OBSERVATION OF MAGNETOROTATIONAL INSTABILITY IN A LIQUID METAL TAYLOR-COUETTE EXPERIMENT

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1. Introduction

Magnetic fields play a double role in the cosmos: First, planetary, stellar and galactic fields are produced by the homogeneous dynamo effect in moving electrically conducting fluids. Second, magnetic fields accelerate the formation of stars and black holes, by enabling outward transport of angular momentum in accretion disks by virtue of the so-called magnetorotational instability (MRI). This instability had been discovered as early as 1959, when Velikhov showed that a Taylor-Couette flow in its hydrodynamically stable regime (i.e. with outward increasing angular momentum) can be destabilized by an applied axial magnetic field [1]. But it was only in 1991, that the relevance of MRI for accretion disks in the vicinity of young stars and black holes was realized in a seminal paper by Balbus and Hawley [2].

The last decades have seen remarkable theoretical and computational progress in understanding the dynamo effect and the MRI. The dynamo effect has even been verified experimentally, in large-scale liquid sodium facilities in Riga and Karlsruhe, and is presently studied in laboratories around the world [3]. In contrast, obtaining the MRI experimentally has been less successful thus far [4]. Recently, an MRI-like instability has been observed on the back-ground of a turbulent spherical Couette flow [5], but the genuine idea that MRI would destabilize an otherwise stable flow was not realized in experiment up to present.

If only an axial magnetic field is externally applied, the azimuthal field that is necessary for the occurrence of the MRI must be produced by induction effects, which are proportional to the magnetic Reynolds number (Rm) of the flow. But why not substitute this induction process simply by externally applying an azimuthal magnetic field as well? This question was addressed by Hollerbach and Rüdiger [6], who showed that the MRI is then possible with far smaller Reynolds (Re) and Hartmann (Ha) numbers. In this paper, we report the first experimental verification of this idea, presenting evidence of the MRI in a liquid metal co-rotating Taylor-Couette cell with externally imposed axial and azimuthal (i.e., helical) magnetic fields. Further details can be found in [7,8].

2. The experimental facility

The basic part of our facility "PROMISE" (Potsdam Rossendorf Magnetic InStability Experiment) is a cylindrical containment vessel V made of copper (see Fig. 1). The use of copper was motivated by the finding that the instability usually occurs at lower Reynolds and Hartmann numbers for the case of ideally conducting boundaries than for non-conducting boundaries [6,9]. The inner wall of the vessel V is 10 mm thick, and extends in radius from 22 to 32 mm; the outer wall is 15 mm thick, extending from 80 to 95 mm. This vessel is filled with the eutectic alloy $Ga^{67}In^{20.5}Sn^{12.5}$, which has the advantage of being liquid at room temperatures.

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The copper vessel V is fixed, via an aluminium spacer D, on a precision turntable T; the outer wall of the vessel thus serves as the outer cylinder of the Taylor-Couette cell. The inner copper cylinder I of the Taylor-Couette flow is fixed to an upper turntable, and is immersed into the liquid metal from above. It has a thickness of 4 mm, extending in radius from 36 to 40 mm, thus leaving a gap of 4 mm between this immersed cylinder I and the inner wall of the containment vessel V. The actual Taylor-Couette cell then extends in radial direction over a cylindrical gap of width $d = r_{out} - r_{in} = 40$ mm, and in axial direction over the liquid metal height of $0 \text{ mm} < z < 410$ mm.

In the present configuration, the upper endplate is a Plexiglas lid P fixed to the frame F. In contrast, the bottom is simply part of the copper vessel, and hence rotates with the outer cylinder. There is thus a clear asymmetry in the endplates, with respect to both their rotation rates and electrical conductivities.

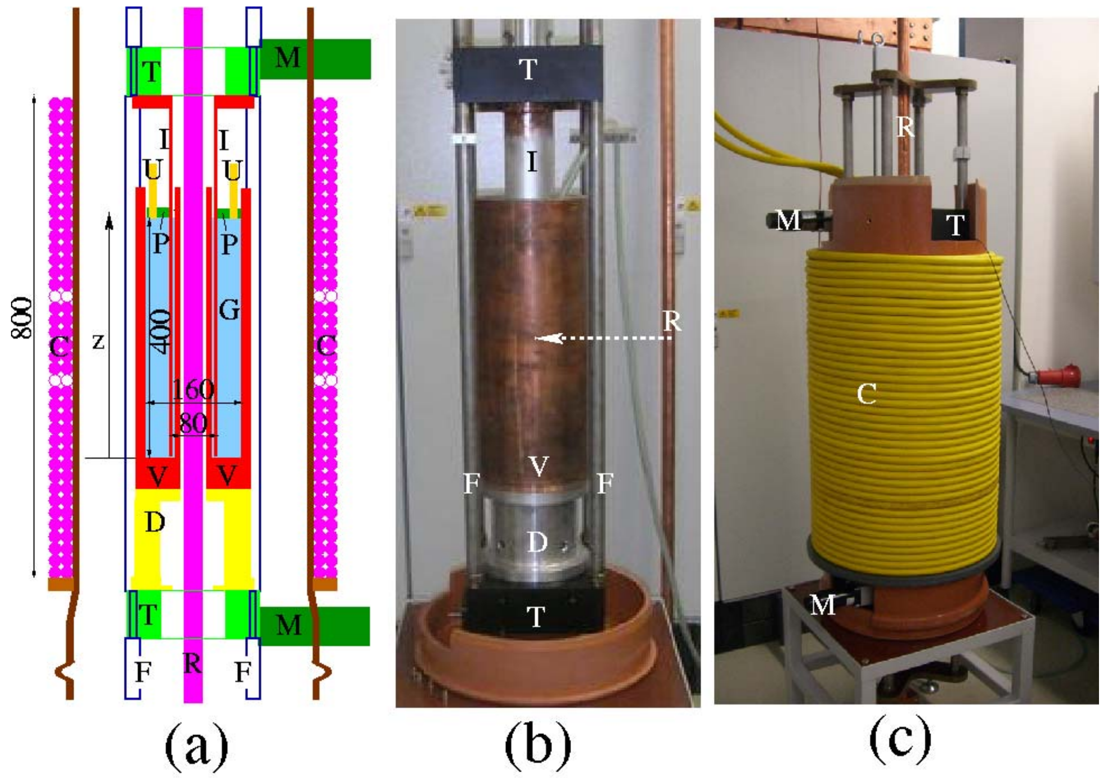

Fig. 1: The PROMISE experiment. (a) - Sketch. (b) - Photograph of the central part. (c) - Total view. V - Copper vessel, I - Inner cylinder, G - GaInSn, U - Two ultrasonic transducers, P - Plexiglas lid, T - High precision turntables, M - Motors, F - Frame, C - Coil, R - Copper rod. The dimensions are in mm.

It should be mentioned that the tolerance of the key components of the apparatus is not quite at the 10^{-2} mm level that can be achieved in ordinary, hydrodynamic Taylor-Couette experiments, e.g. [10]. One of the reasons is that it is necessary to intensively rub the fluid into the copper in order to ensure a well-defined electrical contact between the GaInSn and the walls. Due to the resulting abrasion, the accuracy of the copper cylinders is certainly not better than 10^{-1} mm. Note also that the typical Reynolds number $Re = 2 \cdot \pi \cdot f_{in} \cdot r_{in} \cdot d/\nu$ in the experiment is $O(10^3)$, which is considerably greater than the critical value $Re_c = 68$ for the transition to the Taylor vortex flow in the non-magnetic Taylor-Couette problem (for this radius ratio, with stationary outer cylinder).

Axial magnetic fields of order 10 mT are produced by a double-layer coil with 76 windings (C in Fig. 1). The omission of windings at two symmetric positions close to the middle, as seen in Fig. 1a, resulted from a coil optimization to maximize the homogeneity of the axial field throughout the volume occupied by the liquid. The coil is fed by currents up to approximately 200 A. Beyond this value the coil is significantly heating up. The azimuthal field, also of order 10 mT, is generated by a current through a water-cooled copper rod R of radius 15 mm. The power supply for this axial current delivers up to 8000 A.

In the present set-up, the measuring instrumentation consists exclusively of two ultrasonic transducers with a working frequency of 4 MHz; these are fixed into the Plexiglas lid, 15 mm away from the outer copper wall, flush mounted at the interface to the GaInSn. Using Ultrasound Doppler Velocimetry (UDV) [11], they provide full profiles of the axial velocity v_z along the beam-lines parallel to the axis of rotation. The spatial resolution in axial direction is 0.685 mm, the time resolution is 1.84 sec. It should be mentioned that the necessary velocity resolution in the order of 0.1 mm/s is at the margin of applicability of the UDV system. The width of the beam (over which v_z is averaged) is approximately 8 mm, according to the diameter of the ultrasonic transducers. The comparison of the two signals from two opposite sensors is important in order to clearly distinguish between the expected axisymmetric ($m = 0$) MRI mode [9] and certain $m = 1$ modes which also play a role in some parameter regions of the experiment.

3. Experiments and their main results

We have carried out a large number of experimental runs in order to cover a wide range of parameter dependencies. Typically, the duration of an experimental run was 1900 sec, after a waiting time of one hour. Such a long waiting time was chosen not only due to the hydrodynamic gap diffusion time $\tau_{gap} = d^2/\nu$, but also due to our numerical predictions of rather small growth rates of the MRI mode in helical fields.

In pre-experiments, both with a water-glycerol mixture and with GaInSn, we had clearly identified the expected Taylor vortex flow in the hydrodynamically unstable regime (e.g. with stationary outer cylinder). As a consequence of the reflection symmetry breaking under the influence of a helical magnetic field [12], the Taylor vortex flow is replaced by an oscillatory axi-symmetric vortex flow that propagates in a unique direction along the vertical axis [6]. This direction depends on the screw-sense of the magnetic field and the direction of the flow rotation (all results presented in the following are for an upward travelling wave). This travelling wave appears already with a stationary outer cylinder, i.e at $\mu := f_{out}/f_{in} = 0$, although with a very low frequency. With increasing μ, the wave frequency increases and typically reaches a value of $0.2 \times f_{in}$ at the Rayleigh value $\mu_{Ray} = (r_{in}/r_{out})^2$ (we have $r_{in}/r_{out} = 0.5$ here, hence $\mu_{Ray} = 0.25$). The crucial point now is that under the influence of helical magnetic fields the critical Reynolds numbers remain relatively small increasingly far above the Rayleigh value which looses its character as a sharp boundary between unstable and stable flows. Typically, this shift to larger values of μ becomes larger for increasing values of the ratio of azimuthal field to axial field, $\beta := B_\phi(r = r_{in})/B_z$.

Another typical feature of the MRI is that, for fixed Re and fixed azimuthal field, it sets in at a certain critical value of the Hartmann number $Ha = B_z \cdot (r_{in} \cdot d \cdot \sigma \cdot \rho \cdot \nu)^{1/2}$, and disappears again at some higher value. This appearance and disappearance of a travelling mode is a suitable indicator for the existence of the proper MRI mode and its distinction from other possible flow structures. In the following, we focus exclusively on experimental results which substantiate this behaviour.

All results presented in the following are for rotation rates of $f_{in} = 0.06$ Hz and $f_{out} = 0.0162$ Hz, i.e. for $\mu = 0.27$ which is above the Rayleigh value $\mu_{Ray} = 0.25$. Figure 2 documents a selection of four experimental runs for coil currents I_{coil} of 10, 50, 75 and 120 A. In each case, the axial current I_{rod} was fixed to 6000 A. The colour coding of the plots indicates the axial velocity component v_z measured along the ultrasound beam, from which we have subtracted the z- dependent time average in order to filter out the two Ekman vortices which appear already without any magnetic field. These vortices, characterized by inward radial flows close to the upper and lower endplates, show up in our UDV data in the form of positive and negative axial velocities, separated approximately at mid-height by a rather sharp boundary. This boundary most likely corresponds to a jet-like radial outflow in the centre of the cylinder, as discussed in [13].

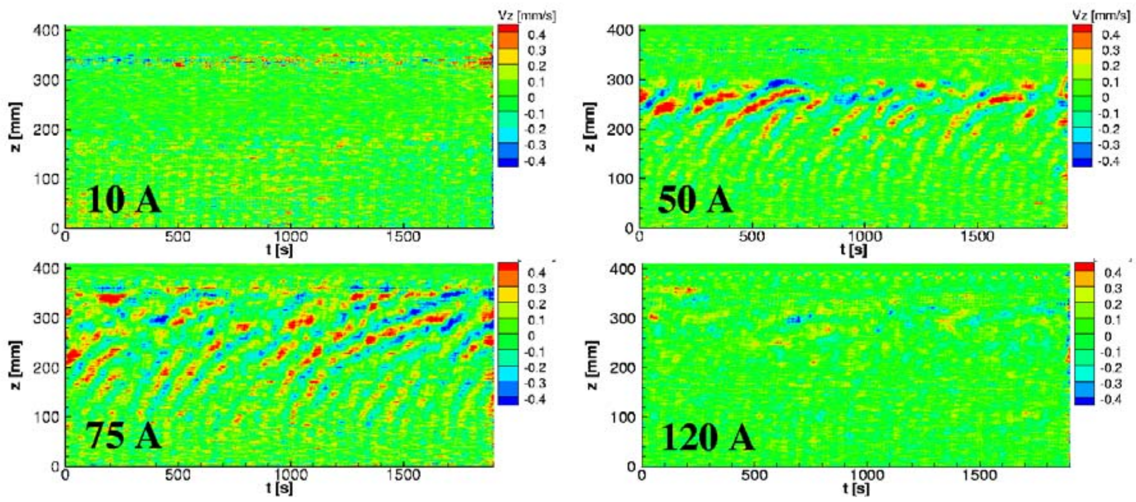

Fig. 2: Measured axial velocities in dependence on the vertical position and the time, for different coil currents. The appearance and disappearance of a travelling wave mode for increasing coil current (i.e., Hartmann number) is clearly visible.

For $I_{coil} = 50$ A we already observe a travelling wave-like structure which is, however, still restricted to the middle part of the cylinder. One might speculate that, due to the (jet-like) radial outward flow there, fluid with lower angular momentum is transported outward, which leads to a locally steeper decrease of v_ϕ than what would be expected from the rotation ratio μ of outer to inner cylinder [13]. At z~300 mm, i.e. closely above the radial outflow region, the wave dies away.

For $I_{coil} = 75$ A this travelling wave fills the entire cylinder. Some refraction of the wave remains visible approximately at mid-height. Increasing I_{coil} even further though, at 120 A this wave ceases to exist, and we again have a rather featureless flow. This is shown more quantitatively in Fig. 3, where we exhibit the power spectral density (PSD) for the axial velocity for 5 selected values of I_{coil}. A feature common to all five curves is the appearance of the rotation rates of the inner and outer cylinders. This reflects certain geometric imperfections of the facility, probably in the form of cylinder eccentricities or metal oxides sticking to some parts of the walls. There is also a peak approximately at the mean of the inner and outer cylinder frequencies, which, on closer inspection of the two transducer signals, turns out to be a non-axisymmetric m = 1 mode. Most interesting for us here, however, is the MRI mode at f/f_{in} ~ 0.1-0.2. These frequencies have been analysed in detail for all values of I_{coil} chosen in the experiment.

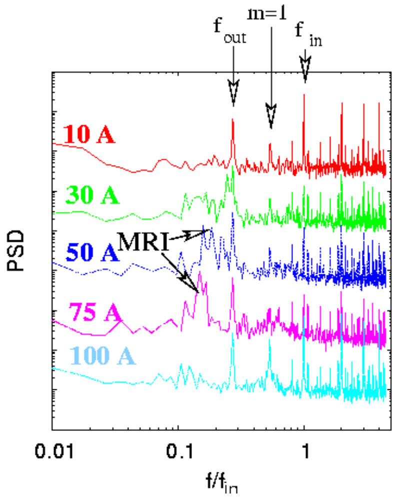

Fig. 3: Power spectra of the measured v_z fluctuations for 5 different coil currents.

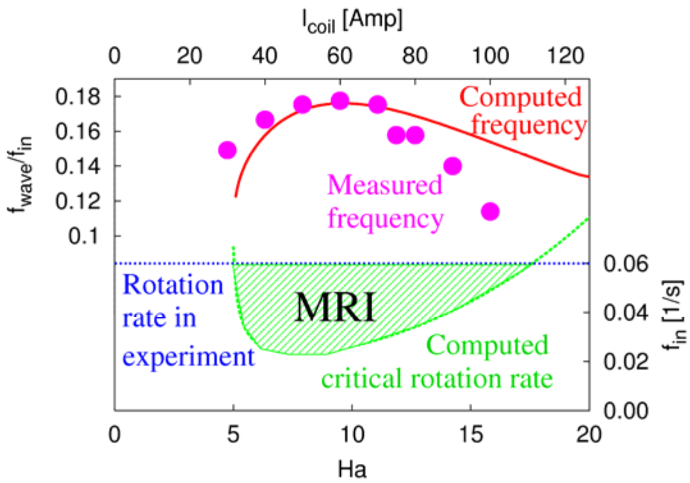

Fig. 4: Computed critical rotation rate for varying I_{coil} (and hence Ha), for fixed I_{rod} = 6000 A, and measured and computed frequency f_{wave} of the travelling MRI wave normalized to f_{in} .

We now interpret these experimental data in the context of numerical predictions, obtained and cross-checked by various independent codes [6, 9, 14] for the solution of the linear eigenvalue problem in unbounded cylinders with ideally conducting radial boundaries. First, the lower part of Fig. 4 shows the computed critical rotation rate of the inner cylinder, compared with the rotation rate of 0.06 Hz chosen in the experiment. According to this plot, MRI should be expected approximately between 30 A < I_{coil} < 110 A, were the experimental rotation rate is above the critical one. Actually, MRI appears in this region, although it is hard to indicate precise boundaries. This is typical for an imperfect bifurcation in finite systems for which the instability sets in gradually instead of abruptly.

In the upper part of Fig. 4, we compare the measured frequencies of the travelling wave, normalized to the rotation rate of the inner cylinder, with the frequencies computed for the infinite cylinder. Evidently, the measured frequencies are in reasonable correspondence with the computed ones, and show a similar dependence on Ha, with a maximum close to Ha = 10.

4. Summary and outlook

We have obtained experimental evidence for the existence of the MRI in current-free helical magnetic fields, by showing its appearance in a certain interval of Hartmann numbers, in good agreement with numerical predictions. Certainly, much numerical work remains to be done, including the treatment of the non-linear regime, a more realistic handling of the magnetic radial boundary conditions, and a detailed investigation into the role of the magnetic axial boundary conditions. For later experiments, a symmetrization of the axial boundaries is envisioned. Connected with this, a suppression of the Ekman vortices by means of split rings (proposed in [13]), may also help to avoid artefacts in the mid-height of the cylinder.

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