Geoid Modelling with Point Masses

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Abstract

The computation of a German GPS/levelling quasigeoid in connection with a GPS network based on the European Terrestrial Reference Frame (ETRF) 1989 and the new German 1st Order Height Network in normal heights (DHHN92) will be described. The GPS measurements are carried out by 2 x 24 hours, the height accuracy in relation to the GPS reference network DREF91 is one centimeter and better. The normal heights are derived by precise connection levellings to the DHHN92 points.

After removing the long wavelength influences by the EGM96 the quasigeoid modelling is carried out by point mass adjustment. As observations GPS/levelling quasigeoidal heights in distances of about 25 km and mean 5 km x 5 km gravity anomalies were used. After completing this project it is possible using the modelled quasigeoid to transform GPS heights to levelling heights with a precision of about 1 - 2 cm per 100 km.

1 Objectives

On the 34th Meeting of the AdV Working Group of Vertical Control Networks and Gravity Control Networks (AK-Niv) in Potsdam in 1992 the Bundesamt für Kartographie und Geodäsie (BKG) made a project proposal to determine a GPS/levelling quasi- geoid for Germany.

Based on these conditions the quasigeoid was created for the direct transition from ellipsoidal GPS heights in ETRS89 to normal heights in DHHN92.

In connection with the project to determine a GPS/levelling quasigeoid for Germany which is based on the ETRS 89 and DHHN 92, the connection to the GPS densification networks planned by the states was designed. For the necessary height accuracy the measuring time for quasigeoid determination is longer than for the densification networks which are more oriented towards position determination. Therefore, cooperation and use of common capacities for planning and performing the campaigns were agreed between the BKG and the State Survey Agencies (LVA). For capacity reasons, and due to the political necessities, the efforts focused at first at the new German States. The average point-to-point distance is about 25 km. Most of the new stations in the densification networks were three-dimensional suitably marked for the quasigeoid determination.

2 Principles

The role of the geoid as reference surface for physical heights has fundamentally changed with the growing practical use of three-dimensional satellite methods. As long as planimetric and height measurements were done by separate processes, the quasigeoid was necessary for the reduction of observations of the first order planimetric networks from the earth surface to the ellipsoid as computation surface. Accordingly, the accuracy demands of some decimeters for the distances between first order points were sufficient.

Heights from levelling observations relate to the geoid, but for performing the measurements and for the use of the levelling heights the knowledge of the geoid itself is not necessary. So, in the past only geodetic scientists came into contact with the geoid.

The three-dimensional coordinate determination using GPS allows with a short measuring time a direct interpolation of coordinates over great distance ranges. The heights which are derived by GPS relate to a reference ellipsoid. The physical heights which are classically determined by levelling, will also in future keep their importance as official heights because they are more indispensable than gravity field-related heights for extensive engineering tasks.

Whereas the GPS positioning connected to the GPS permanent stations requires only short measuring periods, the distance-integrating technique of levelling cannot be designed more efficiently for technical reasons. But if in addition to the ellipsoidal GPS height the levelling height is necessary, one has to take another approach if one does not want to lose the operative character of the GPS measurements. This can be realized by directly converting GPS heights into official heights using the geoid. For this a quasigeoid is necessary which fits to the GPS and levelling network with the accuracy of the height determination. As official heights step by step normal heights referred to the gauge Amsterdam were introduced for Germany. So, a quasigeoid ζ_{ETRS}^{DHHN} by the theory of Molodenski must be determined which fits to the GPS heights in ETRS89 and to the normal heights H_n of DHHN92. Using the quasigeoid GPS heights h can be converted into levelling heights of the official national height system DHHN92

$$H_n^{DHHN} = h^{ETRS} - \zeta_{ETRS}^{DHHN}$$

in any point.

According to the boundary value problem of physical geodesy geoidal heights are derived from globally distributed gravimetric measurements using the Stokes formula. Because of the datum difference the gravimetric geoid does not fit a priori to the GPS and levelling heights. On the other hand, the gravimetric geoid has long-wave errors. For these two reasons additional geodetic measurements and measurements which are connected with the earth’s gravity field have always been included into the determination of the geoid. In the age of classical geodesy these were the astrogeodetic deflections of the vertical which were derived from the geodetic coordinates and from the astronomical latitude and longitude determinations in the so-called astronomic points.

With the possibility of the three-dimensional geodesy this function can be adopted by geoidal or quasigeoidal heights which were determined by GPS/levelling. The geoid heights form like the astrogeodetic deflections of the vertical control values for the transition from the physical system to the geodetic reference system. For this purpose the GPS/levelling quasigeoid serves in Germany. Considering the accuracy of the levelling heights and of GPS heights as well as the geoid modelling, an optimal density for the GPS/levelling points is between 25 km and 30 km; a realistic objective of the accuracy of 1 cm to 2 cm per 100 km results. For many practical purposes the accuracy of the conversion of GPS heights into official heights is sufficient.

At the control points now GPS heights based on ETRS89 and levelling heights in the system of the DHHN92 must be available to which the gravimetric quasigeoid can be referred. To the gravimetric quasigeoid which is available as continuous area a correction $\Delta\zeta$ from the control points is added

$$\zeta_{ETRS}^{DHHN} = \zeta_{grav} + \Delta\zeta .$$

For the project an alternative method was developed at the BKG which is based on the modelling with point masses and where no separate purely gravimetric quasigeoid determination is required. As observations the GPS/levelling quasigeoidal heights and gravity anomalies are introduced. The formulas which define the relations between the masses m on the one hand and the height anomalies

$$\zeta' = \frac{k}{\gamma_0} \sum \frac{m}{l_0}$$

and gravity anomalies

$$\Delta g' = k \sum \frac{m(H - H')}{l_0^3}$$

on the other hand are the original correction equations with the Newton’s gravitational constant k and the distance

$$l_0 = R\sqrt{2(1-\cos\psi)} \,.$$

(R ... mean Earth radius, ψ ... geocentric distance angular).

The basic set-up corresponds to that of the Remove Restore Technique (RRT). The observations are reduced by the longwave parts with a current geopotential model ($\zeta_{GPM}, \Delta g_{F_{GPM}}$), the shortwave influences of the topography with the topographic reduction ($\zeta_{DTM}, \Delta g_{F_{DTM}}$) :

$$\Delta g' = \Delta g_F - \Delta g_{F_{GPM}} - \Delta g_{F_{DTM}}$$

and

$$\zeta' = \zeta_{SN} - \zeta_{GPM} - \zeta_{DTM} \,.$$

With the remaining residual quantities $\Delta g'$ and ζ' the point masses adjustment is performed. The hierarchically arranged point masses shall approximate the density distribution within the earth's crust. For points for which the positions in space are given, the point masses are determined by adjustment from the gravity and height anomalies which are taken as observations. The mutual accuracy of both observation types is there considered by a weight matrix.

Results of the quasigeoid determination are available for the new German States with an areal size of about 100,000 km^2.

3 Available data

For the quasigeoid determination 4 groups of data are available:

- height anomalies from GPS and levelling (ζ_{SN})
- terrestrial gravity anomalies (Δg_F)
- geopotential models (*GPM*)
- digital terrain models (*DTM*)

Height anomalies from GPS and levelling

Height anomalies from GPS and levelling are available for altogether 196 points (Figure 1). The computation of the GPS heights based on the European Terrestrial Reference System 1989 (ETRS89) h^{ETRS}. The connection levellings to the re-adjusted DHHN92 were performed by the State Survey Agencies. All heights H_n^{DHHN} were determined with the precise levelling ($1\,mm/\sqrt{km}$). Considering the accuracy of the DHHN92, an accuracy of about 20 mm is obtained in the north-south direction. Considering the accuracy of the levelling heights of about 1 cm and better related to the error-free introduced DREF heights, for the GPS/levelling height anomalies $\zeta_{SN}=h^{ETRS}-H_n^{DHHN}$ follows an estimated a priori accuracy of about 1.5 cm.

Gravity anomalies

For the processed area more than 70,000 point gravity values and about 10,000 mean gravity anomalies are available. In addition to that, there are gravity values on the territory of Gemany from the BKG data base and gravity values from the data base of the International Gravity Bureau. For the territory of Poland mean gravity values could be made available by exchange. The gravity values were consistently predicted over Bouguer anomalies to mean anomalies of 5 km x 5 km resp. 2 km x 2 km at the hilly part of the new German States (Figure 2).

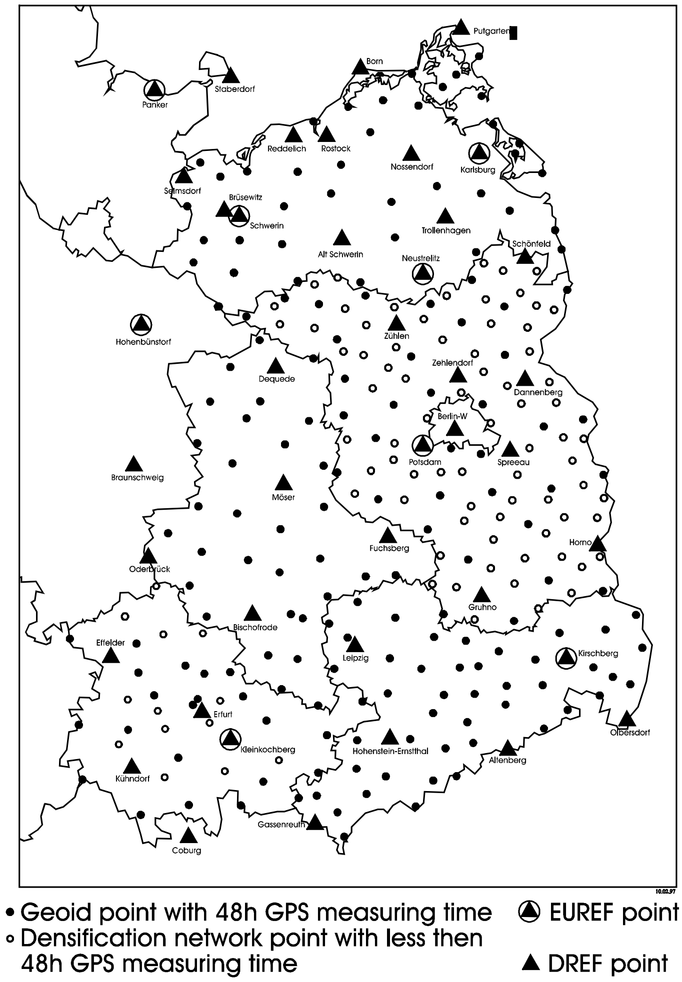

Figure 1: Densification networks for quasigeoid determination in the New German States

Geopotential model

The computations were based on the geopotential model EGM96 (LEMOINE, F.G., et al.). The EGM96 is the result of the cooperation of the NASA GSFC (Goddard Space Flight Center) and the DMA (Defense Mapping Agency, from 1 October 1996 on National Imagery and Mapping Agency - NIMA). The DMA used here new and improved gravity data from regions of the earth which were not available before, so for example from China, the former Soviet Union, South America and Africa.

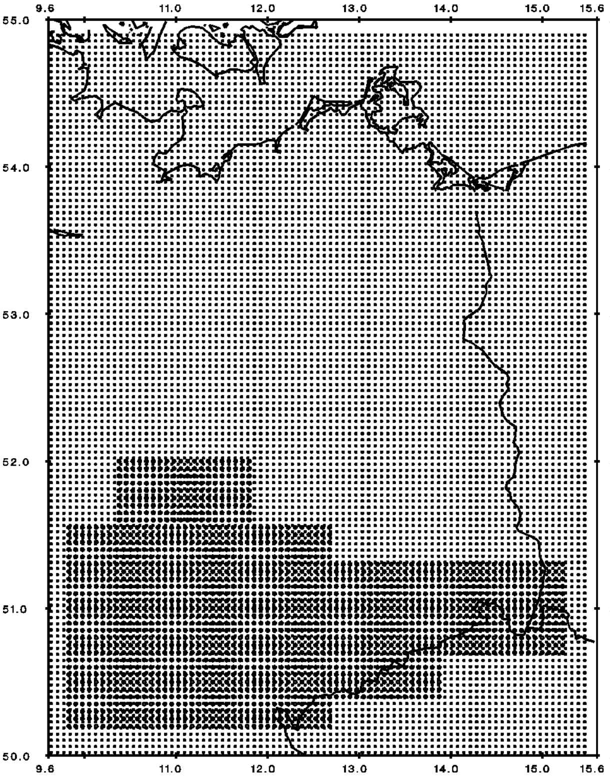

Figure 2: Distribution of mean gravity anomalies 5 km x 5 km and 2 km x 2 km

Altimeter data of the GEOSAT were used to help to realize the longwave part. In addition, data of airborne gravimetric campaigns over Greenland and the Arctic were included. Additionally, compared to OSU91A, among others new satellite tracking data - so the GPS data of the satellites EUVE, GPS/MET and two years direct altimetry in connection with GPS/SCR/DORIS tracking data from TOPEX were used. Therefore, the EGM96 is a considerable progress compared to the previous models. For this reason the computation was performed with the EGM96 (Figure 3) .

Digital terrain model

The influence of the topography was computed for the territory of the Federal Republic of Germany using a digital terrain model 30´´ x 50´´. Investigations for hilly areas with a higher resolving terrain model (0.1 km x 0.1 km) lead to the result that no significant differences exist regarding the influence on the quasigeoid determination.

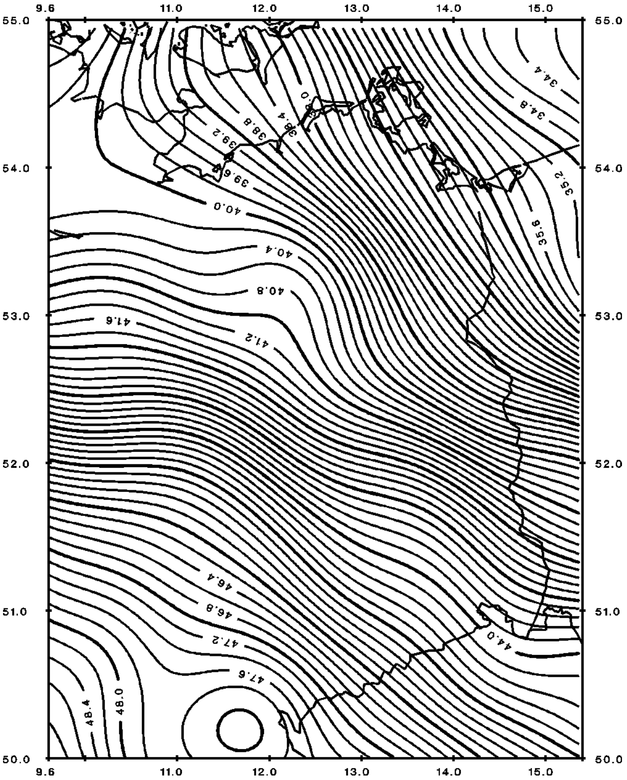

Figure 3: EGM96 geoid

4 Point masses adjustment

Investigations showed that a hierarchical arrangement of the point masses lead to optimal results (ratio of observations, unknowns and accuracy). The point masses, arranged in a depth of 10 km and with a distance of 0.1° x 0.15°, are adjusted to the mean gravity anomalies and shall approximate the shortwave parts of the quasigeoid. The point masses which are situated in a depth of 30 km with a distance of 0.2° x 0.3° cover those frequencies which are determined by the height anomalies from GPS and levelling. 8 point masses in a depth of 200 km shall compensate for long wavelength and slope influences. In addition we arranged in a depth of 5 km in hilly areas point masses in a distance of 0.05° x 0.075°. (Figure 4)

The bias between the observed GPS/levelling height anomalies and the EGM96 quasigeoid is about 70 cm and follows mainly from the difference between the GRS80 ellipsoid and the mean earth ellipsoid which is approximated within a global quasigeoid determination.

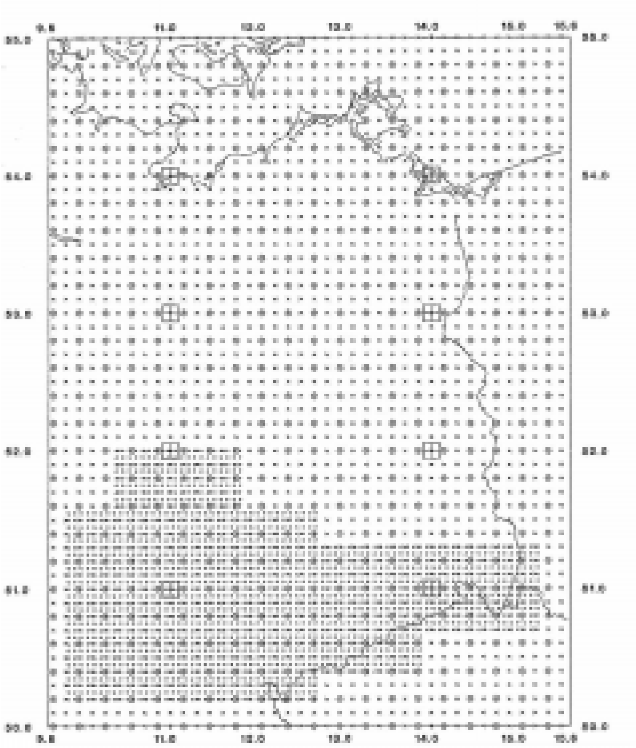

Figure 4: Distribution of point mass positions

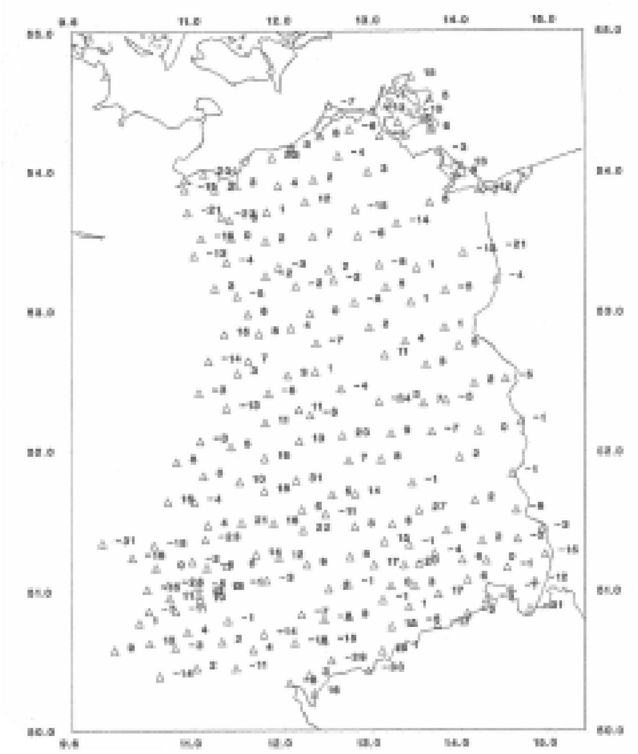

Figure 5: Differences between measured GPS/levelling height anomalies and modelled quasigeoidal heights in mm

As a priori accuracies 15 mm were introduced for height anomalies and 1 mgal for the mean gravity anomalies. Considering the marginal overlapping 4450 point masses were estimated from 196 height anomalies and 25700 mean gravity anomalies.

There, a mean residual deviation between the model quasigeoid from point masses and the more precise values of 11.4 mm was obtained. A separate processing of the northern flat country area and the hilly area showed the expected result of 9.5 mm residuals and 12.3 mm residuals, respectively (Figure 5 and Table 1).

	bias	bias corr. rms
Northern area	.893	9.5
Southern area	.619	12.3
whole area	.056	11.4

Table 1: Differences after the point mass adjustment in 196 points between height anomalies from GPS (ETRS89) and levelling (DHHN92) and the point mass modelled quasigeoid in mm

To reduce the shortwave parts of the earth's gravity field from the gravity anomalies and height anomalies before the point mass adjustment, investigations with digital terrain models were performed. It was shown that with a preceding reduction of the topography no significant improvements in the point mass modelling could be obtained. From this it has been concluded that already with the distribution of the point masses a good approximation of the earth's gravity field could be obtained. For this reason the final quasigeoid consists of the GPM EGM96, the bias of 71.1 cm and the point mass model (PMM):

$$\zeta_{ETRS}^{DHHN} = \zeta_{EGM96} + \bar{\zeta}_{PMM} - 71.1\text{cm}.$$

The bias reduced (71.1 cm) local quasigeoid in relation to the EGM96 quasigeoid as result of point mass adjustment is shown in Figure 6.

The quasigeoid was approximated within a grid space of 1' x 1.5' in which for arbitrary points the geoidal height can be interpolated within the considered area (third-degree polynomial). The standard deviation of the reproducibility of the quasigeoid model from the grid due to numerical limitation is 1.2 mm.

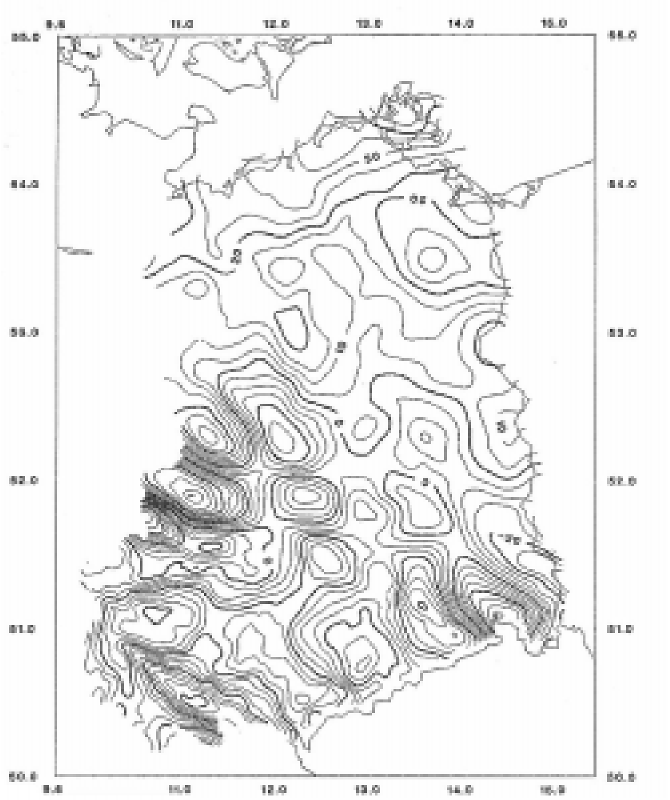

Figure 6: Height anomalies related to EGM96 as result of the point mass adjustment in cm (Bias 71.1 cm)

5 Conclusions

The computations showed that the applied method of the point mass modelling on the basis of the Remove Restore Technique is a robust technique. The deciding factor is that height anomalies from GPS and levelling (25 km) and gravity anomalies (2 km) are available with a high density as measurement data and the position of the point masses is determined a priori. On the territory of the new German States thus an accuracy of about 1 - 2 cm per 100 km was obtained.

Larger residuals are to be seen in single points which could not yet be assigned to a measurement group. Considering the experience we gained up to now it can be concluded that the point mass model ensures even in hilly country a mm-precise approximation of the quasigeoid. Reasons for larger residual deviations can be:

- height inaccuracies in the GPS measurements
- differences between the measuring epochs of the GPS observations and the levelling (here 20 years) and
- influences of not identified eccentricities.

The investigations to clarify local differences are still continuing.

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