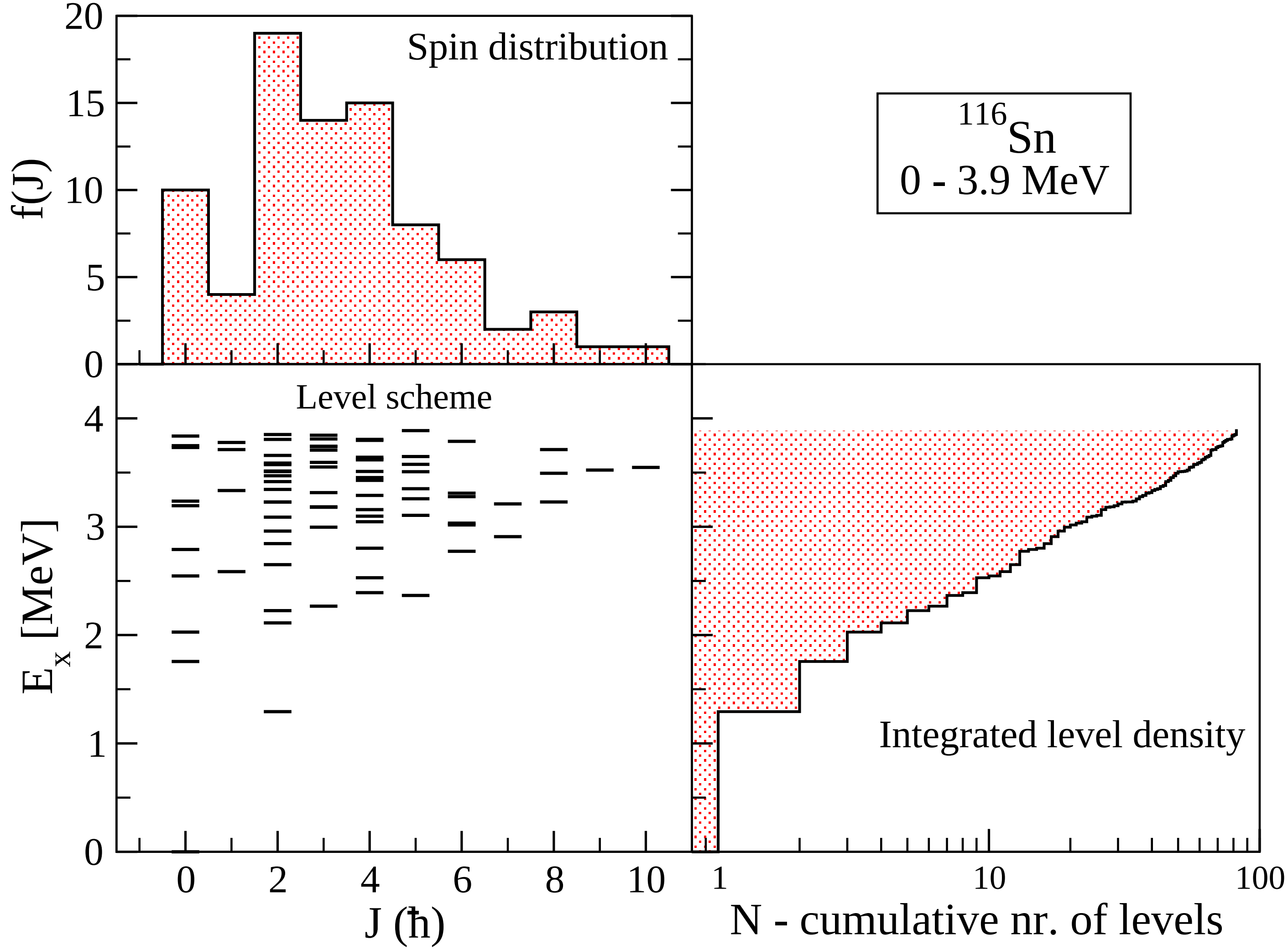
Mass and energy dependence of nuclear spin distributions

Till von Egidy
Physik-Department, Technische Universität München, Germany
Dorel Bucurescu
National Institute of Physics and Nuclear Engineering, Bucharest, Romania



Spin distribution
f(J)
20
15
10
5
0
116Sn
0 - 3.9 MeV
Level scheme
E_x [MeV]
4
3
2
1
0
0
2
4
6
8
10
J (ħ)
Integrated level density
1
10
100
N - cumulative nr. of levels

Formulas for Level Densities

$$\rho(E,J) = f(J)\rho(E)$$

Spin Distribution with σ = spin-cutoff parameter :

$$f(J,\sigma) = e^{-J^2/2\sigma^2} - e^{-(J+1)^2/2\sigma^2}$$

Back-shifted Fermi Gas Formula : free parameters a, E_1

$$\rho_{BSFG} = \frac{e^{2\sqrt{a(E-E_1)}}}{12\sqrt{2}a^{1/4}(E-E_1)^{5/4}}$$

Constant Temperature Formula : free parameters T, E_0

$$\rho_{CT} = \frac{1}{T} e^{(E-E_0)/T}$$

Spin Distribution and Spin Cut-off Parameter σ^2

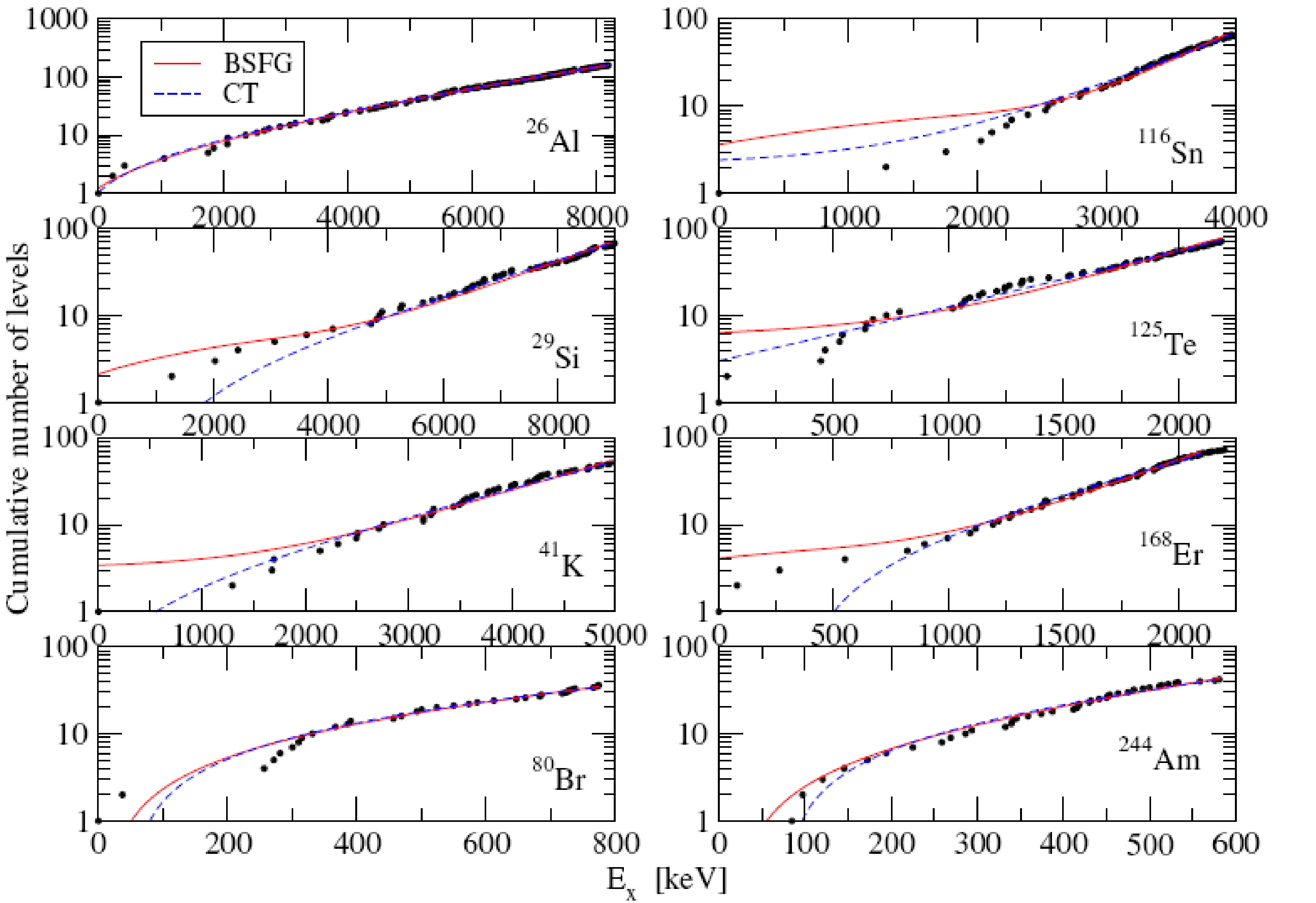
$$f(J,\sigma) = \exp(-J^2/2\sigma^2) - \exp (- (J+1)^2/2\sigma^2)$$

- σ^2 is expected to depend on mass A, level density parameter a, temperature T, moment of inertia Θ, deformation β and excitation energy E.
- $\sigma^2 = g \langle m^2 \rangle t$; $a = \pi^2 g /6$; $t = \mathrm{sqrt}(U/a)$; $\langle m^2 \rangle = 0.146 A^{2/3}$
- Gilbert, Cameron : $\sigma^2 = 0.0888\ a\ t\ A^{2/3}$
- Dilg, Vonach et al.: $\sigma^2 = 0.0150\ t\ A^{5/3}$
- Iljinov et al. : $\sigma^2 = T\ \Theta / \hbar^2$; $R = r_0 A^{1/3}$; $\Theta = {}^2/_5\ M R^2$
- Rauscher, Thielemann, Kratz: $\sigma^2 = \Theta_{rigid}/\hbar^2\ \mathrm{sqrt}(U/a)$, $U=E - \delta$
- Huang Zhonfu et al.: $\sigma^2 = 0.0073\ A^{5/3}\ (1+\mathrm{sqrt}(1+4aU))/2a$

- Which formula is correct? Which dependence on A, a, T, Θ, β, E?
- What is the influence of the shell structure?
- Rigid Moment of inertia Θ_{rigid} ?

Experimental Cumulative Number of Levels N(E)

Resonance density is included in the fit



T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

Methods for determination of nuclear level density and spin distribution

- Determination of total nuclear level density $\rho(E)$:

  fit $D(E)=1/\rho(E)$ to exp. spacings S_i ***and*** *average n-resonance* spacing D_{res} (PRC 72(2005)044311); 310 nuclei from ^{18}F to ^{251}Cf

  BSFG and CT model parameters.

- Determination of average low-energy spin-cutoff parameter σ:

  fit to experimental spin distributions (counting of levels in spin groups in each nucleus) (PRC 78(2008)051301R): 8116 levels in 1556 spin groups

  - simple A-dependence: $\sigma^2 = 2.61\ A^{0.28}$;
  - even-odd spin staggering of spin distribution in even-even nuclei.

- Determination of energy and A-dependence of σ:

  new method of moments *of nuclear level schemes in the* ***(I,E_x)****-plane* (PRC 80(2009)054310); 7202 levels in 227 nuclei (with > 18 levels/known spin)

  $$\sigma^2 = 0.391\ A^{0.675}\ (E-0.5\ Pa')^{0.312}$$

Completeness of nuclear level schemes

Concept in experimental nuclear spectroscopy:
"All" levels in a given ***energy range*** and ***spin window*** are known.
A confidence level has to be given by experimenter: e.g., "less than 5% missing levels".

We assume no parity dependence of the level densities.

Experimental basis:

(n,γ), ARC : non-selective, high precision;
(n,n'γ), (n,pγ), (p,γ);
(d,p), (d,t), (^{3}He,d), … , (d,pγ), …
β-decay;
(α,nγ), (HI,xnypzα γ), HI fragmentation reactions;

* Comparison with theory: one to one correspondence;
* Comparison with neighbour nuclei;
* Much ***experience*** of the experimenter.

Low-energy discrete levels: ***Firestone&Shirley, Table of isotopes (1996); ENSDF database.***

Neutron resonance density: ***RIPL-2 database; http://www-nds.iaea.org***

New method to determine σ(A,E) with moments in the (E,J) plane of level schemes

Experimental moments with levels *i* of nucleus *k* : $M_{m.n}^{k,\exp} = \sum_i (J_i^m E_i^n)$

Used moments:

$$J, J^2, J^3, JE, J^2E, J^3E, JE^2, J^2E^2, J^3E^2$$

Calculated moments:

$$M_{m,n}^{k,calc} = \sum_{E,J} \rho_k(E,J,\sigma) J^m E^n$$

χ^2 of the fit:

$$\chi^2 = \sum_k \sum_{m=1}^{3} \sum_{n=0}^{2} [(M_{m,n}^{k,\exp} - M_{m,n}^{k,calc}) / dM^k]^2$$

227 nuclei with at least 18 levels with known J^π (7202 levels)

Main result of our investigation

$$\sigma^2 = 0.391 A^{0.675} (E - 0.5 Pa')^{0.312}$$

The energy is back-shifted by the pairing energy

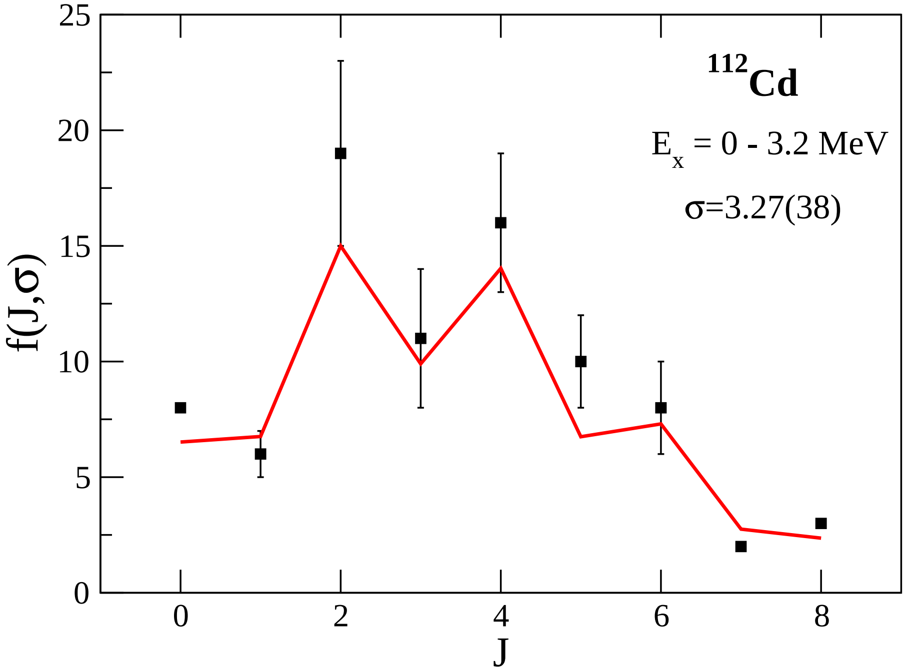
deuteron pairing Pa´, calculated from experimental mass values:

$$Pa'_k = \frac{1}{2}[M(A+2, Z+1) - 2M(A,Z) + M(A-2, Z-1)]$$

Spin staggering of spin

$$f_{ee}(J,\sigma) = (1+x)\, f($$

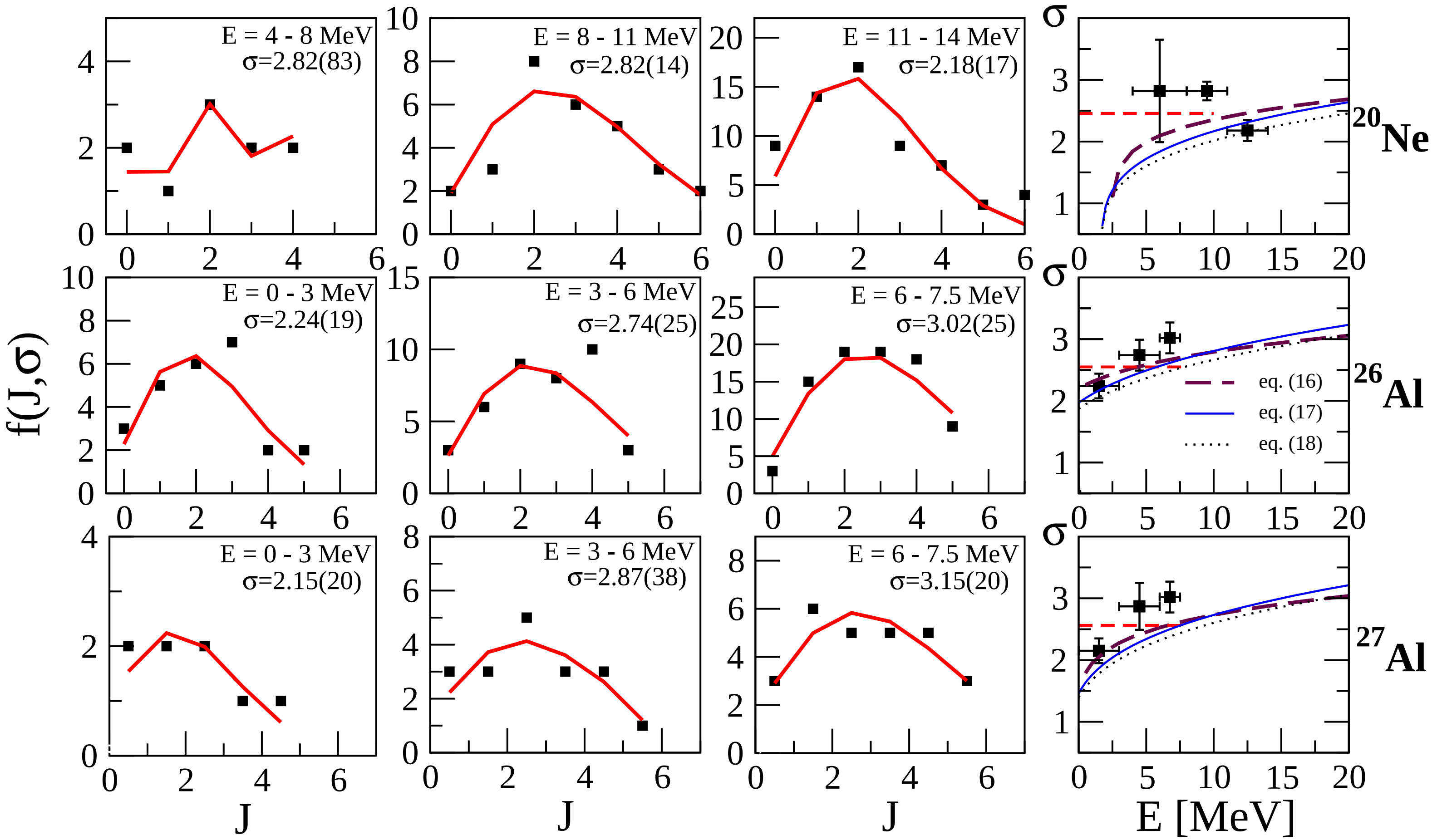
$$x = \begin{cases} +0.227, & even\ \ spin \\ -0.227, & odd\ \ spin \\ +1.02, & zero\ spin \end{cases}$$



The spin staggering is largely independent of mass A

T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

Comparison of our new formula with experimental results



Exp.: from discrete levels

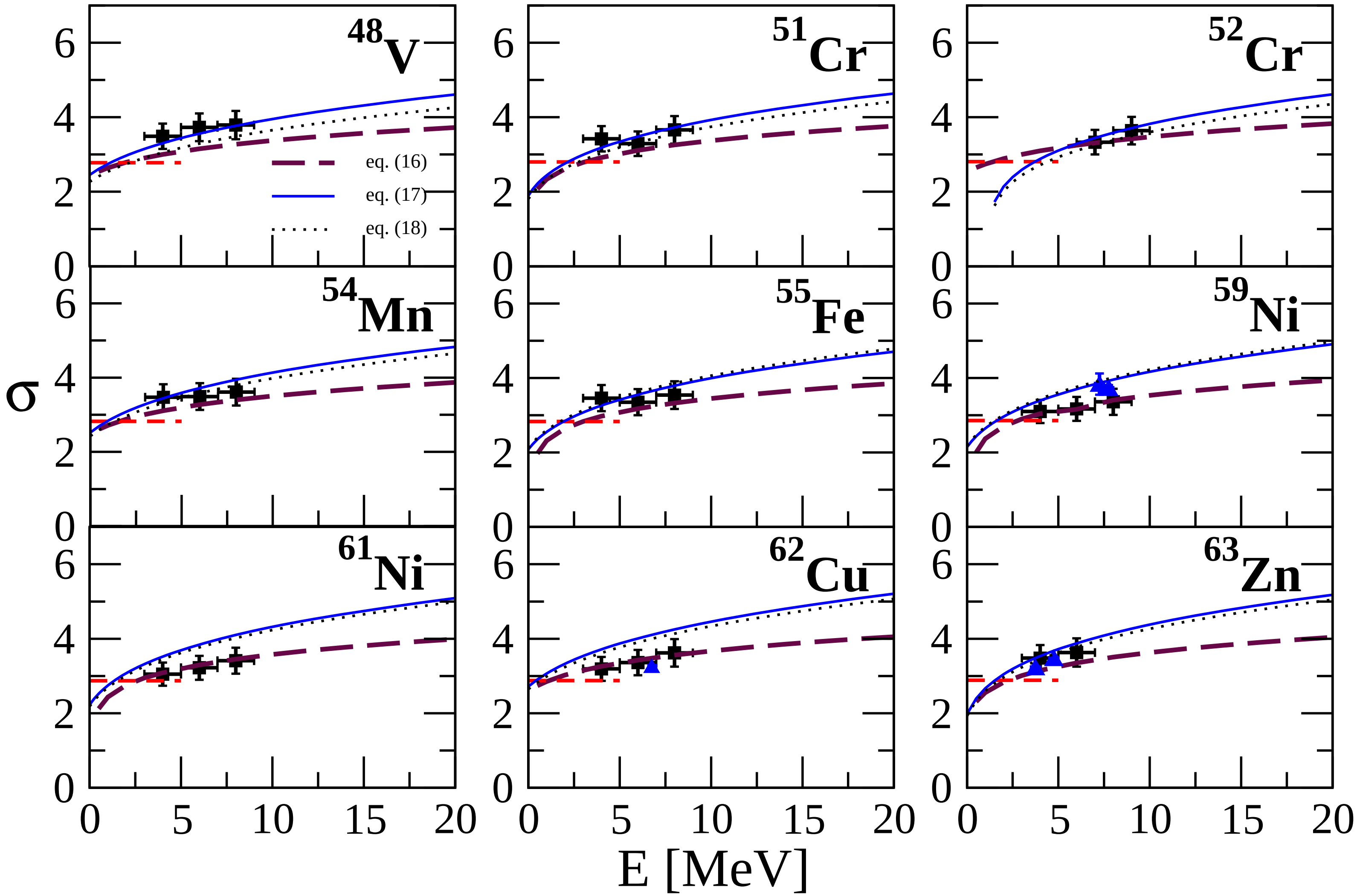
present

statist. mech. calc.

rigid body

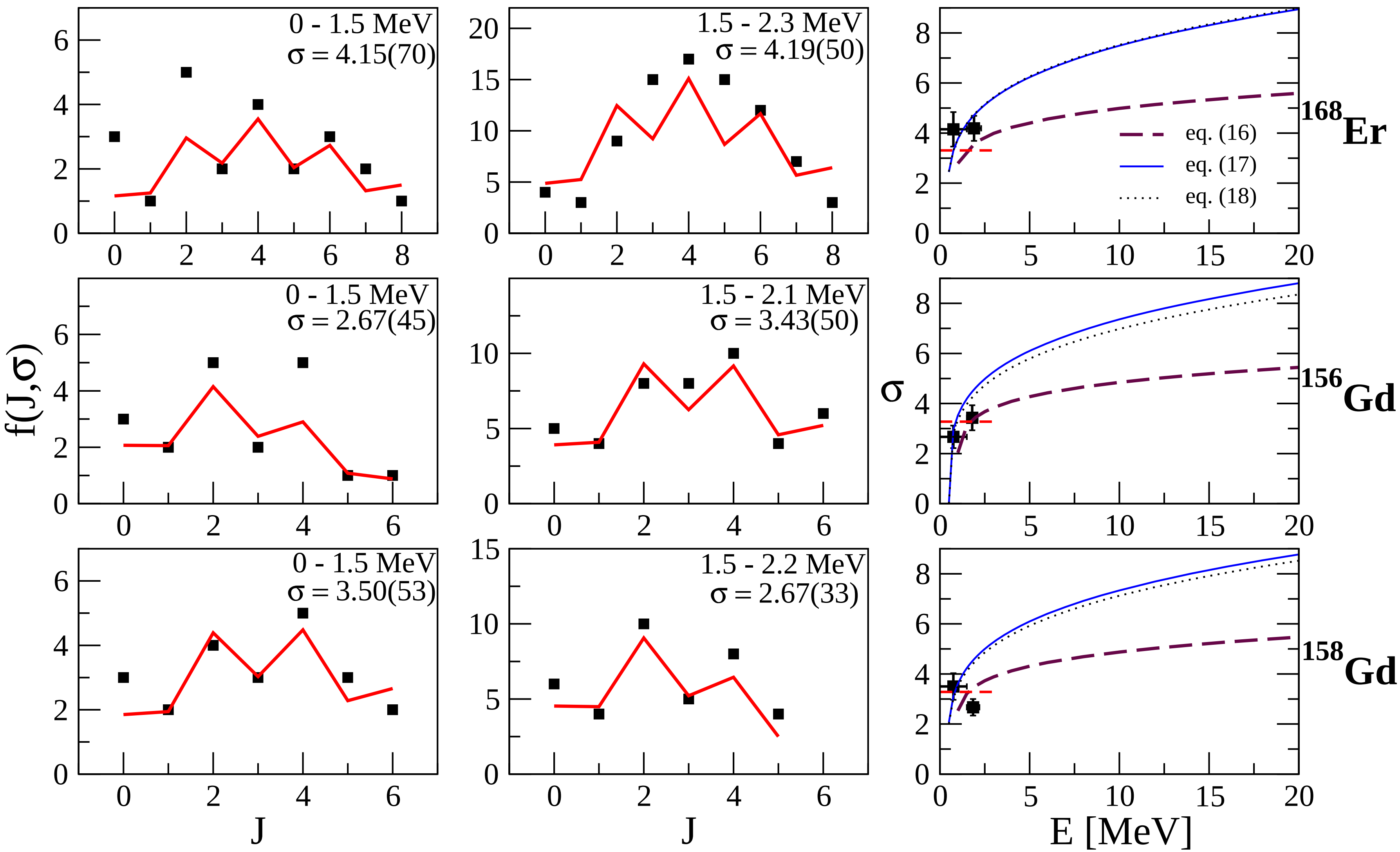
T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

Comparison of our new formula with experimental results

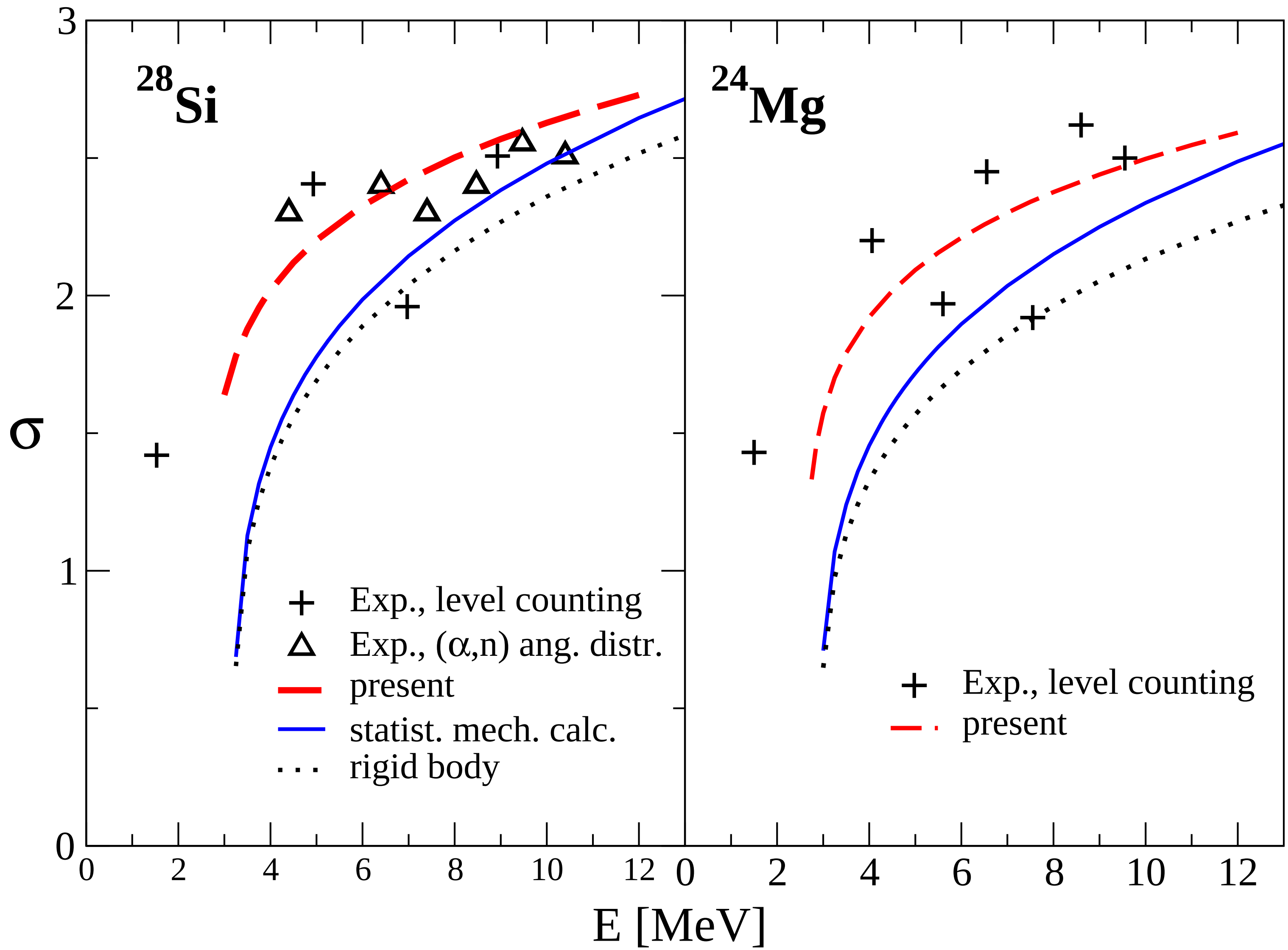


Exp: from evaporated particle angular distributions

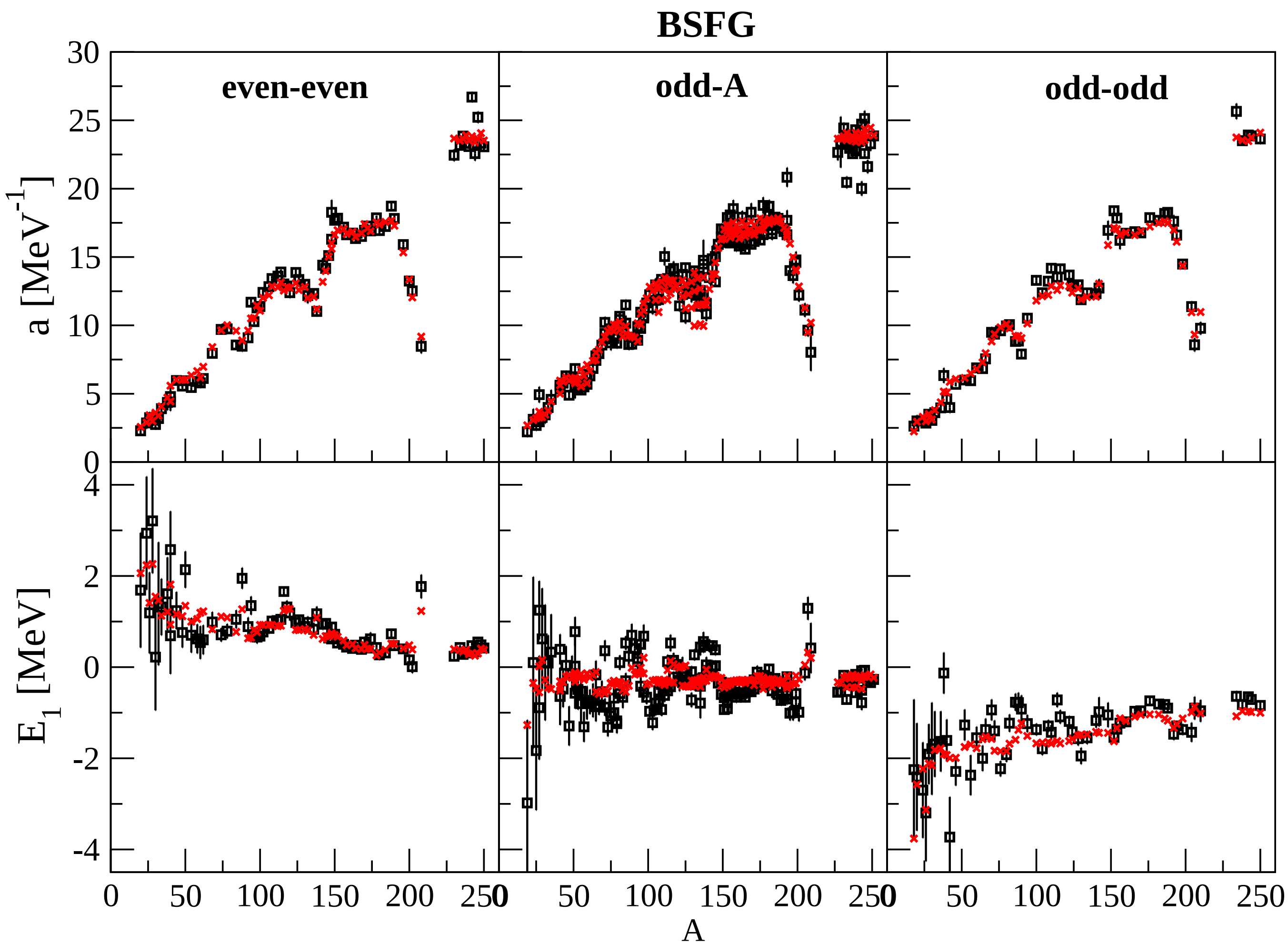
Comparison of our new formula with experimental results



Comparison of new formula with experiment and calculations



New determination of level density parameters a and E_1 for BSFG

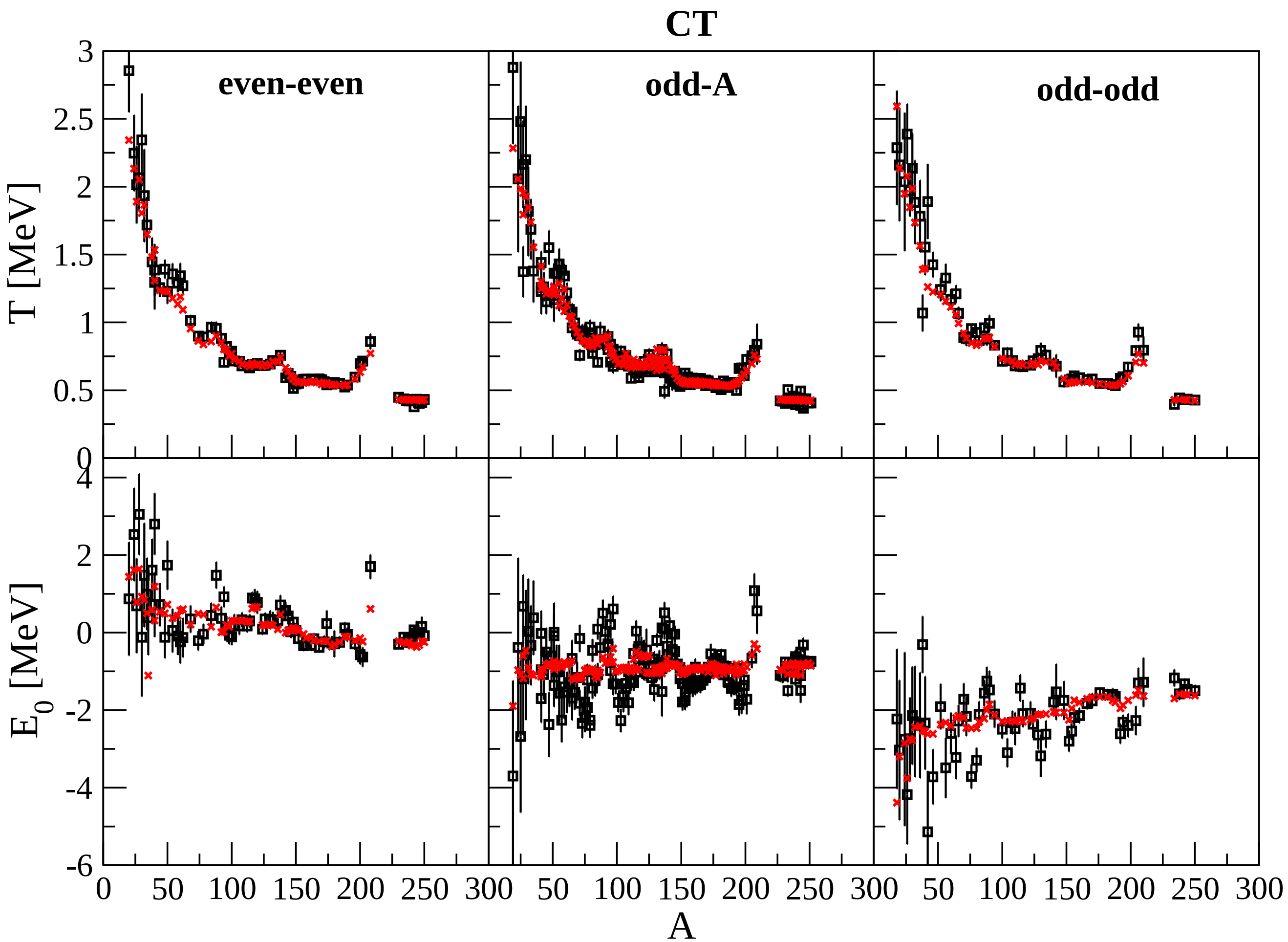


$$a = (0.199 + 0.0096S')A^{0.869} \qquad E_0 = -0.381 + 0.5Pa'$$

$$S' = S + 0.5Pa'$$ shell effect $S = M_{exp} - M_{Weiz}$

T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

New determination of level density parameters T and E_0 for CT

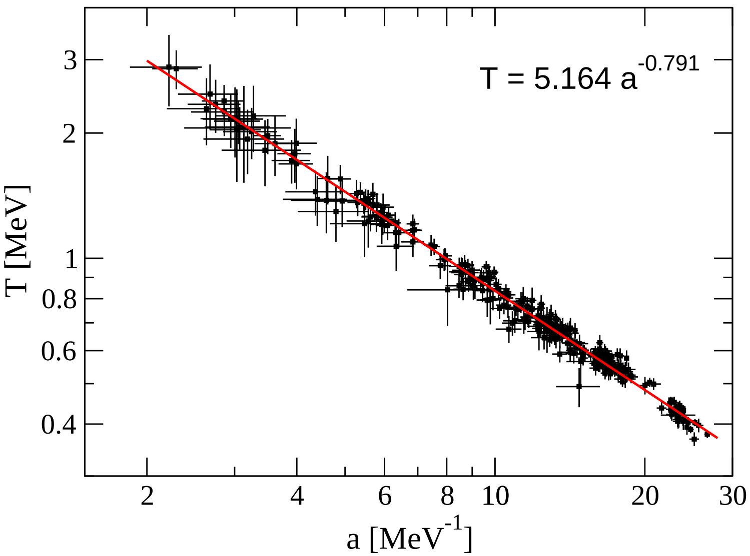


$$T = A^{-2/3} / (0.0597 + 0.00198 S')$$

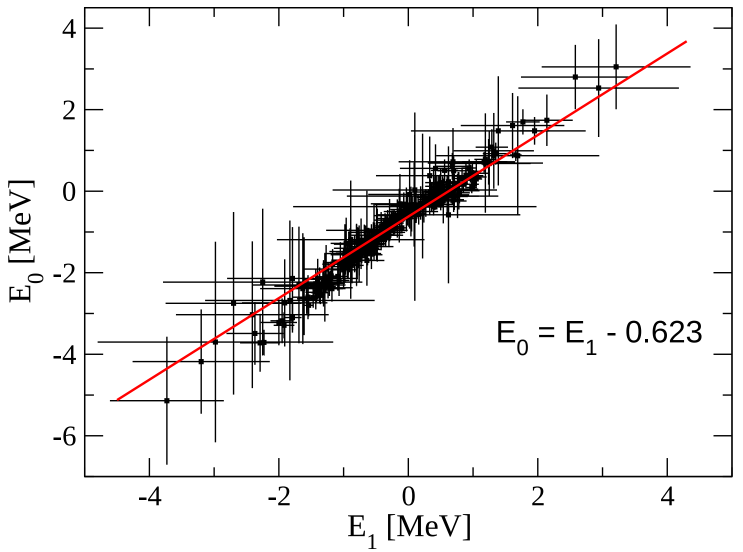
$$E_0 = -1.004 + 0.5 P a'$$

T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

Correlation of level density parameters



$$T = 5.164 a^{-0.791}$$



$$E_0 = E_1 - 0.623$$

T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

Conclusion

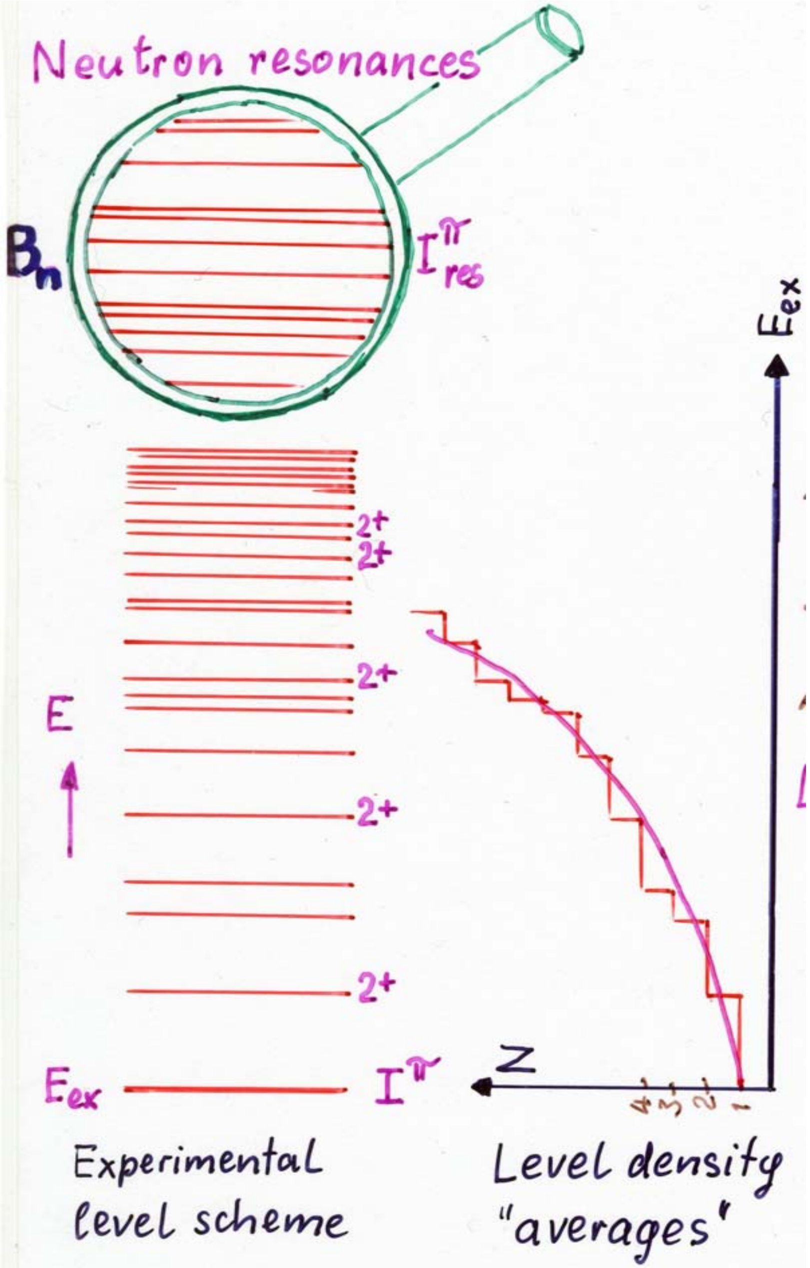
This very simple formula is

a challenge for theoretical calculations

$$\sigma^2 = 0.391 A^{0.675} (E - 0.5 Pa')^{0.312}$$

Valid for nuclei from F to Cf up to about 15 MeV

Published in: Physical Review C80, 054310 (2009)



Level densities: **averages**

Average level density $\boldsymbol{\rho(E)}$**:**

$$\rho(E) = dN/dE = 1/D(E)$$

Cumulative number **N(E)**

Average level spacing **D**

Level spacing $S_i = E_{i+1} - E_i$

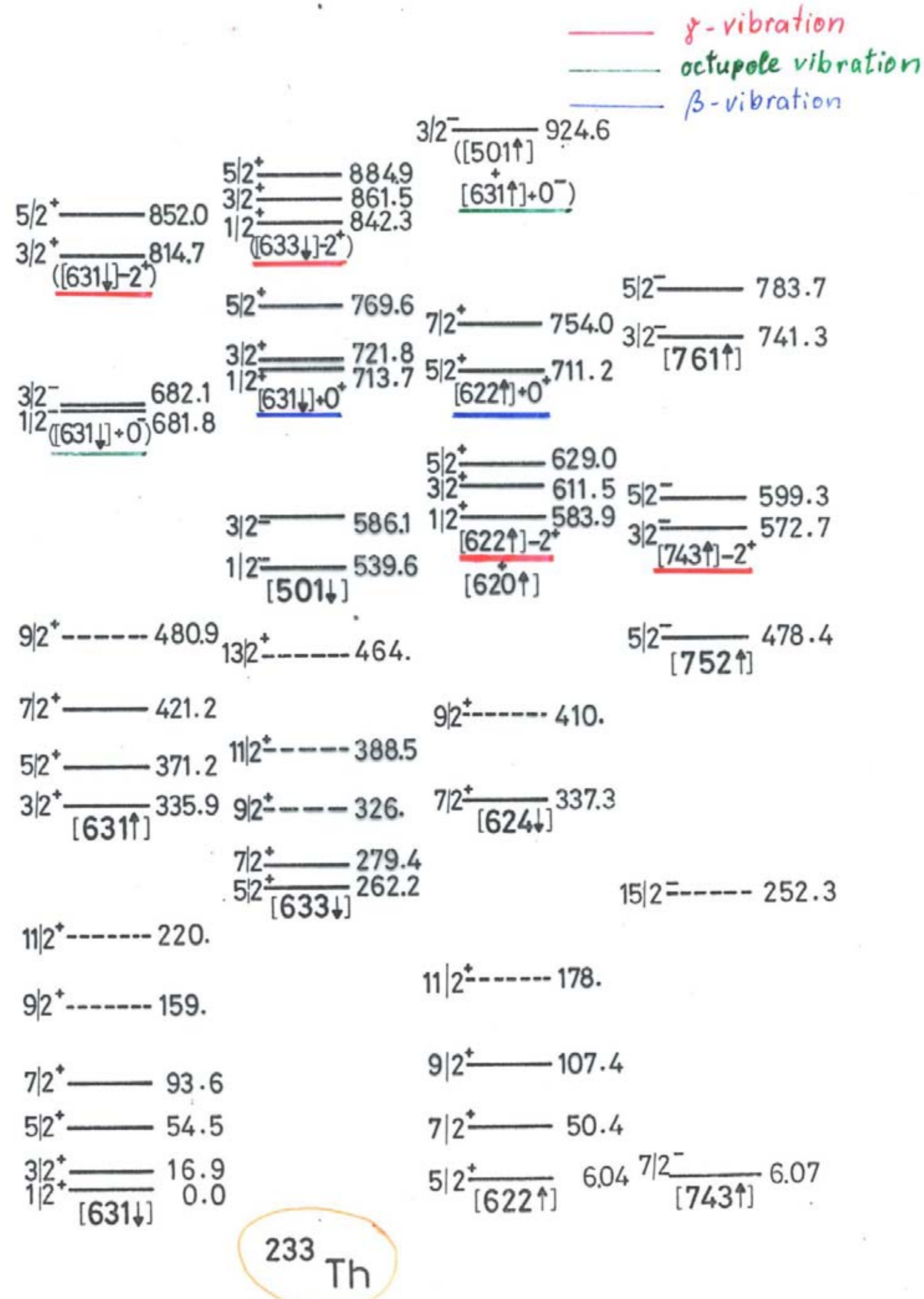
D(E) determined by a fit to the individual level spacings S_i

Level spacing correlation:

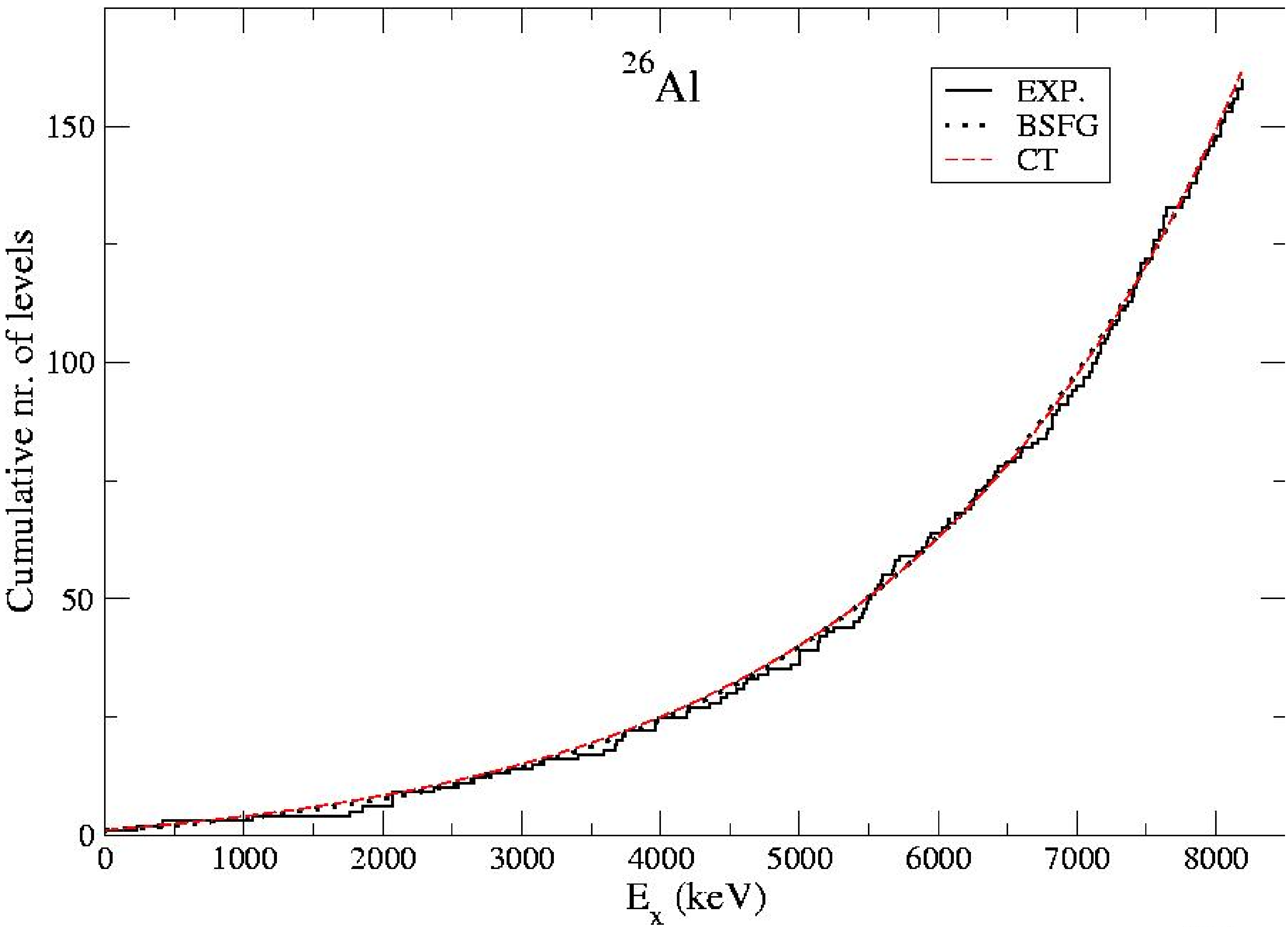
Chaotic properties determine fluctuations about the averages and the errors of the LD parameters.

T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

^{233}Th: Example of a complete low-energy level scheme



T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010



T. von Egidy, D. Bucurescu, Mass and energy dependence of spin distributions, Rossendorf 2010

shell correction **S(Z,N)** $= M_{exp} - M_{liquid\ drop}$

Macroscopic liquid drop mass formula (Weizsäcker):

J.M. Pearson, Hyp. Inter. 132(2001)59

$$\mathbf{E_{nuc}/A = a_{vol} + a_{sf}A^{-1/3} + (3e^2/5r_0)Z^2A^{-4/3} + (a_{sym}+a_{ss}A^{-1/3})J^2}$$

$$\mathbf{J} = (N-Z)/A; \quad A = N+Z$$

$$[\ E_{nuc} = -B.E. = (M_{nuc}(N,Z) - NM_n - ZM_p)c^2\ ]$$

From fit to 1995 Audi-Wapstra masses:

$a_{vol} = -15.65$ MeV; $a_{sf} = 17.63$ MeV;

$a_{sym} = 27.72$ MeV; $a_{ss} = -25.60$ MeV;

$r_0 = 1.233$ fm.

Nuclear Temperature at low excitation energy

- Thermodynamical definition of temperature

$$T = dE / d \log \rho(E)$$

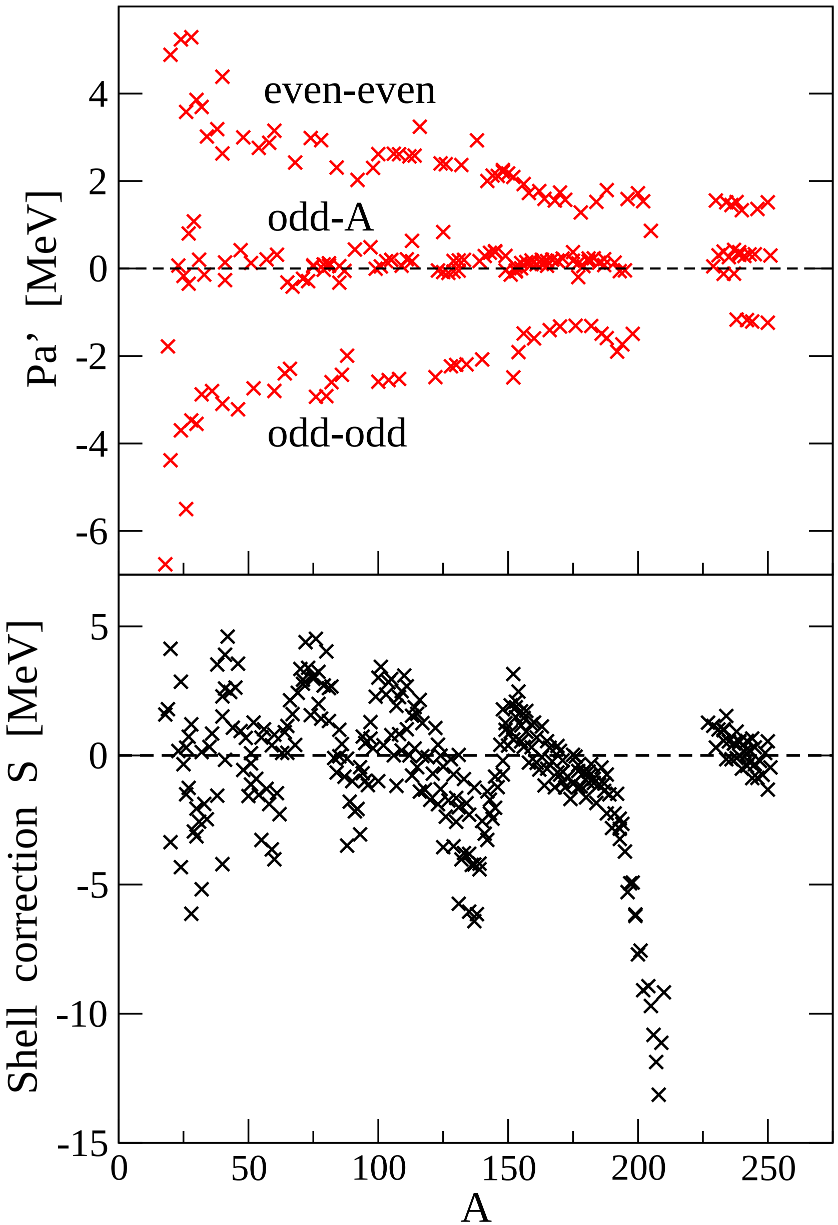
Integration with T = ***constant*** yields

$$\rho(E) = e^{(E - Eo) / T} / T$$

The agreement of this formula with experimental $\rho(E)$ shows that T = const. at low energy in spite of increasing energy.

This is similar to melting ice where temperature is constant during heating.

$\mathbf{T \sim E/n_{ex} \sim A^{-2/3}}$ indicates that the nucleus is melting from the surface.



T. von Egidy, D. Bucurescu

BSFG with energy-dependent „a“ (Ignatyuk)

$$a(E,Z,N) = \tilde{a}\,[1 + S'(Z,N)\, f(E - E_2) / (E - E_2)]$$

$f(E - E_2) = 1 - e^{-\gamma\,(E - E_2)}$; $\gamma = 0.06\ \mathrm{MeV}^{-1}$

$\tilde{a} = 0.1847\, A^{0.90}$

$E_2 = E_1$

This formula reduces the shell effect very slowly: 10% at 10 MeV, 50% at 30 MeV.

Comparison with previous level density calculations

- A. S. Iljinov et al. and T. Rauscher et al. have the following differences:

- 1. Data Set: Iljinov: Low levels, resonance densities, reaction data.
  Rauscher: Only resonance densities.
  We: Low levels and resonance densities.
- 2. Backshift: Iljinov: Fixed $\Delta = c\ 12\ A^{-1/2}$, $c = 0, 1, 2$ for o-o, odd, e-e.
  Rauscher: Fixed $\Delta = \frac{1}{2}(\Delta_n + \Delta_p)$ from mass tables.
  We: Independly fitted and calculated with deuteron pairing.
- 3. Formulas: Iljinov: Different formulas, also rotation and vibration.
  Rauscher: Ignatyuk's formulas
  We: CT, BSFG, Ignatyuk, different shell and pairing corr.
- 4. Fit Procedure: Iljinov: Calc. a for each data point, global fit of $\tilde{a}$(A).
  Rauscher: Global fit of $\tilde{a}$(A) to resonance densities.
  We: First individual fit of a,E_1, $\tilde{a}$,E_2 and T,E_0, then f (A).