}$

Hartmann number: $Ha = \dfrac{r_{out} B_z}{\sqrt{\mu_0 \rho_0 \nu \eta}}$ (ρ_0 - density)

Lundquist number: $S = Ha \cdot Pm^{1/2} = \dfrac{r_{out} B_z}{\eta \sqrt{\mu_0 \rho_0}}$

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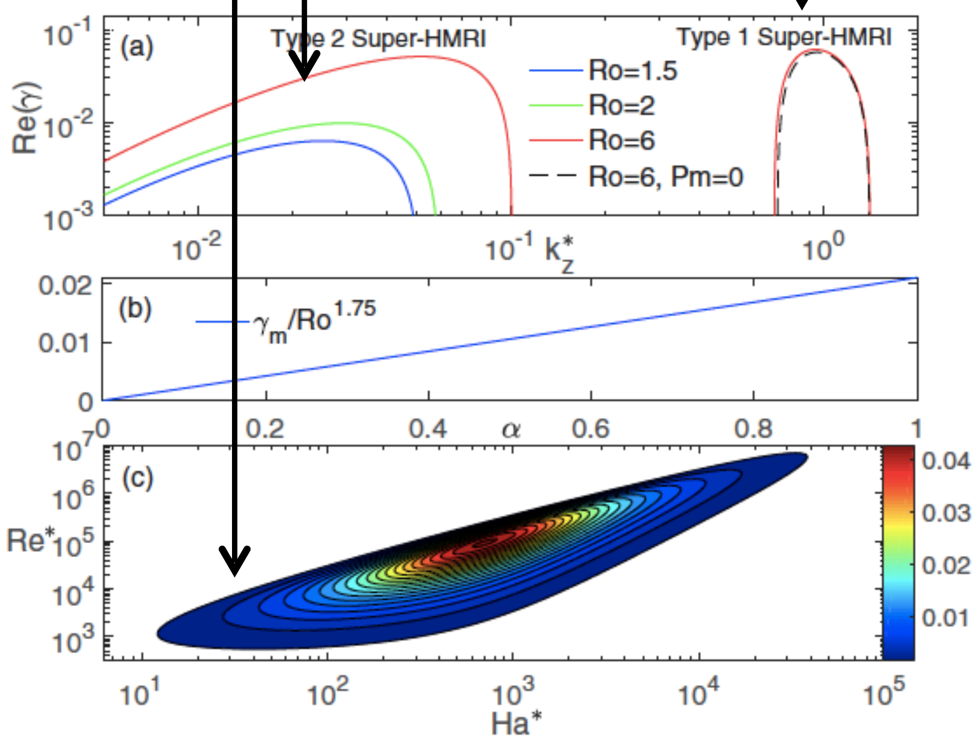
Solution: The usual Type 1 Super-HMRI plus a new Type 2 Super-HMRI

New Type 2 Super HMRI **Type 1 Super- HMRI**



$Ha = 90,$

$\mathrm{Re} = 8000,$

$\alpha = 1, \beta = 1$

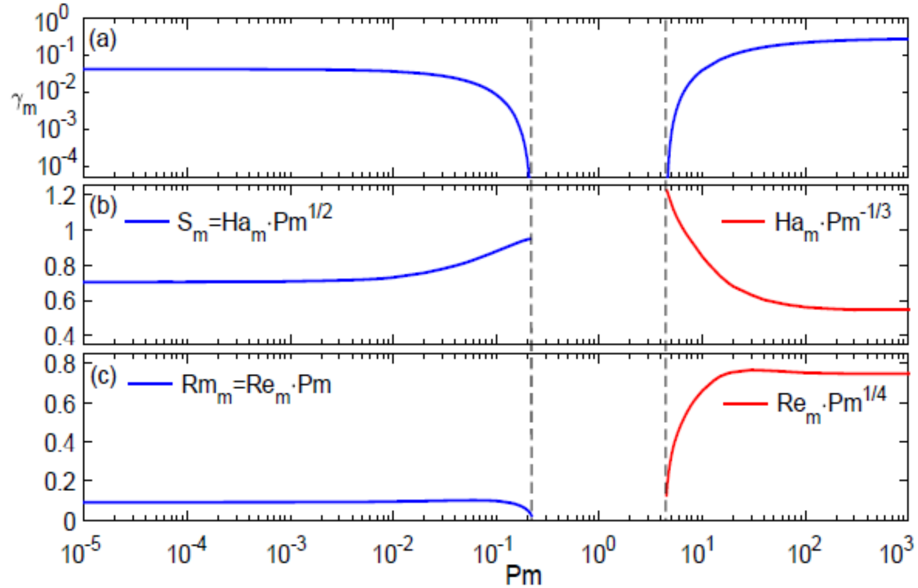
$Pm = 10^{-6}$

1. A new Type 2 Super-HMRI: at small k_z , all Ro and non-zero (finite) $Pm \neq 0$

2. Usual Type 1 Super-HMRI: at larger k_z, high shear $Ro > Ro_{ULL}$, down to the inductionless limit $Pm = 0$ (dashed black line)

Scaling behaviour of this new double-diffusive instability

γ_m -- the growth rate optimized over k_z, Ha and Re , which is achieved at some Ha_m and Re_m



$Ro = 1.5$

γ_m tends to constant values at $Pm << 1$ and $Pm >> 1$

At small $Pm << 1$, the instability is determined by S_m and Rm_m, which are constant

At large $Pm >> 1$, obeys scalings $Ha_m \propto Pm^{1/3}$ and $\mathrm{Re}_m \propto Pm^{-1/4}$

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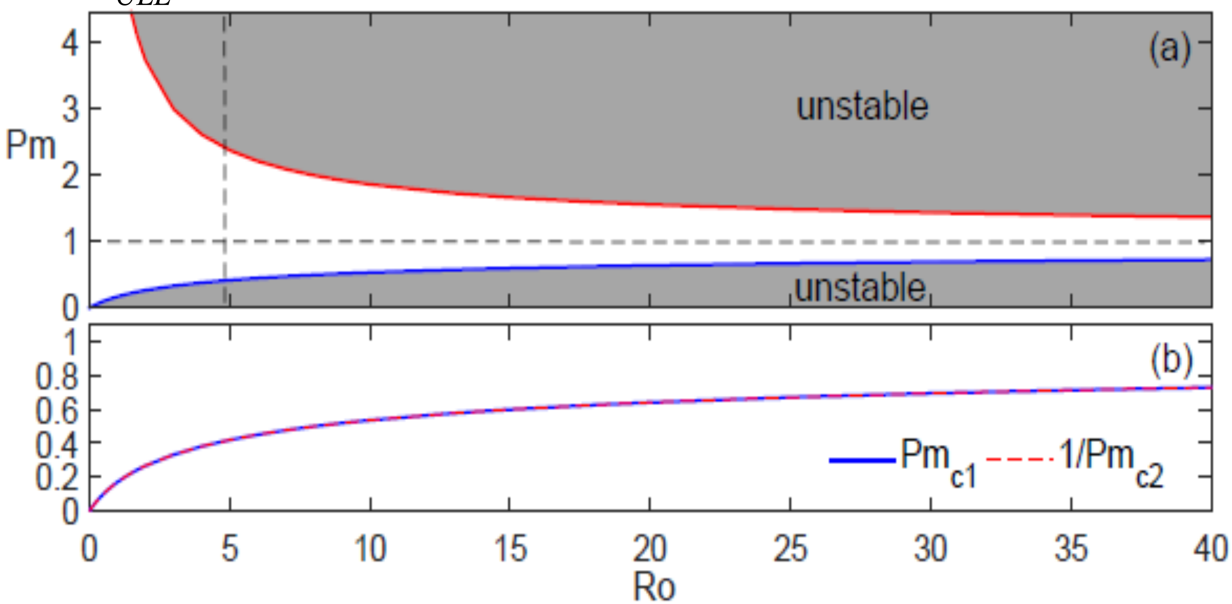
The instability domain in the Ro-Pm plane

$Ro_{ULL} = 4.828$



The stability boundaries at $Pm_{c2}(Ro) > 1$ (red) and $Pm_{c1}(Ro) < 1$ (blue) are related by

$$\boxed{Pm_{c1} = \frac{1}{Pm_{c2}}}$$

The new double-diffusive Type 2 Super-HMRI is not constrained by the upper Liu limit

There is no instability at Pm=1 .

Second method: 1D stability analysis

Numerical method:

The radial structure of the velocity and magnetic fields are expanded in Chebyshev polynomials (up to $N = 30 - 40$). The governing equations of **non-ideal MHD** then reduce to a large ($4N \times 4N$) matrix eigenvalue problem (Hollerbach & Rüdiger 05)

Boundary conditions on the inner and outer cylinders:

No-slip for velocity and conducting or insulating for the magnetic field

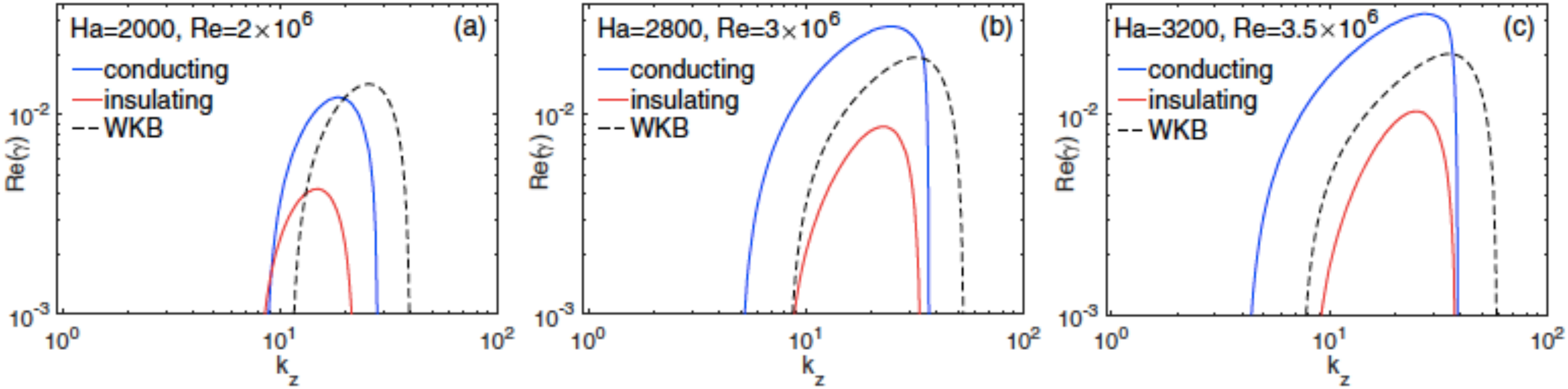
Second method: 1D stability analysis

Calculations in the narrow gap: $\hat{\eta} = r_i/r_o = 0.85.$

The Rossby number: $\mathrm{Ro} = 3.5 < \mathrm{Ro}_{ULL}$

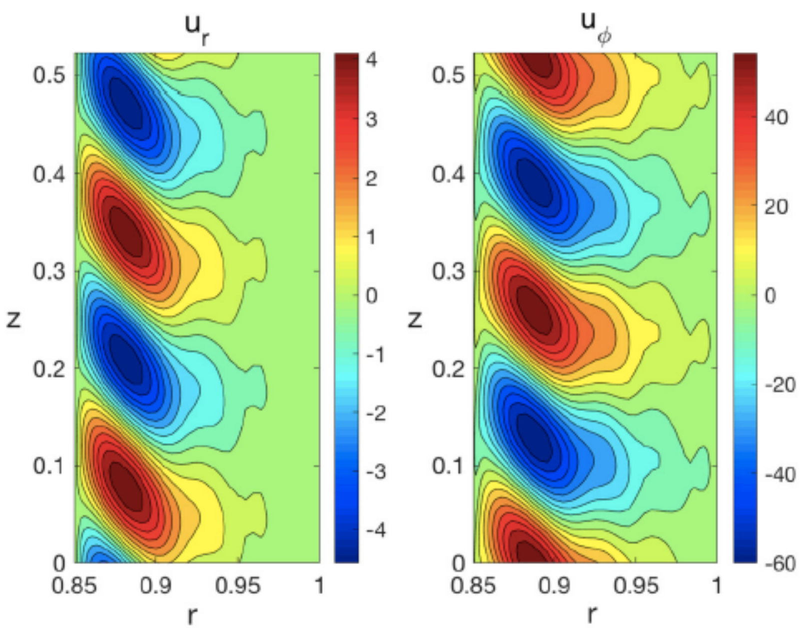
Angular velocity profile: $\Omega(r) = \Omega_o \left(\frac{r}{r_o}\right)^{2\mathrm{Ro}}$

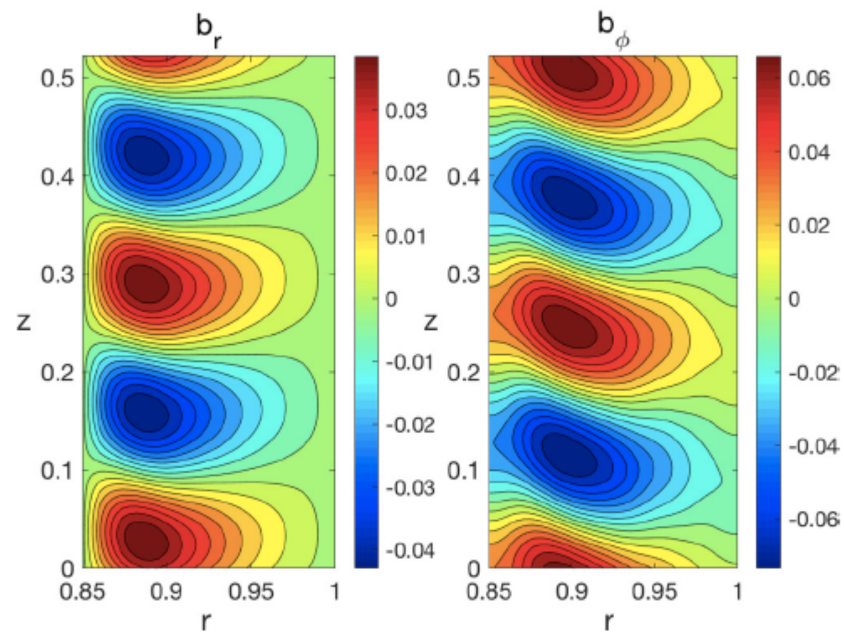
Small $\mathrm{Pm} = 10^{-3}$, large $\beta = 100$.



Typical eigenfunctions of the modes

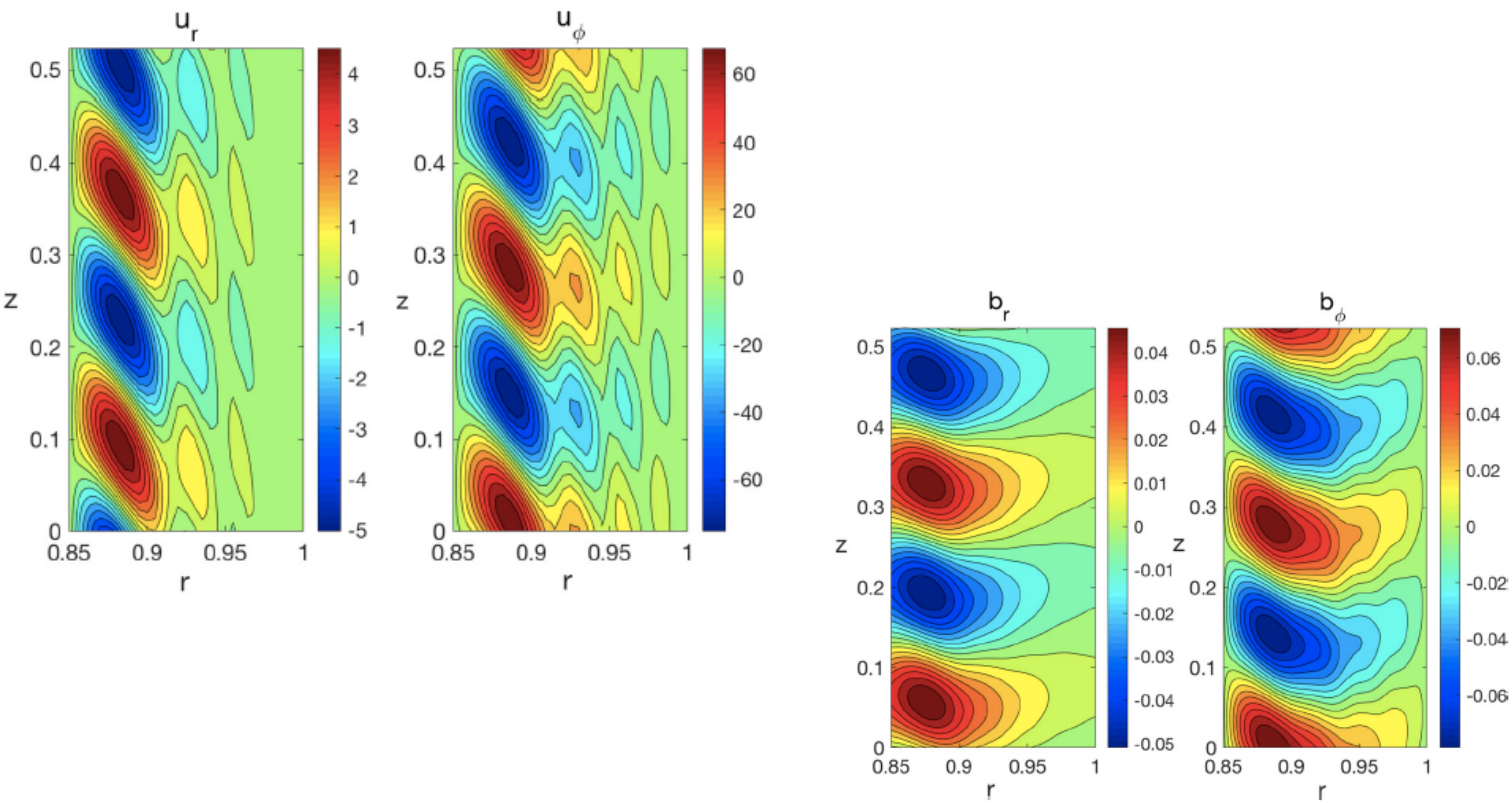
Conducting BC





Typical eigenfunctions of the modes

Insulating BC



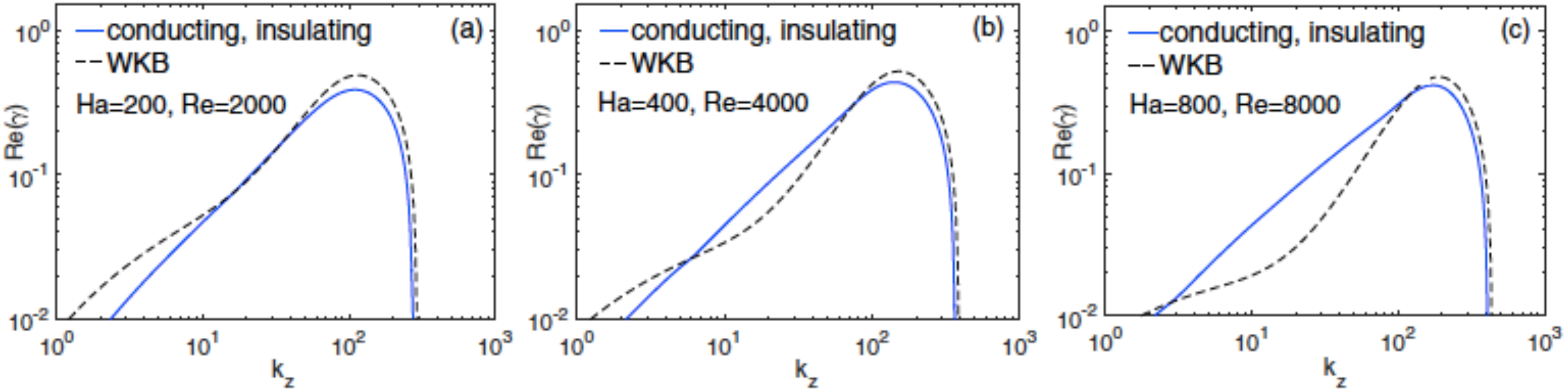
Second method: 1D stability analysis

Calculations in the narrow gap: $\hat{\eta} = r_i/r_o = 0.85.$

The Rossby number: $\mathrm{Ro} = 3.5 < \mathrm{Ro}_{ULL}$

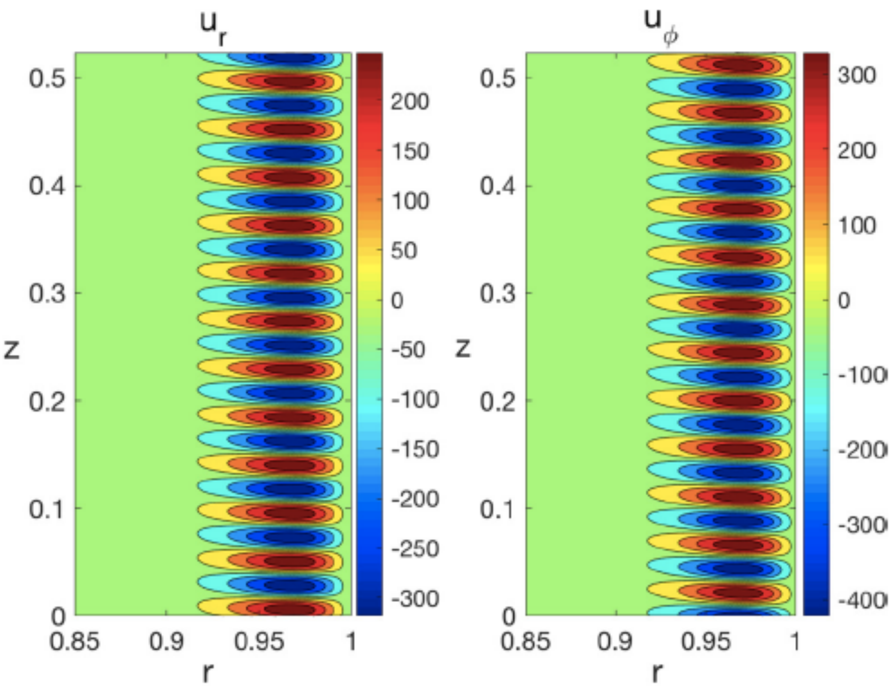
Angular velocity profile: $\Omega(r) = \Omega_o\left(\frac{r}{r_o}\right)^{2\mathrm{Ro}}$

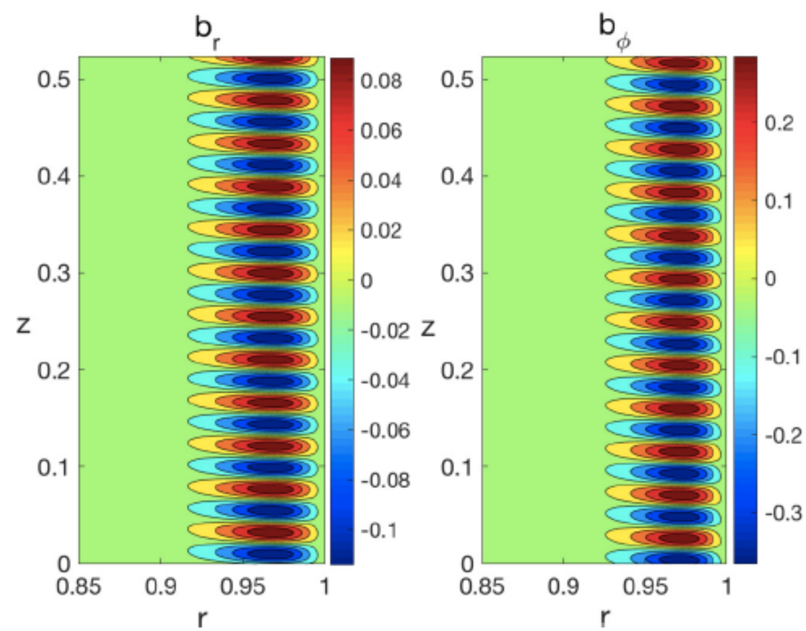
Large $\mathrm{Pm} = 100$, $\beta = 100.$



Typical eigenfunctions of the modes

Insulating/conducting BC *(no difference)*





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Second method: 1D stability analysis for Taylor-Couette flow

Calculations in the wide gap: $\eta = r_{in} / r_{out} = 0.5$

Taylor-Couette flow profile: $\Omega(r) = \dfrac{\Omega_o}{1-\hat{\eta}^2}\left(1 - \dfrac{r_i^2}{r^2}\right),$

Small $Pm = 10^{-6}$

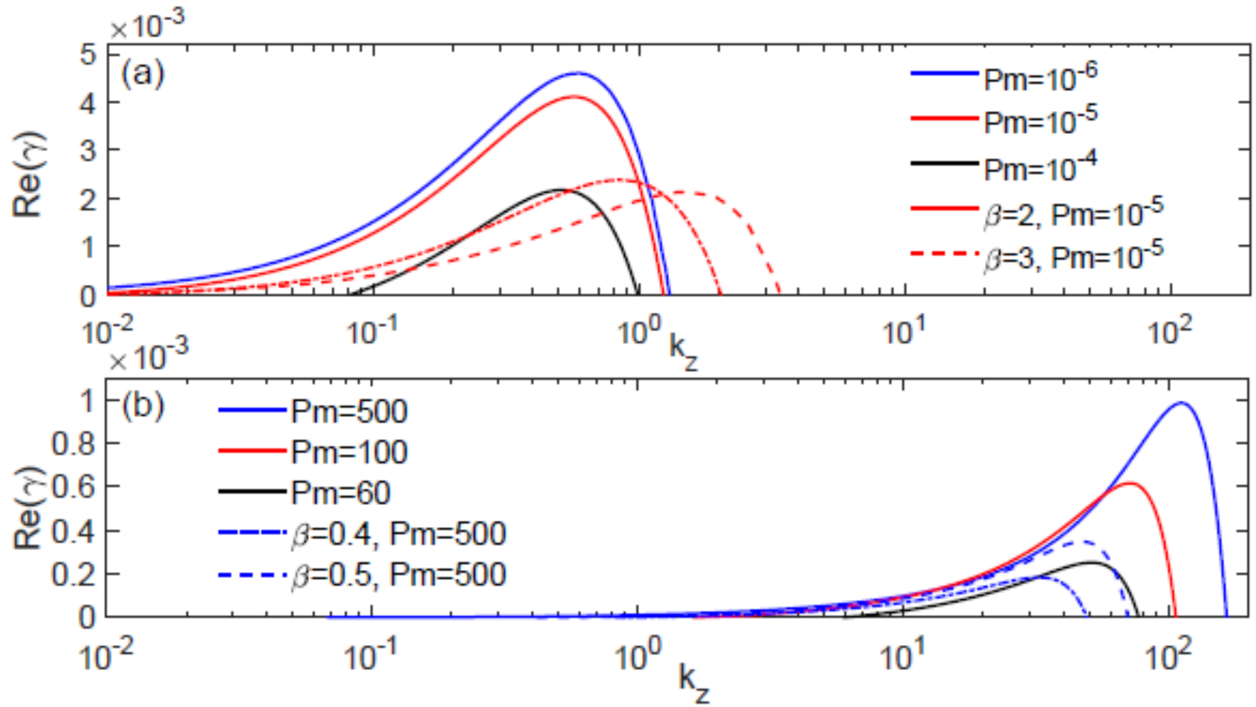
$\beta = 1$

Boundary conditions on the inner and outer cylinders:

No-slip for velocity and conducting for the magnetic field

Similar to **PROMISE** and **DRESDYN** facilities

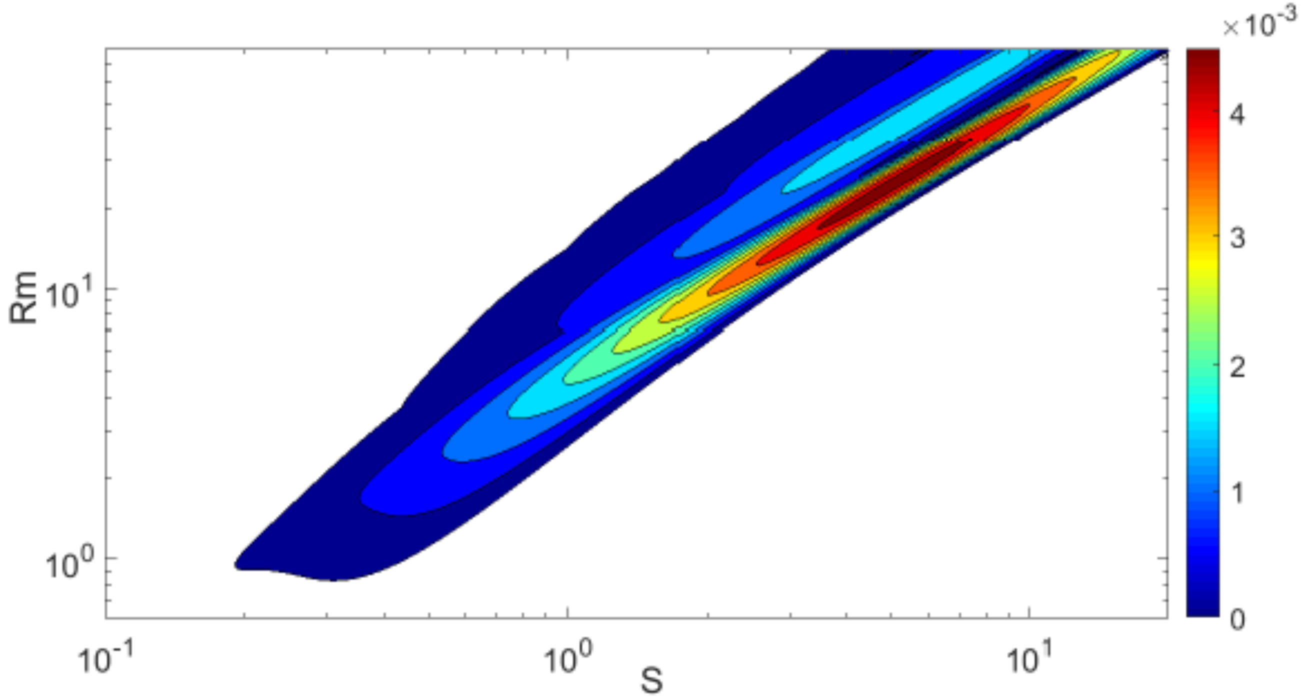
Results: Confirmation of WKB result for Pm<<1 and Pm>>1



1. Ha and Re (or S and Rm), as well as k_z, increase with β in qualitative agreement with the scalings in the WKB analysis

2. At small $Pm << 1$, relevant parameters are again the Lundquist, S, and magnetic Reynolds Rm numbers, as in the WKB case

Results: Growth rate (optimized over k_z) in the (S-Rm) plane



The unstable region is localized, as in the local analysis, reaching a maximum value

$\gamma_m = 4.8 \cdot 10^{-3}$ at $S_m = 5.2,\ Rm_m = 25$

The instability first emerges at the critical $Rm_c = 0.9$ and $S_c = 0.3$

Prospects for an experiment

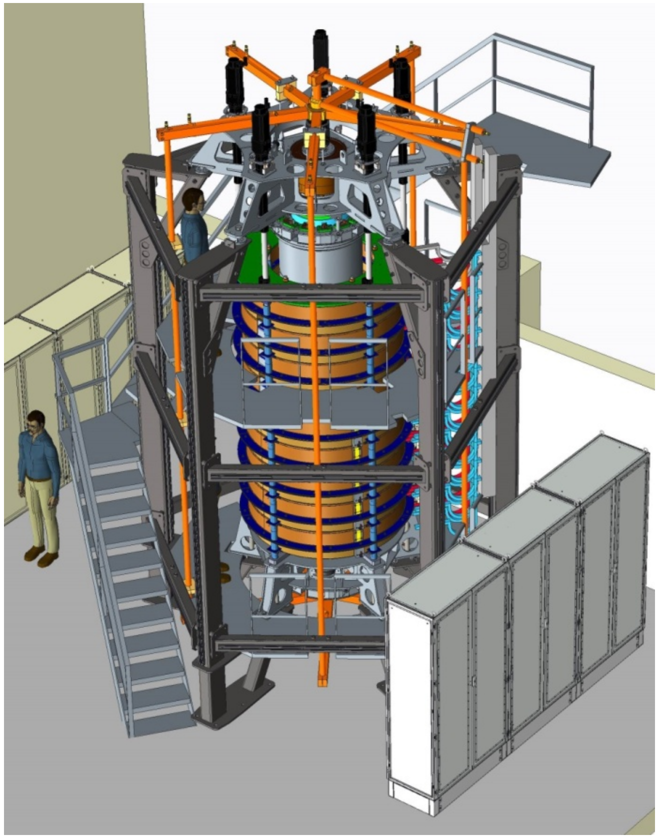
Those values S~5 and *Rm~25* are well within the capabilities of the new Taylor-Couette device being currently built within DRESDYN in HZDR

- r_{in}=0.2 m
- r_{out}=0.4 m
- h=2 m

- f_{in}=20 Hz
- f_{out}=6 Hz

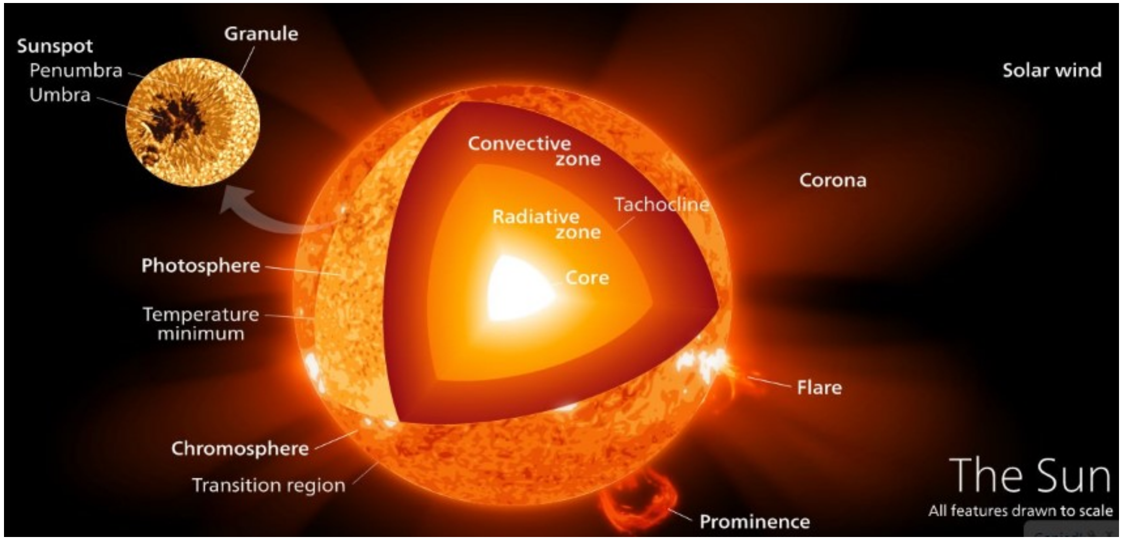
- B_z=120 mT

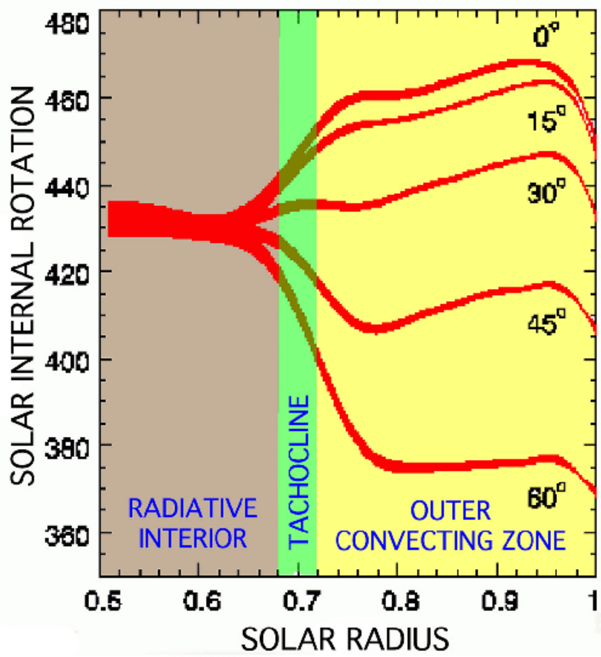
- Rm = 40
- S = 8



...offering a realistic prospect for experimental realization of this new double-diffusive, positive shear Type 2 Super-HMRI

Possible application of new Type 2 Super-HMRI to the solar tachocline





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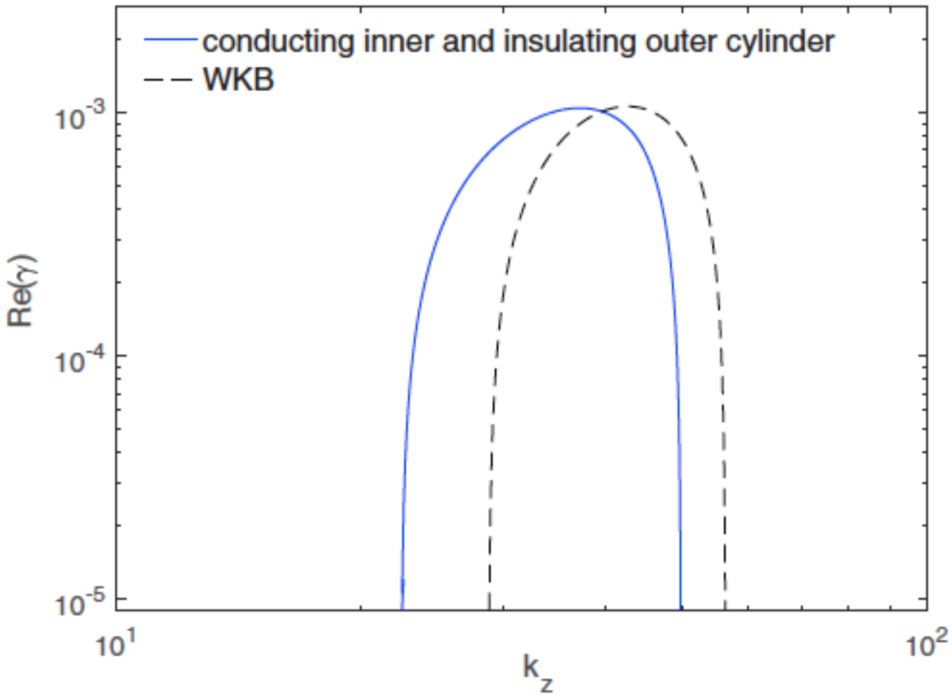
Type 2 Super-HMRI growth rate in a TC flow, for the parameters close to those in the solar tachocline (e.g., Garaud 2007, Arlt 2009)



$$\hat{\eta} = 0.94$$
$$Ro = 0.7$$
$$\mathrm{Ha} = 2500$$
$$\mathrm{Re} = 1.5 \times 10^6$$
$$\beta = 500$$
$$\mathrm{Pm} = 0.01$$

Conducting inner cylinder and insulating outer cylinder

The corresponding characteristic growth time of the most unstable mode is only ~ 10 years (!) , which is comparable to the solar cycle

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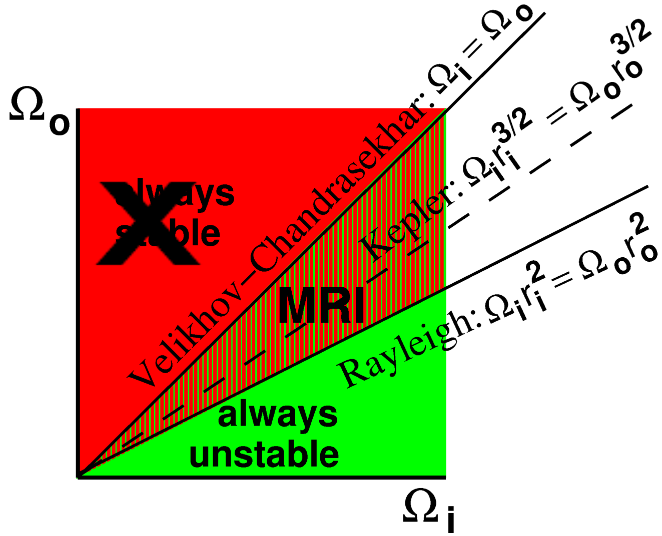
Main results and implications

We have uncovered and analysed a new type of double-diffusive axisymmetric type 2 Super-HMRI in rotating flows with arbitrary positive shear, including $Ro < Ro_{ULL}$, where any MRIs were previously unknown



The only prerequisites: $Pm \neq 1$ and imposed magnetic field with both axial and azimuthal components

Apart from this, the scaling is similar as for standard MRI (with Rm and S) at small Pm

1. **One of the astrophysical domains of applicability: the near-equatorial regions of the solar tachocline**

2. **This new type 2 Super-HMRI should also be observable in the DRESDYN MRI experiment**

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Possible implications for the solar dynamo?

This new type of HMRI could also revive the idea of a subcritical Tayler-Spruit solar dynamo (Spruit 2002):

Its axisymmetric (m = 0) nature can overcome the difficulties in getting the Tayler-Spruit dynamo to form a closed loop from the combination of the non-axisymmetric (m=1) Tayler instability and the axisymmetric (m = 0) Ω-effect (although Cowling's theorem should be kept in mind…)

To ascertain this, further analysis is needed:

1. Linear dynamics of non-axisymmetric perturbations

***2.* Nonlinear development**

Thank you for attention

More details can be found in our recent paper

G. Mamatsashvili et al., 2019, *Phys. Rev. Fluids*, 4, 103905

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