Nonstatistical Effects in Neutron Resonance Parameters

P. E. Koehler[1], F.Bečvář[2], J. A. Harvey[1], M. Krtička[2], and K. H. Guber[3]

[1]Physics Division, Oak Ridge National Laboratory
[2]Charles University, Faculty of Mathematics and Physics
[3]Nuclear Science and Technology Division, Oak Ridge National Laboratory

- **Neutron widths**
- **Radiation widths**

This work was supported by the U.S. DOE Office of Nuclear Physics, by the NNSA Nuclear Criticality Safety Program, and by Czech Research Plans MSM-021620859 and INGO-LA08015.

Consensus View from Last ~50 Years:

Reduced Neutron Widths Follow at Porter-Thomas Distribution (PTD)

- **PTD derived from 3 fundamental assumptions:**

  Time-reversal invariance holds ($\gamma_{\lambda c}$ real).

  Single channel (elastic scattering) for neutrons.

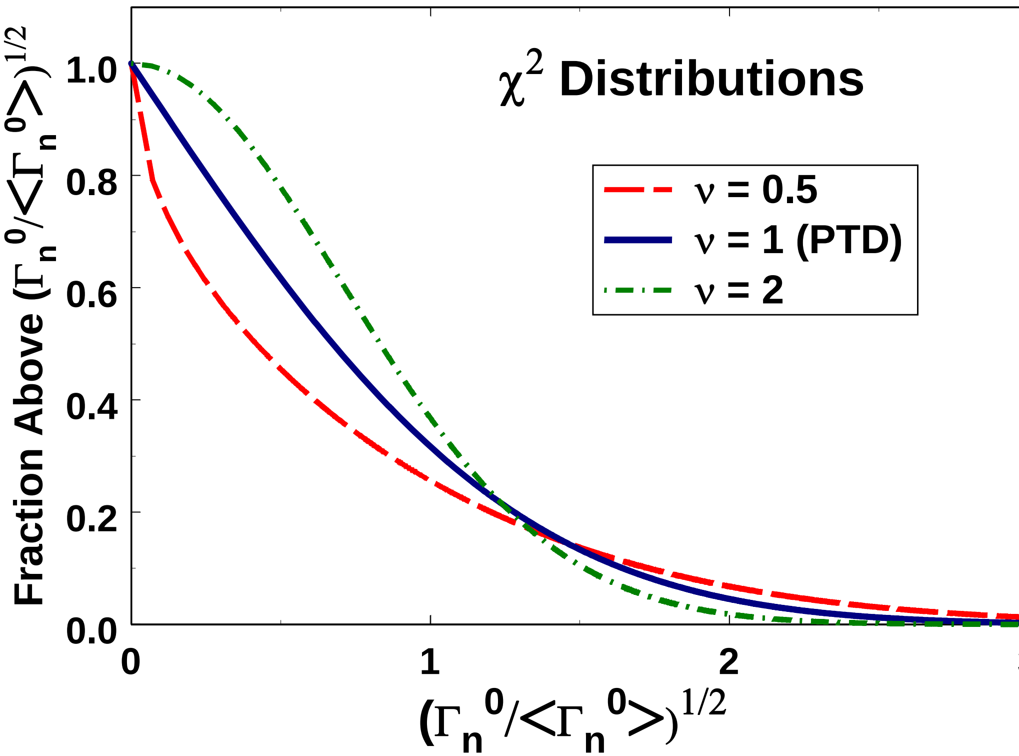
  Widths are "statistical".
  Compound nucleus model, central-limit theorem ⇨ reduced width amplitudes ($\gamma_{\lambda c}$) Gaussian distributed with zero mean ⇨ Reduced neutron widths, $\Gamma_n^0 = 2P\gamma_n^2 = \Gamma_n/\sqrt{E_n}$ (*s* wave), follow a χ^2 distribution with one degree of freedom ($\nu = 1$).

$$P(x,\nu) = \frac{\nu}{2G(\nu/2)}\left(\frac{\nu x}{2}\right)^{\nu/2-1}\exp\left(-\frac{\nu x}{2}\right)$$

$x = \Gamma_n^0/\langle\Gamma_n^0\rangle$

Random Matrix Theory

- Predicts both eigenvector (e.g., Γ_n^0) and eigenvalue (e.g. D, Δ_3) distributions.
- Gaussian orthogonal ensemble (GOE) should apply to highly excited states of heavy nuclides (i.e., near neutron threshold).
- Krieger and Porter showed that "level independence" and "form invariance" could replace "statistical" assumption.



Ideal Data Set for Testing the PTD: Γ_n^0 values for all *s*-wave resonances of a given J^π

Problems

- Purity.
  i.e., no *p*-wave resonances in *s*-wave set.

- Completeness.
  No missing resonances.
  e.g., due to finite detection threshold.

- Limited number of resonances.
  Data spread out over broad distribution.
  Always fighting limited statistical precision.

Effect of Missing Small Widths

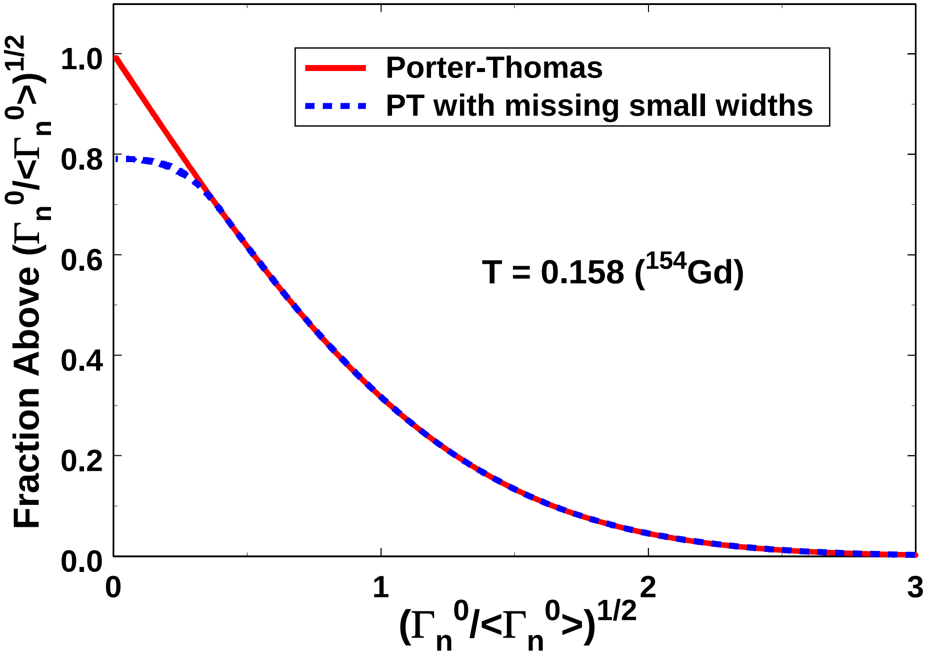
- Missing small widths changes shape of the distribution.

  $\Gamma_n^0/<\Gamma_n^0>=T*E/E_{max}$.

- If it's assumed that all widths were observed, obtain larger ν from maximum-likelihood (ML) analysis.

  $\nu_{true} = 1.0$

  $\nu_{ML} = 1.9$



Experiment threshold must be accounted for in comparison to theory.

Effect of Missing Small Widths

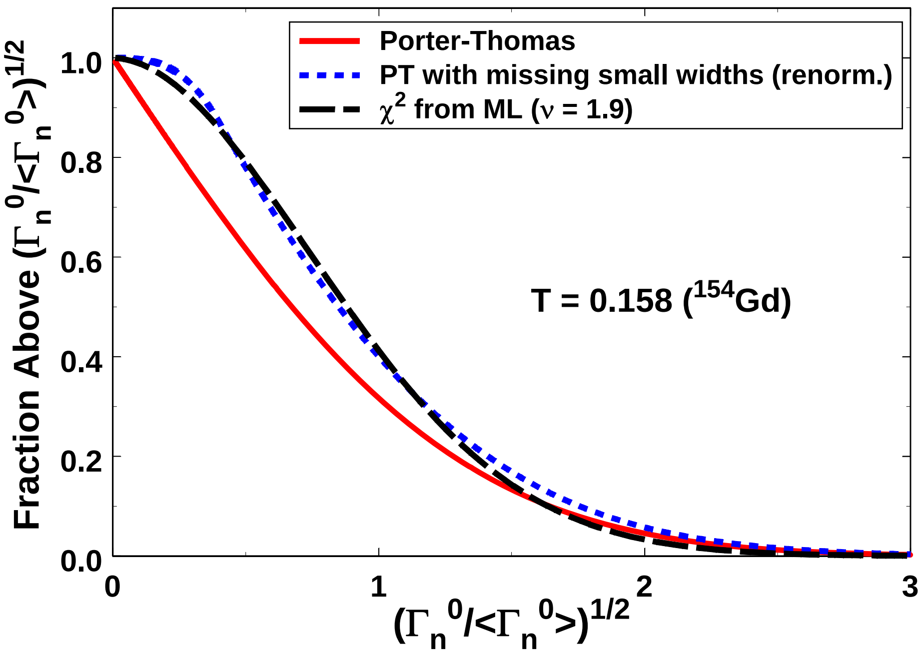
- Missing small widths changes shape of the distribution.

  $\Gamma_n^0/<\Gamma_n^0>=T*E/E_{max}$.

- If it's assumed that all widths were observed, obtain larger ν from maximum-likelihood (ML) analysis.

  $\nu_{true} = 1.0$

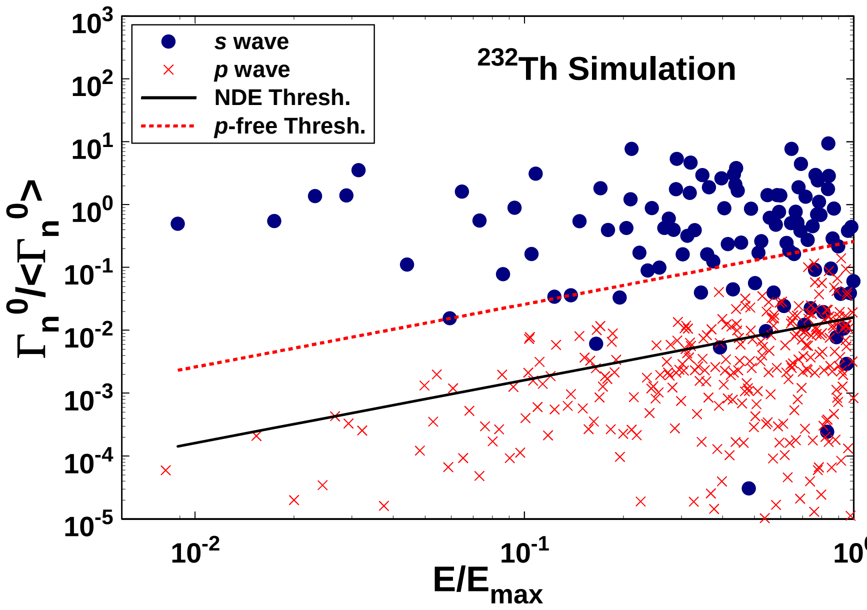
  $\nu_{ML} = 1.9$



Experiment threshold must be accounted for in comparison to theory.

Effect of *p*-wave Contamination

- Added *p*-wave widths smaller than *s*-wave ones.
  Simulation for ^{232}Th (assuming GOE).
  $\Gamma^0_{n,s} \sim$ constant
  $\Gamma^0_{n,p} \sim E$
- Assuming all widths above NDE threshold are *s* wave results in smaller ν from ML analysis.



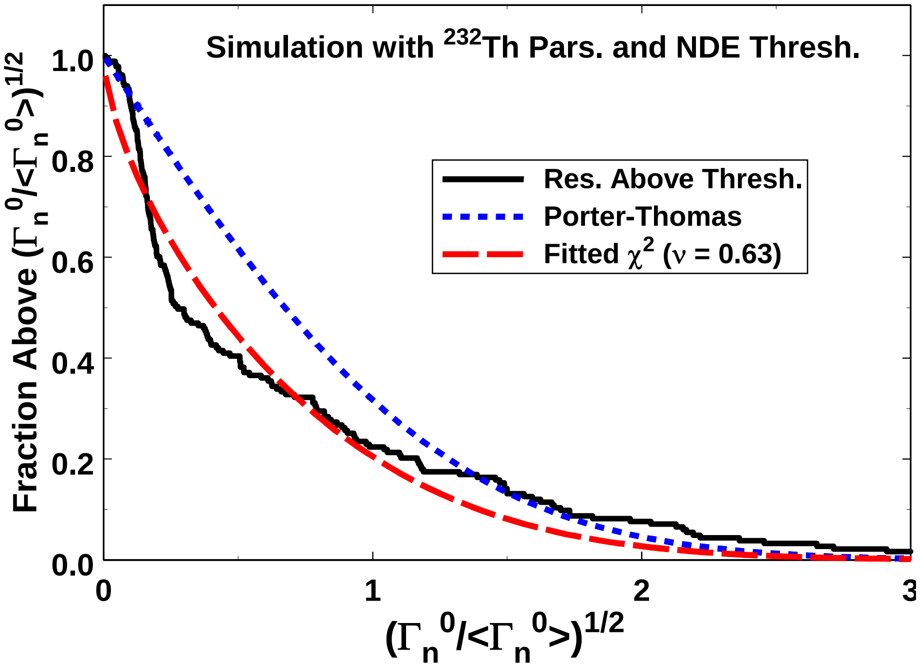
$\nu_{true} = 1.0$

$\nu_{ML} = 0.6$

Comparison to theory must assess purity of the data.

Effect of *p*-wave Contamination

- Added *p*-wave widths smaller than *s*-wave ones.
  Simulation for ^{232}Th (assuming GOE).
  $\Gamma^0_{n,s}$ ~ constant
  $\Gamma^0_{n,p}$ ~ E
- Assuming all widths above NDE threshold are *s* wave results in smaller ν from ML analysis.

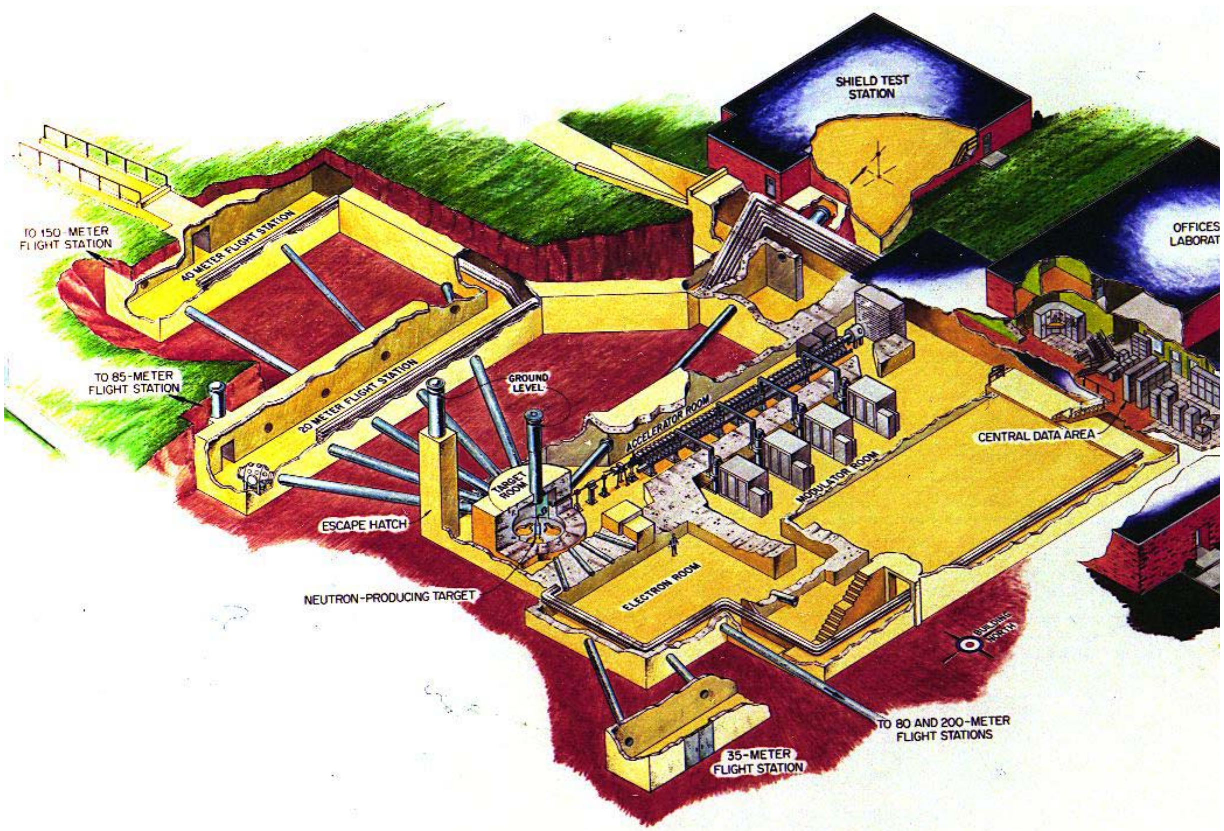


ν_{true} = 1.0

ν_{ML} = 0.6

Comparison to theory must assess purity of the data.

ORELA Measurements



Typical Run Parameters

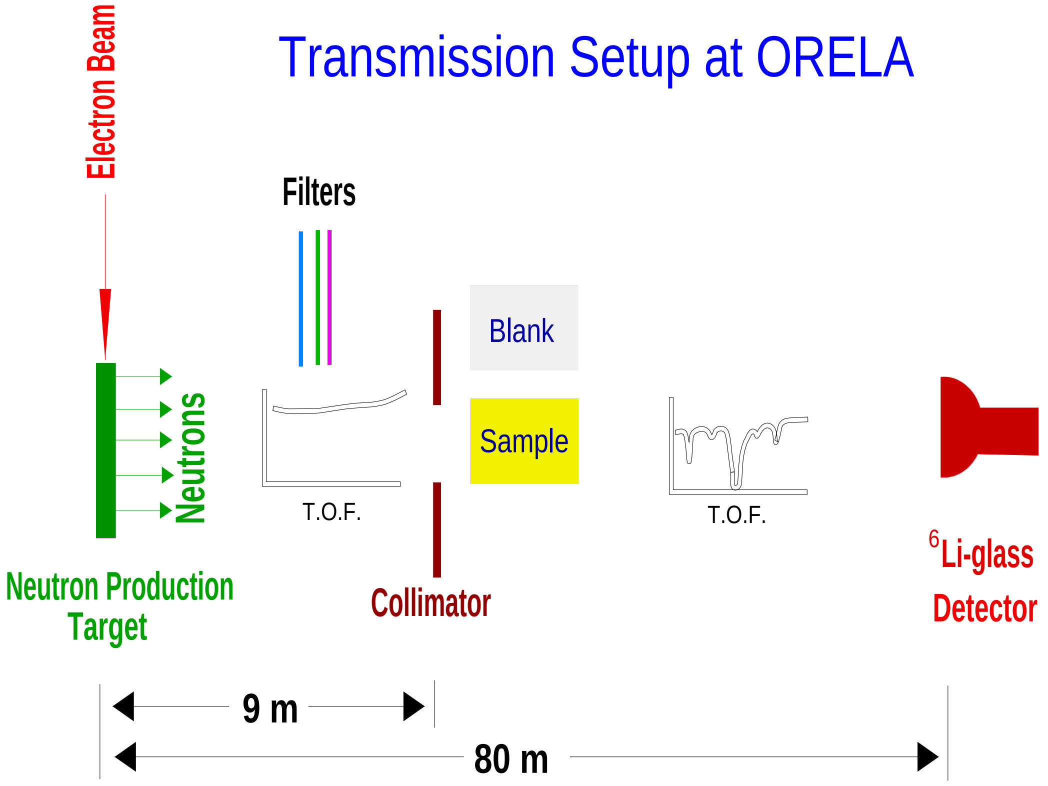
Rep. rate = 525 Hz
Δt = 8 ns
P = 8 kW
E_n=10 eV – 500 keV

(n,γ) with C_6D_6 on F.P. 6 and 7 @ 40 m

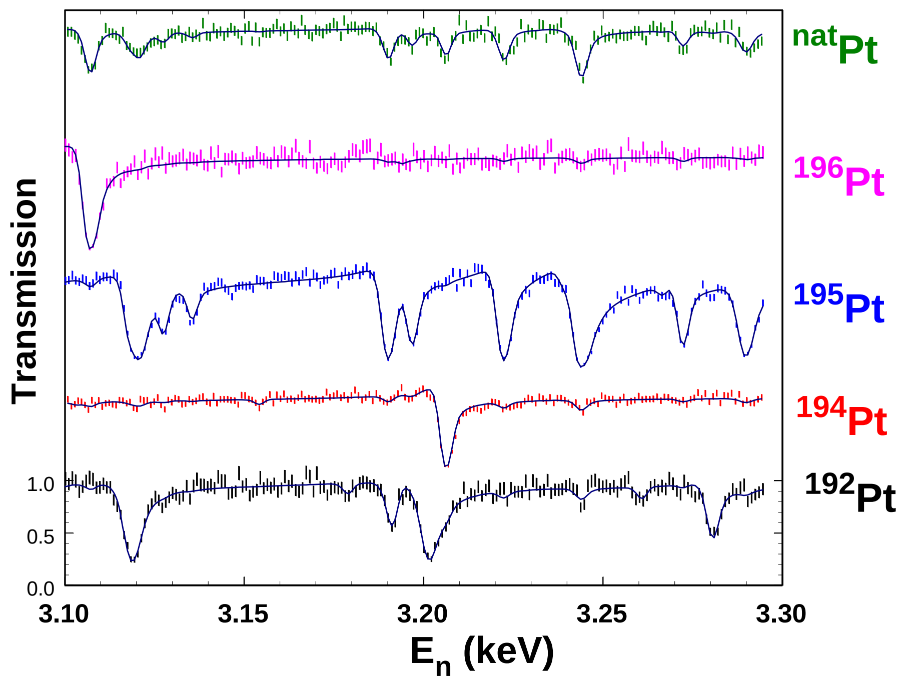
σ_t with ^{6}Li-glass on F.P. 1 @ 80 m

(n,α) with CIC on F.P. 11 @ 10 m

ORELA Measurements



ORELA Measurements

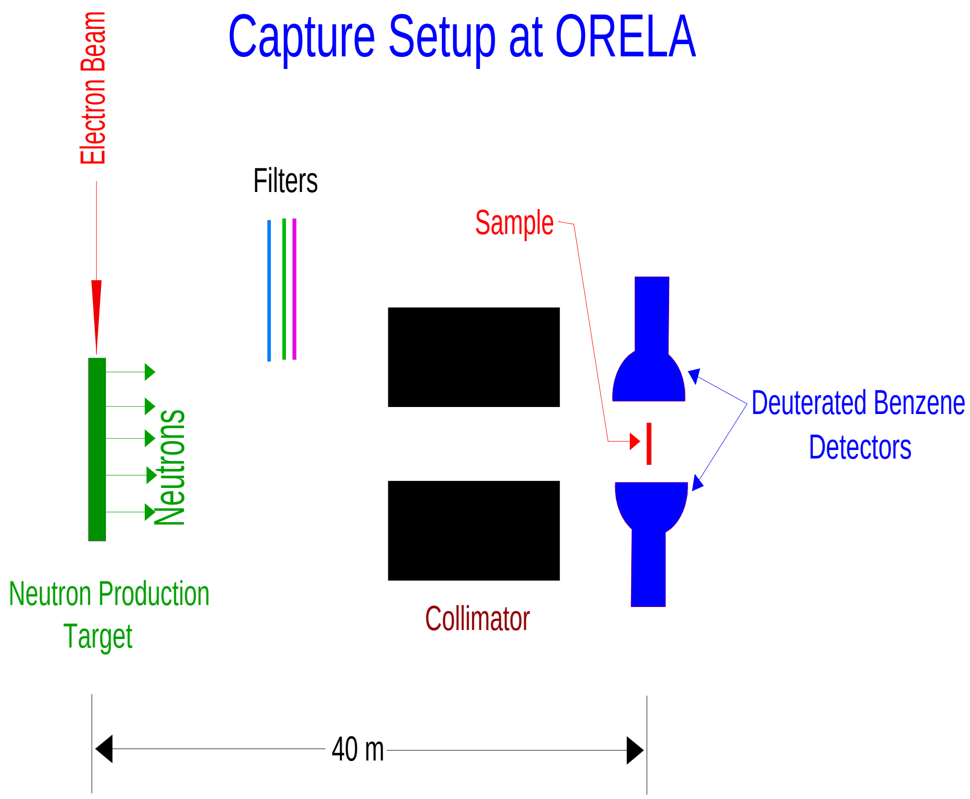


Depth~$g\Gamma_n$

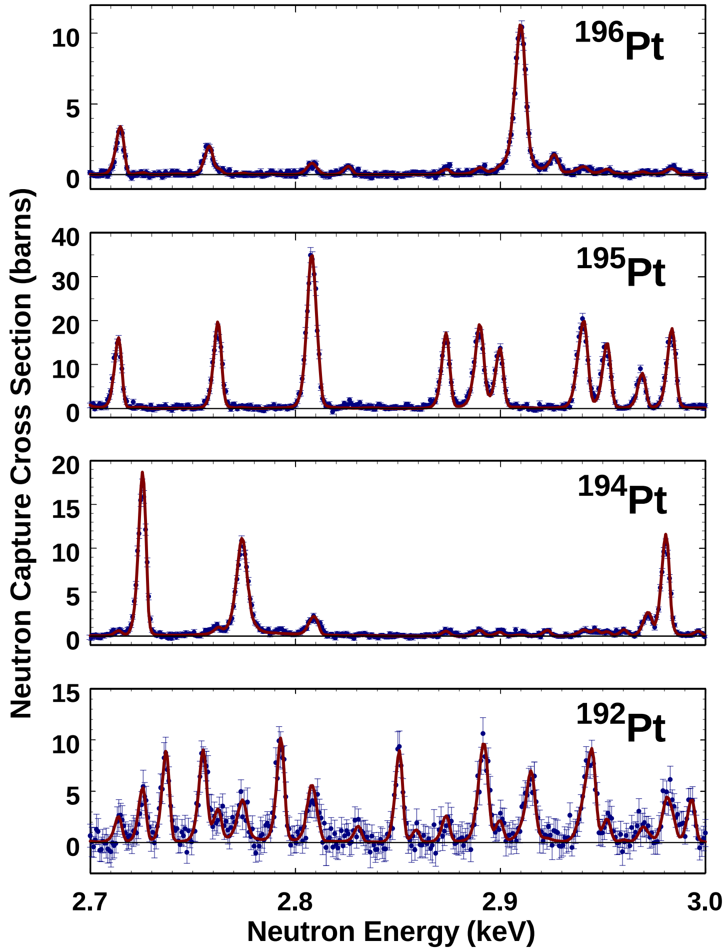
Asymetric shape => *s*-wave

Only in rare cases can both Γ_γ and $g\Gamma_n$ be determined from transmission alone

ORELA Measurements



ORELA Measurements



$A_\gamma = g\Gamma_n\Gamma_\gamma/(\Gamma_n+\Gamma_\gamma)$

Only in rare cases (e.g., $\Gamma_\gamma+\Gamma_n > \Delta E$) can both Γ_γ and $g\Gamma_n$ be determined from (n,γ) alone.

Resonance Parameters from R-Matrix Analysis

To test the PTD, need E and Γ_n for all resonances of a given J^π.

Obtainable from Total Cross-Section Data

Target Spin	Relative size of $g\Gamma_n$			
	Small	Intermediate	Large	Very Large
0	Nothing	E, $g\Gamma_n$	E, $g\Gamma_n$, (g), (Γ_γ),$(J)^\pi$	E, $g\Gamma_n$, (g), $(J)^\pi$
>0	Nothing	E, $g\Gamma_n$	E, $g\Gamma_n$, (Γ_γ),π	E, $g\Gamma_n$, $(J)^\pi$

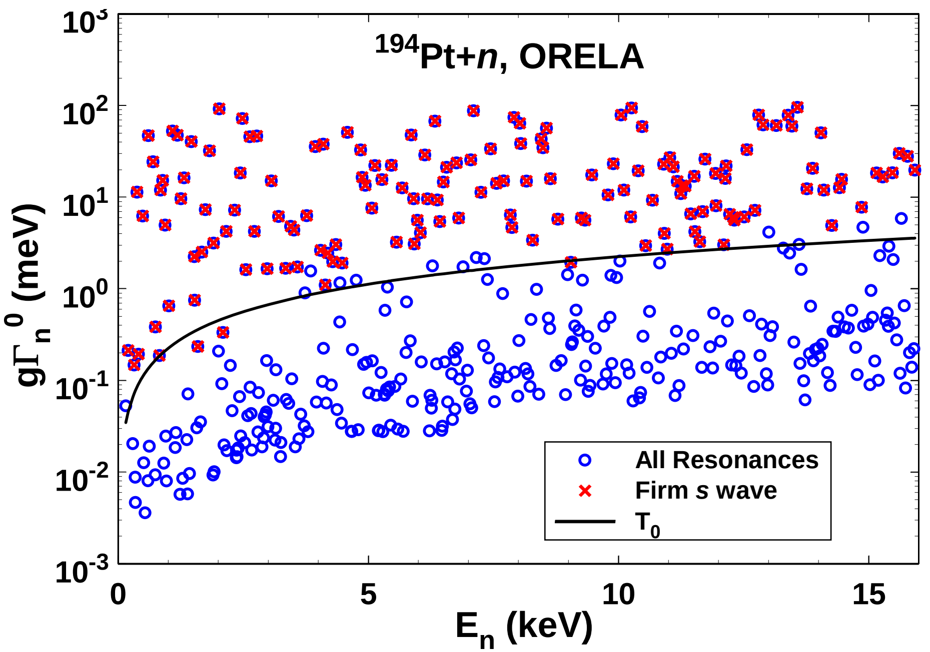
Obtainable from Capture Cross-Section Data

Target Spin	Relative size of $g\Gamma_n$			
	Small	Intermediate	Large	Very Large
0	E, $g\Gamma_n$	E, A_γ	E, A_γ	E, $g\Gamma_n$, (g), (Γ_γ),$(J)^\pi$
>0	E, $g\Gamma_n$	E, A_γ	E, A_γ	E, $g\Gamma_n$, (Γ_γ),π

Typical case: Can obtain Γ_n^0 only for subset of *s*-wave resonances, only if have both capture and total cross-section data, and only for zero-spin targets.

Testing the PTD Using ^{192,194}Pt+n ORELA Data

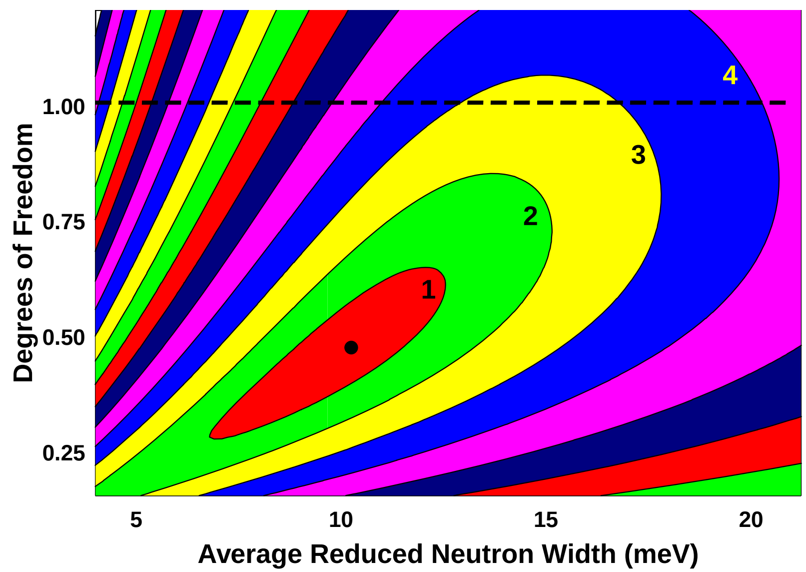
- ^{192,194,196}Pt+n ORELA data better in many ways.
  More resonances.
  Better sensitivity (~10x).
  Better separation of *s* and *p* waves ($S_0 \approx 10 S_1$).
  Better J^{π} assignments.
- Improved Maximum-Likelihood analysis.
  Used energy-dependent threshold.
  Maximizes statistical significance while eliminating *p*-wave contamination.
  Analysis threshold T_0 much higher than experimental one.



Testing the PTD using ^{192,194,196}Pt+n ORELA Data
(Phys. Rev. Lett. 105, 072502 (2010))

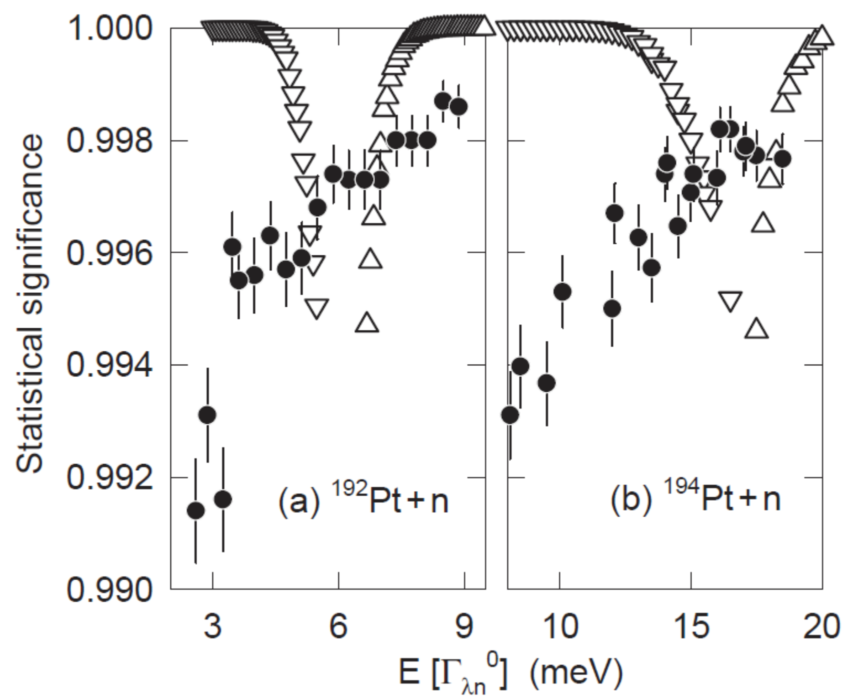
- Maximum-Likelihood (ML) analysis.
  ^{192}Pt: ν = 0.57±0.16
  ^{194}Pt: ν = 0.47±0.19
  ^{196}Pt: ν = 0.60±0.28
- Additional calculations to determine confidence level (CL) for rejecting PTD.
  Monte Carlo simulation to determine CL as function of $\langle\Gamma_n^0\rangle$.
  Two new statistics to limit range of $\langle\Gamma_n^0\rangle$.
  Auxiliary ML analysis to verify that *p*-wave contamination is negligibly small (0.069 for ^{192}Pt, 0.0047% for ^{194}Pt).

$$z\left(\nu, \mathrm{E}[\Gamma^0_{\lambda\mathrm{n}}]\right) = 2^{\frac{1}{2}}\left[\ln L_{\max} - \ln L\left(\nu, \mathrm{E}[\Gamma^0_{\lambda\mathrm{n}}]\right)\right]^{\frac{1}{2}}$$



Testing the PTD using ^{192,194,196}Pt+n ORELA Data
(Phys. Rev. Lett. 105, 072502 (2010))

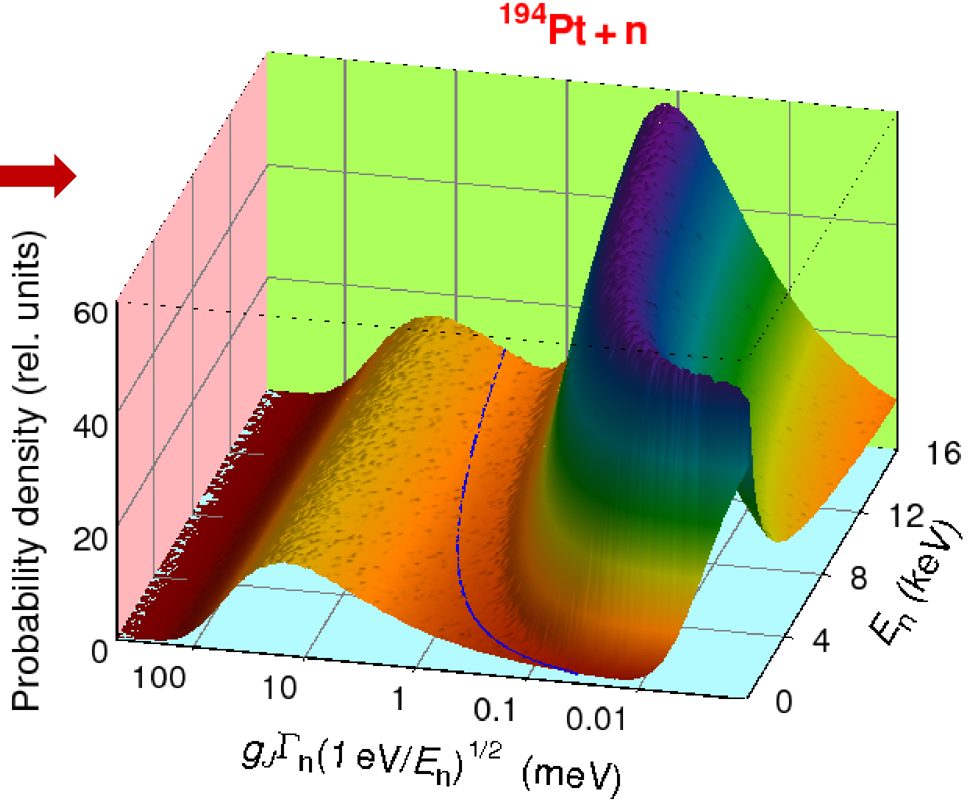
- Maximum-Likelihood (ML) analysis.
  ^{192}Pt: ν = 0.57±0.16
  ^{194}Pt: ν = 0.47±0.19
  ^{196}Pt: ν = 0.60±0.28
- Additional calculations to determine confidence level (CL) for rejecting PTD.
  Monte Carlo simulation to determine CL as function of $\langle\Gamma_n^0\rangle$.
  Two new statistics to limit range of $\langle\Gamma_n^0\rangle$.
  Auxiliary ML analysis to verify that *p*-wave contamination is negligibly small (0.069 for ^{192}Pt, 0.0047% for ^{194}Pt).



Testing the PTD using ^{192,194,196}Pt+n ORELA Data

(Phys. Rev. Lett. 105, 072502 (2010))

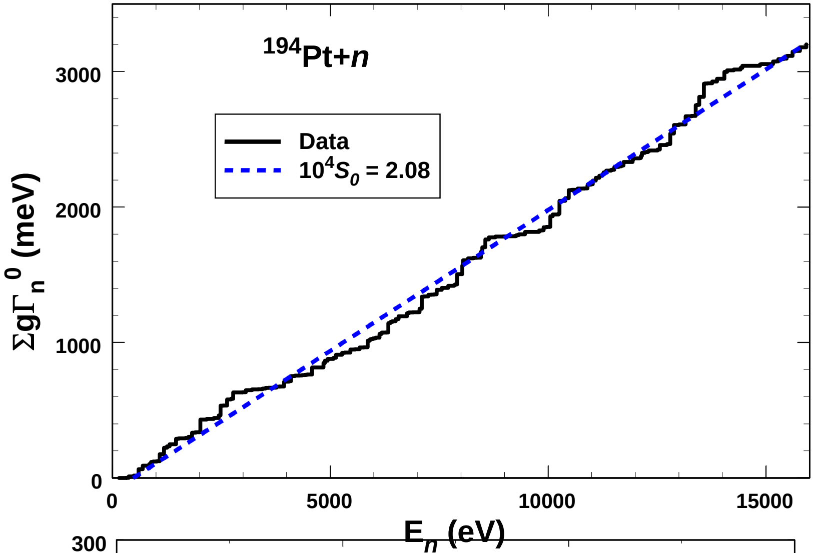
- Maximum-Likelihood (ML) analysis.
  ^{192}Pt: ν = 0.57±0.16
  ^{194}Pt: ν = 0.47±0.19
  ^{196}Pt: ν = 0.60±0.28
- Additional calculations to determine confidence level (CL) for rejecting PTD.
  Monte Carlo simulation to determine CL as function of $\langle\Gamma_n^0\rangle$.
  Two new statistics to limit range of $\langle\Gamma_n^0\rangle$.
  Auxiliary ML analysis to verify that *p*-wave contamination is negligibly small (0.069 for ^{192}Pt, 0.0047% for ^{194}Pt).

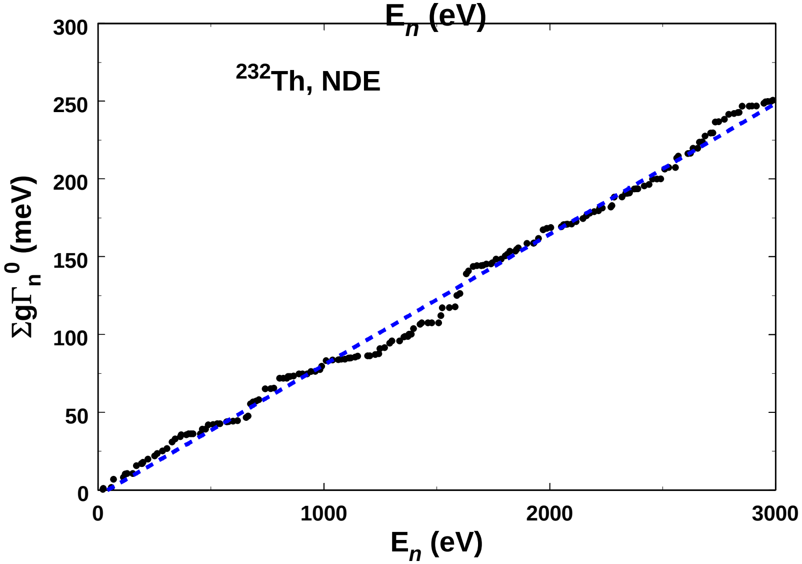


PTD rejected at 99.997% confidence level

Possible explanations

- TRIV and unknown (e.g. inelastic) extra neutron channel ruled out. Lead to $\nu>1$, but $\nu<1$ observed.
- Widths not statistical. But typical nonstatistical signatures absent in data (e.g., steps in $\Sigma\Gamma_n^0$ vs. E_n).
- Might be signature of collective effect (e.g., Y. Alhassid and A. Novoselsky, Phys. Rev. C 45, 1677 (1992)). Model calculations for low excitations yielded transition strength distributions with $\nu<1$ as system became more collective. But why would highly excited states in ^{193,195}Pt be collective?



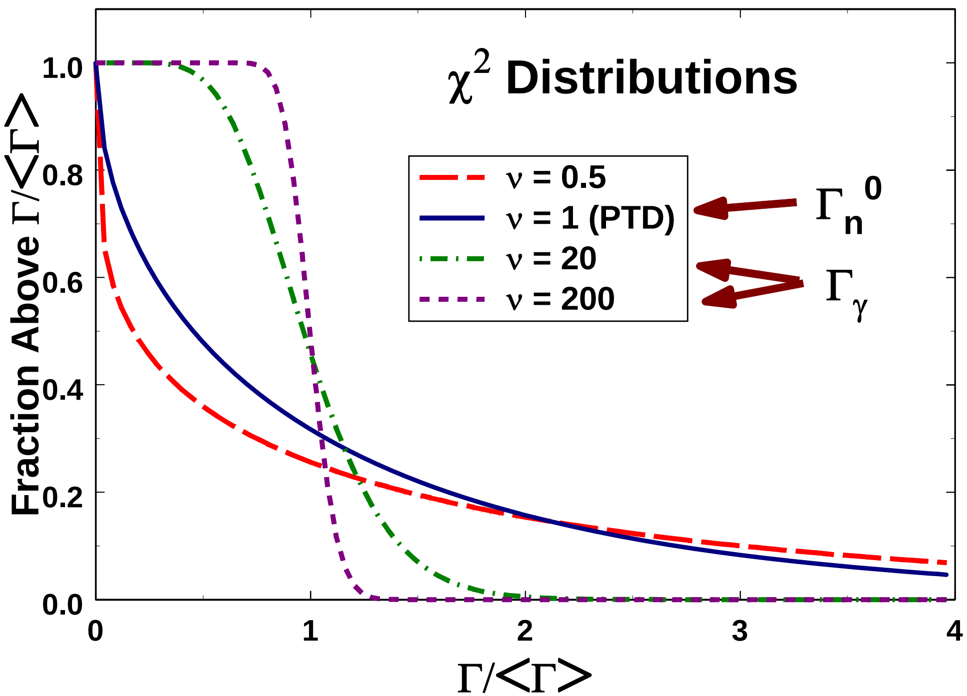


Form invariance or level independence assumptions violated?

- Level independence and form invariance assumptions shown to yield same results (and more) as "statistical" assumption.
- But, what does violating these assumptions mean and how could they cause ν=0.5?

Γ_γ Distributions

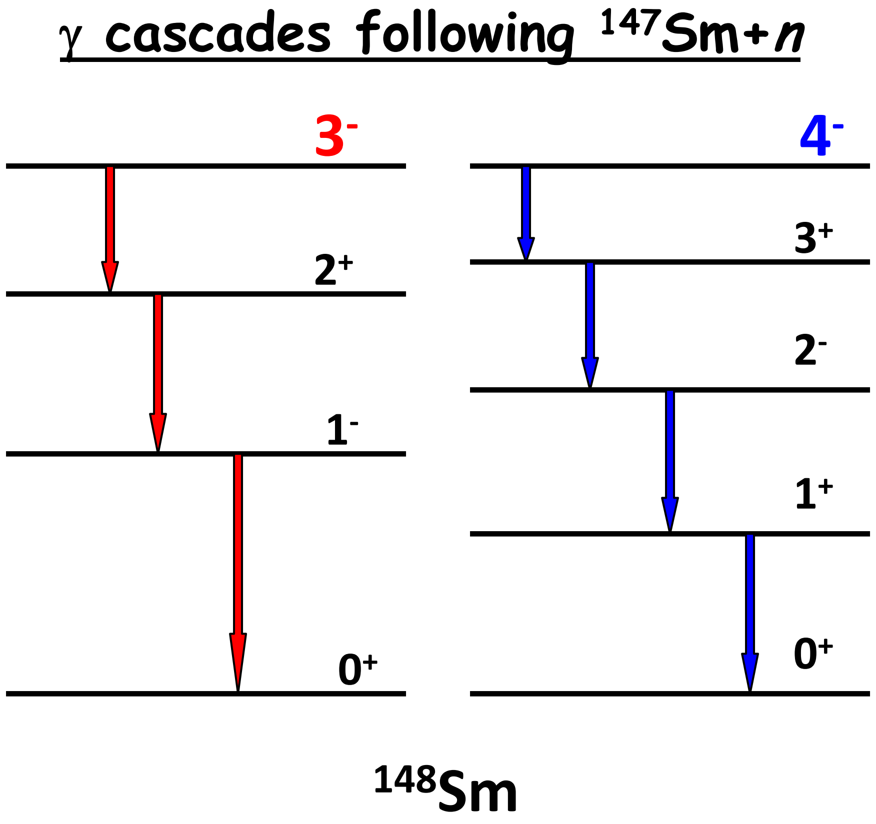
- Partial widths $\Gamma_{\gamma i}$ follow PTD.
- Obtain $\Gamma_\gamma = \sum \Gamma_{\gamma i}$ from *R*-matrix analyses.
- Assuming averages the same, Γ_γ follows χ^2 dist. with ν equal to number of independent transitions.



Γ_γ dists. very narrow.

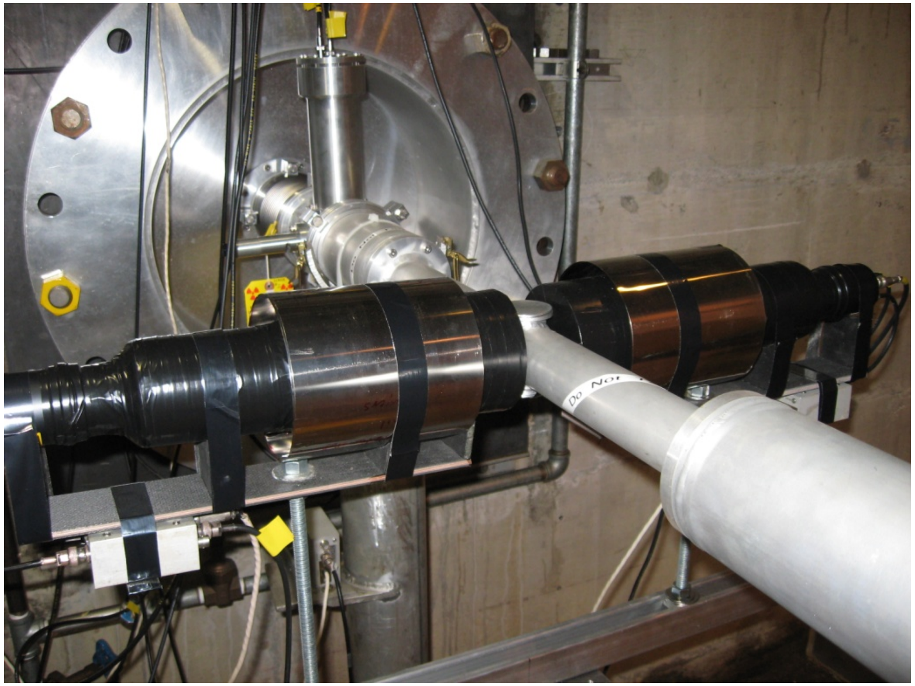
Using γ-cascade Information to Assign J^{π}

- Expect: Higher J ⇨ larger M ⇨ more coincidences and softer singles spectrum.
- First demonstrated by Coceva *et al.*, Nucl. Phys. A117, 586 (1968).
- Implemented with improvements at ORELA.
  New C_6D_6 CINDORELA apparatus.
  Gates optimized during replay.
  Multiple gates on singles and coincidences.
  Parities better separated using overall singles pulse-height shapes.



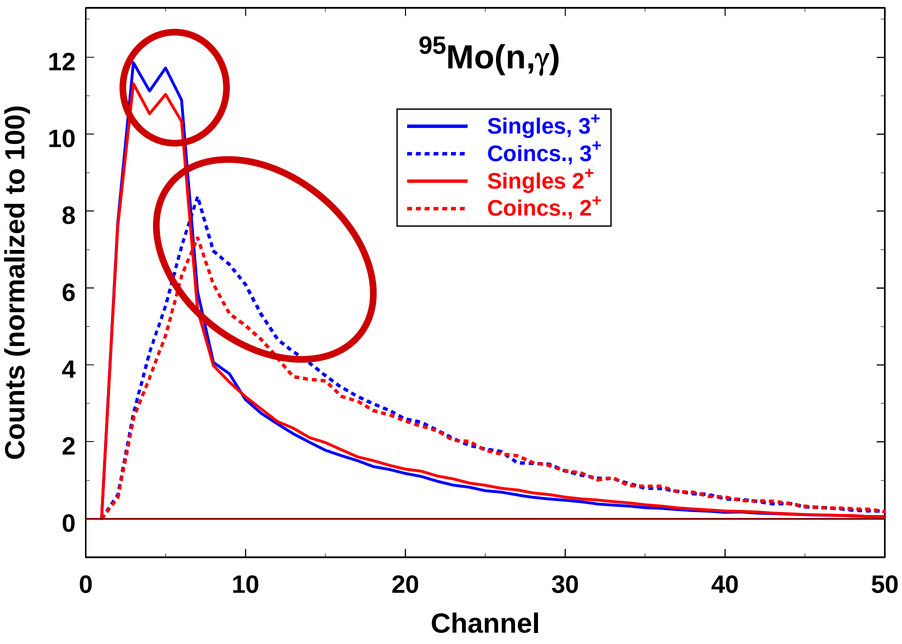
Using γ-cascade Information to Assign J^π

- Expect: Higher J ⇨ larger M ⇨ more coincidences and softer singles spectrum.
- First demonstrated by Coceva *et al.*, Nucl. Phys. A117, 586 (1968).
- Implemented with improvements at ORELA.
  New C_6D_6 CINDORELA apparatus.
  Gates optimized during replay.
  Multiple gates on singles and coincidences.
  Parities better separated using overall singles pulse-height shapes.



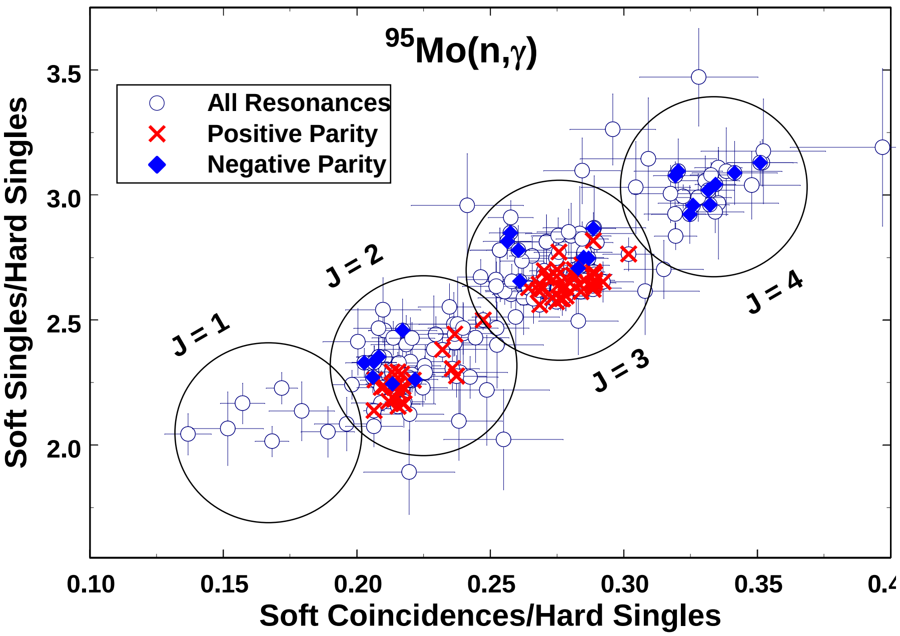
Using γ-cascade Information to Assign J^{π}

- Expect: Higher J ⇨ larger M ⇨ more coincidences and softer singles spectrum.
- First demonstrated by Coceva *et al.*, Nucl. Phys. A117, 586 (1968).
- Implemented with improvements at ORELA.
  New C_6D_6 CINDORELA apparatus.
  Gates optimized during replay.
  Multiple gates on singles and coincidences.
  Parities better separated using overall singles pulse-height shapes.



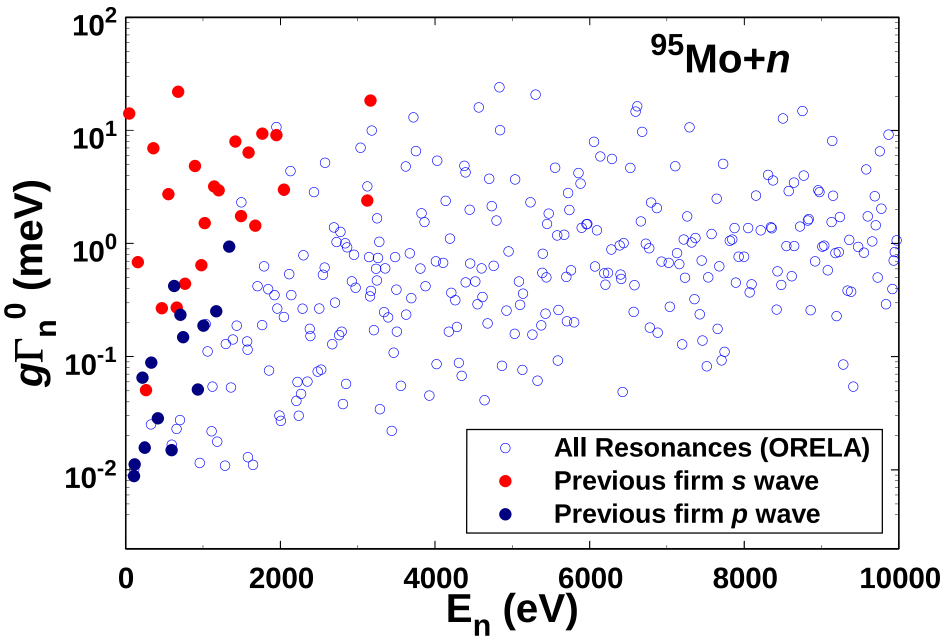
Using γ-cascade Information to Assign J^{π}

- Expect: Higher J ⇨ larger M ⇨ more coincidences and softer singles spectrum.
- First demonstrated by Coceva *et al.*, Nucl. Phys. A117, 586 (1968).
- Implemented with improvements at ORELA.
  New C_6D_6 CINDORELA apparatus.
  Gates optimized during replay.
  Multiple gates on singles and coincidences.
  Parities better separated using overall singles pulse-height shapes.



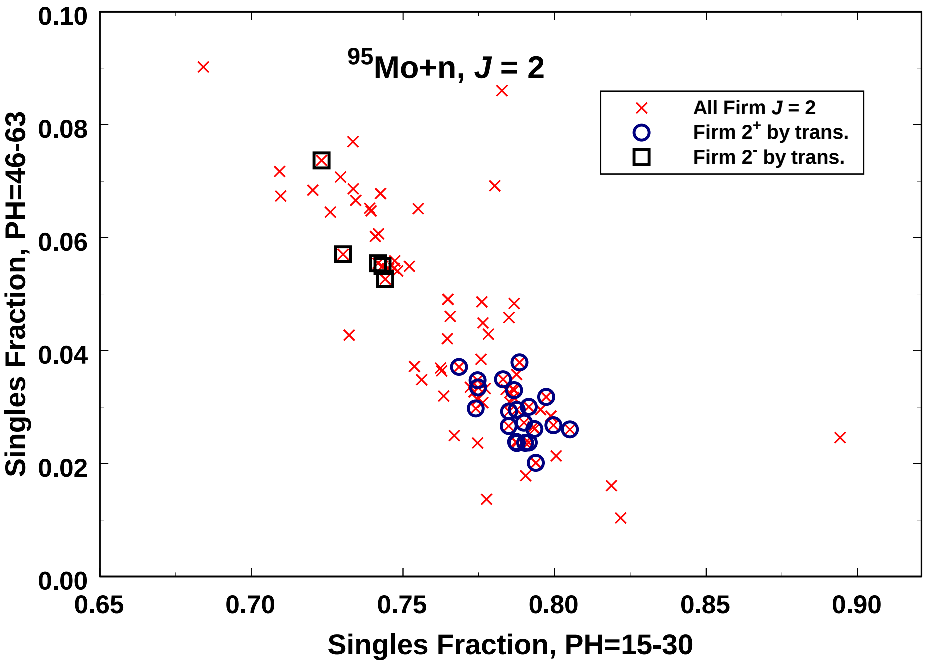
Using γ-cascade Information to Assign J^{π}

- Expect: Higher J ⇨ larger M ⇨ more coincidences and softer singles spectrum.
- First demonstrated by Coceva *et al.*, Nucl. Phys. A117, 586 (1968).
- Implemented with improvements at ORELA.
  New C_6D_6 CINDORELA apparatus.
  Gates optimized during replay.
  Multiple gates on singles and coincidences.
  Parities better separated using overall singles pulse-height shapes.



Using γ-cascade Information to Assign J^{π}

- Expect: Higher J ⇨ larger M ⇨ more coincidences and softer singles spectrum.
- First demonstrated by Coceva *et al.*, Nucl. Phys. A117, 586 (1968).
- Implemented with improvements at ORELA.
  New C_6D_6 CINDORELA apparatus.
  Gates optimized during replay.
  Multiple gates on singles and coincidences.
  Parities better separated using overall singles pulse-height shapes.



CINDORELA Results

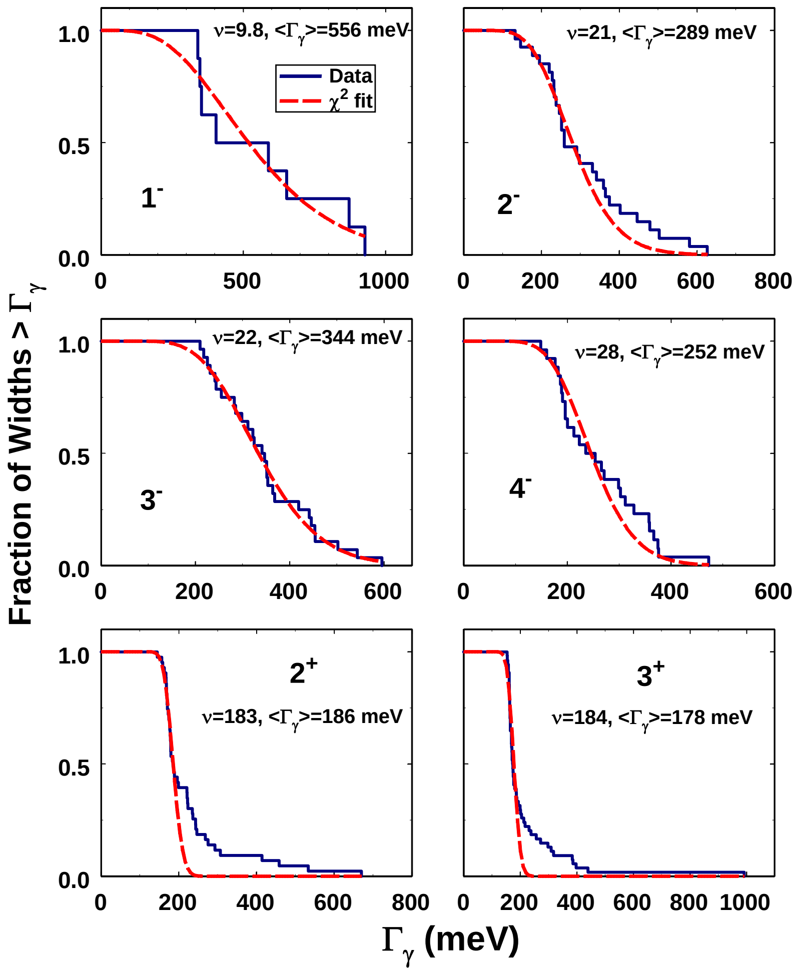
- Firm J^{π}'s for 220 of 314 resonances observed for En<10 keV.

  Previously: 32 of 107.

  ^{95}Mo very difficult test case: Peak of *p*- and minimum of *s*-wave neutron strength functions, so six J^{π}'s possible.

- Separate Γ_γ and Γ_n distributions for 1^-, 2^-, 2^+, 3^-, 3^+, and 4^- resonances.

  Best Γ_γ data ever obtained.



^{95}Mo Average Resonance Parameters from ORELA

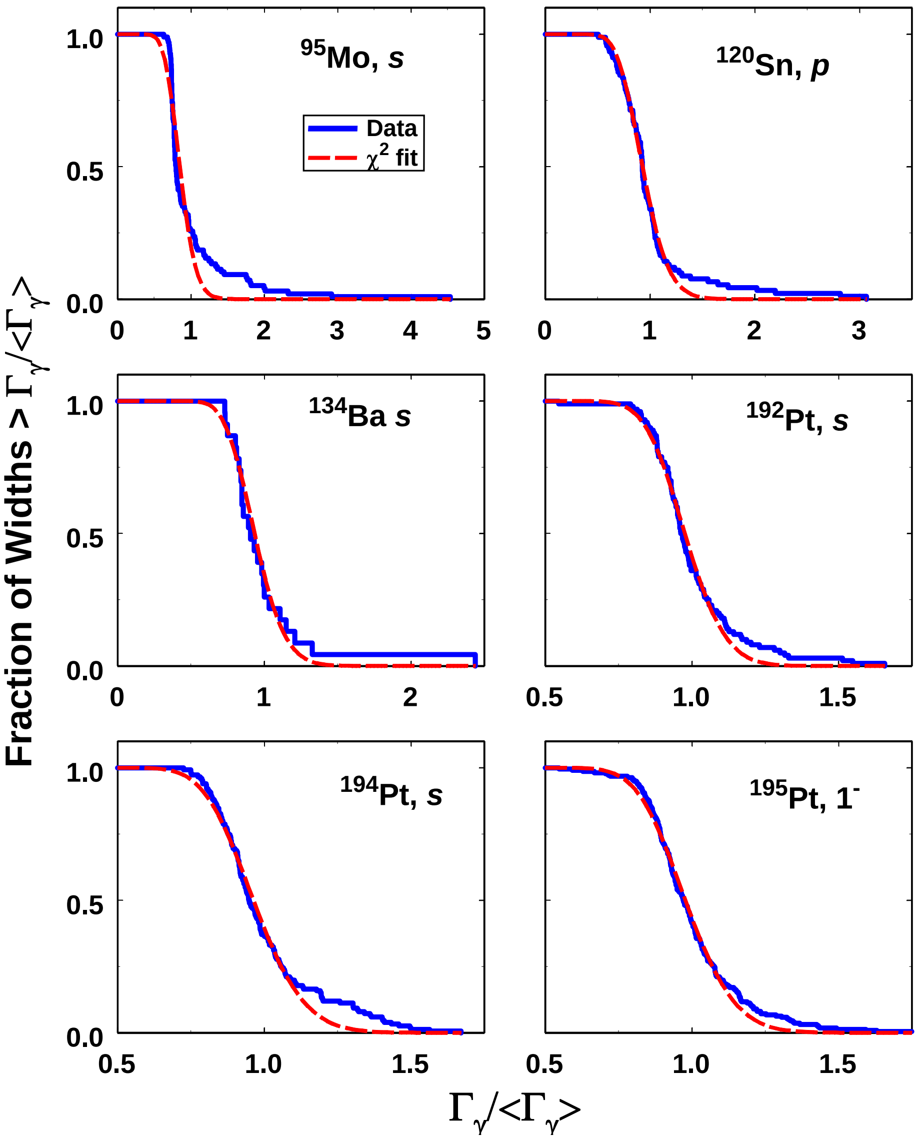
J^{π}	D_{lJ}(eV)	$10^4 S_{lJ}$	$<\Gamma_{\gamma lJ}>$ (meV)
1^-	627	0.17	556
2^-	153	0.98	289
2^+	134	0.17	186
3^-	148	1.05	344
3^+	120	0.32	178
4^-	148	0.80	252

	Mughabghab	ORELA
D_0 (eV)	81±14	63.3
D_1 (eV)	37.7±4.3	46.2
$10^4 S_0$	0.47±0.17	0.49
$10^4 S_1$	6.89±1.77	3.0
$<\Gamma_{\gamma 0}>$ (meV)	162±7	120 - 134
$<\Gamma_{\gamma 1}>$ (meV)	210±40	148 - 627

Extra Component to Γ_γ Distributions?

- Simplest model: Γ_γ follow a χ^2 distribution with ν~20-200.
  ν = number of channels.
- Data for several cases have extra tail at larger Γ_γ.
  Size of tail decreases with A.
- Data for several other cases shows no need for extra tail.

Data from ORELA



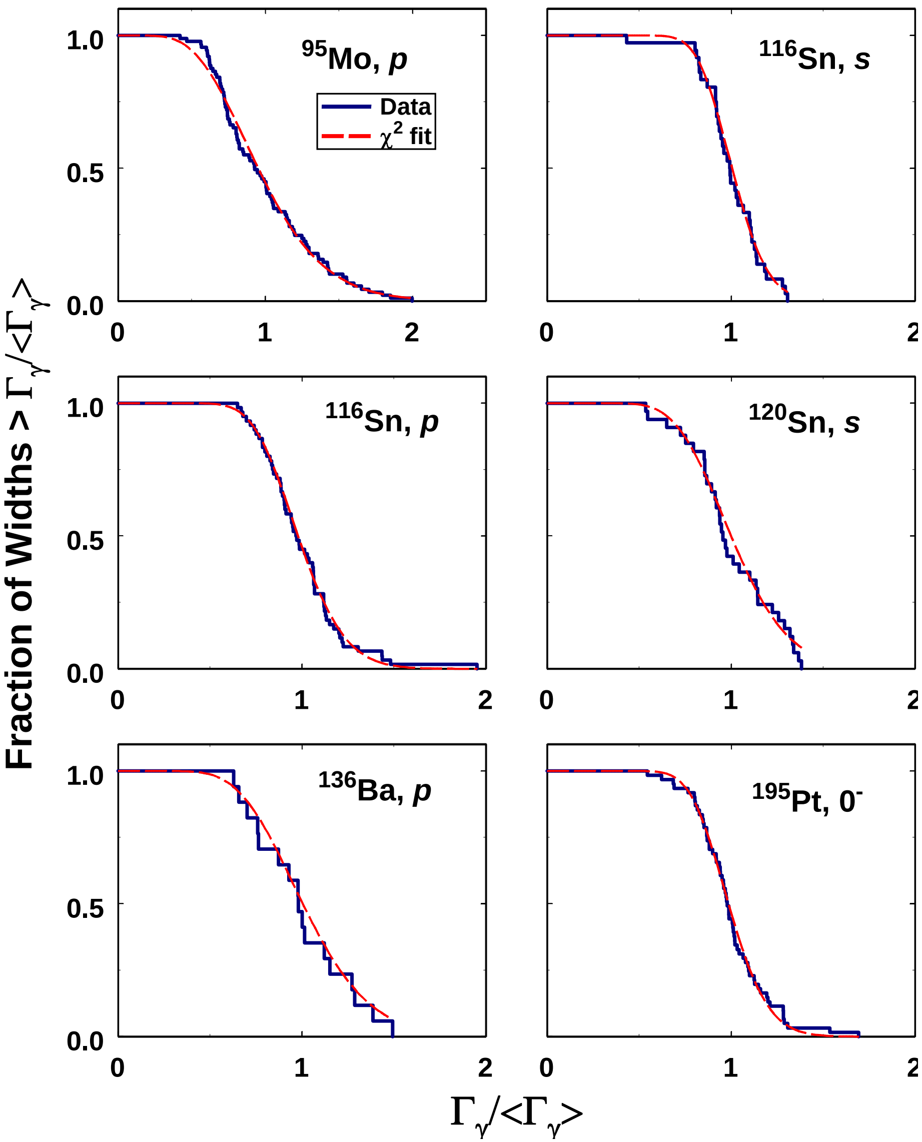
Extra Component to Γ_γ Distributions?

- Simplest model: Γ_γ follow a χ^2 distribution with ν~20-200.
  ν = number of channels.

- Data for several cases have extra tail at larger Γ_γ.
  Size of tail decreases with A.

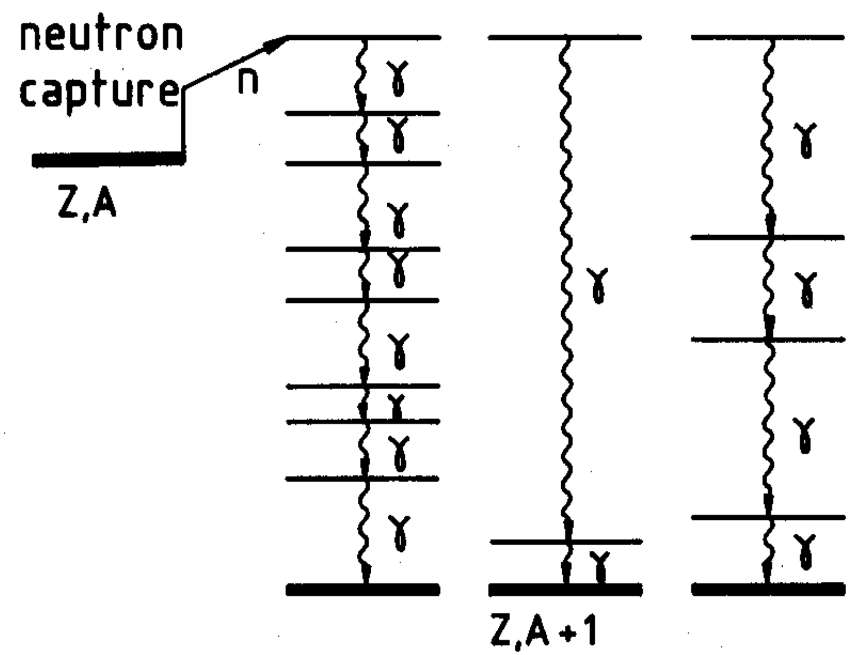
- Data for several other cases shows no need for extra tail.

Data from ORELA



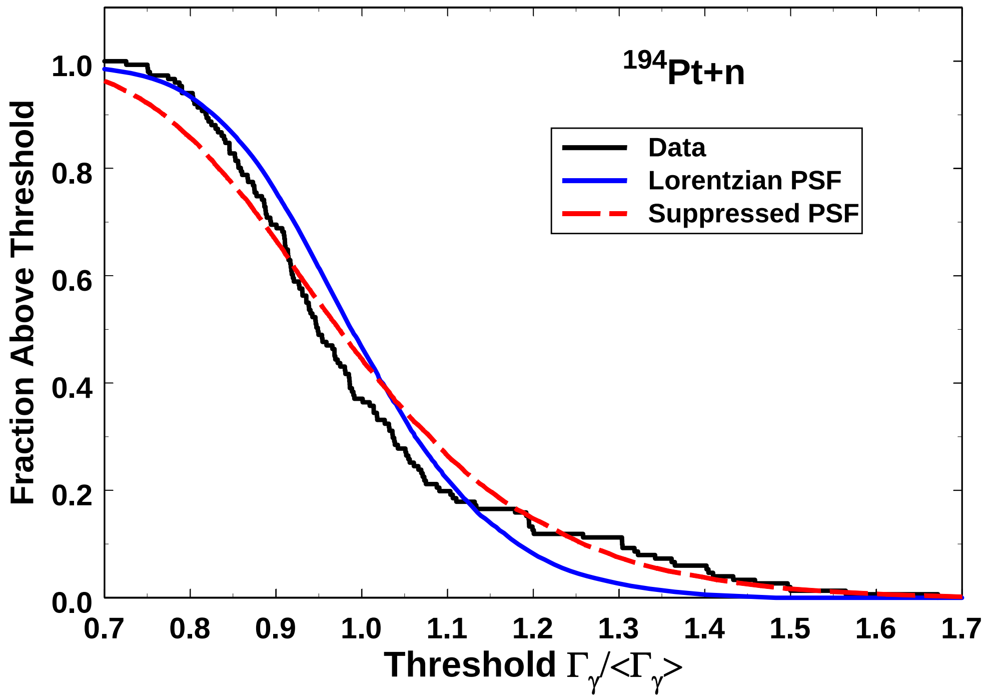
Improving PSF Models Using Neutron Resonance Γ_γ Data

- Compare measured Γ_γ distributions to DICEBOX simulations.
  Vary PSF and LD models to obtain agreement with data.
- Must agree with both distribution shape as well as its average.
- Avoids some confounding uncertainties.



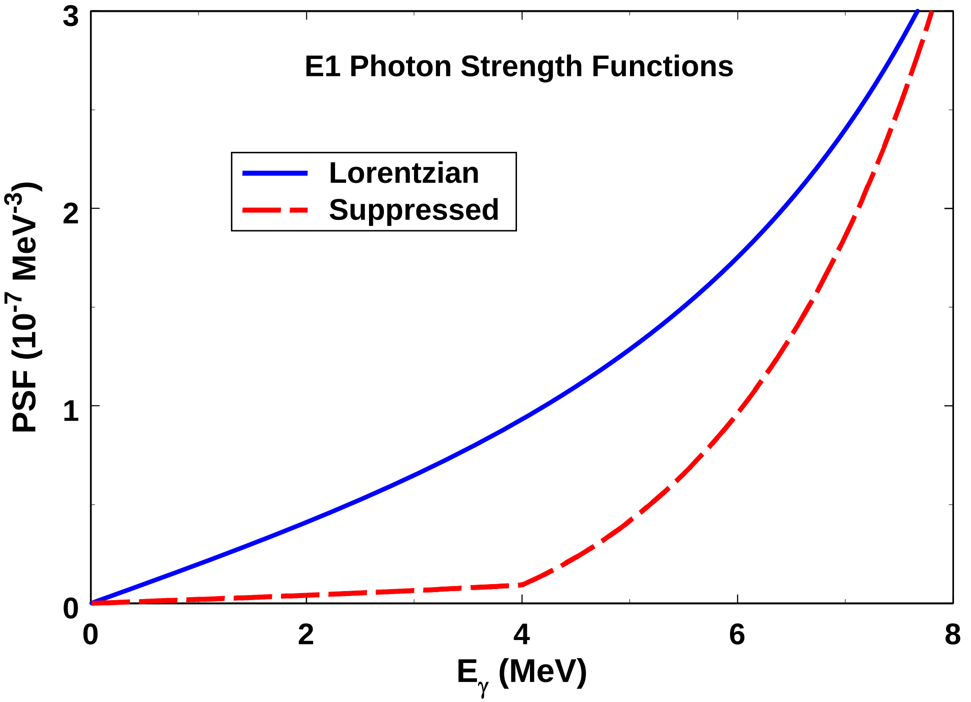
Improving PSF Models Using Neutron Resonance Γ_γ Data

- Compare measured Γ_γ distributions to DICEBOX simulations.
  Vary PSF and LD models to obtain agreement with data.
- Must agree with both distribution shape as well as its average.
- Avoids some confounding uncertainties.



Improving PSF Models Using Neutron Resonance Γ_γ Data

- Compare measured Γ_γ distributions to DICEBOX simulations.
  Vary PSF and LD models to obtain agreement with data.
- Must agree with both distribution shape as well as its average.
- Avoids some confounding uncertainties.



Improving PSF Models Using Neutron Resonance Γ_γ Data

- Compare measured Γ_γ distributions to DICEBOX simulations.
  Vary PSF and LD models to obtain agreement with data.
- Must agree with both distribution shape as well as its average.
- Avoids some confounding uncertainties.

A*	$<\Gamma_\gamma>$ (meV)		
		PSF	
	Meas.	Lorentzian	Suppressed
^{193}Pt	62±6	108	22
^{195}Pt	77±10	330	32
^{196}Pt (0^-)	109±16	160	17
^{196}Pt (1^-)	128±15	180	27
^{197}Pt	85±12	330	47

Conclusions 1

- Assumptions and ingredients of the statistical model (SM) can be tested and improved using Γ_n and Γ_γ distributions from neutron resonance measurements.

- Γ_γ distributions sensitive to photon strength function (and level density) model used in the SM.

  Detailed simulations of cascade needed (DICEBOX).
  Model must reproduce both $< \Gamma_\gamma >$ and fluctuations.
  Indication of an extra component in Γ_γ distributions for some nuclides.

Conclusions 2

- SM assumption that reduced widths follow a Porter-Thomas distribution (PTD) shown to be incorrect in several cases.

- Best case so far: Γ_n^0 data for $^{192,194,196}Pt$.
  PTD excluded at 99.997% confidence level.
  $\nu \approx 0.5$ (PTD has $\nu = 1$).
  Might be signature of collective effect (e.g., Y. Alhassid and A. Novoselsky, Phys. Rev. C 45, 1677 (1992)).

- Other cases (e.g., ^{147}Sm and ^{232}Th), ν changes from 1 to ≈ 2.
  No known model, but perhaps related to doorway effects.

- Is RMT wrong, or are these just special cases (e.g., unusual nuclear structure).

Impact On Applications
(e.g., Astrophysics, Nuclear Energy)

- PSF has large impact on calculated cross sections of nuclides beyond measurement.
- ν of $\Gamma_n{}'$ distribution affects calculated cross sections and important parameters for applications.

  Width fluctuation correction depends on ν.
  For $\Gamma_{c'}/\Gamma_c=1$, $S_{cc'}=\nu/(\nu+1)$.

  Effects self shielding correction for reactors, etc.

Future Prospects

- **Need high quality neutron capture and total cross sections.**

  New resonance parameters should be used to test theory instead of using theory to correct data.

- **Need careful R-matrix analysis .**

  Very important to indicate which J^{π} assignments are firm (independent of theory being tested).

- **New techniques for determining J^{π}'s should be extremely valuable (and are not too difficult to implement).**