

# Evolution of low-lying M1 modes in germanium isotopes

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**hZDR**

 **HELMHOLTZ  
ZENTRUM DRESDEN  
ROSSENDORF**

## Gamma-ray strength functions

- Gamma-ray strength functions describe average electromagnetic transition strengths at high excitation energy and high level density of nuclear states:

$$f_{fiL}(E_\gamma) = \overline{\Gamma}_{fiL} \rho(E_i, J_i) / E_\gamma^{2L+1} \quad E_\gamma = E_i - E_f \quad J_i = 0, \dots, J_{\max}$$

- Photoabsorption:

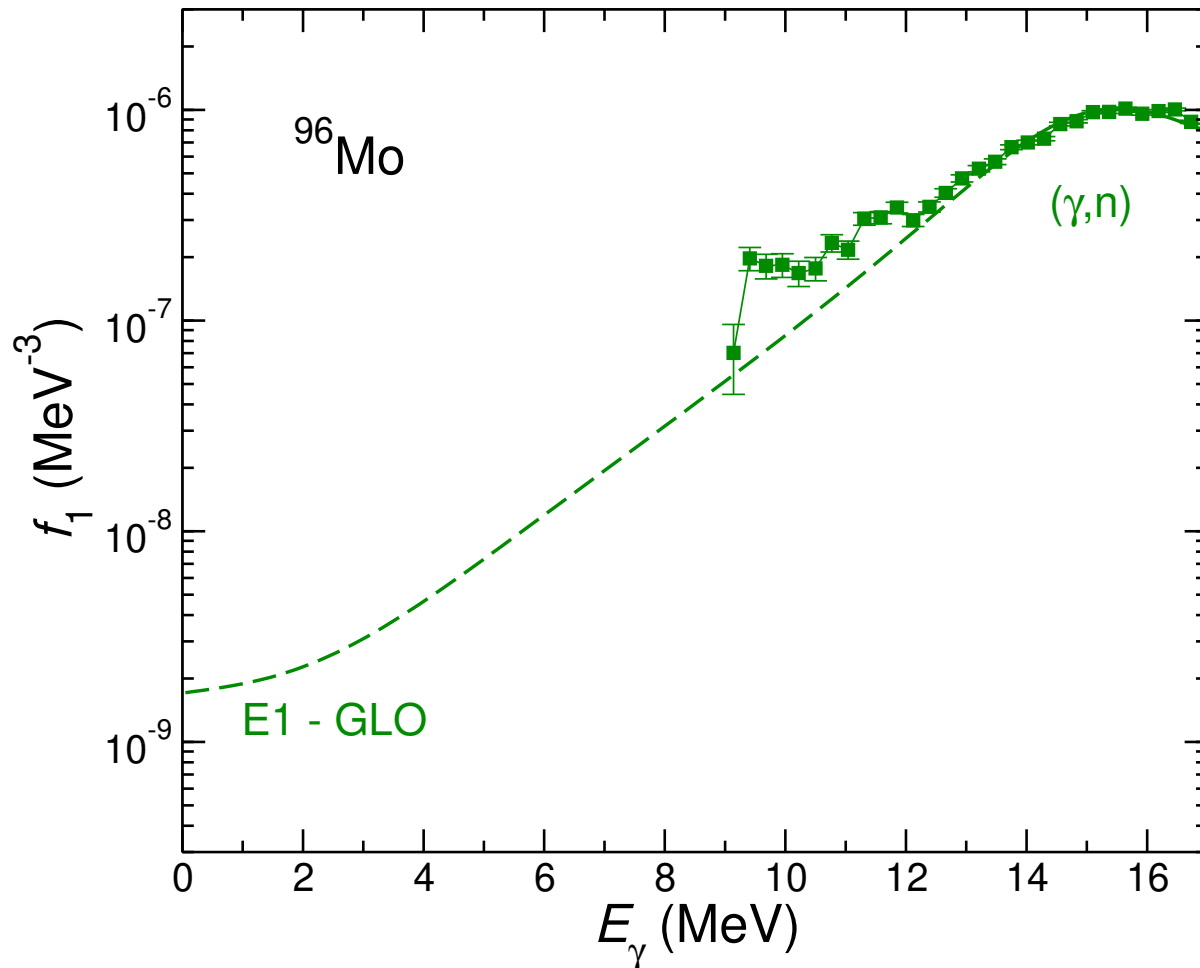
$$f_L = \sigma_\gamma / [(2J_i+1)/(2J_0+1) (\pi\hbar c)^2 E_\gamma^{2L-1}] \quad E_\gamma = E_i \quad J_i = 1, (2)$$

- Brink-Axel hypothesis:

The strength function does not depend on the excitation energy.

The strength function for excitation is identical with the one for deexcitation.

# Dipole strength functions in $^{96}\text{Mo}$

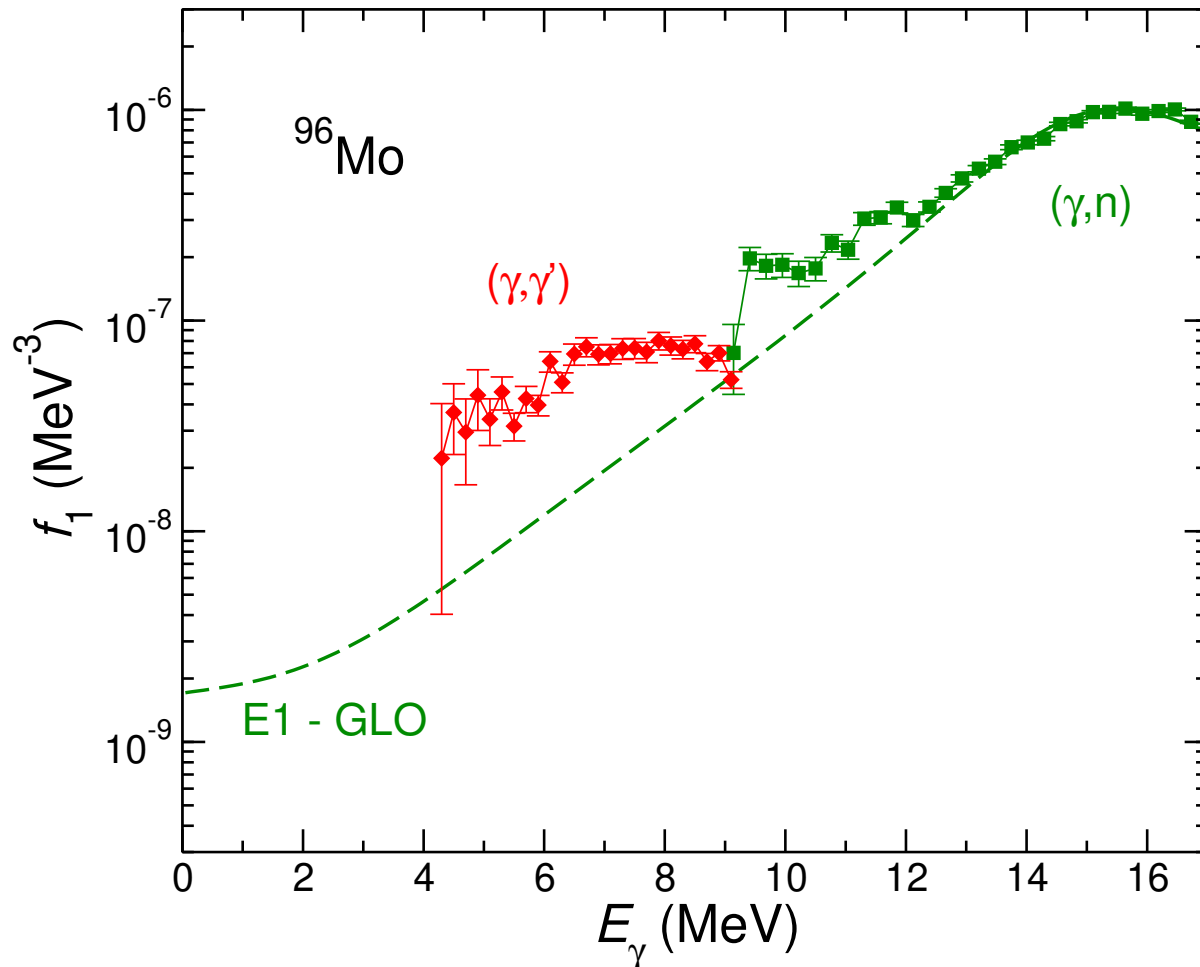


$(\gamma, n)$  data:

H. Beil et al., NPA 227, 427 (1974).

GLO: RIPL data base.

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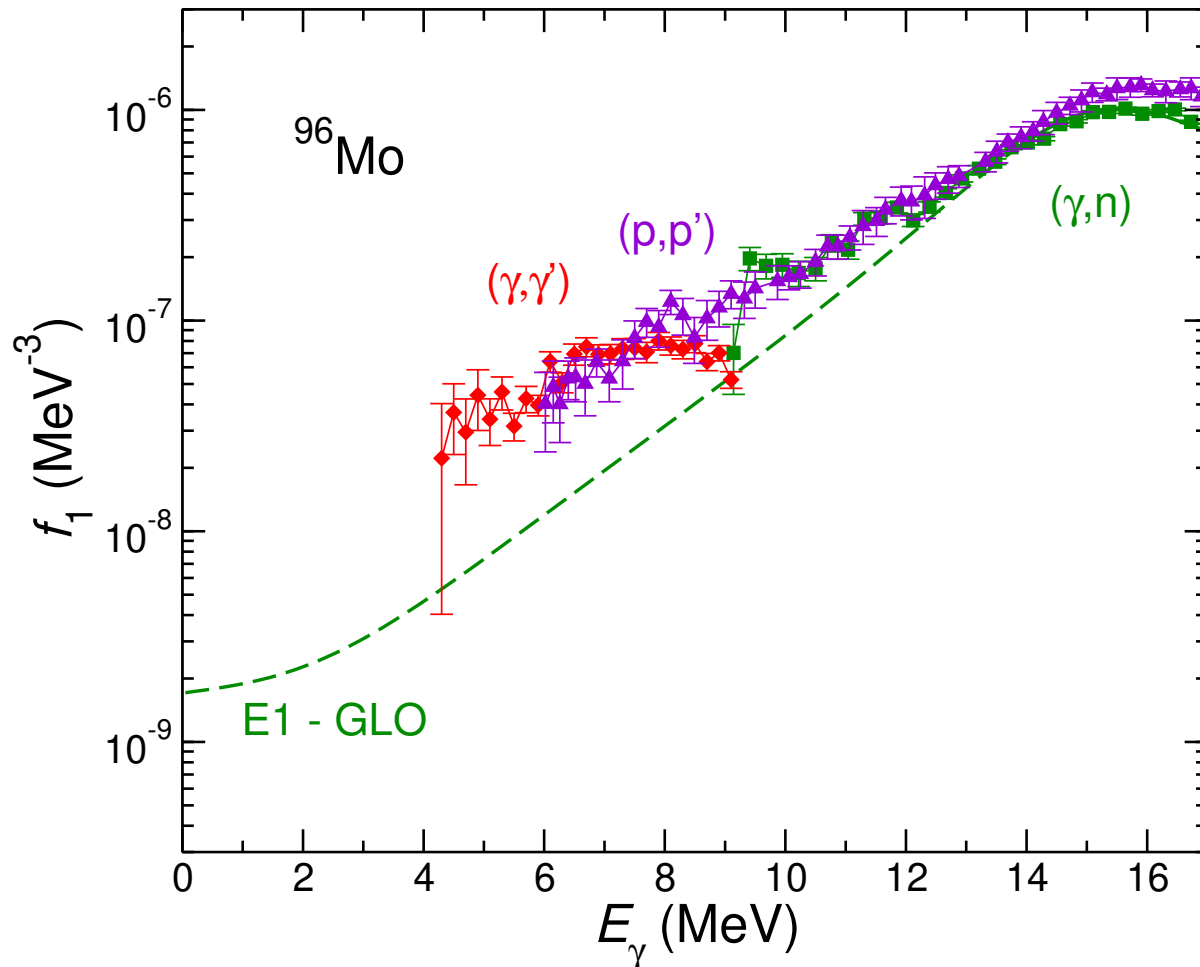
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GLO: RIPL data base.

$(\gamma, \gamma')$  data from  $\gamma\text{ELBE}$  (HZDR):

G. Rusev et al., PRC 79, 061302 (2009).

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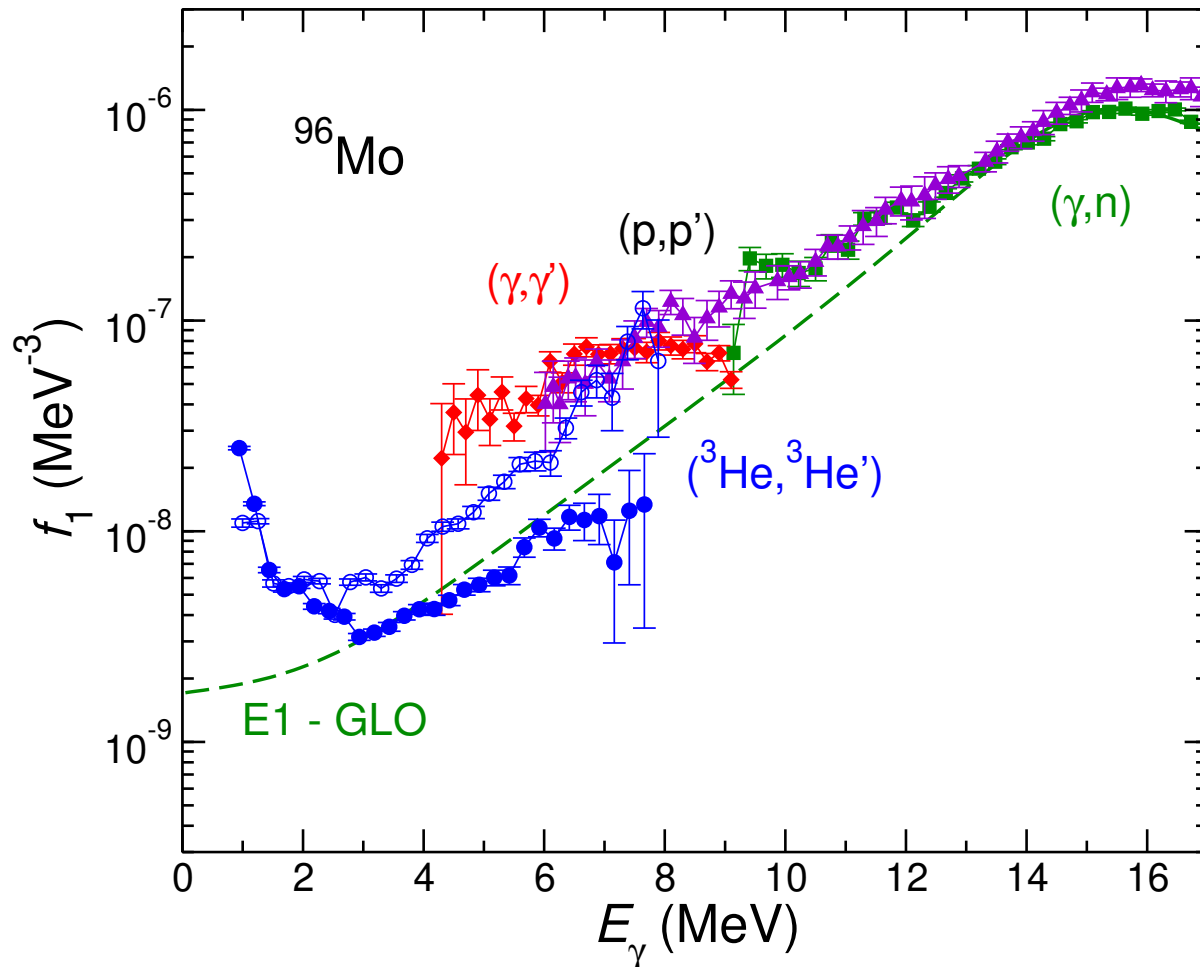
$(\gamma, \gamma')$  data from  $\gamma\text{ELBE}$  (HZDR):

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$(p, p')$  data from RCNP (Osaka):

D. Martin et al., PRL 119, 182503 (2017).

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D. Martin et al., PRL 119, 182503 (2017).

$(^3\text{He}, ^3\text{He}')$  data from OCL (Oslo):

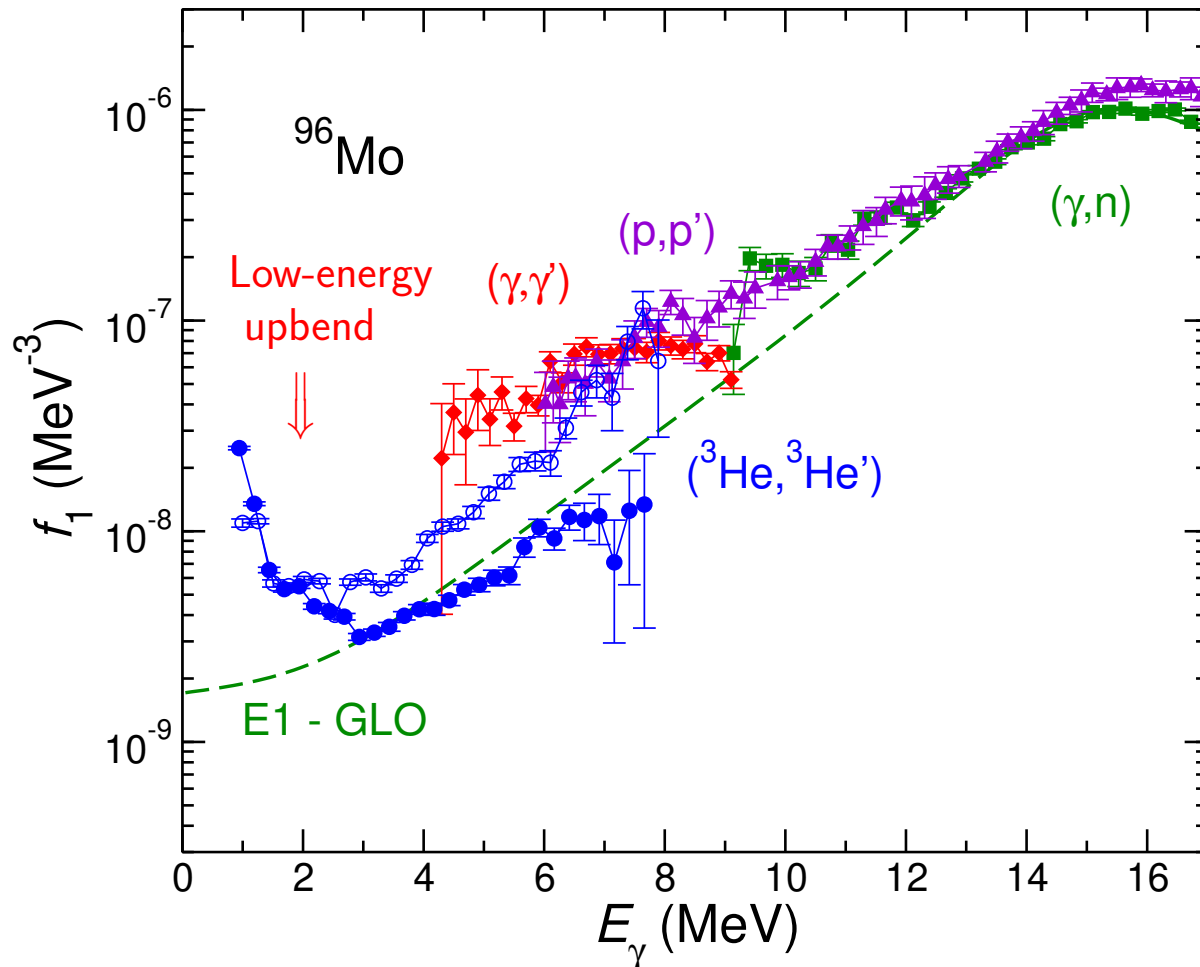
M. Guttormsen et al.,

PRC 71, 044307 (2005).

H. Utsunomiya et al.,

PRC 88, 015805 (2013).

# Dipole strength functions in $^{96}\text{Mo}$



What are origin and consequences of the upbend?

$(\gamma, n)$  data:

H. Beil et al., NPA 227, 427 (1974).

GLO: RIPL data base.

$(\gamma, \gamma')$  data from  $\gamma\text{ELBE}$  (HZDR):

G. Rusev et al., PRC 79, 061302 (2009).

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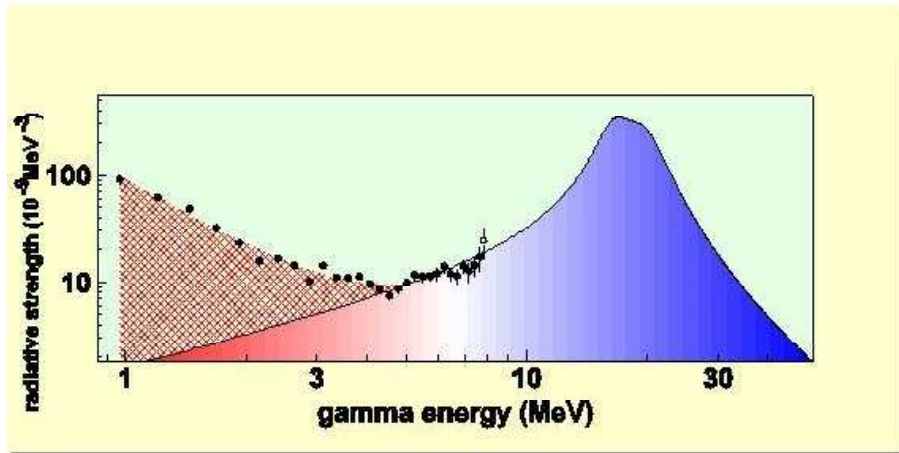
D. Martin et al., PRL 119, 182503 (2017).

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# Effect of enhanced low-energy dipole strength in the r-process

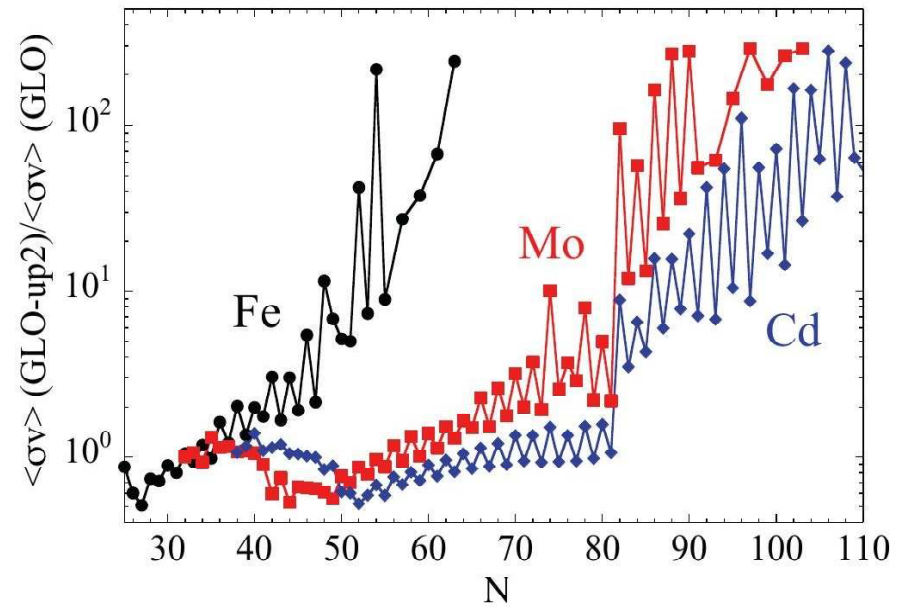


“The nucleus goes red.”

Unexpected upbend of dipole strength toward low energy observed in Fe, Mo, Cd isotopes.

Courtesy of M. Guttormsen.

⇒ Big influence of low-energy strength on neutron-capture rates of very neutron-rich nuclei in the astrophysical r-process.



Ratios of Maxwellian-averaged  $(n, \gamma)$  reaction rates at  $T = 10^9$  K for isotopic chains up to the neutron drip line using strength functions with and without low-energy upbend of dipole strength.

A.C. Larsen, S. Goriely, PRC 82, 014318 (2010)



# Shell-model calculations of M1 strength functions

## *Determination of average quantities:*

- 40 levels of each spin from 0 to 10 for each parity.
- All possible transitions between these 440 states - about 24000 M1 transitions.
- Average  $B(M1)$  values in bins of  $E_\gamma = E_i - E_f$ .
- M1 strength function:

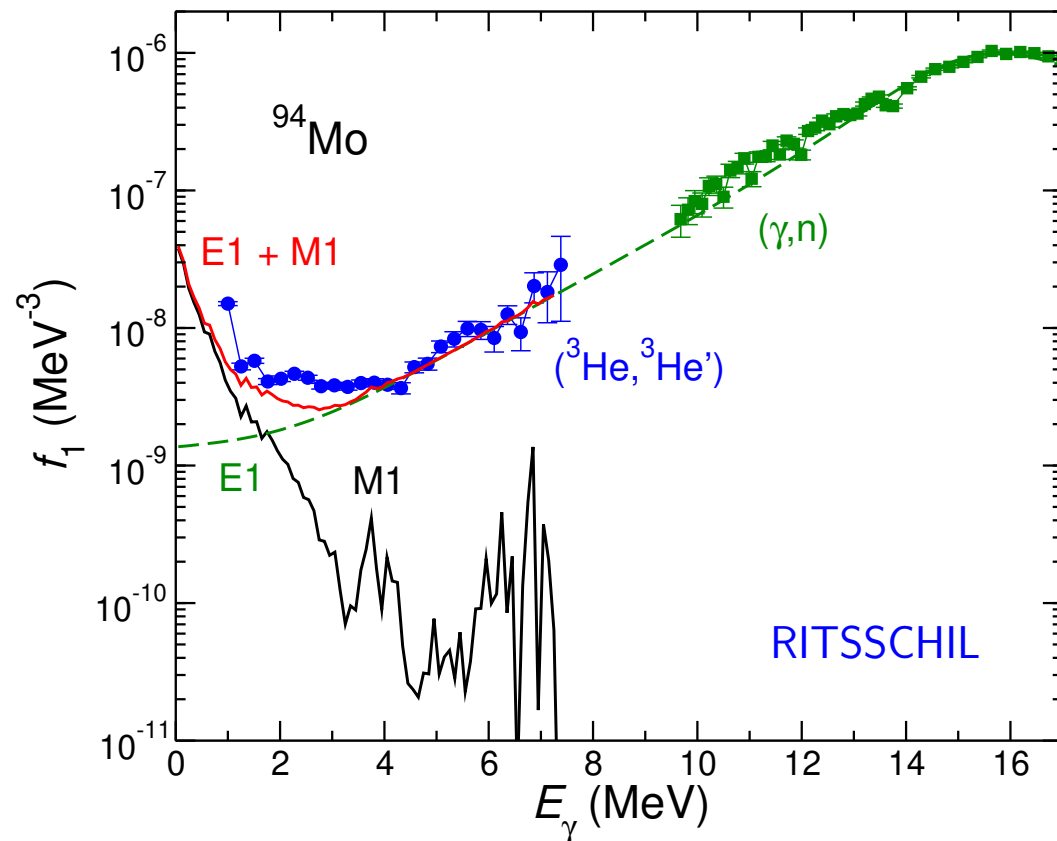
$$f_{M1}(E_\gamma, E_i, J_i, \pi) = 16\pi/9 (\hbar c)^{-3} \overline{B}(M1, E_i \rightarrow E_f, J_i, \pi) \rho(E_i, J_i, \pi).$$

$f_{M1}(E_\gamma)$  obtained by averaging over  $E_i, J_i,$  and  $\pi$ .

## *Codes:*

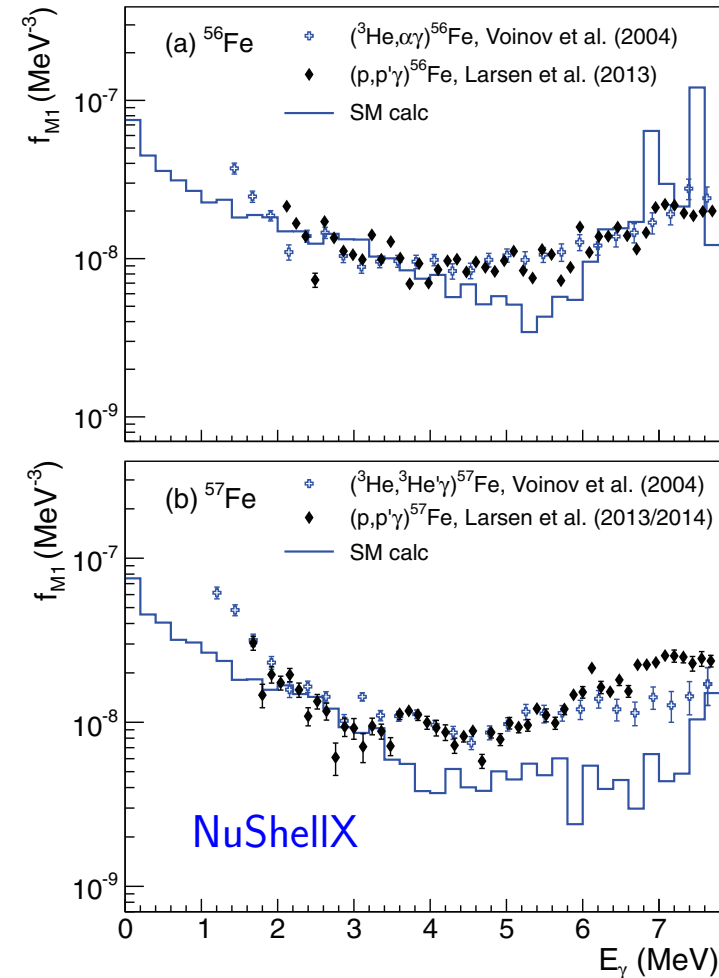
- NuShellX@MSU  
B.A. Brown and W.D.M. Rae, Nucl. Data Sheets 120, 115 (2014).
- RITSSCHIL  
D. Zwarts, Comput. Phys. Commun. 38, 365 (1985).

# Shell-model calculations of M1 strength functions



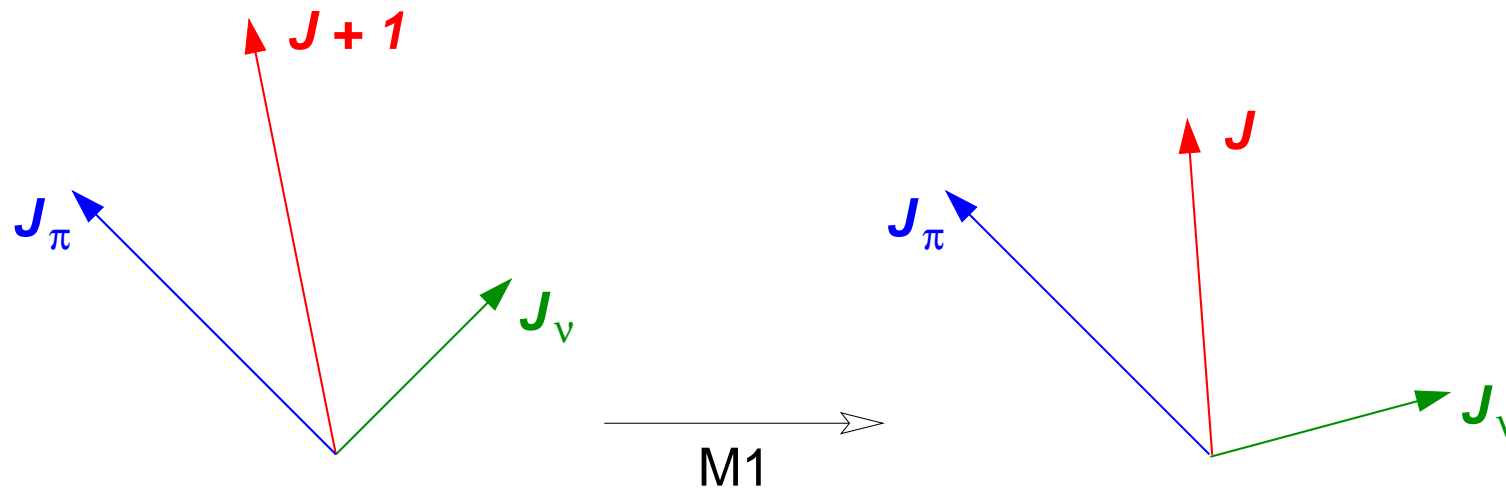
Low-energy enhancement of M1 radiation

R. Schwengner, S. Frauendorf, A.C. Larsen  
PRL 111, 232504 (2013)



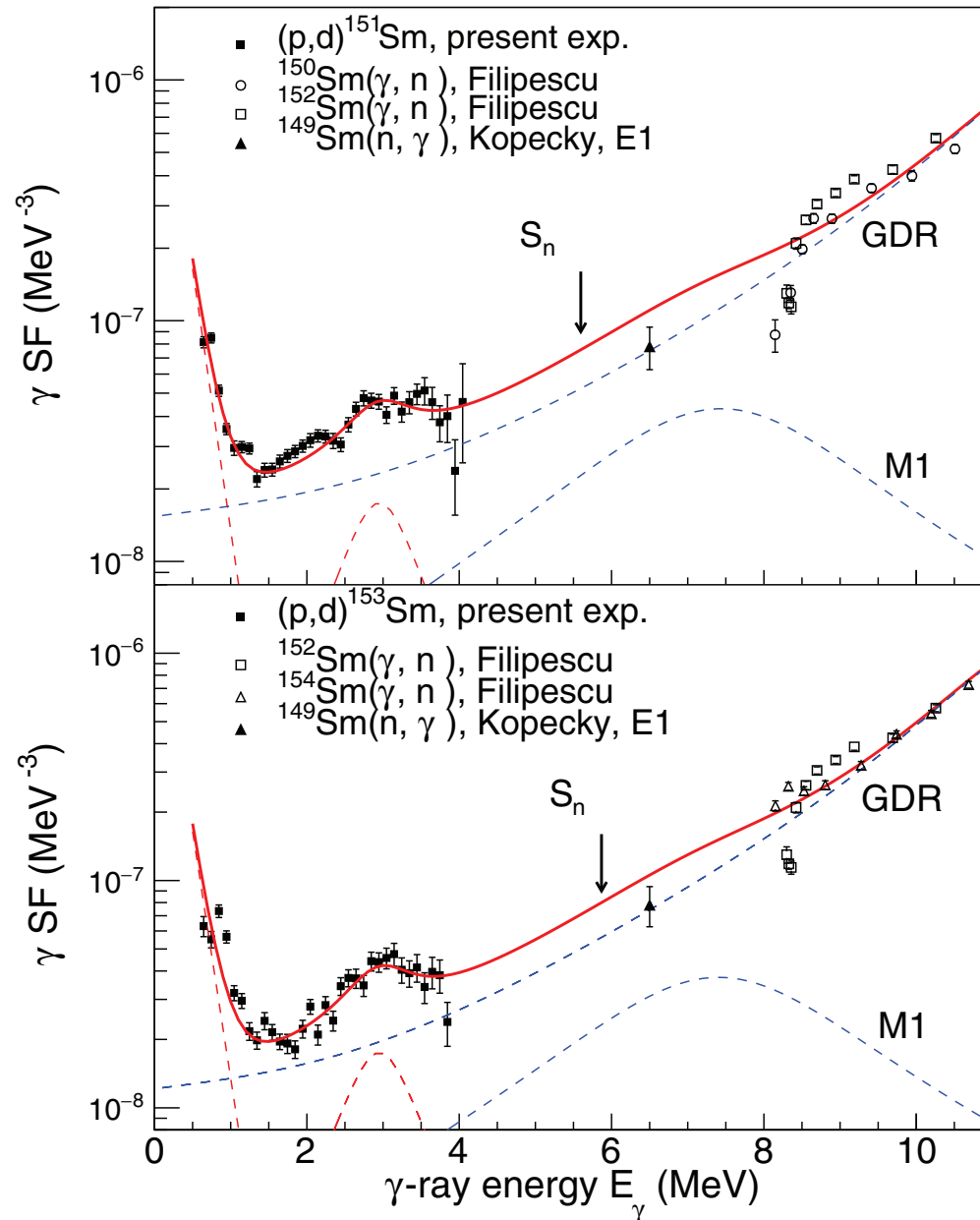
B.A. Brown, A.C. Larsen  
PRL 113, 252502 (2014)

## Generation of large M1 strengths



- ⇒ Large M1 strengths appear between close-lying states with equal configurations (multiplets) by a reorientation of proton and neutron spins.
- ⇒ Important role of configurations including protons and neutrons in specific high- $j$  orbitals (e.g.  $f_{7/2}$ ,  $g_{9/2}$ ,  $h_{11/2}$ ).

# Dipole strength functions in $^{151}\text{Sm}$ and $^{153}\text{Sm}$



(p,d) data:

A. Simon et al.

PRC 93, 034303 (2016)

⇒ First observation of upbend and scissors resonance in one nuclide.

⇒ Strength in the scissors region about three times that found in  $(\gamma, \gamma)$  experiments.

## Shell-model calculations for $^{60}\text{Fe}_{34}$ , $^{64}\text{Fe}_{38}$ , $^{68}\text{Fe}_{42}$

*Code:* NuShellX@MSU

[B.A. Brown and W.D.M. Rae, Nucl. Data Sheets 120, 115 (2014)]

*Model space:* CA48PN with CA48MH1 Hamiltonian ( $^{48}\text{Ca}$  core)

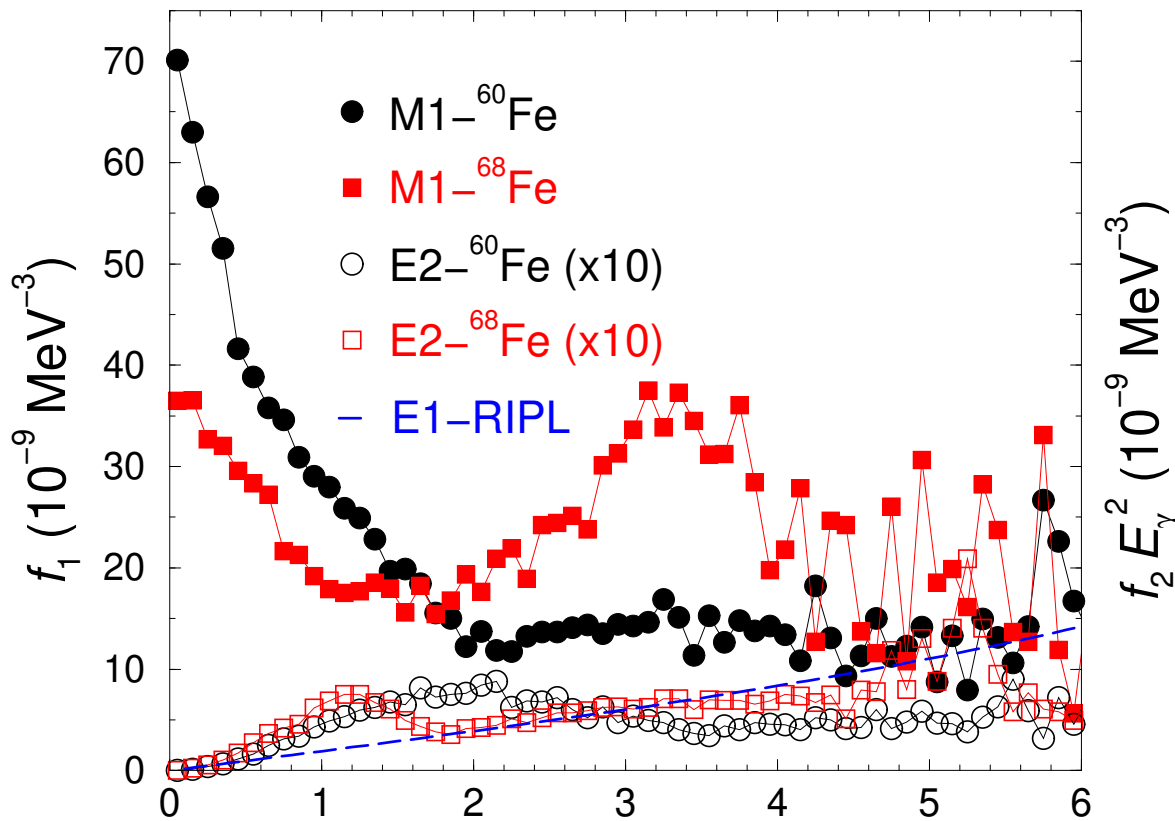
[M. Hjorth-Jensen, T.T.S. Kuo, and E. Osnes, Phys. Rep. 261, 125 (1995)]

*Orbitals:*  $\pi(0f_{7/2}, 0f_{5/2}, 1p_{3/2}, 1p_{1/2}) \quad \nu(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2})$

*Calculations:* 40 levels of each spin from 0 to 10 and of each parity.

*Transition strengths:*  $e_{\pi} = 1.5 e$ ,  $e_{\nu} = 0.5 e$ ;  $g_s = 0.9 g_s^{\text{free}}$

# Strength functions from shell-model calculations



Dipole strength function

$$f_1 = 16\pi/9 (\hbar c)^{-3} \overline{B(M1)} \rho(E_x, J)$$

$\rho(E_x, J)$  - level density of the shell-model states, includes  $\pi = +, \pi = -$ , all spins from 0 to 10.

R. Schwengner, S. Frauendorf, B.A. Brown

PRL 118, 092502 (2017)

	$E_\gamma$ (MeV)		Sum
	$E_\gamma < 2$ MeV	$2 \leq E_\gamma \leq 5$ MeV	
$^{60}\text{Fe}_{34}$	5.67	3.52	9.19
$^{64}\text{Fe}_{38}$	4.46	5.13	9.59
$^{68}\text{Fe}_{42}$	3.98	6.63	10.61

$$B(M1)_{\text{tot}} (\mu_N^2)$$

$\Rightarrow$  Sum of strengths at low energy and in the scissors region stays nearly constant.

## Evolution of M1 strength functions from open to closed shells

- Occurrence of low-lying M1 modes:
    - Enhanced strength near zero transition energy.  
→ decreases from  $^{60}\text{Fe}$  to  $^{68}\text{Fe}$ .
    - Strength in the scissors region.  
→ develops toward the midshell nucleus  $^{68}\text{Fe}$ .
  - Is this correlated evolution of the two M1 modes a general feature in nuclei?
- ⇒ Calculations in another mass region using another Hamiltonian.

# Shell-model calculations for $^{62}\text{Ge}_{30}$ , $^{64}\text{Ge}_{32}$ , $^{66}\text{Ge}_{34}$ , $^{70}\text{Ge}_{38}$ , $^{74}\text{Ge}_{42}$ , $^{78}\text{Ge}_{46}$ , $^{80}\text{Ge}_{48}$

*Code:* NuShellX@MSU

[B.A. Brown and W.D.M. Rae, Nucl. Data Sheets 120, 115 (2014)]

*Model space:* jj44pn with the jj44bpn Hamiltonian ( $^{56}\text{Ni}$  core)

[M. Honma, T. Otsuka, T. Mizusaki, and M. Hjorth-Jensen, Phys. Rev. C **80**, 064323 (2009).]

[B. A. Brown and A. F. Lisetskiy, unpublished.]

[A. F. Lisetskiy, B. A. Brown, M. Horoi, and H. Grawe, Phys. Rev. C **70**, 044314 (2004).]

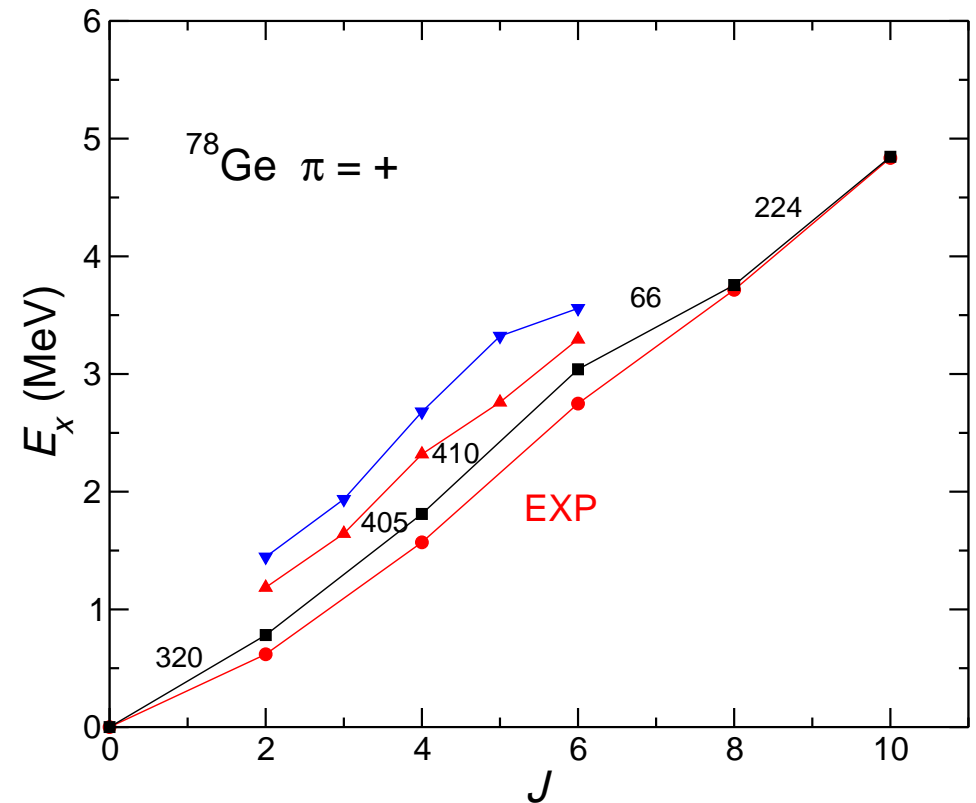
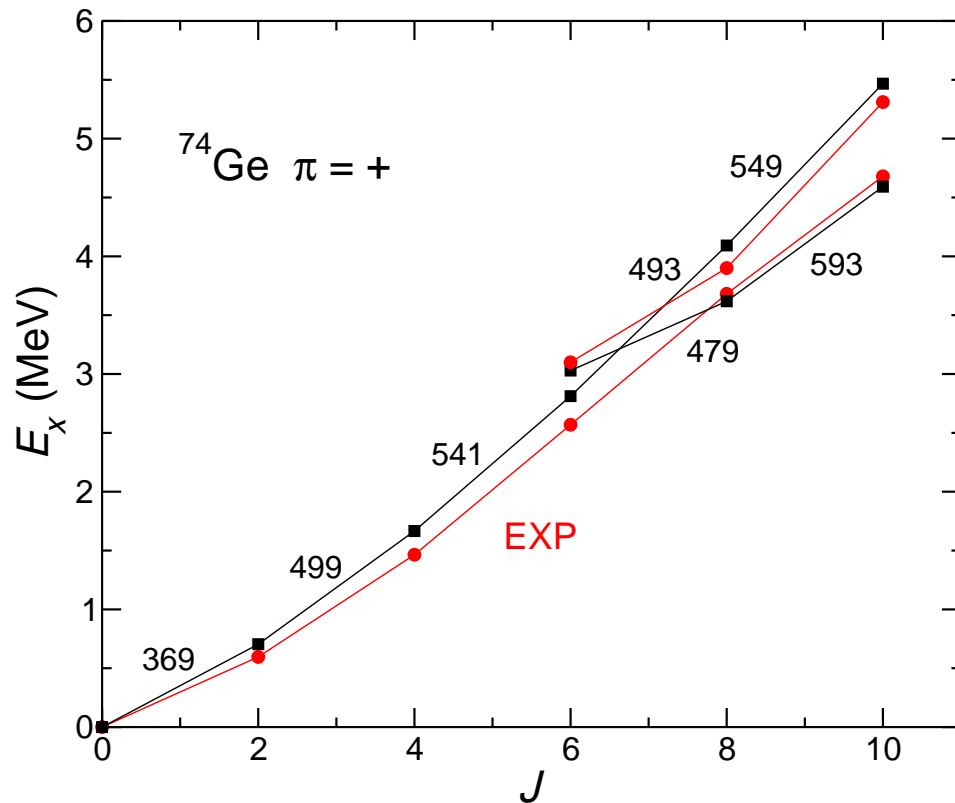
*Orbitals:*  $\pi(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}) \quad \nu(0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2})$

*Calculations:* 40 levels of each spin from 0 to 10 and of each parity.

*Transition strengths:*  $e_{\pi} = 1.5 e$ ,  $e_{\nu} = 0.5 e$ ;  $g_s = 0.7 g_s^{\text{free}}$

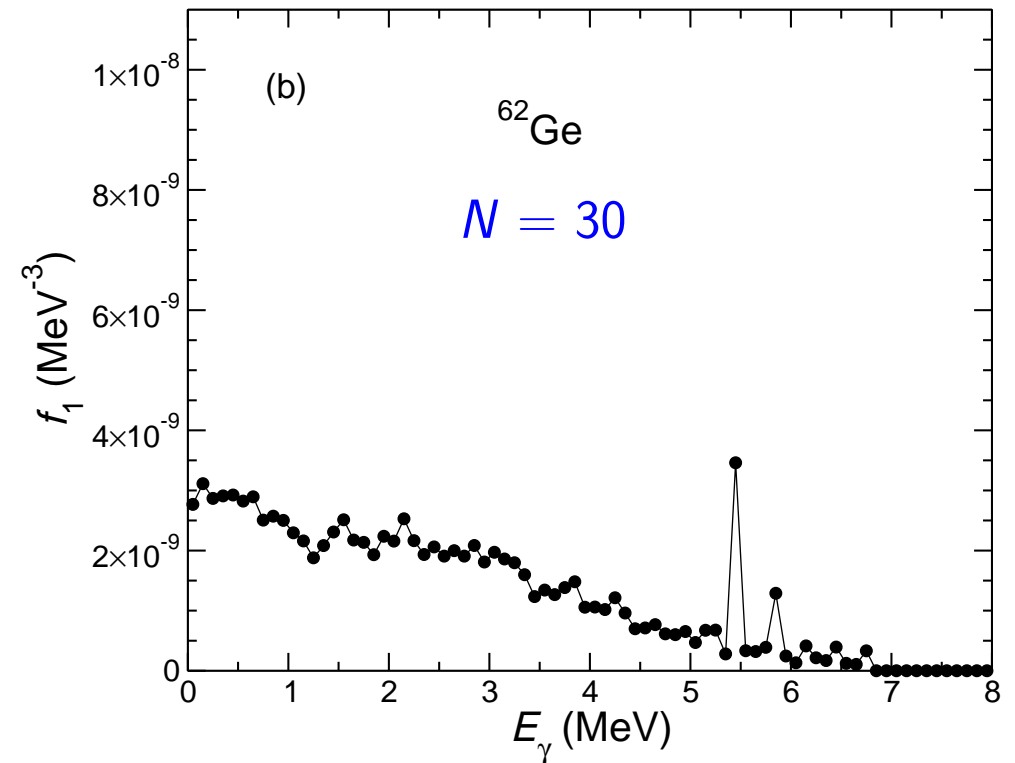
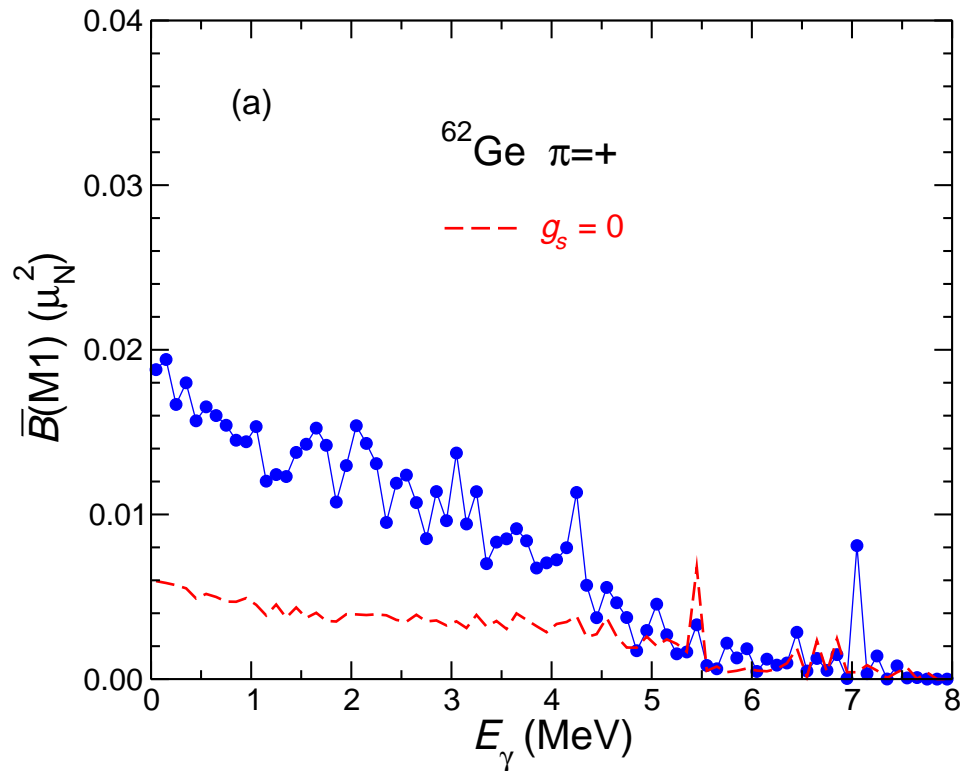


# Level sequences in the yrast region



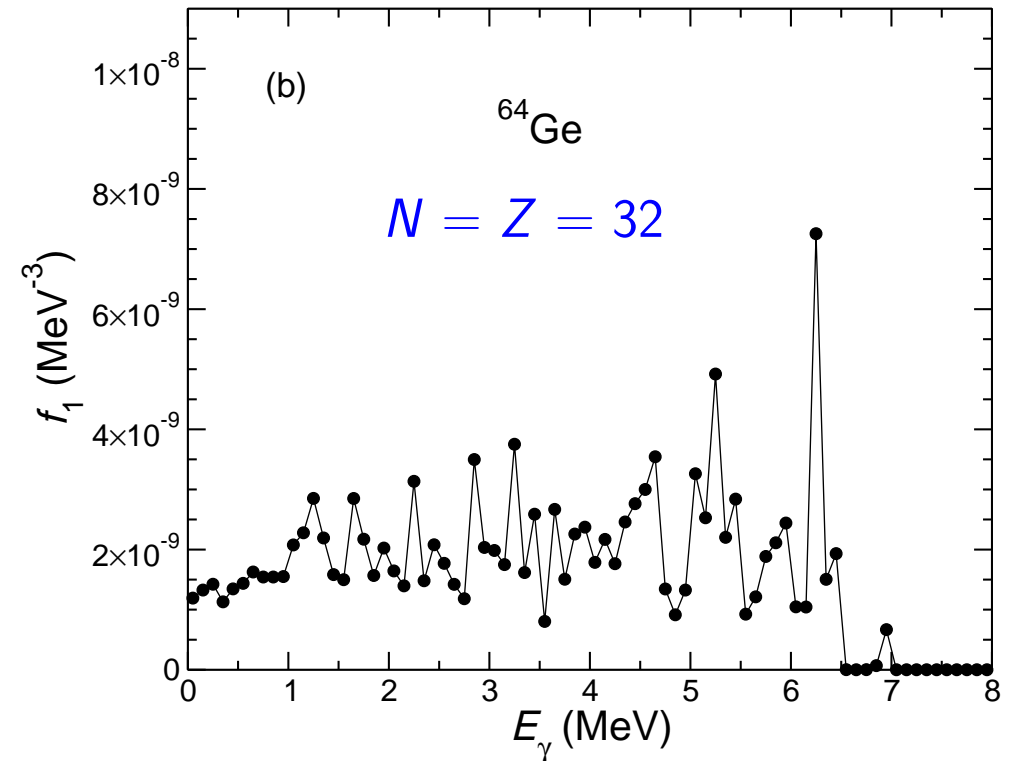
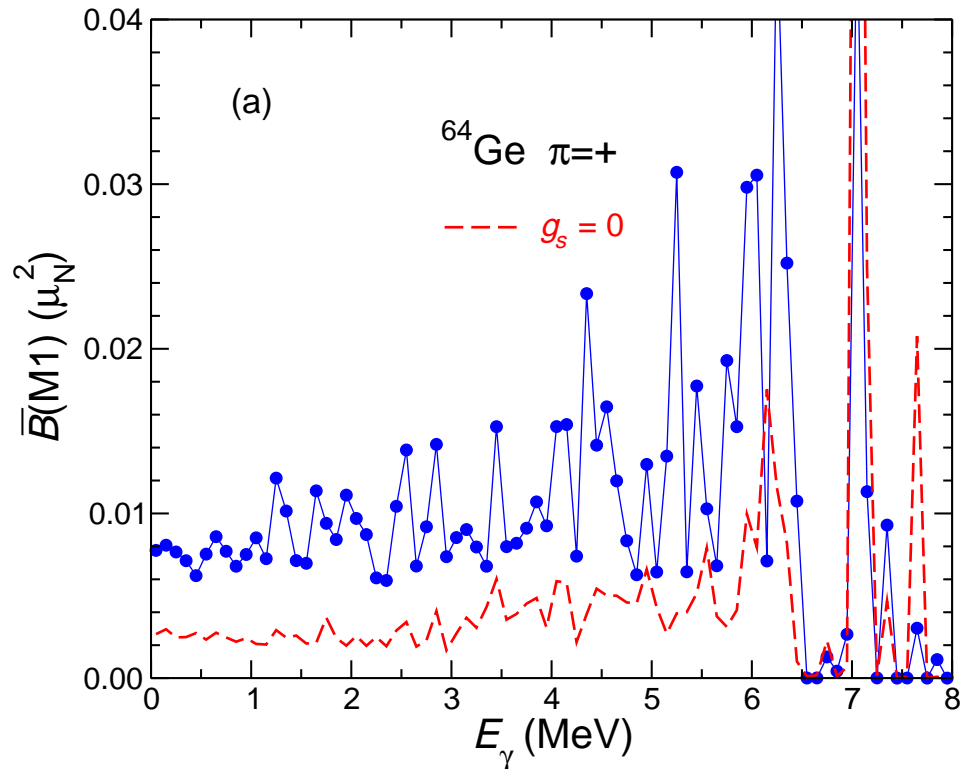
Black numbers: Calculated  $B(E2)$  in  $e^2\text{fm}^4$ .

# M1 strength functions in Ge isotopes with $N = 30$ to 48



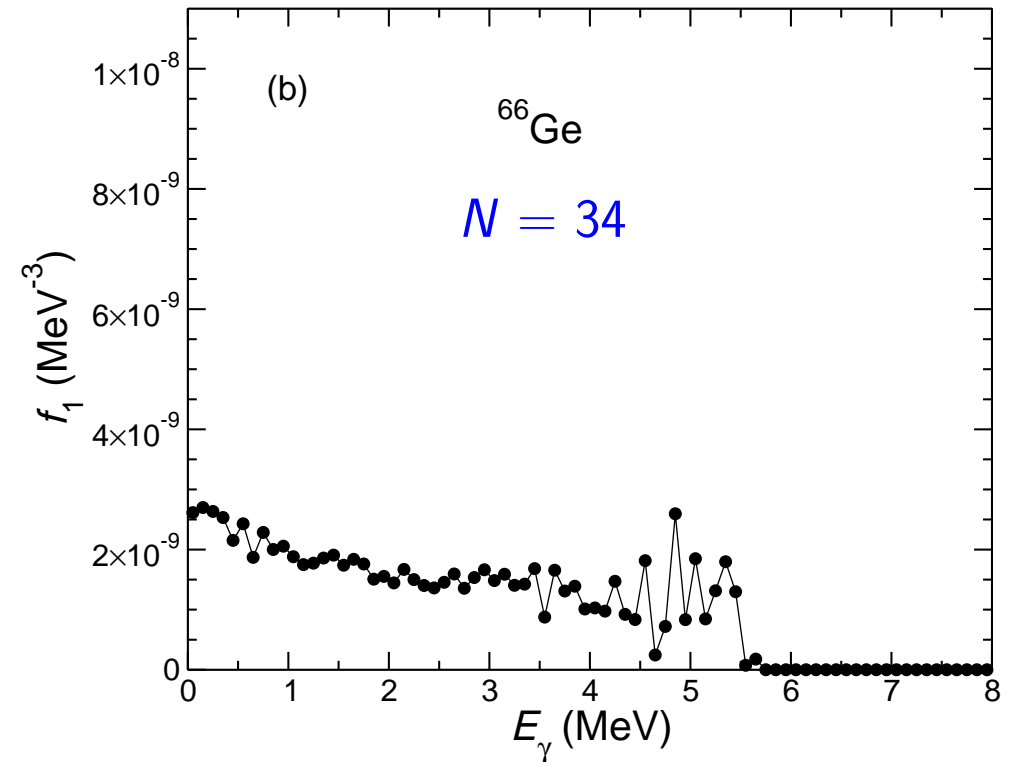
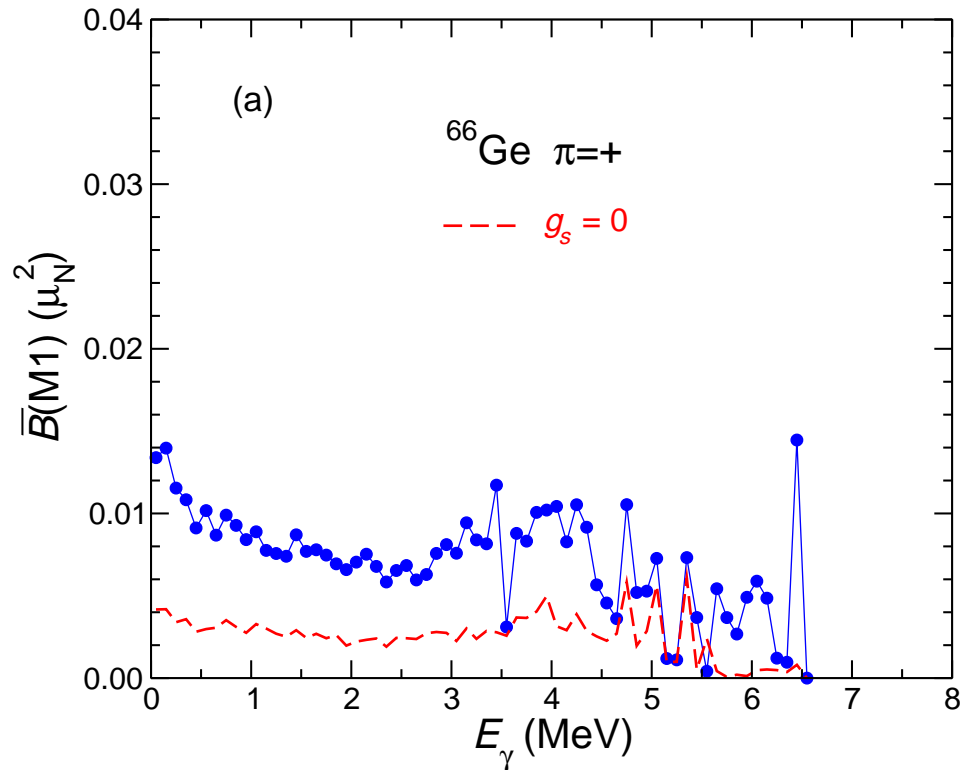
⇒ Steady increase of M1 strength toward low transition energy.

# M1 strength functions in Ge isotopes with $N = 30$ to 48



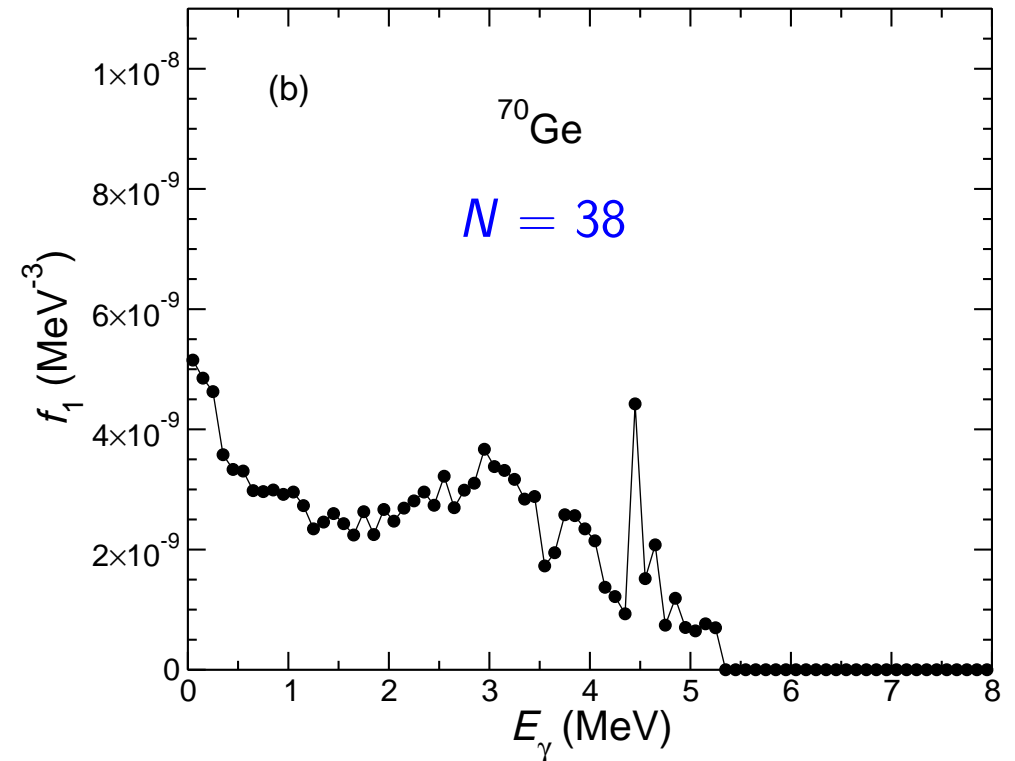
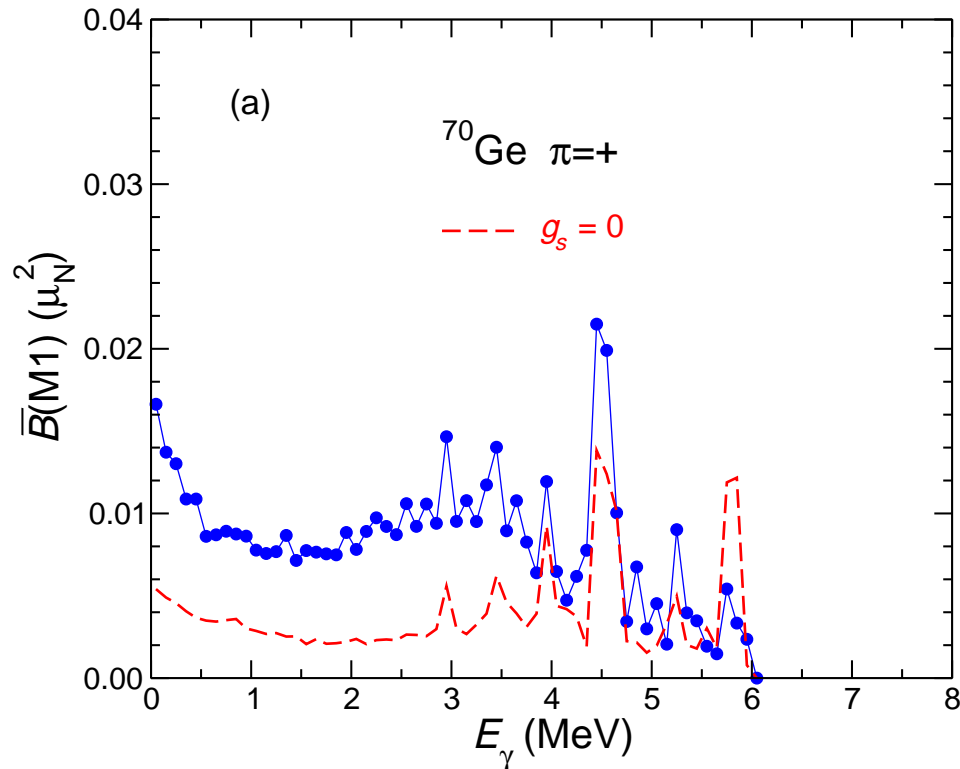
⇒ No enhancement of M1 strength toward low transition energy.  
Suppressed M1 strength between  $T = 0$  states may weaken the average M1 strength.

# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Moderate increase of M1 strength toward low transition energy.

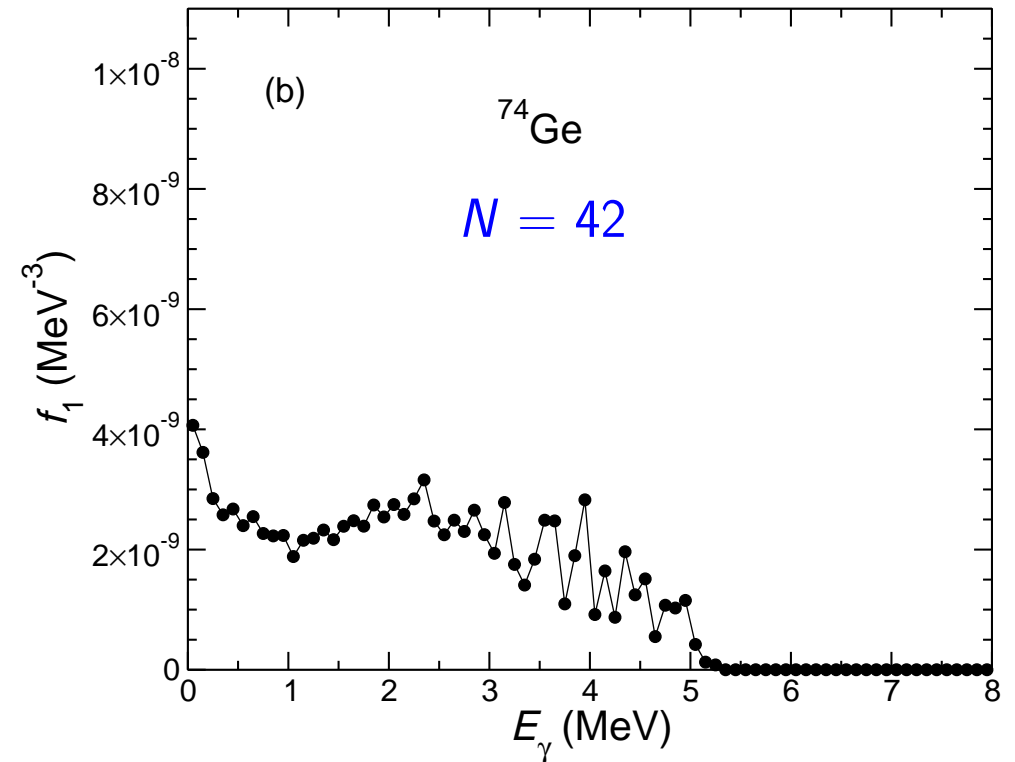
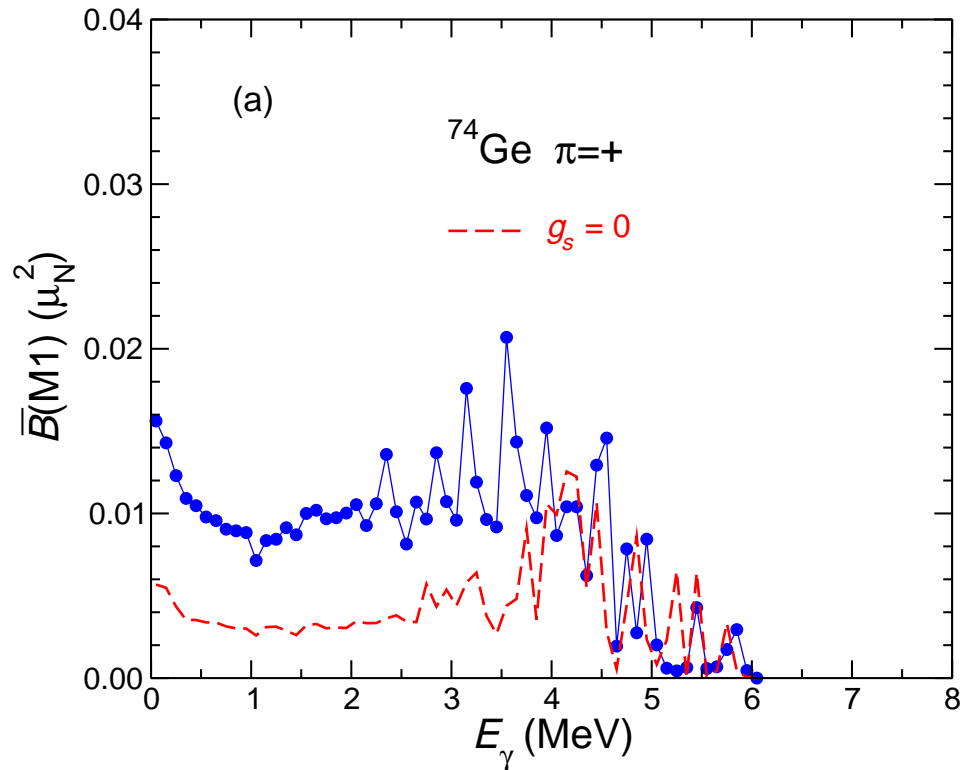
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Low-energy enhancement of M1 strength.

⇒ Development of a bump in the scissors region around 3 MeV.

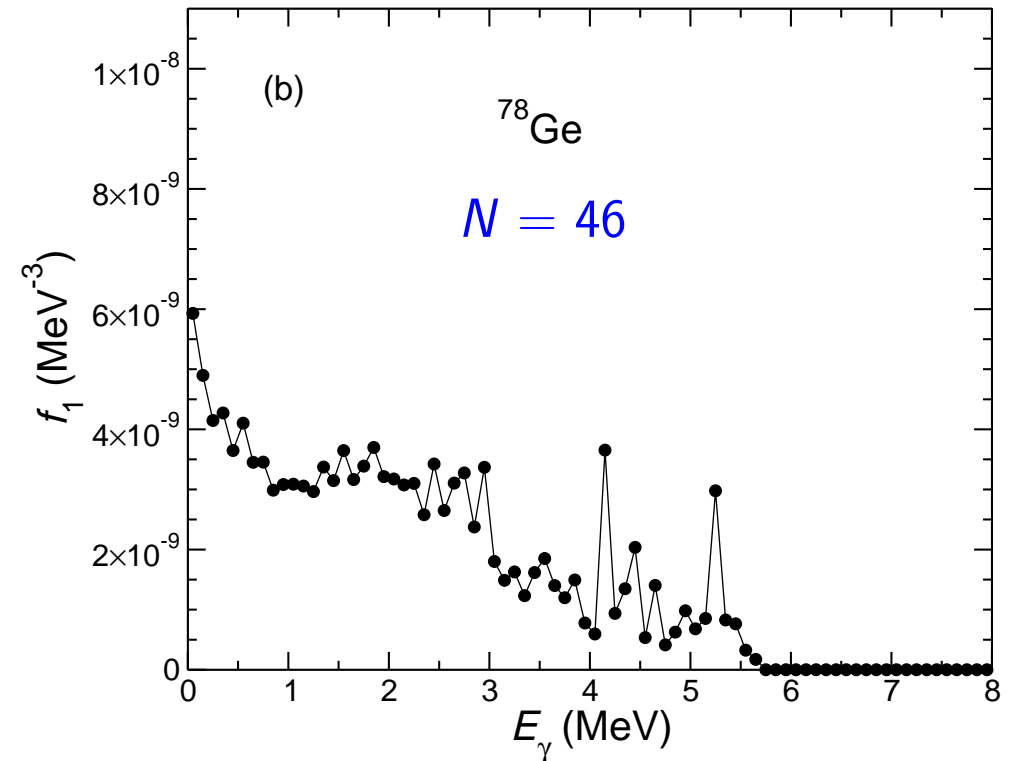
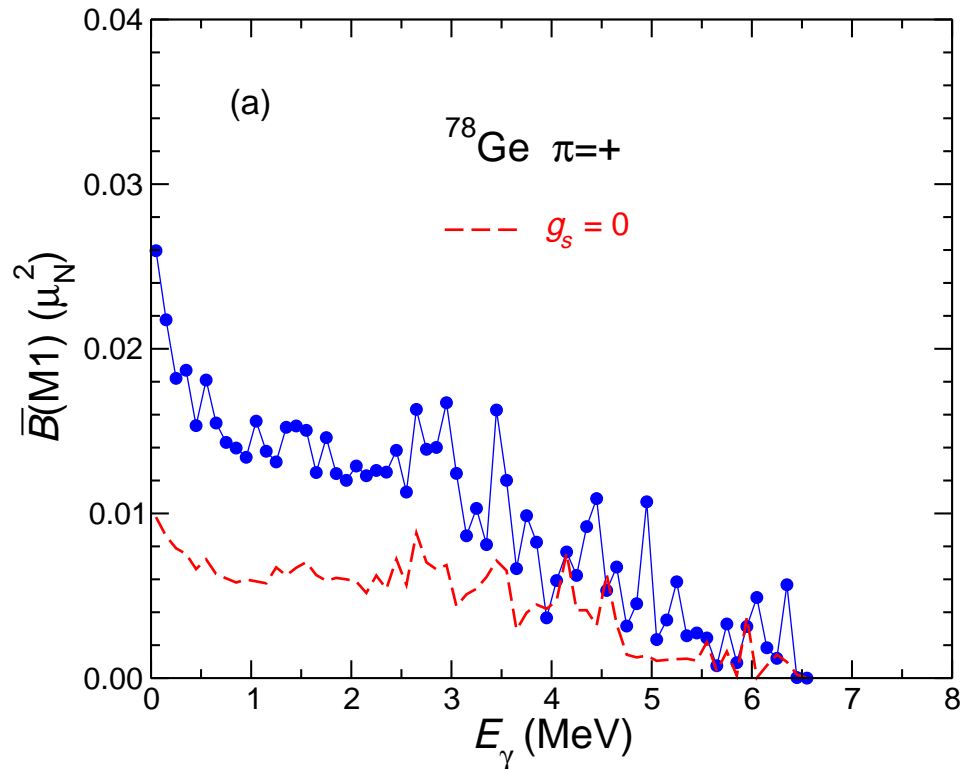
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Low-energy enhancement of M1 strength.

⇒ Broad bump in the scissors region around 3 MeV.

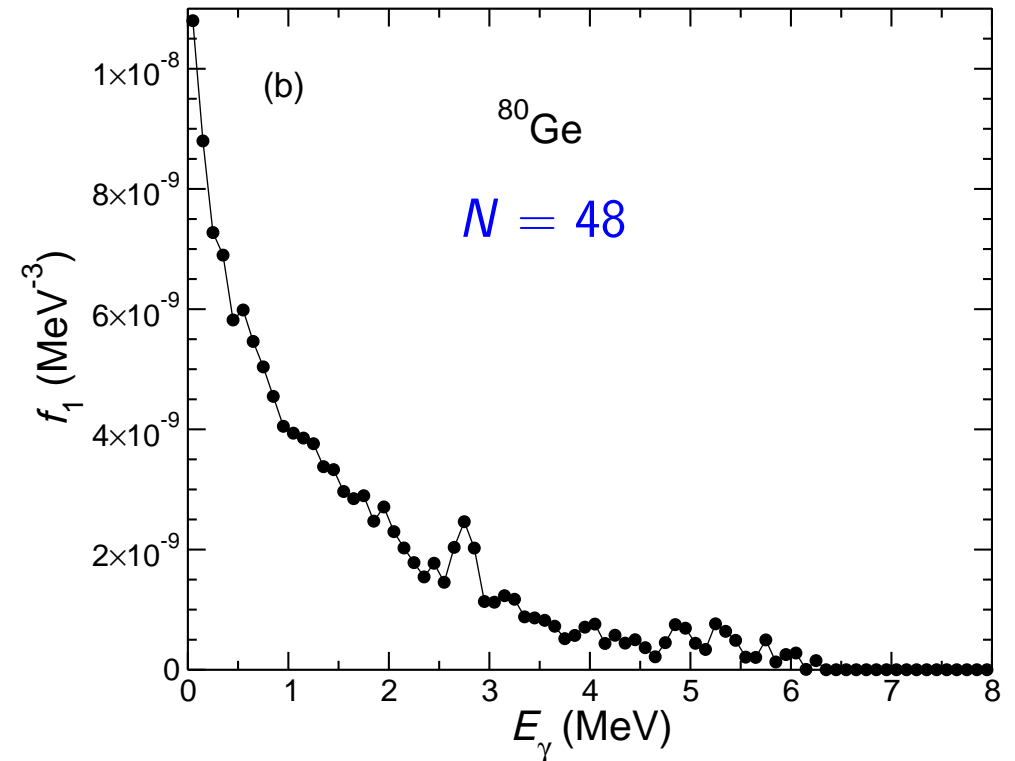
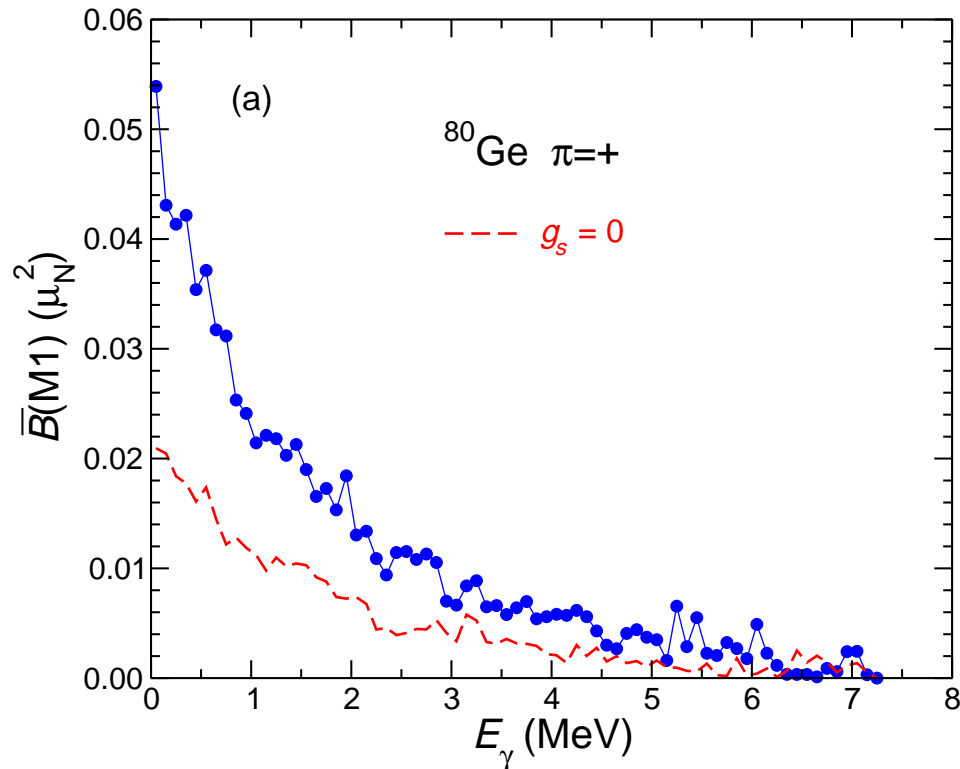
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Increase of the low-energy enhancement of M1 strength.

⇒ Decrease of the bump in the scissors region around 3 MeV.

# M1 strength functions in Ge isotopes with $N = 30$ to $48$

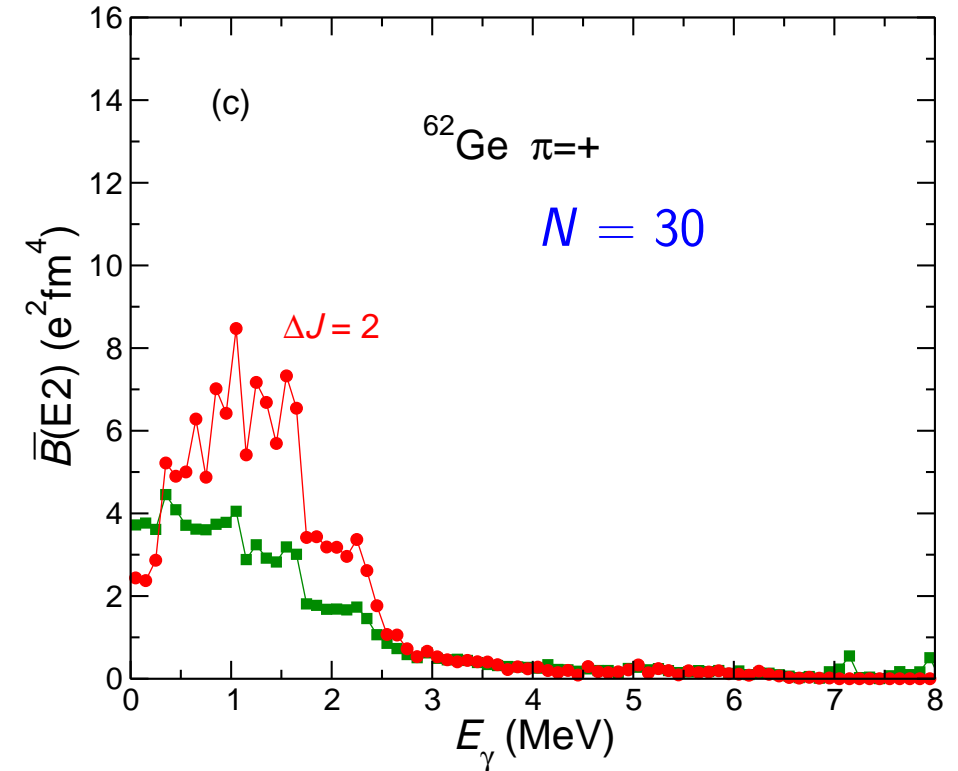
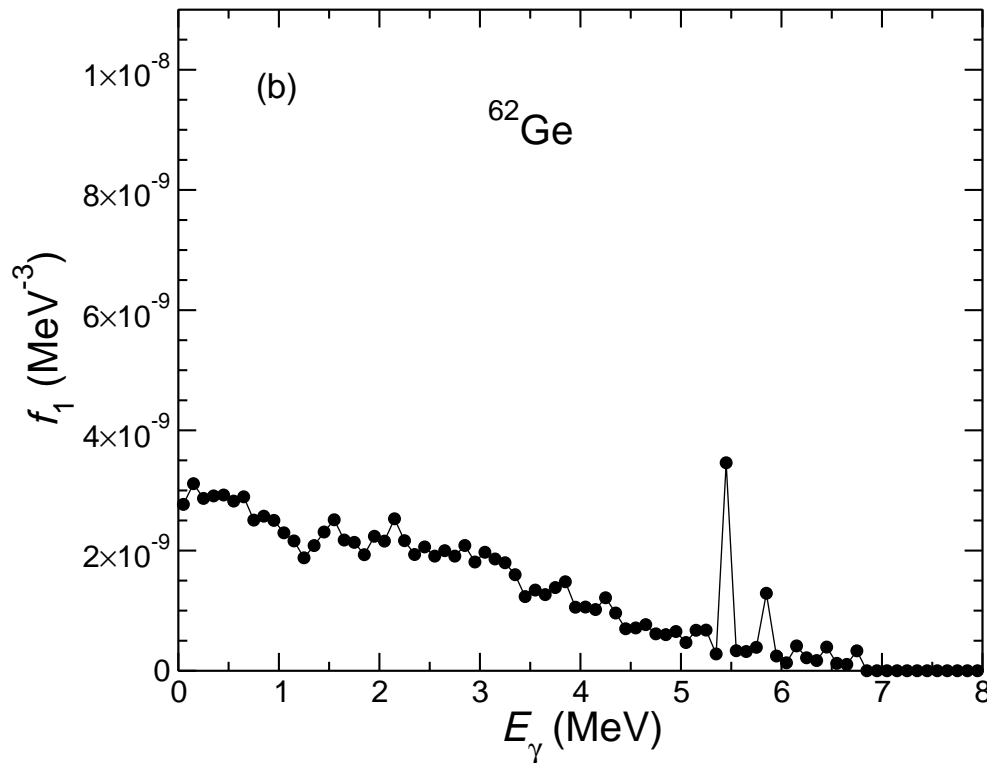


⇒ Steep increase of the low-energy enhancement of M1 strength.

⇒ Vanishing bump in the scissors region around 3 MeV.



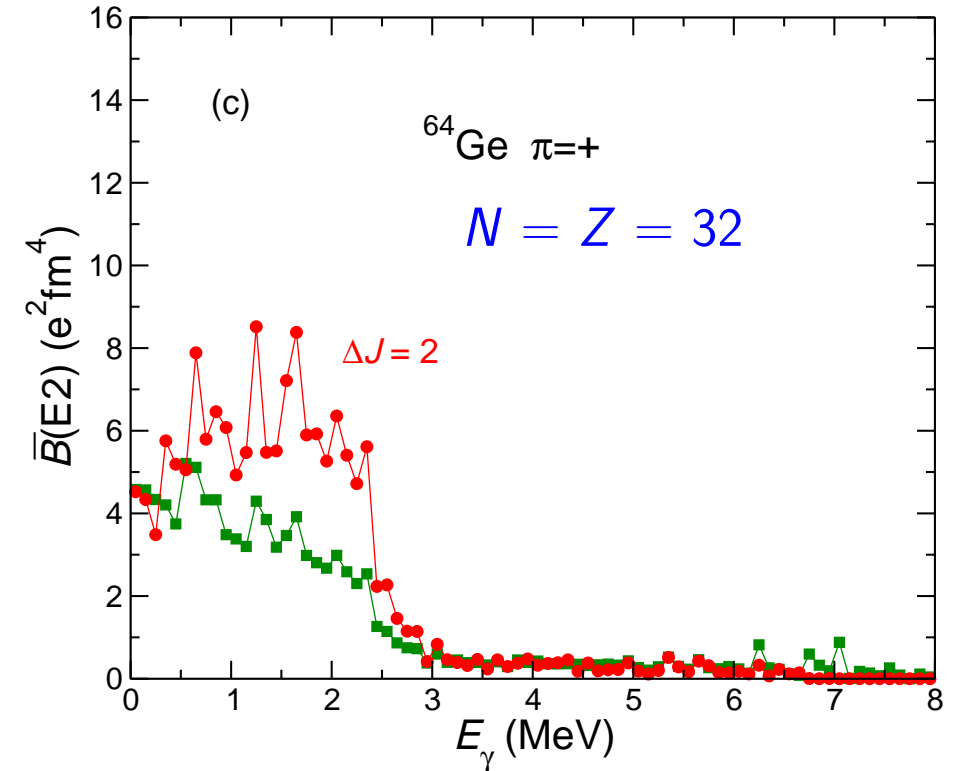
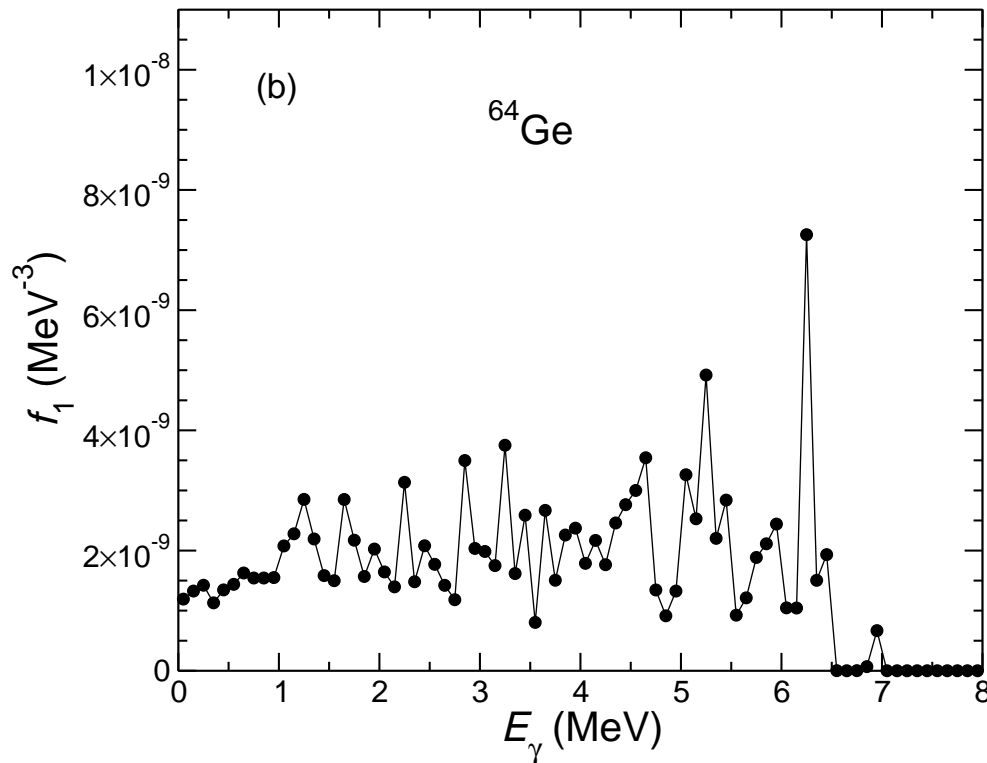
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Steady increase of M1 strength toward low transition energy.

⇒ Moderate strength of stretched E2 transitions.

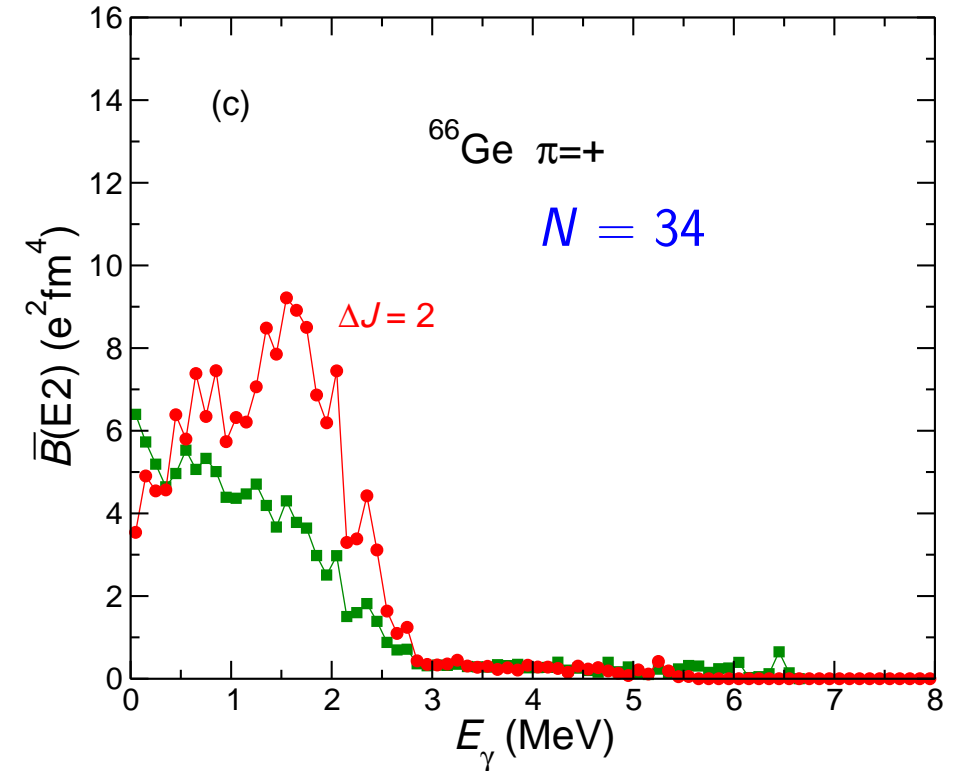
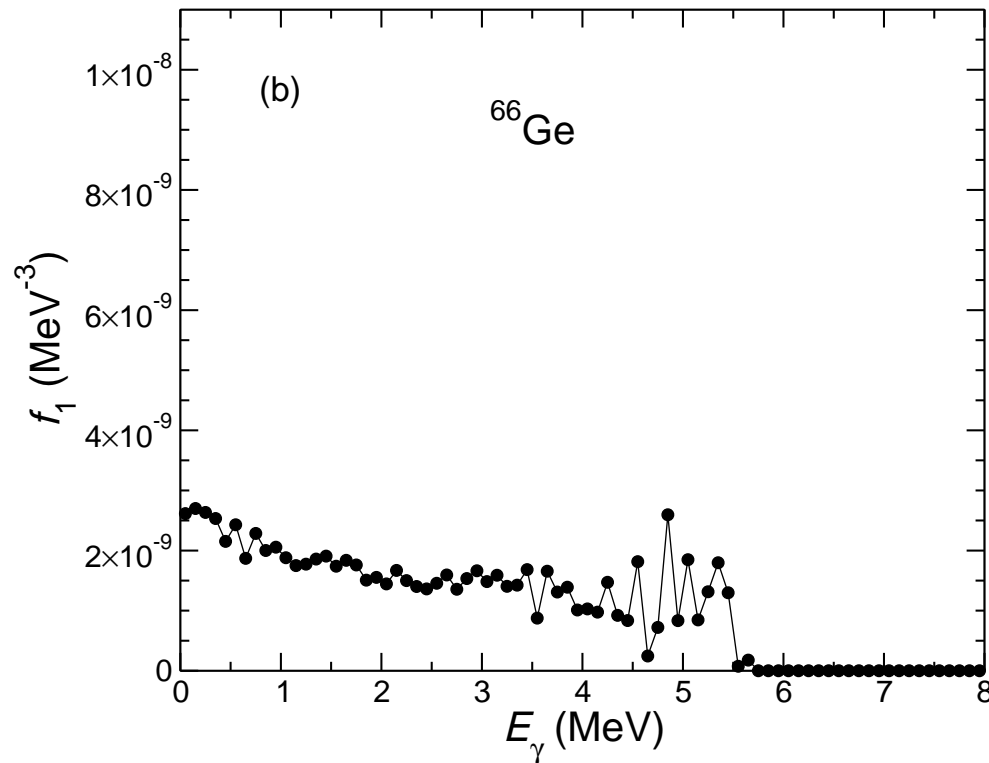
# M1 strength functions in Ge isotopes with $N = 32$ to 48



⇒ No enhancement of M1 strength toward low transition energy.

⇒ Moderate strength of stretched E2 transitions.

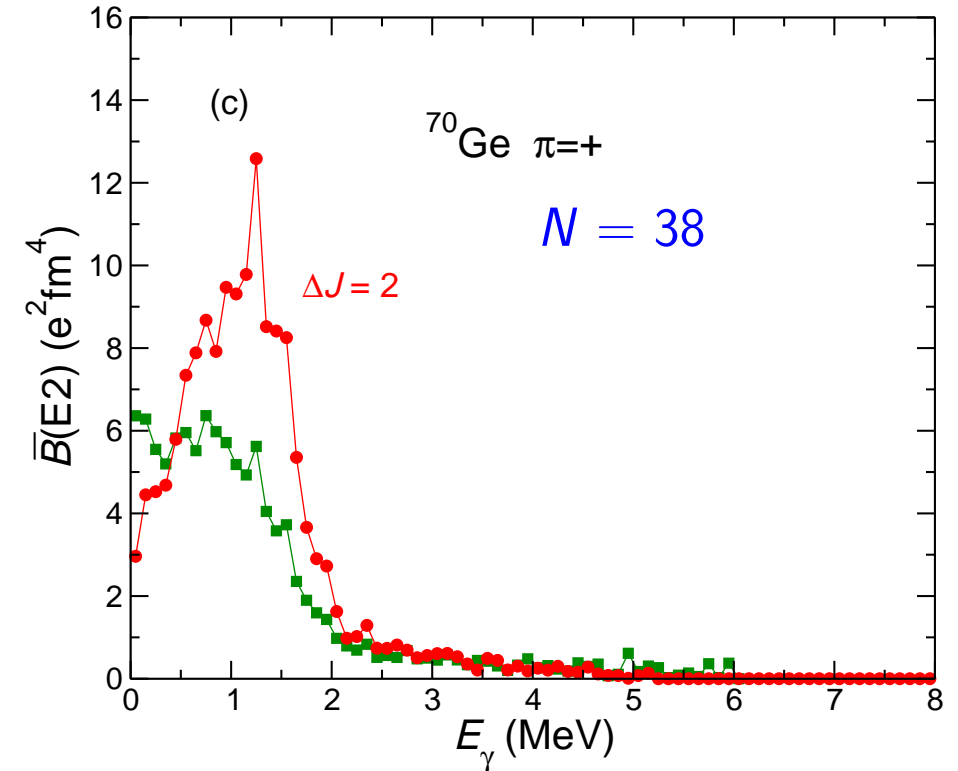
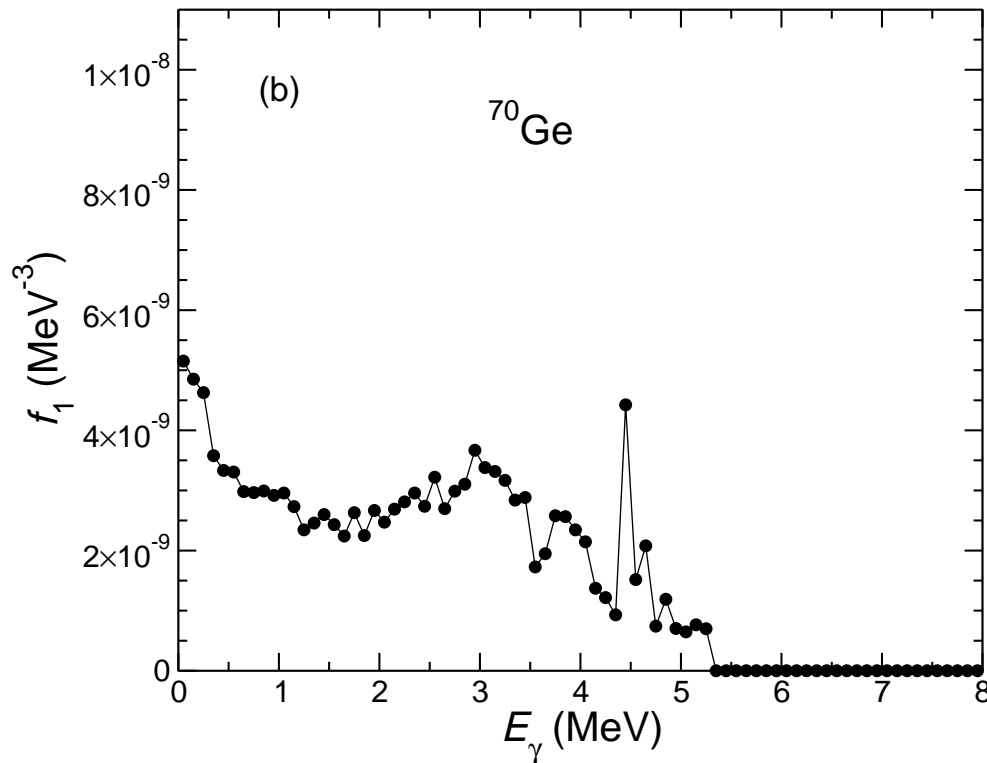
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Steady increase of M1 strength toward low transition energy.

⇒ Moderate strength of stretched E2 transitions.

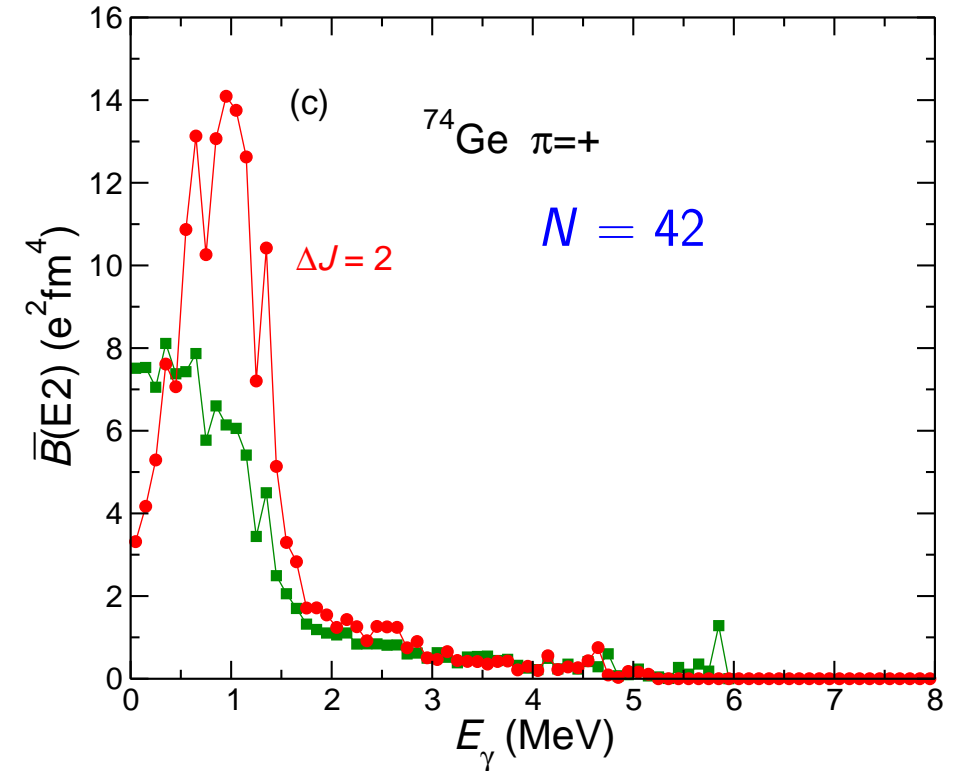
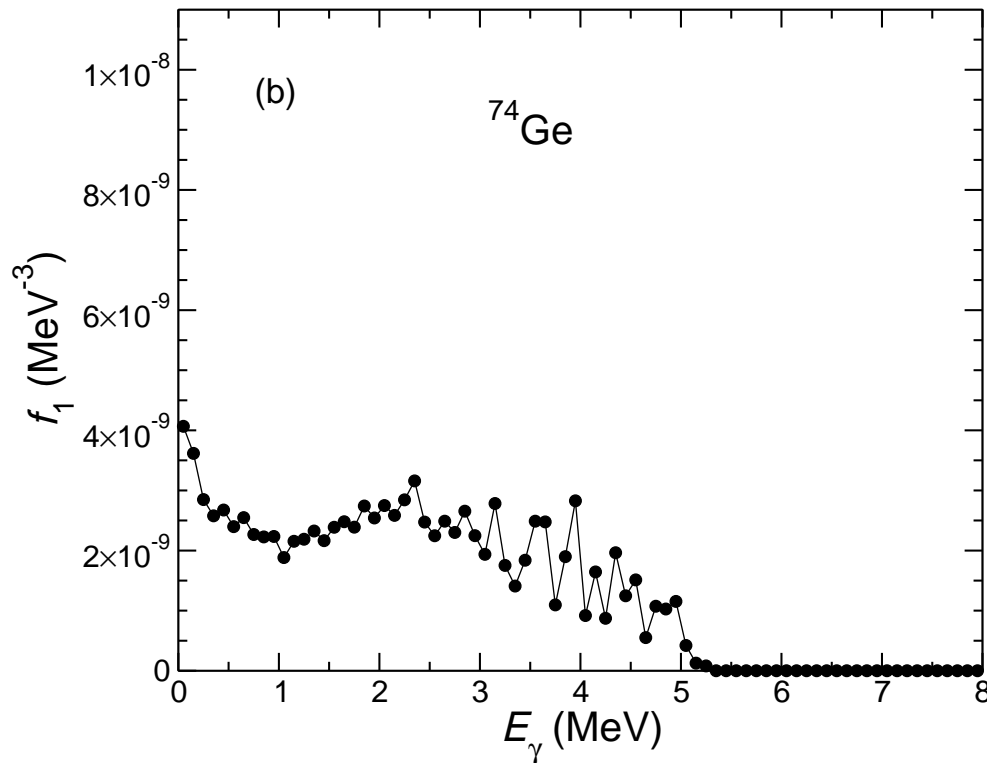
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Development of a bump in the scissors region around 3 MeV.

⇒ Increase of E2 strength of stretched transitions.

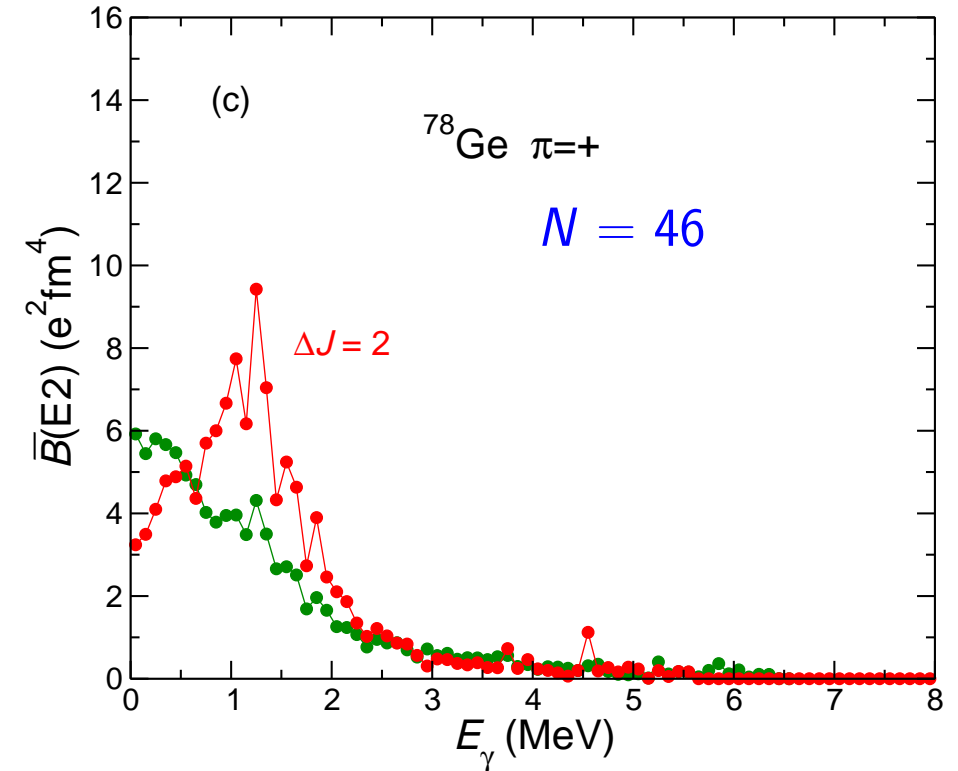
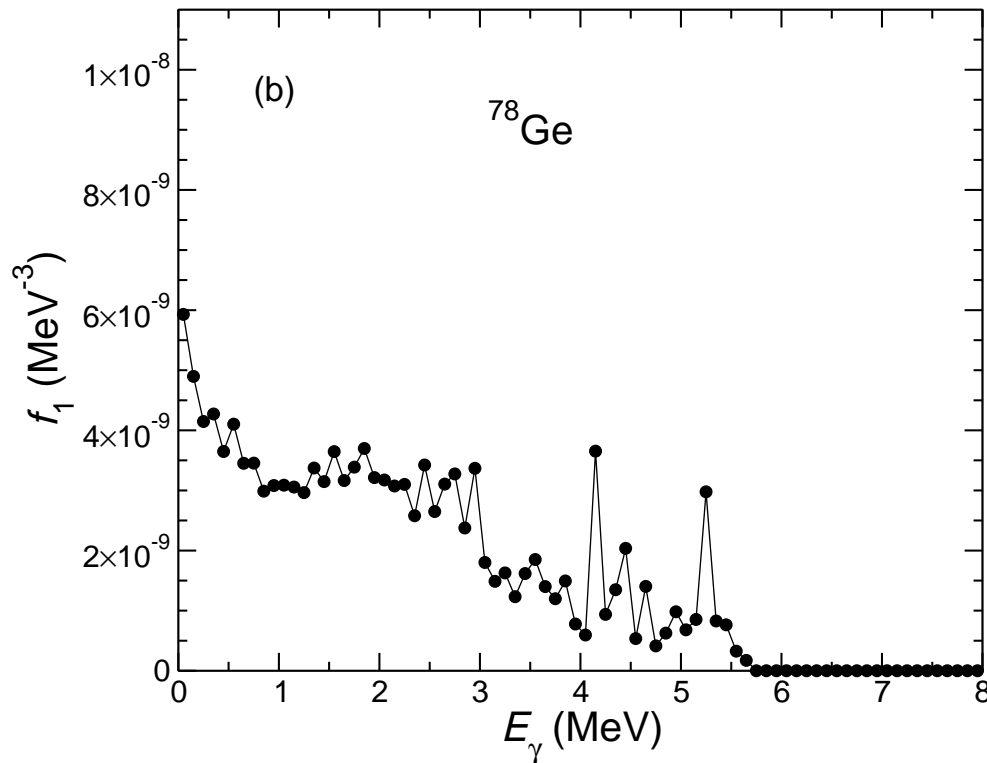
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Broad bump in the scissors region around 3 MeV.

⇒ Increased E2 strength of stretched transitions.

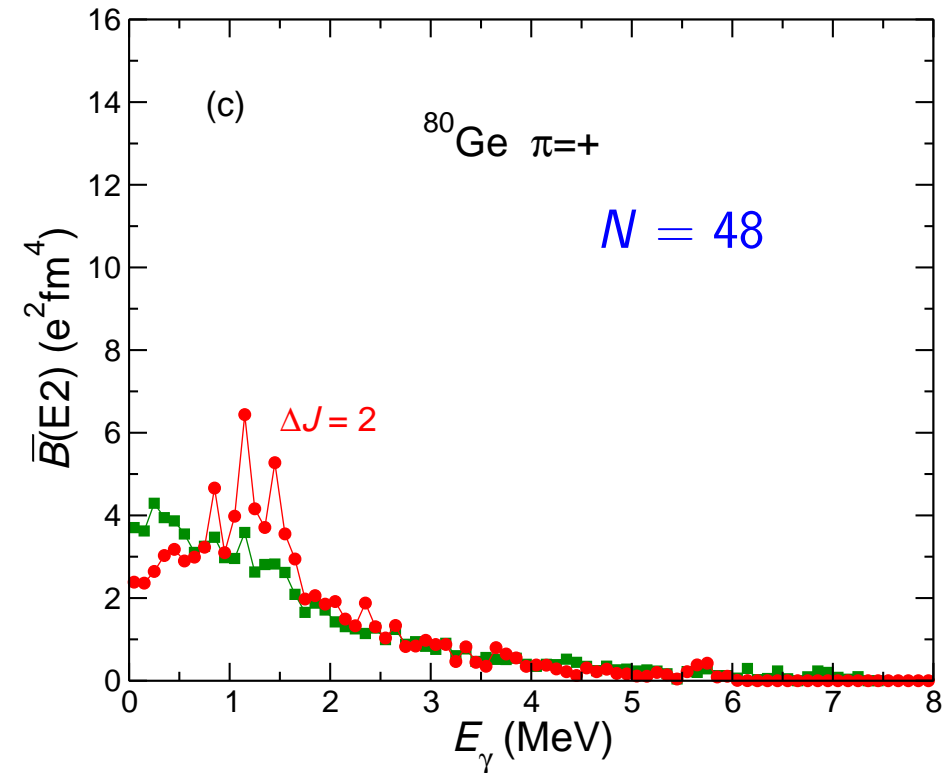
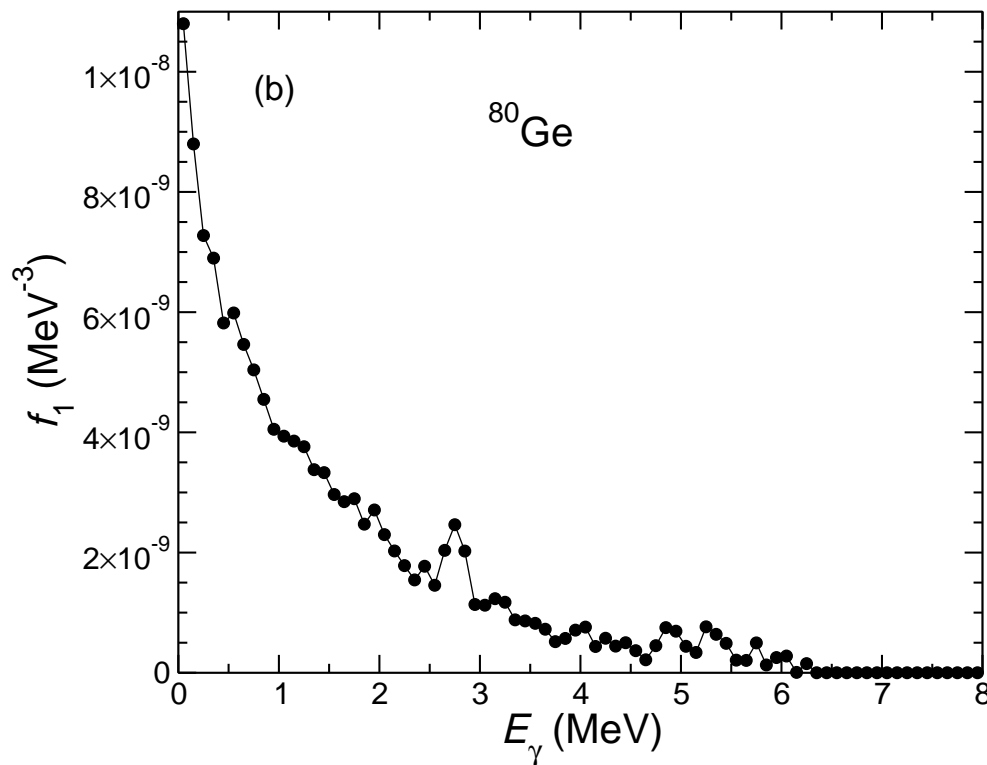
# M1 strength functions in Ge isotopes with $N = 30$ to 48



⇒ Decrease of the bump in the scissors region around 3 MeV.

⇒ Decrease of E2 strength.

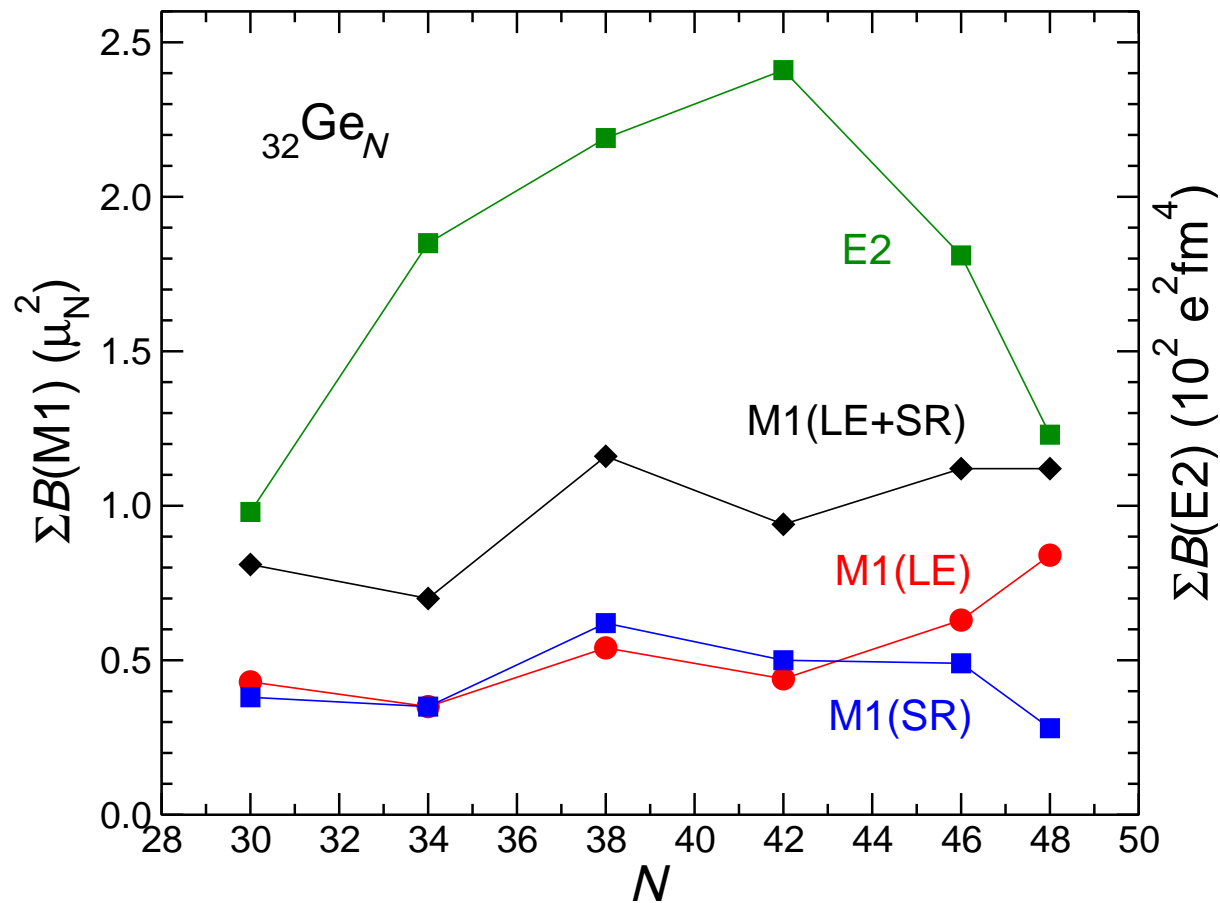
# M1 strength functions in Ge isotopes with $N = 30$ to $48$



⇒ Vanishing bump in the scissors region around 3 MeV.

⇒ Little E2 strength.

## Summed strengths in Ge isotopes with $N = 30$ to 48



- ⇒ Low-energy M1 strength increases with filling the  $g_{9/2}$  orbital toward the shell closure.
- ⇒ M1 strength in the scissors region correlates with collectivity.



## Summary

- Shell-model calculations of a large number of M1 and E2 transition strengths in the series of germanium isotopes with  $N = 30$  to 48.
- Derivation of average quantities  $\Rightarrow$   $\gamma$ -ray strength functions.
- Occurrence of low-lying M1 modes:
  - Enhanced strength near zero transition energy develops with filling neutrons into the  $g_{9/2}$  orbital and becomes strongest near the shell closure at  $N = 50$ .
  - Strength in the scissors region appears in the midshell nuclei and correlates with the quadrupole collectivity.
- These characteristics are consistent with those earlier found in calculations for iron isotopes and with experimental observations in samarium isotopes.
- The present study of low-lying M1 strength in a relatively long isotopic series demonstrates that the appearance of the two correlated M1 modes is a global phenomenon across various mass regions.
- Published in Phys. Rev. C **105**, 034335 (2022).