On phases volume flows evaluations in upward gas liquid flow

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Evaluation of phases volume flows in vertical cylindrical channels based on the measurements of local flow conditions is possible only subject to adequate notions on the two-phase flow structure, correctness of the corresponding computational model and correctness of averaging local velocity values and flow gas content over the channel cross-section. In this case satisfactory results can be obtained for bubble flow and gas piston flow.

Bubble flow. In case of a bubble flow, a matter of interest is a collection of independent measurements of flow parameters in two pipe segments connected in series and differing in cross-section area. The true volume gas content of the flow in the *i*-th section ($i = 1, 2$) averaged by pipe cross-section is

$$\varphi_i = \frac{F_{g,i}}{F_i} \ , \tag{1}$$

where $F_i = (\pi/4)\cdot D_i^2$ is a pipe cross-section area at the *i*-th segment, $F_{g,i} = Q_g/u_{g,i}^{true}$ is cross-section area occupied by gas. The corresponding cross-section area occupied by liquid is $F_{l,i} = (1-\varphi_i)F_i = Q_l/u_{l,i}^{true}$. Here $u_{g,i}^{true}$ and $u_{l,i}^{true}$ are true gas and liquid velocities respectively.

Using in addition the following relationships $F_i = Q_g/u_{g,i}^{reduc} = Q_l/u_{l,i}^{reduc} = u_{l,i}^{reduc}/(1-\varphi_i)$, $u_{g,i}^{true} = u_{l,i}^{true} + u_{g,i}^{rel.}$, as well as a dependency corresponding to the experimental data [1] $u_{g,i}^{rel} = u_{g,\infty}/(1-\varphi_i)$, where $u_{g,\infty}$ is emersion velocity of single gas bubbles in infinite medium, it is possible to derive the following expression for liquid volume flow:

$$Q_l = F_i\left[u_{g,i}^{true}(1-\varphi_i) - u_{g,\infty}\right]. \tag{2}$$

By eliminating value $u_{g,\infty}$ from (2), we will get:

$$Q_1 = \frac{\kappa}{1-\kappa} F_1 \left[u_{g,2}^{true}(1-\varphi_2) - u_{g,1}^{true}(1-\varphi_1) \right], \qquad (3)$$

where $\kappa = F_2/F_1$.

From equations (2) and (3), we can derive a checking expression for average emersion velocity of single gas bubbles:

$$u_{g,\infty} = \frac{1}{1-\kappa}(1-\varphi_1) u_{g,1}^{true} - \frac{\kappa}{1-\kappa}(1-\varphi_2) u_{g,2}^{true} = \frac{\kappa}{1-\kappa}\left(\frac{\varphi_2}{\varphi_1} - 1\right) u_{g,2}^{true}. \qquad (4)$$

If we introduce notations [2,3,4]

$$\eta = \frac{\overline{R}}{R_c}, \quad \overline{R} = \frac{\overline{d}_b}{2}, \quad R_c = \sqrt{\frac{\sigma_1}{\rho_1 \cdot g}}, \quad \eta_0 = \frac{1}{R_c}\left(\frac{\mu_1^2}{\rho_1^2 \cdot g}\right)^{1/3}, \quad \eta_m = 2.22 \cdot \eta_0^{0.438},$$

$$v_0 = \sqrt{2gR_c} \cdot \left[0.47 + 0.53 \cdot \exp(-3.4 \cdot \frac{\overline{d}_b^2}{D^2}) \right], \qquad (5)$$

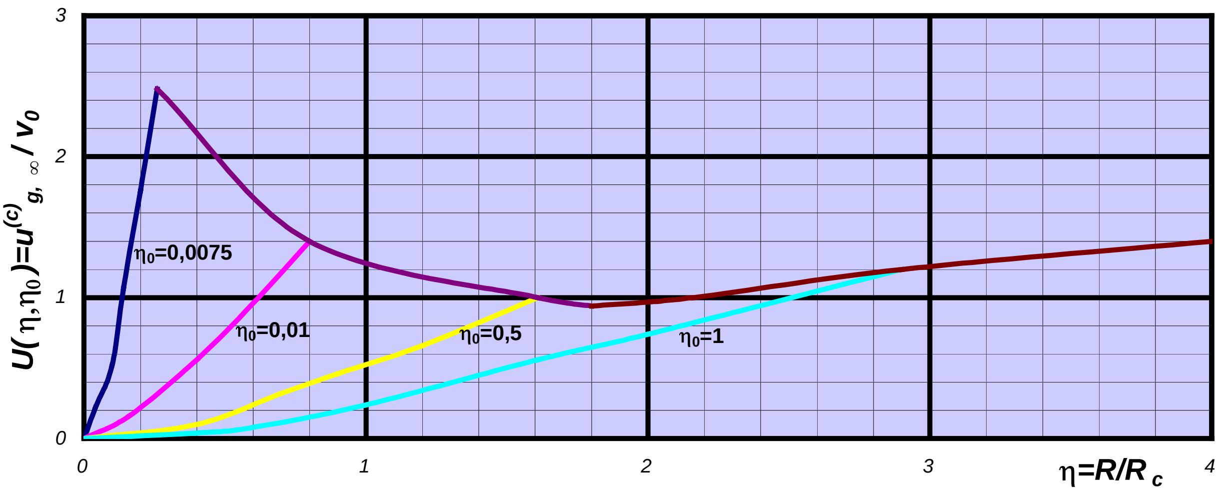
the following expression can be used for calculated value of emersion velocity of bubbles with diameter d_n :

$$u_{g,\infty}^{(c)} = v_0 \cdot \begin{cases} 0,236\eta_0^{-1,5} \cdot \eta^2, & 0 \le \eta \le 1,44\eta_0; \\ 0,307\eta_0^{-0,78} \cdot \eta^{1,28}, & 1,44\eta_0 \le \eta \le \begin{cases} \eta_m, \text{ if } \eta_0 \le 0,617, \\ 2,914\eta_0, \text{ if } \eta_0 \ge 0,617; \end{cases} \\ 1,271\eta^{-0,5}, & \eta_m \le \eta \le 1,8, \text{ if } \eta_0 \le 0,617; \\ 0,707\eta^{0,5}, & \eta \ge \begin{cases} 1,8, \text{ if } \eta_0 \le 0,617, \\ 2,914\eta_0, \text{ if } \eta_0 \ge 0,617. \end{cases} \end{cases} \qquad (6)$$

Value $\eta = \eta_m$ is corresponding to maximum emersion velocity of bubbles (on condition that $\eta_m < 1$).

Figure 1 presents a dependence between dimensionless velocity $U = u^{(c)}_{g,\infty}/v_0$ and relative bubble radius $\eta = R/R_c$ for various η_0 values.

Figure 1.



Average gas bubble radius value is determined from formula:

$$\overline{R} = \frac{1}{R} \cdot \int_0^R P(r) r^*(r) dr, \qquad (7)$$

where $r^* = (R^2 - r^2)^{½}$, $P(r)$ is distribution density of probability that a bubble sticks on sensor at a distance r from its center. Assuming $P(r)=1$, we shall get the following expression for determining the average value of radius r^* being fixed:

$$\overline{r}^* = \frac{1}{R} \cdot \int_0^R \sqrt{R^2 - r^2} \cdot dr = \frac{1}{2R} \left(r\sqrt{R^2 - r^2} + R^2 \cdot \arcsin\frac{r}{R} \right)_0^R = \frac{\pi}{4} R \approx 0{,}785 \cdot R\ . \qquad (8)$$

Hence, the actual bubble radius will be equal to:

$$R = ({}^4/_\pi)\, r^* \approx 1.27 r^*. \qquad (9)$$

In view of a limited measurement accuracy of the true volume gas content φ_1 and φ_2, a value u_g may distinctly differ from a calculated value $u_{g,\infty}^{(c)}$, determined with formula (6). In this case it is necessary to make corrections of measured values φ_1, φ_2, $u_{g,1}^{true}$ and $u_{g,2}^{true}$ included in expression (3) for Q_l, using the following scheme:

$$\hat{u}_{g,1}^{true} = \kappa \cdot u_{g,2}^{true} + (1 - \kappa) \cdot u_{g,\infty}^{(c)}, \qquad (10)$$

$$\hat{\varphi}_1 = \varphi_2 \cdot \kappa \cdot \frac{u_{g,2}^{true}}{\hat{u}_{g,1}^{true}}, \tag{11}$$

$$Q_l^{(1)} = \frac{\kappa}{1-\kappa} \cdot F_1 \cdot \left[u_{g,2}^{true} \cdot \left(1-\varphi_2\right) - \hat{u}_{g,1}^{true} \cdot \left(1-\hat{\varphi}_1\right) \right], \tag{12}$$

$$\hat{u}_{g,2}^{true} = \frac{1}{\kappa} \cdot u_{g,1}^{true} - \frac{1-\kappa}{\kappa} \cdot u_{g,\infty}^{(c)} \tag{13}$$

$$\hat{\varphi}_2 = \varphi_1 \cdot \frac{1}{\kappa} \cdot \frac{u_{g,1}^{true}}{\hat{u}_{g,2}^{true}}, \tag{14}$$

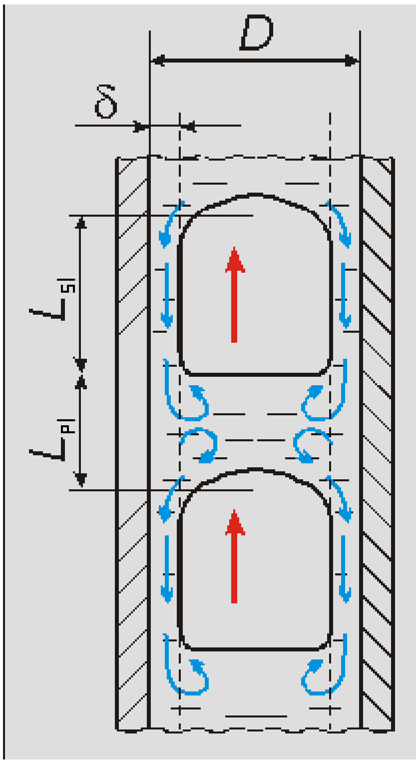
$$Q_l^{(2)} = \frac{\kappa}{1-\kappa} \cdot F_1 \cdot \left[\hat{u}_{g,2}^{true} \cdot \left(1-\hat{\varphi}_2\right) - u_{g,1}^{true} \cdot \left(1-\varphi_1\right) \right]. \tag{15}$$

The value of a corrected liquid phase volume flow is defined by expression:

$$Q_l = \frac{1}{2} \cdot \left(Q_l^{(1)} + Q_l^{(2)}\right). \tag{16}$$

Gas piston flow. A distinctive feature of the gas piston flow is successive travel through a channel of large gas bubbles (slugs) occupying a significant part of its cross-sectional area and so called liquid plugs located between gas slugs (see Fig.2).

Figure 2.



At a channel section in the spot of gas slug moving there is an opposing motion of phases: a gas slug moves upward through an effective channel with diameter $D_o = D - 2\delta$, and a liquid film flows downward on the channel walls. In so doing, in case of bubbling there will be equality of the volume flows of bubbling gas $Q_g^{\uparrow} = u_{sl} \cdot F_o$ and flowing liquid film $Q_l^{\downarrow} = u_{film} F_{film}$, where $F_{film} = F - F_o$, $F = (\pi/4) \cdot D^2$, $F_o = (\pi/4) \cdot (D - 2\delta)^2$.

The gas slug rise velocity in the general case (taking into account the inertial, viscous, and surface forces acting on the slug) can be described by expression [1]:

$$u_{sl} = k \cdot \rho_l^{-1/2} \cdot [gD(\rho_l - \rho_g)]^{1/2}, \tag{17}$$

where

$$k = 0{,}345 \cdot \left(1 - e^{-\frac{0{,}029}{\mu_l^*}}\right)\left(1 - e^{\frac{3{,}37 - \text{Eö}}{n}}\right), \tag{18}$$

$$n = \begin{cases} 10, & \mu_l^* \le 0{,}004, \\ 69(\mu_l^*)^{0{,}35}, & 0{,}004 \le \mu_l^* \le 0{,}056, \\ 25, & \mu_l^* \ge 0{,}056, \end{cases} \tag{19}$$

$$\text{Eö} = \frac{gD^2(\rho_l - \rho_g)}{\sigma} = 4\text{Bo}, \tag{20}$$

$$\mu_l^* = \mu_l [gD^3 \rho_l (\rho_l - \rho_g)]^{-1/2}, \tag{21}$$

Here μ_l^* is dimensionless viscosity, $E\ddot{o}$ – Eotvos number, Bo – Bond number. For Eotvos numbers $E\ddot{o} > 70$ the second parenthesis in expression (18) is near unity (for example, when $D = 0.06\ m$, $\rho_l = 1000\ kg/m^3$, $\sigma = 25\cdot 10^{-3}\ N/m$ $E\ddot{o} = = 1430$), so that when $\rho_g \ll \rho_l$ we can assume:

$$k = 0{,}345\left(1 - e^{-\frac{0{,}029}{\mu_l^*}}\right). \tag{22}$$

Work [2] in the framework of a two-layer flow model gives derivation of the approximated relationships for stabilized values of the thickness and velocity for a liquid film flowing over a vertical surface, the curvature of which can be neglected. The thickness of a laminar film, the velocity profile of which is near unity, equals

$$\delta_{lam} = \frac{\mu_l a}{\rho_l u_*}\ , \tag{23}$$

where $u_* = (\tau_w/\rho_l)^{½} = (g\delta)^{½}$, $a = 11.5 \ldots 11.6$ [5].

Assuming $\delta_{lam}/\delta = m$ we will get the following expression for the flowing film thickness:

$$\delta = \left(\frac{a^2\mu_l^2}{\rho_l^2 g m^2}\right)^{1/3} \approx \frac{5{,}1}{m^{2/3}}\left(\frac{\mu_l^2}{\rho_l^2 g}\right). \tag{24}$$

Liquid film thickness in a laminar-turbulent flow area $(m < 1)$ is determined through the liquid's volume flow in the film $Q_l^{\downarrow}$ by expression: $\delta = Q_l^{\downarrow}/P\cdot u_{film}$, where u_{film} is the average film flow velocity, P – the perimeter of surface, over which it is flowing. In case of a round pipe with diameter D the perimeter $P = \pi\cdot D$.

Let's introduce into consideration a dimensionless parameter $\gamma = Q_l^{\downarrow}\rho_l / a\mu_l P$. Then the maximum laminar film thickness $\delta = \delta_{lam}^{max}$ will correspond to value $m = 1$ and in this case $\gamma_{lam}^{max} = \delta_{lam}^{max}\cdot u_{film}^{lam}\cdot\rho_l / a\cdot\mu_l = a/3 \approx 3.83$.

Reynolds number calculated for the average liquid film flow velocity $u_{film}^{lam} = \gamma_{lam}^{max} \cdot a \cdot \mu_l / \rho_l \cdot \delta_{lam}^{max}$, $Re_{lam}^{max} = u_{film}^{lam} \cdot \rho_l \cdot \delta_{lam}^{max} / \mu_l = a \cdot \gamma_{lam}^{max} \approx 44.1$. For the relation of m and γ values in a range of $3.83 < \gamma < 10^4$ we can derive the following expression:

$$m = 403{,}4 \cdot \exp\left(-5{,}319 \cdot \gamma^{0{,}0898}\right). \tag{25}$$

As we already mentioned above, in case of bubbling when a bubble (slug) is moving through an arbitrary cross-section of the channel, the following condition should be met: $u_{sl} \cdot F_0 = u_{film} \cdot F_{film}$. Hence we get the following relation between the average liquid film flow velocity and gas slug movement velocity:

$$u_{film} = \frac{F_0}{F_{film}} u_{sl} = \frac{(1-2\Delta)^2}{4\Delta(1-\Delta)} u_{sl} \tag{26}$$

Here $\Delta = \delta / D$ is the dimensionless film thickness and the slug rise velocity taking into account expression (22) can be written as follows:

$$u_{sl} = 0{,}345 \cdot \left(1 - e^{-\frac{0{,}029}{\mu_l^*}}\right) \cdot \sqrt{gD}\ . \tag{27}$$

Besides, from the given relationships follows that

$$\gamma = \frac{0{,}0075}{\mu_l^*} \left(1 - e^{-\frac{0{,}029}{\mu_l^*}}\right) \cdot (1-2\Delta)^2\ . \tag{28}$$

We can derive the following expressions for dimensionless film thickness Δ and its dimensionless velocity $u_{film}^* = u_{film} / (gD)^{1/2}$:

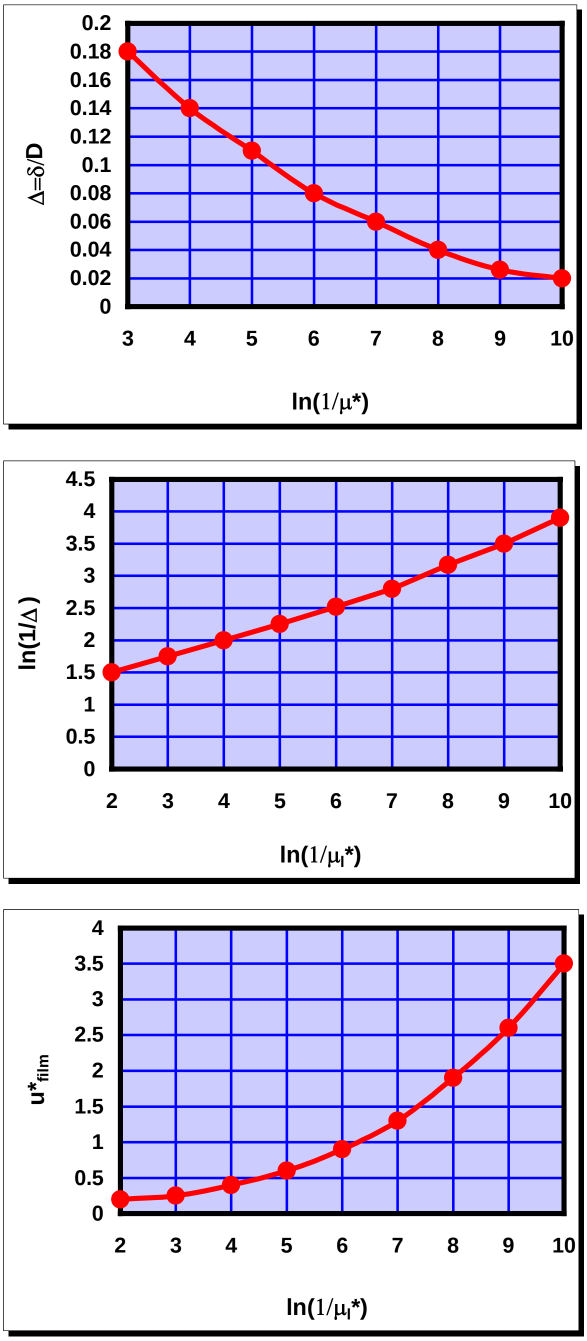
$$\Delta = 14{,}3 \cdot \exp\left[-3{,}69 \cdot \left(\mu_l^*\right)^{-0{,}0567}\right], \tag{29}$$

$$u_{film}^* = 6{,}61 \cdot \left(\mu_l^*\right)^{-0{,}8} \cdot \exp\left[-4{,}86 \cdot \left(\mu_l^*\right)^{-0{,}0567}\right] \tag{30}$$

The above formulas can be used for liquids, the dimensionless viscosity of which falls within a range of $2{\cdot}10^{-5} < \mu_l^* < 10^{-2}$.

Figure 3 presents dependences between dimensionless film thickness, dimensionless film velocity, and logarithm of inverse liquid dimensionless viscosity.

Fig. 3.



In this case (in slug flow regime) liquid and gas volume flow values can be evaluated using the following formulas:

$$Q_l^{(c)} = \overline{u}F, \tag{31}$$

$$Q_g^{(c)} = \overline{\varphi}F\left(\frac{\overline{u}}{1-\overline{\varphi}} + u_{sl}\right). \tag{32}$$

Here $\overline{u}$ - reduced liquid velocity averaged by time interval and referred to full pipe cross-section; $\overline{\varphi} = \overline{\varphi}_0 \cdot F_0/F$, $\overline{\varphi}_0$ – true gas content value average by time interval and referred to cross-section F_0 occupied bubble and equal to $F_0 = {}^{\pi}/_{4}(D - 2\delta)^2$.

Since ideal slug flow regime doesn't occur in various technical devices values $\overline{u}$ and $\overline{\varphi}_0$ can be expressed by functions of measured parameters u and φ satisfying known experimental values Q_l and Q_g if they are put into the relationships (31) and (32).

Conclusions

1. Method and relationship for determination liquid volume flow are worked out and proposed for bubble flow regime. They are based on measurement of local values of true gas velocity and of true volume gas content in two pipe segments connected in series and differing in cross-section area.
2. Easy in practice universal expression for calculation of emersion velocity of bubbles varying in diameter in liquids with different physical properties is proposed.
3. The scheme for correction of measured values φ_1, φ_2, $u_{g,1}^{true}$, $u_{g,2}^{true}$ and the expression for Q_l using approbated analytical relationships for $u_{g,\infty}^{(c)}$ is presented.
4. Formulas for computation dimensionless film thickness Δ and its dimensionless velocity u_{film}^* for slug flow regime are proposed. They can be used for liquids, the dimensionless viscosity of which falls within a range of $2\cdot 10^{-5} < \mu_l^* < 10^{-2}$.
5. Relationships for determination of liquid and gas volume flow values are derived using calculated film thickness in slug flow regime.

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