INVESTIGATIONS ON THE STABILITY OF A BUBBLE COLUMN

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1. Introduction

Bubble columns are widely used in industrial applications since they enable an effective mass transfer between the gaseous and liquid phase, e.g. for heterogeneous chemical reactions. The performance of a bubble column strongly depends on the characteristics of the flow. There is a number of possibilities to characterise the flow pattern, e.g. depending on the gas volume flow rate or on horizontal or vertical positions. At least two basic flow patterns occur, mainly in dependence on the gas volume flow rate [1]. For low gas volume flow rates a more or less homogeneous flow (no large vortexes of the liquid, flat profile of the gas volume fraction, bubbles rise uniformly in nearly straight lines and have similar size) was observed. With increasing gas volume flow rates a transition to a heterogeneous regime, which is characterised by a non-regular flow pattern, that means non-regular vortexes of the liquid flow occur, the gas volume fraction is centre peaked, large bubbles are generated by coalescence and bubble rise velocities vary in a wide range (for details see e.g. [2]). If large bubbles are injected (e.g. big orifices) also a 'pure' heterogeneous regime was observed [1]. For this case the heterogeneous regime also exists for low gas volume flow rates.

The dependency of the transition on different parameters as e.g. gas flow rate, gas volume fraction, gas density, liquid viscosity or column dimensions was investigated. There are also a number of investigations on the stability of bubble columns, which aim to predict the transition between the homogeneous and heterogeneous flow regimes (e.g. [3] and [4]). Different mechanisms as the formation of vertical gas fraction waves or the maximum gas flow rate, which enables the backflow of the liquid carried up with the bubbles are considered. As a result these analyses give criteria of stability in dependence on the gas volume fraction or geometric parameters. In [3] also the influence of the deformation of large bubbles on the stability is considered by a modification of the added mass coefficient. There is no general criterion for stability, which really fits the broad spectrum of experimental data. None of the models gives an acceptable explanation for the existence of a 'pure' heterogeneous regime. The transversal lift force was not considered in any of these investigations, but should be important for the stability as shown below.

2. Influence of the lift force on stability

The transversal lift force was studied in detail for flows with strong gradients of the liquid velocity, e.g. for vertical pipe flow, since it is assumed to be proportional to the gradients of the components of the liquid velocity field. It is given by

$$\vec{F}_L = -C_L \rho_l \alpha (\vec{w}_g - \vec{w}_l) \times rot(\vec{w}_l) . \qquad (1)$$

In bubble columns with stagnant liquid or low liquid volume flow rates this gradient is rather small. Nevertheless recent investigations [5, 6, 7] show, that it has an important influence on the flow pattern. In case of a positive lift coefficient C_L (as originally proposed) the bubbles migrate from the higher to the lower liquid velocity region, e.g. from the centre towards the wall in case of co-current upward vertical pipe flow. This force is the reason for the wall peak in the radial profile of the gas volume fraction, which is observed for small bubbles.

In a bubble column, the role of the lift force is less obvious, since it is absent in a uniform flow field. However, it can appear when a local disturbance of the gas fraction distribution is assumed. Thus, a local increase of the gas volume fraction (=bubble density) causes a local acceleration of the liquid in the upward direction, i.e. a gradient of the vertical component of the liquid velocity is generated. Such gradient is directed in the horizontal plane and according to the lift force the bubbles start to migrate in lateral direction from that region of locally increased gas volume fraction to regions with lower gas volume fraction. This is clearly a stabilising effect and simulations with CFD codes show that the lift force causes an even spreading of the bubbles over the cross section of the bubble column [5, 6]. Even if plumes of small bubbles are generated by the gas injection, they are spread uniformly over the cross section of the column [7].

Theoretical [8] and experimental [9] investigations showed, that for bubbles with large deformations a force occurs, which can be modelled with the same approach as the classical lift force, but with a negative sign of the lift force coefficient. For this reason Tomiyama et al. [9] derived a correlation of the lift force in dependence on the bubble size and material parameter according to their experimental investigations on single bubbles:

$$C_L = \begin{cases} \min[0.288\tanh(0.121\,\mathrm{Re}), f(Eo_d)] & Eo_d < 4 \\ f(Eo_d) & \text{for} \quad 4 < Eo_d < 10 \\ 0.27 & Eo_d > 10 \end{cases}$$
$$\text{with} \quad f(Eo_d) = 0.00105Eo_d^3 - 0.0159Eo_d^2 - 0.0204Eo_d + 0.474. \tag{2}$$

Here Eo_d is the modified Eötvös number calculated with the bubble diameter in the horizontal plane d_h:

$$Eo_d = \frac{g(\rho_l - \rho_g)d_h^2}{\sigma} \tag{3}$$

For air-water flow at ambient conditions the coefficient changes its sign at an equivalent bubble diameter (diameter of a spherical bubble for a given bubble volume) of about 5.8 mm. That means bubbles larger than such a critical diameter migrate into regions with higher liquid velocity, i.e. towards the centre of the pipe in case of co-current upward vertical pipe flow. In previous investigations this correlation of the lift force coefficient was successfully applied to vertical pipe flow (e.g. [10, 11]).

Since with a negative sign of the lift force coefficient the bubbles migrate into regions with higher liquid velocity, the lift force has a destabilising effect when bubbles large enough are present in the bubble column. Again a local increase of the gas volume fraction locally increases the vertical component of the liquid velocity. According to the lift force the large bubbles migrate into the region of increased gas volume fraction, what causes a further increase of the gas volume fraction and with that also a further increase of the liquid velocity. Thus, there is positive feedback for an initial disturbance of the local gas volume fraction on itself. The amplitude of an initial small perturbation grows and the system is thus unstable.

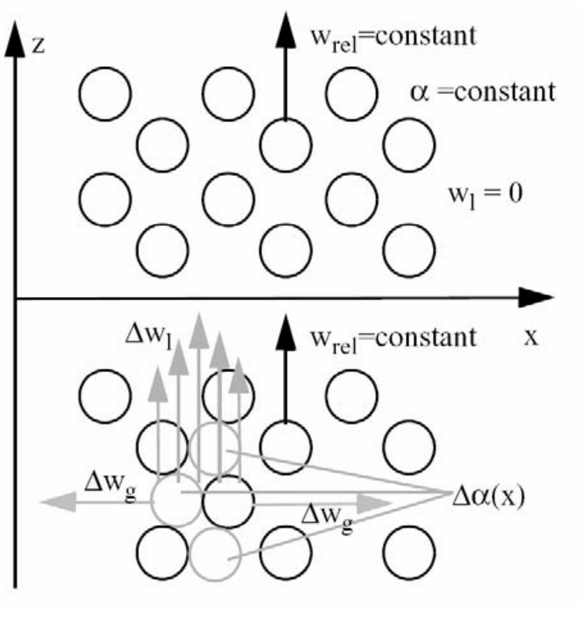
The only mechanism, which can counteract the growing non-uniformity of the gas fraction distribution is turbulent dispersion, which acts to smooth the gas fraction profile. It is modelled by a so called turbulent dispersion force. This force therefore always contributes to stabilise a bubble column.

From these considerations it is clear, that a bubble column with a mono-dispersed bubble flow is stabilised by the lift force – i.e. a homogeneous flow can be observed – if the bubble size is less than the critical one, at which the lift force changes its sign. For larger bubbles it is not clear whether the turbulent dispersion force is able to compensate the destabilising action of the lift force. To answer this question a stability analysis was done. In a first step, a mono-dispersed bubbly flow was assumed. Then the analysis was extended to two bubble classes – one representing bubbles lower than the critical bubble diameter and the other representing bubbles with an equivalent diameter larger than the critical diameter. Finally a simplified criterion for an estimation of the stability in case of N bubble classes or for a given continuous bubble size distribution was proposed. It is valid, if the turbulent dispersion is negligible.

3. Outline of the linear analysis of stability

In this chapter only the main ideas of the stability analysis are given. The complete analysis can be found in [12]. A homogeneous flow pattern of a bubble column without any consideration of the influence of the wall or the gas injection zone is considered (valid at an appropriate distance from the walls and the injection device). The net liquid superficial velocity of the initial state is assumed to be zero, the gas bubbles rise with a constant velocity in vertical direction z, which only depends on the bubble size and material parameter (see Fig. 1). The following approximations and assumptions are made to establish the set of partial differential equations describing the system illustrated in Fig. 1:

- the liquid velocity has only a component in the vertical direction,
- the gradients of the liquid velocity occur only in x-direction,
- pressure gradients are neglected
- the only driving force is assumed to be a local change of the averaged density.



(a) undisturbed system

(b) disturbances

Fig. 1 Simplified model for the stability analysis (from [12]).

The analysis of stability takes into account the consequences of a local disturbance of the gas volume fraction $\delta\alpha(x,t)$ at a horizontal line, i.e. it is one-dimensional. Such a disturbance accelerates the liquid in vertical direction because of the lower local density. As a result disturbances of the liquid velocity $\delta w_l(x,t)\vec{e}_z$ and of the gradient of the liquid velocity $\delta \frac{dw_l}{dx}(x,t)$ occur. Because the transversal lift force is proportional to this gradient bubbles start to migrate in the x-direction $\delta w_g(x,t)\vec{e}_x$. Depending on the sign of the lift force coefficient this migration can act to decrease (positive coefficient for small bubbles) or to increase the initial disturbance (negative coefficient for large bubbles) of the gas volume fraction. In addition the turbulent dispersion force always acts to flatten the profile of the gas volume fraction, i.e. it stabilizes the system.

Basing on the above mentioned assumptions a simplified Navier-Stokes equation for the liquid, which reflects the time and space dependent change of the vertical liquid velocity caused by a local disturbance of the gas volume fraction, can be obtained. An equation for the lateral bubble velocity is derived from the balance of the forces acting on the bubbles in lateral direction. In this balance the drag force, virtual mass force, lift force and turbulent dispersion force are considered. Finally, the set of equations is completed by the 1D continuity equation for the gas phase.

In the frame of linear stability analysis the feedback of a small initial disturbance on itself is investigated. The transfer functions characterize the effect of a Dirac-shaped disturbance of a given input parameter on the corresponding output parameter. For a compact presentation these transfer functions are derived in the Laplacian space as usual in automatic control theory. Fig. 2 shows the system considered for the analysis in case of a mono-dispersed bubble flow.

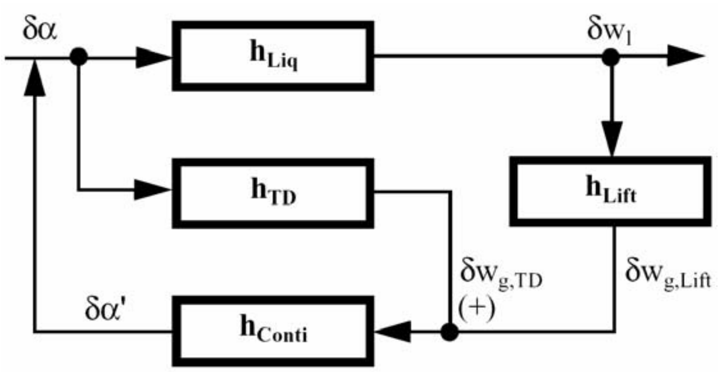


Fig. 2 Propagation and feedback of a disturbance δα. The transfer functions h characterize the effects of a disturbance of an input parameter on the disturbance of the output parameter (from [12]).

The transfer function h_{Liq} describes the variation of the liquid velocity as a consequence to a perturbation of the local gas volume fraction and can be derived from the Navier-Stokes equation of the liquid, h_{TD} gives the modification of the lateral bubble velocity caused by a local modification of the gas volume fraction by the action of the turbulent dispersion force and can be deduced by the balance of forces acting on the bubbles in lateral direction, the transfer function h_{Lift} represents the modification of the lateral bubble velocity caused by a local modification of the gradient of the liquid velocity by the action of the lateral lift force, and h_{Conti} gives the modification of the local gas volume fraction caused by a modification of the lateral bubble velocity. According to the methods usual in automatic control theory a transfer function for the open system is given by

$$H^{open} = -(h_{Liq}h_{Lift} + h_{TD})h_{Conti} \tag{4}$$

As shown in [12] the feedback of an initial disturbance on itself has the same spatial

dependency as the initial disturbance itself. This result justifies the chosen one-dimensional modelling and the consideration of the amplitudes only for the transfer functions.
The system is stable, if all the poles of the closed-loop transfer function (i.e. the roots of the characteristic polynomial $1+H^{open}$) have negative real parts. To analyse the stability of the system it is not necessary to find all the roots of the characteristic polynom. The Routh-Hurwitz criterion can be applied instead. It states, that the system is stable, if all coefficients of the characteristic equation $1+H^{open} = 0$ and the so called Hurwitz determinants are positive (or have the same sign). This criterion is used for the present analysis.

4. Results of the analysis

In the result of this analysis the following condition for stability was found for mono-dispersed flow:

$$C_L > -4.44 \frac{C_D w_{rel} \nu_l}{gL^2} \tag{5}$$

Here C_D is the drag force coefficient, w_{rel} the relative velocity between gas and liquid, ν_l the kinematic viscosity, g the acceleration due to gravity and L a typical length scale for possible disturbances. For an air-water system at ambient conditions the absolute values for this expression is very small because of the small values of ν_l. The stabilising action of the turbulent dispersion force is very weak. That means that for all mono-dispersed bubble columns with a low liquid viscosity, the change of the sign of lift force can be assumed as the criterion for stability.

The analysis was extended for two bubble classes and by neglecting the turbulent dispersion force also to N bubble classes. For the latter case the following criterion was found:

$$\sum_{i=1}^{N} \alpha_i \frac{C_L^i w_{rel}^i d_b^i}{C_D^i} > 0. \tag{6}$$

This condition is proposed as an approximated, generalized criterion for the stabilizing effect of the lift force. It should be a good approximation for all cases with low liquid turbulence. For a given continuous bubble size distribution, defined on the basis of the differential volume fraction according to:

$$h(d_b) = \frac{d\,\alpha}{d\,d_b} \tag{7}$$

it can be rewritten as:

$$\int_0^\infty \frac{C_L(d_b) w_{rel}(d_b) d_b h(d_b)}{C_D(d_b)} d\,d_b > 0 . \tag{8}$$

5. Conclusions

The investigations presented have shown, that the influence of the lateral lift force on the stability of the flow is much stronger compared to the bubble dispersion. In case of a positive sign of the lift force coefficient, as observed for small bubbles, it stabilizes the flow. For a negative sign, i.e. in case of large bubbles, it destabilizes the flow. There is a strong connection between bubble size and other parameters, e.g. the gas volume fraction. In case of a transition from the homogeneous to the heterogeneous flow regime large bubbles are generated by coalescence. As the onset of bubble coalescence is strongly connected with the local gas volume fraction, the generation of vertical gas volume fraction waves has an important influence on the local bubble size distributions. For a general prediction of stability a combination of the different analyses including modelling of bubble coalescence and break-up is necessary.

When the gas fraction distribution has once become non-uniform, there may soon be an onset of strong coalescence in regions of increased gas fraction, which can further destabilize the column. This makes it very difficult to perform detailed measurements of the bubble size distributions in the transition region between the two regimes. For this reason there are up to now no data available, which give an experimental confirmation of the theory. A hint for the correctness of the result of the analysis, is the agreement with the findings in [1], that a ‘pure’ heterogeneous regime exists for gas injection devices, which produce very large bubbles.

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