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Originally published:

July 2018

Applied Thermal Engineering 143(2018), 293-301

DOI: https://doi.org/10.1016/j.applthermaleng.2018.07.067

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Electromagnetically driven convection suitable for mass transfer enhancement in liquid metal batteries

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Abstract

Liquid metal batteries (LMBs) were recently proposed as cheap large scale energy storage. Such devices are urgently required for balancing highly fluctuating renewable energies. During discharge, LMBs tend to form intermetallic phases. These do not only limit the up-scalability, but also the efficiency of the cells. Generating a mild fluid flow in the fully liquid cell will smoothen concentration gradients and minimise the formation of intermetallics. In this context we study electro-vortex flow numerically. We simulate a recent LMB related experiment and discuss how the feeding lines to the cell can be optimised to enhance mass transfer. The Lorentz forces have to overcome the stable thermal stratification in the cathode of the cell; we show that thermal effects may reduce electrovortex flow velocities considerable. Finally, we study the influence of the Earth magnetic field on the flow.

Keywords: liquid metal battery, mass transfer, electro-vortex flow, swirl, Rayleigh-Bénard convection, OpenFOAM

Preprint submitted to Journal of Heat and Mass Transfer

January 15, 2018

1 1. Introduction

Integrating highly fluctuating renewable energies (such as photovoltaics and wind power) into the electric grid calls for large scale energy storage. Such storage must be, first of all, safe and cheap. The liquid metal battery (LMB) promises both. After being intensively investigated in the 1960s, and abandoned later, LMB research experienced a renaissance some ten years ago. For an overview of the pioneering work, see [1–3] (recommended [4]) and for the recent work [5] and [6].



Figure 1: Sketch of a typical Li||Bi liquid metal battery with an intermetallic phase forming in the cathode (left) and vertical temperature distribution in the three layers for pure diffusion (right).

Fig. 1a shows a sketch of a typical LMB. A dense metal on the bottom q (cathode, positive electrode) is separated by a liquid salt from a lighter metal 10 at the top (anode, negative electrode). All three phases float above each other; 11 the salt acts as the electrolyte. The word "liquid metal battery" names only a 12 type of battery (which may consists of many different active metals). Typical 13 cells include Ca||Bi [7, 8], Ca||Pb [9], K||Hg [10, 11], Li||Bi [1, 12–16], Li||Pb 14 [1, 17], Li||Sb [5, 17], Li||Sn [1, 14, 18–20], Li||Zn [1], Mg||Sb [5, 21, 22], Na||Bi 15 [1, 14, 18, 20, 23–27], Na||Hg [5, 28, 29], Na||Pb [1, 14, 20, 27, 30], Na||Sn 16 [1, 11, 18, 20, 31, 32] and Na||Zn [33, 34] as well as exotic ones such as Li||Se 17 [1, 35, 36] or Li||Te [1, 14, 15, 35, 36]. 18

During discharge, the anode metal is oxidised, crosses the electrolyte layer 19 and alloys in the bottom layer with the dense metal ("concentration cell"). 20 Commonly, the ohmic resistance of the electrolyte layer represents the most im-21 portant overvoltage. However, at higher discharge currents concentration polar-22 isation enters the field [5, 11, 22, 28, 32, 37]. Example: when discharging a LillBi 23 cell, Li-rich alloy will concentrate at the cathode-electrolyte interface. When a 24 certain local concentration is exceeded, a solid intermetallic phase (Li_3Bi) will 25 form (fig. 1a) [1, 24]. Such intermetallics often float on the cathode metal [38]. 26 Sometimes they expand during solidification. As the walls impede a lateral 27 expansion, the intermetallic will form a dome until finally short-circuiting the 28 electrolyte. Especially in Ca based cells, locally growing dendrites may addi-29 tionally short-circuit the cell [7]. Besides of all the mentioned drawbacks, the 30 formation of intermetallics has one advantage: it removes anode metal from the 31 melt and keeps thereby the voltage constant. It should be also mentioned that 32 some intermetallics have high resistances while others are good conductors. 33

When *charging* the cell of fig. 1a, the cathode-electrolyte interface will deplete of Li and a similar concentration gradient may develop [24]. This effect is undesirable, too. Finally, all the same effects may theoretically happen in the anode compartment, too, if an alloyed top electrode is used (e.g. CaMg [8, 21]). However, such effects were not reported, yet.

It was early proposed that a mild fluid flow may counterbalance concentra-39 tion gradients and increase thereby the efficiency of LMBs [1, 24, 37]. While 40 "mechanical stirring" [1, 37] seems difficult to realise, a localised heating or cool-41 ing inducing thermal convection may be a very good option [39, 40]. Electro-42 vortex flow (EVF) may be used for an efficient mass transfer enhancement, too 43 [41–43]. Simply saying, EVF always may develop when current lines are not in 44 parallel. It can therefore easily be adjusted by choosing the diameter/geometry 45 of the current collectors and feeding lines appropriately. EVF drives a jet away 46 from the wall, forming a poloidal flow [44]. For a classical example of the origin 47 of EVF, see Lundquist [44] and Shercliff [45], for a good introduction David-48 son [46] and a detailed overview including many experiments Bojarevics et al. 49

[47]. Its relevance for LMBs is outlined by Ashour et al. [43]. It should also
be mentioned that other flow phenomena like the Tayler instability [48–56] ,
Rayleigh-Bénard convection [57, 58] or interface instabilities [59–63] may enhance mass transfer in LMBs, as well.

This article is dedicated (mainly) to electro-vortex flow. It's aim is twofold: first, we will show how the connection of the supply lines to the cell influences the flow. Second, we study how electro-vortex flow and thermal convection interact. For this purpose we combine numerical simulation with a simple 1D conduction model. These models – and the experiment which inspired our studies – are described in the following section.

60 2. Physical, mathematical and numerical model

In this section we will first present the experiment [64] which inspired this article. Thereafter we explain the way in which we estimate the temperature gradient appearing in the cathode of a liquid metal battery (LMB). Finally, we give an introduction to the 3D numerical models used.



Figure 2: Dimensions of the experiment and simulation model (in mm). The grey values are not exactly known; they are estimated from the sketch in [64]. The wires are assumed to be made of copper.

Fig. 2 illustrates the mentioned experiment, conducted by Kelley & Sadoway 65 [64]. A cylindrical steel vessel contains a melt of eutectic lead-bismuth at 150 °C. 66 An electric current (up to $0.375 \,\mathrm{A/cm}^2$) is applied between a bottom and top 67 electrode. The bottom electrode may be attached centrically or laterally. The 68 upper electrode consists of a nickel-iron foam; such foam is often used in LMBs 69 to contain the anode metal [6]. As the setup is heated from below, Rayleigh-70 Bénard cells appear. If an internal current is applied, the flow becomes much 71 more regular at $0.05 \,\mathrm{A/cm}^2$. It is deduced that convection cells align with the 72 magnetic field. It is further claimed that the copper plate which placed between 73 the bottom electrode and the the vessel, "ensures a uniform current density" in 74 the melt. We will show that this is not exactly true; we will further demonstrate 75 how electro-vortex flow may give an alternative explanation for the increase in 76 order. 77

We use the following material properties of eutectic PbBi at 150 °C [43]: 78 a kinematic viscosity of $\nu = 2.7 \cdot 10^{-7} \,\mathrm{m}^2/\mathrm{s}$, a thermal expansion coefficient 79 of $\beta = 1.3 \cdot 10^{-4} \,\mathrm{K}^{-1}$, an electric conductivity of $\sigma = 9 \cdot 10^5 \,\mathrm{S/m}$, a density 80 of $\rho = 10505 \text{ kg/m}^3$, an isobaric heat capacity of $c_p = 148 \text{ J/kg/K}$, a heat 81 conductivity of $\lambda = 10 \,\mathrm{W/m/K}$, a thermal diffusivity of $\alpha = 6 \cdot 10^{-6} \,\mathrm{m^2/s}$, a 82 Prandtl number of Pr = 0.04 and a sound velocity of $u_s = 1.765 \text{ m/s} [65-67]$. 83 The electric conductivity of the vessel is assumed to be $\sigma = 1.37 \cdot 10^6 \,\text{S/m}$ and 84 of the wires and copper plate $\sigma = 58.1 \cdot 10^7 \, \text{S/m}$. The electric conductivity of 85 the Fe-Ni foam is not easy to determine; we use a value of $\sigma = 1.37 \cdot 10^6 \, \text{S/m}$ 86 without further justification. 87

Geometrically, the described experiment perfectly represents a liquid cath-88 ode of an LMB. However, the temperature gradient in a working LMB depends 89 on the boundary conditions. For a single cell in an environment at room tem-90 perature it will rather be opposite to that in the experiment. As the electrolyte 91 layer has the highest resistance (four orders larger than the metals), most heat 92 will be generated there [57]. Fig. 1b shows a typical vertical temperature profile 93 through all three layers. If no thermal management system induces additional 94 temperature gradients (as suggested in [21, 40]) a stable thermal stratification 95



Figure 3: Temperature difference in the cathode for pure conduction in a Li||Bi cell. The same model and material properties as in [68] are used.

⁹⁶ is expected in the cathode. To drive a flow there, any force has to overcome
⁹⁷ first this stable stratification.

The temperature difference between top and bottom of the cathode can be 98 estimated using the simple 1D heat conduction model developed by Personnettaz 99 et al. [68] (for a 3D study of heat transfer in a Li||PbBi cell, see [69]). He 100 considers a Li|LiCl-KCl|Bi cell operating at 450°C. Although our cathode is 101 made of eutectic PbBi (and not Bi), we use the same model to get a rough 102 estimate of the temperature gradient in the cathode. The Li-layer is assumed 103 to be 32 and the Bi-layer 16 mm thick; for the material parameters, see [68]. 104 Depending on the current density and thickness of the electrolyte, the ΔT over 105 the cathode changes as illustrated in fig. 3. In our numerical simulation we will 106 assume the electrolyte to be $5 \,\mathrm{mm}$ thick (realistic values are $3-15 \,\mathrm{mm}$ [70]). We 107 will use the temperature difference of fig. 3 as boundary condition as 108

$$\Delta T = \frac{h_{\rm Bi} h_{\rm salt} q \left(2 h_{\rm Li} \lambda_{\rm salt} + h_{\rm salt} \lambda_{\rm Li}\right)}{2 h_{\rm Bi} \lambda_{\rm Li} \lambda_{\rm salt} + 2 h_{\rm Li} \lambda_{\rm Bi} \lambda_{\rm salt} + 2 h_{\rm salt} \lambda_{\rm Bi} \lambda_{\rm Li}},\tag{1}$$

with h, λ and q denoting the layer heights, the thermal conductivities and the

volumetric heat source in the electrolyte. We will study, if electro-vortex flowcan overcome the stable stratification.

The numerical model is implemented in OpenFOAM [71]; the electro-vortex flow solver is explained in detail in [72]. Basically, it computes the electric potential ϕ and current density \boldsymbol{J} on a global mesh as

$$\nabla \cdot \sigma \nabla \phi = 0 \tag{2}$$

$$J = -\sigma \nabla \phi \tag{3}$$

with σ denoting the electric conductivity. All conducting regions (of different conductivities) are fully coupled. The results are then mapped on a separate fluid mesh. Induced currents and magnetic fields are neglected, which is justified as long as the velocities are small. On the fluid mesh the following set of equations is solved:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla) \, \boldsymbol{u} = -\nabla p + \nu \Delta \boldsymbol{u} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{\rho} \tag{4}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} dV'$$
(5)

$$0 = \Delta \boldsymbol{B} \tag{6}$$

with t, u, p, ν , ρ , μ_0 , r and V denoting the time, the velocity, the pressure, the kinematic viscosity, the density, the vacuum permeability, the coordinate and the cell volume, respectively. The Biot-Savart integral is only used to determine the magnetic field B on the boundaries. The fluid mesh has at least 200 cells on the diameter, which is fine enough according to [43].

¹²⁵ If thermal effects shall be included, the Oberbeck-Boussinesq approximation ¹²⁶ [73] is used (for its validity, see [43, 74]). The following set of equations is solved

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{u}\boldsymbol{u}) = -\nabla p_d + \nu \Delta \boldsymbol{u} - \boldsymbol{g} \cdot \boldsymbol{r} \nabla \rho_k + \frac{\boldsymbol{J} \times \boldsymbol{B}}{\rho_0}$$
(7)

$$\nabla \cdot \boldsymbol{u} = 0 \tag{8}$$

$$\frac{\partial T}{\partial t} + \nabla \cdot (\boldsymbol{u}T) = \frac{\lambda}{\rho_0 c_p} \Delta T \tag{9}$$

with $\boldsymbol{u}, p, \nu, \boldsymbol{g}, \boldsymbol{r}, T, c_p \boldsymbol{J}$ and σ denoting velocity, pressure, kinematic viscosity, gravity, position vector, temperature, specific heat capacity, current density and electric conductivity, respectively. The density $\rho = \rho_0 \rho_k = \rho_0 (1 - \beta (T - T_{ref}))$ is calculated using the mean density ρ_0 at reference temperature T_{ref} and the coefficient of thermal expansion β ; J and B are determined by the electro-vortex solver as described above. At least 250 cells on the diameter and strongly refined boundary layers are used.

¹³⁴ 3. Results

This section is arranged as follows: firstly, we compare the influence of a
symmetric and asymmetric current supply on pure electro-vortex flow (fig. 4).
Thereafter, we study the influence of the Earth magnetic field and of thermal
stratification on both connection types (fig. 5 and 6). Further, we give estimates of the flow velocity depending on the cell current.



Figure 4: Current path and velocity streamlines for a current supply from the side (a). Velocity on a vertical plane for symmetric (b) and lateral current supply (c). The current is I = 40 A; the results show electro-vortex flow alone.

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Fig. 4a illustrates the current path, streamlines and velocities for a lateral supply line. Electro-vortex flow is simulated alone; the applied current is 40 Å. The flow profile is essentially horizontal forming two kidney-shaped vortices. The velocity reaches 2.5 mm/s. The *horizontal* jet (also shown in 4c) is uncommon for electro-vortex flow, but can easily be explained. As the current flows mainly horizontally through the copper plate, it induces a magnetic field in the fluid. This field points towards the observer (in fig. 4a and c). The current in
the liquid metal flows upwards (vertically) and interacts with the induced field.
Consequently, the Lorentz force points to the right and drives the observed flow
in "prolongation" of the current supply. For similar experiments, see [75, 76].

Fig. 4b shows the flow profile for a symmetric current supply, again for 40 A. 150 A typical poloidal flow develops as it was often observed experimentally [47, 77– 151 81]. Similar flow structures are very well known from vacuum arc remelting and 152 electro-slag remelting [82–89]. However, depending on the exact geometry, the 153 direction of the flow might be reversed [90–93]. In our simulation, the velocities 154 reach $0.6 \,\mathrm{mm/s}$ for the symmetrical setup. This is only $25\,\%$ of the flow velocity 155 observed for a lateral current supply. Due to the shallow liquid metal layer, a 156 poloidal flow will dissipate strongly in the boundary layer. 157

The simulated velocities are not directly comparable to the experiment. The 158 latter was additionally heated from below (vertical temperature difference of ap-159 proximately $\Delta T = 10$ K). As shown numerically by Beltrán, the experimentally 160 observed flow is mainly caused by Rayleigh-Bénard convection. Also he used a 161 volumetric expansion coefficient three times smaller than the real one [64, 94]162 (for the correct value see [43]), his velocity profile and magnitude (3 mm/s)163 matches very well to the experimental results (compare fig. 9 in [94] and fig. 4 164 in [64]). Electro-vortex flow will generally lead to velocities one order of mag-165 nitude smaller (Kelley and Sadoway [64] used currents of 23.3 A at most; our 166 results are for 40 A). However, electro-vortex flow will surely influence the flow 167 structure and may explain the increase in order of the flow which was observed 168 experimentally. 169

In the next step we focus on the symmetric current supply (with the poloidal flow) only, and analyse the influence of a vertical magnetic background field. When we add the magnetic field of the Earth (measured in Dresden as B = $(15 \cdot e_x, 5 \cdot e_y, 36 \cdot e_z) \mu T$) the original poloidal flow (fig. 5a) becomes strongly helical (fig. 5b). The appearance of such azimuthal swirl flow is well known from experiments [43, 77, 95] and can be easily explained. Radial cell currents and a vertical magnetic background field lead to azimuthal Lorentz forces [79, 84, 95].

Compared to a recent experiment by Ashour [43] with a point electrode on the 177 top, we observe considerably stronger swirl (compare fig. 5b with fig. 5 in [43]). 178 We attribute this difference to the location of the azimuthal forcing. Here, the 179 force is well distributed in the whole volume; in [43] it is concentrated only in 180 the centre of the liquid metal "sheet". We suppose the distributed azimuthal 181 Lorentz force to better suppress the poloidal flow by forcing the streamlines into 182 a dissipative Ekman layer [84]. Fig. 5c shows the volume averaged mean velocity 183 of the poloidal and azimuthal flow – with and without the Earth magnetic field. 184 If we add a vertical field, azimuthal swirl appears (compare the dashed curve). 185 At the same time, the poloidal flow is strongly reduced (by a factor of 1/2). This 186 fits nicely to Davidsons "poloidal suppression" model [84]. This is remarkable, 187 because simulations with a point electrode (see [43]) did not show such a strong 188 suppression. 189

Keeping the symmetric current supply, we now focus on the influence of the 190 temperature stratification. During operation of an LMB, the cathode will be 191 heated from above; the temperature stratification will be stable. At first glance, 192 this configuration is similar to arc remelting. There, an electric arc heats the 193 melt from above. However, the bath is cooled rather from the side than from 194 below which leads to strong thermally driven flow [96], but we have a stable 195 thermal stratification instead. Based on the temperature conduction model 196 described in section 2 we apply a vertical temperature gradient of $\Delta T = 0.7 \,\mathrm{K}$ 197 (at 40 A). The stable thermal stratification slows down the electro-vortex flow 198 (compare fig. 5d and e). While the general flow structure does not change, 199 especially the velocity near the bottom wall decreases by a factor of 2/3. This 200 result cannot be compared to the experiment, as Kelley and Sadoway heated 201 from below (and we from above). A temperature gradient as in the experiment 202 is not expected to appear during "normal" operation of an LMB; however, an 203 additional heating or cooling for mass transfer enhancement (as proposed in 204 [39, 40]) can easily lead to similar configurations. 205

We use two quantities to estimate the mass transfer in the cathode: the volume averaged velocity as global measure, and the mean velocity gradient at



Figure 5: Streamlines and velocity without (a) and with the Earth magnetic field (b). Volume averaged mean velocities of the azimuthal and poloidal flow for both cases (c). Velocity on a vertical plane for symmetric current supply without temperature (d) and with a negative temperature gradient of 0.7 K (e). Volume averaged mean velocity (f) and mean velocity gradient (g) of electro-vortex flow alone, with an additional Earth magnetic field (Bz) and with a stabilising temperature gradient. I = 40 A.

the foam-cathode interface as local one. Fig. 5f and g show both quantities for electro-vortex flow alone, with the Earth magnetic field ("Bz") and with a stabilising thermal gradient. The azimuthal flow, caused by the Earth magnetic field, yields the highest velocities. A vertical temperature gradient does barely influence the horizontal flow. The poloidal electro-vortex flow ("EVF alone") is considerably slower – it is strongly dissipated at the bottom wall. The vertical

temperature gradient effectively breaks the downwards flow. Interestingly, a 214 strong flow in the volume leads also to strong velocity gradients at the interface. 215 We now consider the lateral current supply, and study again the influence 216 of temperature and the Earth magnetic field. The prevailing horizontal flow is 217 hardly influenced by a stabilising vertical temperature gradient. The flow struc-218 ture changes only slightly; the velocities with and without temperature gradient 219 are almost the same (compare fig. 6c and d). Taking into account the Earth 220 magnetic field changes the flow much more (compare fig. 6a and b). The hori-221 zontal current and vertical magnetic background field generate a Lorentz force 222 which deflects the jet in clockwise direction. Presumably the stronger dissipa-223 tion in the boundary layers decreases the velocity slightly. Most importantly, 224 the Earth magnetic field does not lead to swirl flow in this configuration - the 225 jet is only deflected. Fig. 6e and f show the mean velocity and the mean velocity 226 gradient for pure electro-vortex flow, with the Earth magnetic field and with 227 the stabilising temperature gradient. The differences are only marginal. 228

229 4. Summary & outlook

We have discussed, how electro-vortex flow (EVF) has the potential to enhance mass transfer in liquid metal batteries (LMBs). In a first step we discussed why such mass transfer enhancement is important. We emphasised that mostly (but not only) mixing of the cathode during discharge is highly beneficial. We studied the flow structure and magnitude of EVF numerically. Moreover, we discussed the influence of stray magnetic fields, the connection of the supply lines and a stable thermal stratification on electro-vortex flow.

A lateral current supply to the cathode will generate a *horizontal* flow. In contrast, a central current supply below the cathode will induce a *vertical* jet. Looking only on this flow-direction, would expect a vertical flow to be better suited for enhancing mass transfer. It will remove reaction products directly from the cathode-electrolyte interface. However, the vertical (or better: poloidal) flow has three disadvantages: (1) it's mean velocity is much smaller



Figure 6: Electro-vortex flow for a lateral supply wire without (a) and with the Earth magnetic field (b). Flow in the cross section of the jet without (c) and with a stabilising thermal gradient (d). The current for (a)-(d) is 40 A. Volume averaged mean velocity (e) and mean velocity gradient (f) for electro-vortex flow alone, with the additional Earth magnetic field (Bz) and with a stabilising temperature gradient.

²⁴³ compared to the horizontal flow, (2) it is dampened by the stable temperature ²⁴⁴ stratification and (3) it will turn to a swirling flow under presence of the Earth ²⁴⁵ magnetic field. In contrast, the horizontal jet will not be dampened considerably ²⁴⁶ by a temperature stratification nor be strongly influenced by the Earth mag-²⁴⁷ netic field. We believe therefore the lateral supply line to be better suited for ²⁴⁸ enhancing mass transfer. Concerning the swirl flow we could (at least partially) ²⁴⁹ confirm Davidsons model of poloidal suppression.

Our models are strongly simplified: we ignore induced currents and mag-250 netic fields; we ignore internal heating; the simulation of thermal convection 251 and EVF is fully decoupled. A next step would be therefore the development 252 of a fully coupled EVF-thermal convection model as well as it's coupling with a 253 real mass transfer (e.g. Li in Bi) model. Of course, velocity and concentration 254 measurements in a real 3-layer LMB would be a large step forward. Performing 255 Kelley's experiment with an inverse temperature gradient (better at room tem-256 perature) could allow a further experimental study of the interaction between 257 EVF and thermal convection. 258

259 Acknowledgements

This work was supported by the Deutsche Forschungsgemeinschaft (DFG, 260 German Research Foundation) by award number 338560565 as well as the 261 Helmholtz-Gemeinschaft Deutscher Forschungszentren (HGF) in frame of the 262 Helmholtz Alliance "Liquid metal technologies" (LIMTECH). The computa-263 tions were performed on the Bull HPC-Cluster "Taurus" at the Centre for In-264 formation Services and High Performance Computing (ZIH) at TU Dresden and 265 on the cluster "Hydra" at Helmholtz-Zentrum Dresden – Rossendorf. Fruitful 266 discussions with V. Bojarevics, P. Davidson, D. Kelley, F. Stefani and T. Vogt 267 on several aspects of electro-vortex flow and thermal convection are gratefully 268 acknowledged. N. Weber thanks Henrik Schulz for the HPC support. 269

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