HELMHOLTZAI ARTIFICIAL INTELLIGENCE

Inverting the Beamline

a random walk of simulation-based inference using machine learning

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Peter Steinbach HZDR / IKTP Dresden, June 10, 2021

Some context first

A disclaimer



(anything presented below is the result of a long random walk)

2/41

whoami



- until 2012 PhD on $pp \rightarrow Z^0 + b$ cross-section at ATLAS (LHC, CERN)
- 2012-2019 HPC developer and Scientific Software Engineer at Scionics/MPI CBG

since 2019 Helmholtz AI team lead for Matter consulting

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Helmholtz Al



- ORGANISATION Central Unit Local Units Helmholtz Association FELLOWS & PARTNERS Research Center RESEARCH FIELDS Key Technologies Aeronautics, Space, and Transport Earth & Environment
- one central, five local units
- each unit: research + consulting team
- since 2019 for 7+X years
- consulting team: collaboration-as-aservice
- more details: helmholtz.ai

4/41

Why are we doing this?

A common situation with simulations



- simulations used in many domains (physics, biology/medicine, chemistry, epidemiology, ...)
- approaches to simulations vary (mechanistic, agent based, distribution based, ...)

- simulations can be computationally challenging
- here: simulations = forward process

Inversion of Simulations?



- inverse process hard to do (if at all tried)
- often, only single observables "fitted"

- simulations updated based on singular observables
- considerable human tuning involved (heuristics)

What are the Most Important Statistical Ideas of the Past 50 Years? - Andrew Gelman & Aki Vehtari (2021) -



[#]MLCollage - @RobertTLange [17/52]

Simulation-based inference discovered recently

(Cranmer et al, 2020, Gelman & Vehtari, 2021)

 $\rho(\vec{\vartheta}|\vec{x}) = \frac{\rho(\vec{x}|\vec{\vartheta}) \cdot \rho(\vec{\vartheta})}{\int \rho(\vec{x}|\vec{\vartheta})\rho(\vec{\vartheta})d\vec{\vartheta}}$

















likelihood $p(\vec{x}|\vec{\vartheta})$ provided by (forward) simulation

Prior $p(\vec{\vartheta})$ given by how we sample the simulation parameters $\vec{\vartheta}$



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Goal: predict posterior to the best of our abilities!

Why do I care?



Scientific Question

Given a beam profile, what were the beamline optics parameters that likely produced it?

- beamline UE112 PGM-1 at BESSY
- beam characteristics fixed at electron storage ring outlet
- forward simulations by rayUI

Normalizing Flows and INNs

Goal: Learn Invertible Mapping



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basic assumption

f ... invertible mapping

X ... data distribution

$$f: X \to Z$$

What we get ...

$$f^{-1}(z) = \hat{x}_{gen}$$

sample *z* from *Z* with $p_Z(z)$ (*Z* is a normal Gaussian distribution)

"obtain" inverse transformation f⁻¹ from learned forward function f

Normalizing Flows are generative models!

given a random variable X ∈ ℝ^d and Z ∈ ℝ^d of d dimensions
given an invertible function f : X → Z and z = f(x)

For interval β over *X*, there has to be β' over *Z* such that

$$\int_{\beta} p_X dx = \int_{\beta'} p_Z dz$$

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HELMHOLTZAI 14/41

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$$p_X(x) = |\frac{dz}{dx}|p_Z(z)$$
$$p_X(x) = |\frac{df(x)}{dx}|p_Z(f(x))$$

For d > 1 dimensions

given a random variable $X \in \mathbb{R}^d$ and $Z \in \mathbb{R}^d$ of *d* dimensions

given an invertible function $f: X \to Z$ and z = f(x)

$$p_X(x) = |det(\frac{df(x)}{dx})|p_Z(f(x))|$$

Achievement: from *x* to *z* and back



from G. Papamakarios et al, "Normalizing Flows for Probabilistic Modeling and Inference", arXiv:1912.02762, 2019.



Why are normalizing flows called normalizing flows?



Why are normalizing flows called normalizing flows?

They normalize x into z!

■ $X \in \mathbb{R}^d$ and $Z \in \mathbb{R}^d$ of *d* dimensions

sample *z* from *Z* with $p_Z(z)$, *Z* being a (multivariate) normal Gaussian distribution

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requirements of invertibility and efficient Jacobian calculations restrict model architecture

Conditional Invertible Neural Networks
arxiv:1907.02392, Code on github

GUIDED IMAGE GENERATION WITH CONDITIONAL INVERTIBLE NEURAL NETWORKS

Lynton Ardizzone, Carsten Lüth, Jakob Kruse, Carsten Rother, Ullrich Köthe Visual Learning Lab Heidelberg

Abstract

In this work, we address the task of natural image generation guided by a conditioning input. We introduce a new architecture called conditional invertible neural network (cINN). The cINN combines the purely generative INN model with an unconstrained feed-forward network, which efficiently preprocesses the conditioning input into useful features. All parameters of the cINN are jointly optimized with a stable, maximum likelihood-based training procedure. By construction, the cINN does not experience mode collapse and generates diverse samples, in contrast to e.g. cGANs. At the same time our model produces sharp images since no reconstruction loss is required, in contrast to e.g. VAEs. We demonstrate these properties for the tasks of MNIST digit generation and image colorization. Furthermore, we take advantage of our bidirectional cINN architecture to explore and manipulate nDach



INNs



edited from arxiv:1907.02392 (inspired by arxiv:1808.04730)

Forward (f)

$$v_1 = u_1 \otimes exp(s_1(u_2)) + t_1(u_2)$$

 $v_2 = u_2 \otimes exp(s_2(u_1)) + t_2(u_1)$

Backward (f^{-1})

$$u_2 = (v_2 - t_2(v_1)) \oslash exp(s_2(v_1))$$

$$u_1 = (v_1 - t_1(u_2)) \oslash exp(s_1(u_2))$$

20/41

Conditioning



from arxiv:1907.02392

this does not compromise invertibility

concatenate output of conditioning network c to inputs of s_i and t_i, e.g.

$$s_1(u_2) \rightarrow s_1([u_2, c(u_2)])$$

cINN Loss from Maximum Likelihood Optimisation

$$\mathcal{L}_i = \mathbb{E}_i \Big[-\log(p(\vartheta_i | x_i)) \Big]$$

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• using $J_i = det(\frac{df_w}{d\vartheta}|_{\vartheta_i})$

- with $p(\vec{\vartheta}|\vec{x}) = |J_i|p_Z(z = f_w(\vec{\vartheta}; \vec{x}))$ from change of variables
- **p**_z describes unit gaussian distribution, $\mathcal{N}(z|0, l) \approx exp(||-\frac{1}{2}z||_2^2)$

cINN Loss from Maximum Likelihood Optimisation

$$\mathcal{L}_i = \mathbb{E}_i \Big[-\log(p(\vartheta_i | x_i)) \Big]$$

using J_i = det(df_W/dϑ |ϑ_i)
 with p(ϑ | x) = |J_i|p_Z(z = f_W(ϑ; x)) from change of variables
 p_Z describes unit gaussian distribution, N(z|0, I) ≈ exp(||-1/2z||²₂)

$$\mathcal{L} = \mathbb{E}_i \Big[-\frac{||f_w(\vartheta; \boldsymbol{c}_u(\vec{x}))||_2^2}{2} - \log(J_i) \Big]$$

for complete derivation see Radev et al, 2020 (footnote: the paper starts from the Kullback-Leibler (KL) divergence, which is equivalent to a Maximum Likelihood Optimisation)





HELMHOLTZAI 24/41

(c)INNs based on normalizing flows

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 $\vec{\vartheta} \in \mathbb{R}^d \rightleftharpoons \vec{x} = f(\vec{\vartheta}) \in \mathbb{R}^k , (d \neq k)$

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promising avenue beyond/aside VAE and GANs

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Quiz: What are the core assumptions for (c)INNs to work?

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Quiz: What are the core assumptions for (c)INNs to work?

- **f** : $\vartheta \to x$ is invertible
- senerative distribution *z* is drawn from a normal distribution
- tractable jacobian so that det(J_f) can be computed

cINNs in the wild

My data



(forward) simulation parameters $ec{ec{artheta}} \in \mathbb{R}^6$



quality at experimental station, $ec{x} \in \mathbb{R}^{200}$

cINN Inference on the validation set



- cINN provides posterior that can be sampled
- extract Maximum a posteriori estimation (MAP) estimate by mean/median

28/41

cINN MAPs on the validation set



training:

- 30 epochs only
- fixed arch: 8 layers
- 256 units per dense layer
- inference: 256 draws per validation sample, MAP by mean
- good: posterior stays within prior support
- to improve: posterior misses out for some dimensions

29/41

core assumption(s)

- network f is sufficiently expressive
- as $N_{ ext{simulations}} o \infty$, network allows mapping of $ec{x}$ onto $p(ec{ec{ec{v}}} | ec{x})$
- ullet training dataset imposes the entire "truth" through implicit prior $ho_{ ext{simulation}}(ec{ec{ heta}})$

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Let's reconsider

- **g**oal: infer $\vec{\vartheta} | \vec{x_o}$ on observation $\vec{x_o}$
- **but:** the global learned posterior may not be too informative at $p(\vec{\vartheta}|\vec{x_o})$ (as we fixed the prior)

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sequential neural density estimation

30/41

Sequential neural density estimation

Algorithm 1 APT with per-round proposal updates

Input: simulator with (implicit) density $p(x|\theta)$, data x_o , prior $p(\theta)$, density family q_{ψ} , neural network $F(x, \phi)$, simulations per round N, number of rounds R.

$$\begin{split} \tilde{p}_1(\theta) &:= p(\theta) \\ \text{for } r = 1 \text{ to } R \text{ do} \\ \text{for } j &= 1 \text{ to } R \text{ do} \\ \text{Sample } \theta_{r,j} &\sim \tilde{p}_r(\theta) \\ \text{Simulate } x_{r,j} &\sim p(x|\theta_{r,j}) \\ \text{end for} \\ \phi &\leftarrow \underset{\phi}{\operatorname{argmin}} \sum_{i=1}^r \sum_{j=1}^N -\log \tilde{q}_{x_{i,j},\phi}(\theta_{i,j}) \\ \tilde{p}_{r+1}(\theta) &:= q_{F(x_o,\phi)}(\theta) \\ \text{end for} \\ \text{return } q_{F(x_o,\phi)}(\theta) \end{split}$$

- Ioss function has to be adapted
- cINNs can be used as conditional density estimator
- Greenberg et al, 2019:

"Learning with such 'atomic' proposals has an intuitive interpretation: we are training the network to solve multiple choice test problems, of the format "which of these ϑ 's generated this *x*?"

sbi MAPs on the validation set



training:

- >100 epochs only
- fixed arch: 8 layers, neural spine flow
 Durkan et al, 2019
- 256 units per layer
- inference: 256 draws per validation sample, MAP by mean
- good: posterior stays within prior support
- to improve: posterior misses out for two dimensions (expected)

www.mackelab.org/sbi

the real world



reproducibility, openness & team work = key! (all results from above were from toy simulations)

34/41

quality control of predictions:

- sample based metrics (Naeem et al, 2020)
- simulation based calibration (Talts et al, 2018)
- integrating cINNs from above into SNPE
- experiment with gradient based MAP estimation
- uncertainties of MAPs?

Summary



normalizing flows have emerged as a learnable transformation between distributions

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Questions? Feedback? Concerns?

- nice blog post with pytorch code samples
- "Normalizing Flows for Probabilistic Modeling and Inference," G. Papamakarios et al, arXiv:1912.02762, 2019.
- "Normalizing Flows: An Introduction and Review of Current Methods", I. Kobyzev et al, arXiv:1908.09257, 2019.
- "Glow: Generative Flow with Invertible 1x1 Convolutions", Kingma et al, arXiv:1807.03039, 2018.

Backup


Converging to a loss function

$$p_X(x) = |det(\frac{df(x)}{dx})| p_Z(f(x))$$

Going from one *f* to multiple: $z_n = f_n \circ \cdots \circ f_1(x)$:



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$$p_X(x) = \prod_{i=1}^n |det(\frac{dz_i}{dz_{i-1}})| p_Z(f(x)), x = z_0$$

$$\log(p_X(x)) = \sum_{i=1}^n \log |det(\frac{dz_i}{dz_{i-1}})| p_Z(f(x)), x = z_0$$

recall: outputs of conditioning network h concatenated to inputs of s_i and t_i



- **recall:** outputs of conditioning network *h* concatenated to inputs of s_i and t_i
- feeds higher semantic features of data into INN

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 - remove last softmax layer

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 - train h jointly with INN
- for image inputs:
 - train classification network
 - remove last softmax layer
 - use latent features as conditioning