Institut für Theoretische Physik
Fakultät Mathematik und Naturwissenschaften
Technische Universität Dresden

In-Medium QCD Sum Rules for ω Meson, Nucleon and D Meson

QCD-Summenregeln für im Medium modifizierte ω-Mesonen, Nukleonen und D-Mesonen

Dissertation
zur Erlangung des akademischen Grades
Doctor rerum naturalium

vorgelegt von

Ronny Thomas
geboren am 7. Februar 1978 in Dresden

Dresden 2008

To Andrea and Maximilian

Eingereicht am 07. Oktober 2008

1. Gutachter: ______________________________

2. Gutachter: ______________________________

3. Gutachter: ______________________________

Verteidigt am ____________________

Abstract

The modifications of hadronic properties caused by an ambient nuclear medium are investigated within the scope of QCD sum rules. This is exemplified for the cases of the ω meson, the nucleon and the D meson. By virtue of the sum rules, integrated spectral densities of these hadrons are linked to properties of the QCD ground state, quantified in condensates. For the cases of the ω meson and the nucleon it is discussed how the sum rules allow a restriction of the parameter range of poorly known four-quark condensates by a comparison of experimental and theoretical knowledge. The catalog of independent four-quark condensates is covered and relations among these condensates are revealed. The behavior of four-quark condensates under the chiral symmetry group and the relation to order parameters of spontaneous chiral symmetry breaking are outlined. In this respect, also the QCD condensates appearing in differences of sum rules of chiral partners are investigated. Finally, the effects of an ambient nuclear medium on the D meson are discussed and relevant condensates are identified.

Kurzfassung

Die Veränderungen von Hadroneneigenschaften durch ein umgebendes nukleares Medium (Kernmaterie) werden mit der Methode der QCD-Summenregeln untersucht. Dies wird am Beispiel des ω-Mesons, des Nukleons und des D-Mesons vorgeführt. Durch die Summenregeln werden integrierte Spektraldichten dieser Hadronen in Beziehung zu Eigenschaften des QCD-Grundzustandes, quantifiziert in Kondensaten, gesetzt. Diskutiert wird am Beispiel des ω-Mesons und des Nukleons, wie diese Summenregeln eine Einschränkung des Parameterbereiches von wenig bekannten Vierquark-Kondensaten durch Vergleich von experimentellen und theoretischen Erkenntnissen erlauben. Ein Katalog unabhängiger Vierquark-Kondensate wird aufgestellt und Relationen zwischen diesen Kondensaten werden deutlich gemacht. Das Verhalten der Vierquark-Kondensate unter der chiralen Symmetriegruppe und der Zusammenhang mit Ordnungsparametern spontaner chiraler Symmetriebrechung werden behandelt. In dieser Hinsicht werden auch die in Differenzen der Summenregeln chiraler Partner eingehenden QCD-Kondensate untersucht. Schließlich werden die Effekte endlicher Kerndichten beim D-Meson diskutiert und relevante Kondensate identifiziert.

Contents

1 Hadrons in Medium

1.1 Probing Strongly Interacting Matter

Hadrons are the observed physical degrees of freedom of the strong interaction at low temperatures and densities or in vacuum [1]. However, it is necessary to relate the measured properties of hadrons to the fundamental theory of the strong interaction - Quantum Chromodynamics (QCD). The degrees of freedom in this local gauge theory are the quarks and, as gauge bosons, the gluons. These fields are said to be confined in hadrons, since quarks and gluons have not been observed as asymptotically isolated particles (confinement hypothesis). Nevertheless, the mass of a hadron, formed out of quarks and gluons, should be explained by QCD parameters. Since the sum of the current quark masses is orders of magnitude below the hadron masses, the latter must be generated from the interaction energy in the theory of QCD. The hadron masses are thus related to the properties of the QCD ground state, which has a highly non-trivial structure. A relation between QCD parameters and the hadronic spectrum is impeded by the generic problems of low-energy QCD. In the high energy limit, many observables can be calculated within perturbation theory where the running coupling is small enough to allow for a perturbative expansion of scattering probabilities due to asymptotic freedom. While going subsequently to lower energy scales, the running coupling α_s increases and the expansion in powers of the coupling strength becomes invalid.

Suitable tools to explore the structure of low-energy QCD in relation to properties of hadrons are among others ”lattice QCD”, chiral effective field theory, instanton models and QCD sum rules. The former is based on a computational treatment of the theory on a discretized space-time lattice with sophisticated algorithms. With improving computational resources it approaches a realistic prediction of QCD observables. Unfortunately, the numerical results obtained therefrom often have to be extrapolated to physical values of the quark masses. The second approach contains effective fields for the hadrons and suitable interaction terms from which self-energies and scattering processes etc. can be evaluated once a unique fit of unknown constants is at our disposal. Instanton models describe special features of the QCD ground state by the density and size of instanton configurations, which are classical solutions of the equations of motion.

In this work, the approach of QCD sum rules (QSR) is focused on. It was originally formulated by Shifman, Vainshtein and Zakharov [2, 3] to describe for example masses of light vector mesons in vacuum [4]. The method since then gained attention in numerous applications, e.g. to calculate masses and couplings of low-lying hadrons, magnetic moments, etc. (cf. [5–7]). Its particular meaning is that numerous hadronic observables are directly linked to a set of fundamental QCD quantities, the condensates.

Condensates are ground state expectation values of QCD and so they quantify the complexity of the QCD ground state. In QCD sum rules, the condensate terms occur as power corrections of a perturbative expansion and dominate the low-energy behavior of QCD. One expects then that condensates should determine the properties of hadrons as well. Although the method does not claim to be a precision tool it is quite meaningful since it can be applied to numerous hadronic observables using a unique set of condensates. Furthermore some of these condensates are directly linked to symmetries of QCD and can measure symmetry violations.

Especially useful becomes this approach when one is interested in changes of hadron properties. Such modifications can arise in a medium of non-vanishing temperature or density. The investigation of these modifications sheds light on the complex ground state of QCD. Changes in this ground state are expected to be reflected in its excitation spectrum, analogous to the changes of atomic excitation spectra embedded in external electromagnetic fields (Stark and Zeeman effects). The excitations of the QCD ground state are the hadrons, and the corresponding external field is represented by the surrounding nuclear matter. Hadrons in the nuclear medium therefore probe this ground state by the modifications they perceive (see Fig. 1.1).

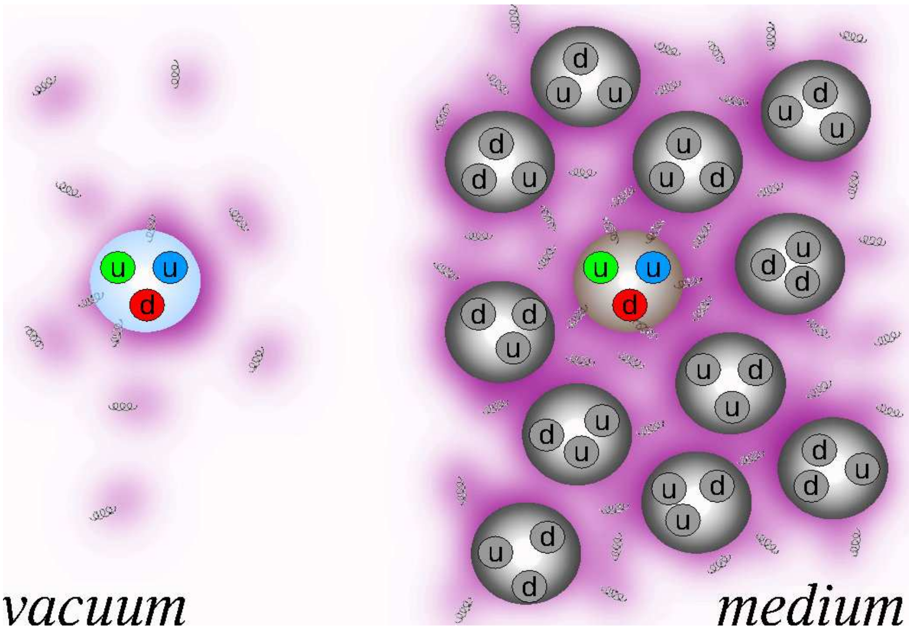

Figure 1.1: Illustration of the modification of hadronic properties in a medium: The hadron, in this case a proton (left panel), characterized by the nucleon mass $M_N = 938\ \mathrm{MeV}$ in vacuum, is injected into nuclear matter of neutrons and protons (right panel). There it probes the finite density state and, vice versa, is modified in the vicinity of surrounding nucleons. One possible consequence is that the probe changes its self-energies (mass) when embedded in such a medium.

The method of QCD sum rules was therefore extended to non-vanishing temperatures [8] and densities for vector mesons [9] and nucleons [10]. With the mapping of hadronic properties onto expectation values of the QCD ground state via QCD sum rules it becomes possible to study the change of the condensates itself and especially of order parameters of characteristic symmetries. The non-zero chiral (two-quark) condensate, for example, exhibits the spontaneously broken chiral symmetry of the QCD Lagrangian. The decrease of this condensate, as expected while approaching the quark-gluon plasma region in the QCD phase diagram at large densities or temperatures, respectively, would indicate the partial restoration of chiral symmetry. Therewith related, a hypothesis of Brown and Rho [11] received much attention. Accordingly, the masses of vector mesons m^V change like the chiral condensate $\langle \bar{q}q \rangle$ as $m^V / m_0^V \propto [\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0]^a$. The qualitative expectation is, that at larger densities or temperatures the decrease of light vector meson masses signals the partial restoration of chiral symmetry. (The subscript "0" denotes the vacuum values, the parameter a has to be determined from theory or experiment [12].) The assumed symmetry restoration is extensively studied because of its role for phase transitions between phases of QCD, especially the phases of hadronic matter versus deconfined matter of quarks and gluons. The chiral condensate $\langle \bar{q}q \rangle$ represents here an important order parameter. Furthermore, the cited scaling hypothesis as well as the simplified formula of Ioffe [13] for the nucleon mass, $M_N \propto \langle \bar{q}q \rangle$, indicate that the creation of hadron masses originates from a symmetry breaking principle as familiar from the Higgs mechanism in the electroweak sector of the Standard Model. The pseudoscalar Goldstone bosons induced by the spontaneous breaking of chiral symmetry are identified with the isotriplet of pions (for $n_f = 2$ flavor degrees of freedom). Although the symmetry is still valid for a massless Lagrangian density, the specification of a ground state restrains the symmetry in the observed hadronic spectrum where, for example, partners of opposite parity exhibit significant mass splittings, e.g. $m_\rho - m_{a_1} \sim 450$ MeV. Therefore, spontaneously broken symmetries are also termed hidden symmetries.

The investigation of hadrons with QCD sum rules however reveals that the relation of masses to QCD condensates is more elaborate than e.g. suggested by Brown-Rho scaling. Especially, the impact of various combined condensates hinders simple scaling arguments. This work discusses such applications for (a) light vector mesons, preferentially the ω meson, (b) the nucleon, i.e. proton and neutron, and (c) the D meson. The chosen examples constitute also three conceptional interesting cases for QCD sum rules. Whereas the ρ and ω mesons (a) have been the initial examples at the event of QCD sum rules [4], baryon sum rules reveal new aspects due to the fermionic character of the considered hadrons. This exemplifies the differences between QCD sum rules for two- and three quark systems. The case of the D meson, on the contrary, is distinguished by the large charm quark mass entering the QSR equations and leads to qualitative new aspects when setting up QCD sum rules. Our investigations will mainly focus on effects in a region of the QCD phase diagram close to the vacuum case, especially at small densities and zero temperatures.

1.2 Experimental Status and Perspectives

The theoretical expectations about modifications of hadrons at finite densities and temperatures have triggered several experiments to explore such effects. In particular the above Brown-Rho relation suggested that measuring a mass shift of a hadron means quantifying a change of the chiral condensate. Generally one can distinguish between experiments where the hadrons in a medium are produced at high energies or at relatively low energies.

In the first case, in high energy collisions of ions, i.e. nucleus-nucleus collisions, the concise identification of medium modifications depends on the complete understanding of the reaction processes. Measured spectra are then integrated over the full space-time evolution of the reaction, partially far from equilibrium, and other effects contributing similar as the impact of medium effects have to be rather well understood and under control. Especially, not only the nucleus-nucleus collisions have to be investigated, but also the corresponding proton-proton as well as proton-nucleus collisions, which, mainly free from medium effects related to compressed nuclear matter, define as normalization what is meant with medium modifications at non-vanishing nuclear densities. Possible changes of vector mesons, e.g., can be measured by their direct electromagnetic decays into di-leptons. These probes have been favored since they only interact electromagnetically and can thus leave the hot and dense interaction zone nearly undisturbed by secondary reactions.

First experimental results for an affected ρ meson mass came from CERES at the CERN SPS where an unexplained excess in low invariant mass di-electron spectra was observed [14]. Improvement of statistical accuracy and mass resolution allowed a more sensitive investigation of the intermediate medium-modified ρ meson. Therewith the measurements of muon pairs at the CERN SPS by the NA60 experiment also revealed an excess at low invariant masses [15, 16]. The reconstructed spectral function of the ρ meson showed a strong broadening, but a significant mass shift could not be discovered. This seems to be incompatible with simple ”moving mass scenarios” as e.g. predicted by the above Brown-Rho scaling of the vector meson mass [11, 12]. Contrary, the E325 experiment at KEK, which also aims on modifications of the in-medium ρ- and ω-spectrum claimed a decrease of the meson mass from measured decay electron pairs [17]. The High Acceptance Di-Electron Spectrometer (HADES) at GSI is designed to measure the electromagnetic decays of light vector mesons with high resolution in proton and heavy ion collisions [18]. It provides a dedicated test of assumptions about medium modifications of spectral functions from the ratios of nucleus-nucleus, proton-nucleus and proton-proton collisions.

Alternatively, one can provide vector mesons by photo-induced production on various targets. In this scenario the experimental situation can be much better prepared and characterized. An observation of additional spectral strength of the ω meson at lower invariant masses in photoproduction experiments was reported by CB-TAPS, identified via the decay channel $\omega \to \pi\gamma$ [19]. The excess of decay strength appeared there when niobium instead of liquid hydrogen was used as target, allowing a comparison between non-vanishing nuclear densities and the vacuum limit. Also CLAS at JLAB studied the photoproduction of vector mesons on various nuclei by the measurement of the di-lepton decay products [20]. Therefore different averaged nuclear densities were provided with the target nuclei carbon, iron, and titanium and compared to spectra obtained with a liquid deuterium target, which represents the nearly unaffected vacuum reference probe. Preliminarily, no indications for a modification of the ρ meson mass have been found there. Instead, both experiments see a significant broadening of vector mesons in a medium, for the ω [21] as well as for the ρ meson [22, 23].

Nevertheless a dedicated analysis of these data is required in both scenarios, hadrons at high or low energies, to disentangle the effects of creation and decay of the investigated hadron and especially the final state interactions of the decay products, since the spectral function is not directly observable. An analysis of the CB-TAPS data suggests some lowering of the ω meson mass when embedded in medium [24]. This is accompanied by collisional broadening, the impact of hadron scatterings on the measured decay spectra. Based on the experimental finding, that the ω meson mass is not increasing, constraints for condensates in medium could already be derived [25], and this derivation will be covered in this thesis.

Also for the formation of nuclei does the in-medium behavior of hadrons play a significant role. A stable description of nuclear matter requires compensating effects of vector and scalar nucleon self-energies. The spectral function of the nucleon or its in-medium self-energies cannot be measured directly, however, they have been determined on the basis of realistic nucleon-nucleon potentials in the framework of chiral effective field theory [26]. The provided density dependent self-energies can be compared to QSR predictions and thereby deliver restrictions on the in-medium behavior of specific condensate combinations [27], as will be demonstrated in the course of this work.

The upcoming experimental possibilities of FAIR at GSI extend the studies of medium modifications from the light-quark sector to hadrons containing heavy quarks as well. Especially, the modifications of the D meson spectra are intended to be investigated. Therefore, the CBM experiment uses the high beam intensities of accelerated heavy ions [28]. The PANDA experiment is designed to use the provided antiprotons for collisions [29]. These explorations and further data from HADES, JLAB, KEK and CERN bear encouraging prospects for an understanding of QCD matter and hadrons at various densities and temperatures.

Structure of the Thesis

The thesis is organized as follows. Hereafter this introductory Chapter 1, we introduce in Chapter 2 the ingredients of QCD sum rules, starting from dispersion relations connecting the hadronic spectral representation of a hadronic correlator with the expressions obtained from an operator product expansion. Here also the QCD condensates and related symmetries of QCD are explained. A separate section is devoted to the discussion of four-quark condensates, which is one result of the present thesis. Chapter 3 contains the analytical and numerical results obtained for the ω meson, with additional remarks on the ρ meson. Also the nucleon and the D meson cases are considered there. Conclusions and summary can be found in Chapter 4. In the appendices, technical details required for the presented calculations are supplemented.

It should be mentioned that parts of the material of this thesis has been published in [25, 27, 30–33]. Yet unpublished material can be found in Chapter 2, where details of the construction of in-medium sum rules are given and the role of four-quark condensates in the context of chiral symmetry breaking is pursued. Furthermore, in Chapter 3 additional details on the evaluation of the sum rule results are collected. Technical details usually not published are also compiled in the appendix to complete the thesis.

2 QCD Sum Rules in Medium

The expected modifications of hadron properties inside nuclear matter are caused by the strong interaction and therefore have to be described within the framework of Quantum Chromodynamics (QCD), if one tries to establish an approach on a fundamental level. A possibility to circumvent the complications with this non-abelian theory with exact local gauge invariance in the non-perturbative sector is provided with QCD Sum Rules (QSR). These have originally been introduced for the description of vacuum masses of vector mesons [3, 4]. Subsequently, numerous applications of these ideas widened the scope of this method, e.g. to baryon masses [13]. Further generalizations of this approach addressed properties as hadronic coupling constants [34] or form factors [35] etc. More recent developments are applications of QCD sum rules to conjectured penta-quarks [36] or tetra-quark states [37]. In our context, the extension of QCD sum rules to properties of individual hadrons embedded into a strongly interacting medium at non-vanishing temperatures and densities [8] is particularly important. The application of the method in the case of finite nuclear density and zero temperature will be covered in this thesis.

2.1 Motivation: The Vacuum Case

For didactic simplicity we follow the historical line and begin with the vacuum case to supply the general ideas and terminology. It is intended as general introduction to the method, and a lot of aspects will be deepened within the course of the generalization to the medium case further below.

QCD sum rules link hadronic observables and parameters of the underlying theory of strong interaction - QCD. This correspondence is achieved by a dispersion relation which connects ranges of distinct hadron momenta. The mass, and this will be the interesting quantity throughout the thesis, is encoded in the hadronic correlation function, i.e. it determines how the hadron propagates. Strictly speaking one shall only refer to the spectral density as introduced later. (For other applications suitable correlation functions can be adopted, for instance three-point functions for hadron couplings [5].)

The correlation function or current-current correlator of the hadron field h is the Fourier transform of the time-ordered expectation value in the physical QCD ground state $|\Psi\rangle$

$$\Pi^h(q) = i \int d^4x \, e^{iqx} \left\langle \Psi \left| T \left[h(x)\bar{h}(0) \right] \right| \Psi \right\rangle \,, \tag{2.1}$$

if we consider the hadrons as fundamental degrees of freedom. It can be expressed in terms of the partonic decomposition of the specific hadron, that means by quarks and gluons. Any interpolating field or current η, built as product of quark and gluon fields, which carries the quantum numbers of the investigated hadron, can lead to a non-vanishing matrix element between vacuum and the hadron state

$$\langle \Psi | \eta(0) | h(p,s) \rangle = \lambda_h \psi(p,s) \,, \quad \lambda_h \neq 0 \,. \tag{2.2}$$

The state $|h(p,s)\rangle$ is a single hadron state with momentum p and spin s. The hadron h is considered here a spin-$\frac{1}{2}$ particle with Dirac spinor $\psi(p,s)$. In principle, the field operators h, η and the correlation function Π^h can have a complex Lorenz structure. This is neglected in the present example but can be found e.g. in Section 3.1 about the ω meson example. (The translation to mesons requires the replacement of $\bar{h}$ by $h^\dagger$ in Eq. (2.1); appropriate wave functions in Eq. (2.2) are then Klein-Gordon fields.)

Practical applications are mostly restricted to simple forms of hadron interpolating fields η, which do not contain derivatives of field operators. The correlator then becomes

$$\Pi(q) = i \int d^4x\, e^{iqx} \langle \Psi \left| T\left[\eta(x)\bar{\eta}(0)\right]\right| \Psi\rangle \ . \tag{2.3}$$

This is the central quantity to be considered. In contrast to Eq. (2.1) the function Π solely contains the quark and gluon degrees of freedom. The coupling factor λ_h between the interpolating field η and the physical hadron state h has to be respected when comparing to hadronic models for Π^h. Eq. (2.3) is the standard form given in the literature as starting point for the QSR derivation.

This ansatz already contains a further severe restriction to the interpretation of QSR results. It includes besides the hadron h also any other hadron configurations with the same quantum numbers, like excitations of h or even multi-particle states. This becomes obvious from the introduction of the spectral function into Eq. (2.3)

$$\Pi(q) = -\int dq'_0 \left[\frac{\rho(q')}{q_0 - q'_0 + i\epsilon} + \frac{\tilde{\rho}(q')}{q_0 - q'_0 - i\epsilon}\right] \ , \tag{2.4}$$

the Källen-Lehmann representation, which follows directly when writing out the time ordering with the integral representation of the Heaviside step-function $\theta(t) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} da\, e^{iat}/(a - i\epsilon)$ and implies the definition of the spectral densities

$$\rho(q) = \frac{1}{2\pi}\int d^4x e^{iqx} \langle \Psi \left|\eta(x)\bar{\eta}(0)\right| \Psi\rangle \ , \tag{2.5}$$

$$\tilde{\rho}(q) = \frac{1}{2\pi}\int d^4x e^{iqx} \left\langle \Psi \left|\bar{\eta}^T(0)\eta^T(x)\right| \Psi\right\rangle^T \ . \tag{2.6}$$

In this example for fermions, the transposed Dirac structures ensure the correct matrix form in $\tilde{\rho}$. In general, the spectral functions for particles ρ and anti-particles $\tilde{\rho}$ are independent. However, for the vacuum case these functions are related as $\rho(\omega) = -\tilde{\rho}(-\omega)$ due to the symmetries of the QCD ground state. The correlation function has thus in vacuum the form

$$\Pi(q) = \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{s - q^2} \ . \tag{2.7}$$

The dependence of Π on q^2 is justified by the Lorentz invariance of the QCD vacuum. By introducing a complete set of hadronic eigenstates into the definition,

$$\rho(q) = (2\pi)^3 \sum_n \delta^{(4)}(q - P_\Psi - P_n)\langle\Psi|\eta(0)|n\rangle\langle n|\bar{\eta}(0)|\Psi\rangle \tag{2.8}$$

contains all configurations which couple with the right quantum numbers $\lambda_n \neq 0$ as claimed.

In vacuum one can choose a suitable frame, where the four-momentum of the ground state P_Ψ is zero. Since the QSR will determine basically the correlator, that means only the integrated spectral strength, the determination of individual hadronic properties also relies on the assumption that the lowest lying hadron dominates the spectral integral. In this simplified case all higher states are negelected in the sum over n and summarized into a (yet to be determined) continuum contribution ρ^{con}. This leads with Eq. (2.2) to

$$\rho(q) = (2\pi)^3 \sum_s \delta^{(4)}(q - p)|\lambda_h|^2 \psi(p,s)\bar{\psi}(p,s) + \Theta(q^2 - s_0)\rho^{con}(q) \ . \tag{2.9}$$

It is common praxis to approximate the continuum part ρ^{con} by the so called semi-local quark-hadron duality hypothesis: The hadronic continuum is substituted by the contribution of quark and gluon degrees of freedom to the correlator at asymptotically large momenta (local hypothesis). Actually, this local demand can be weakened, since only the integrated form is required (semi-local hypothesis). This allows for local oscillations around the asymptotic limit of perturbative QCD.

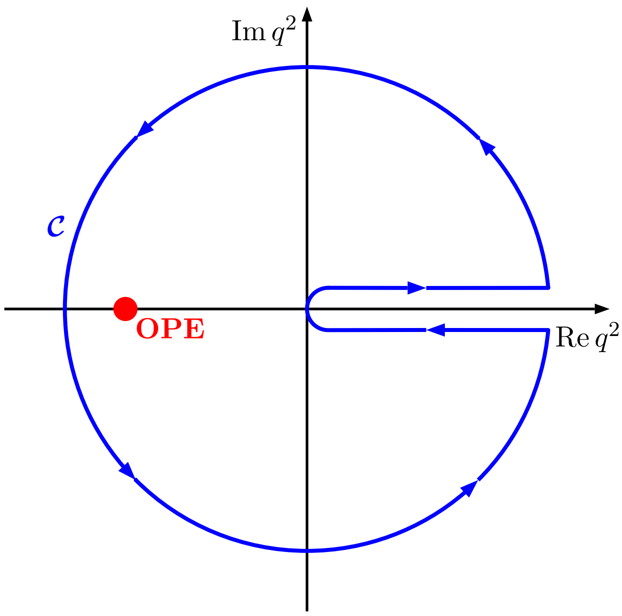

Figure 2.1: In the vacuum case the shown (blue) contour in the complex plane of q^2 is used to derive dispersion relations which relate the operator product expansion (red dot with label OPE) to the hadronic representation of the correlation function on the real axis with $\mathrm{Re}\, q^2 > 0$.

The correlation function Eq. (2.3), so far expressed in hadronic degrees of freedom for physical momenta q^2 (the positive real axis in Fig. 2.1) can on the other hand be evaluated at large Euclidean momenta (the negative real axis in Fig. 2.1). Assuming analyticity of the correlation function, its values on the positive and negative real axis can be related by a dispersion relation. Therefore, consider the Cauchy integral representation of an analytical function

$$\Pi(q^2) = \frac{1}{2\pi i} \int_{\mathcal{C}} ds \frac{\Pi(s)}{s - q^2} \ , \tag{2.10}$$

whereby the integral contour $\mathcal{C}$ is shown in Fig. 2.1. The poles of the integrand on the positive real axis are thereby excluded. A decomposition of the contour $\mathcal{C}$ in the limits of infinite radius R of the circle and infinitesimal exclusion of the positive real axis yields

$$\Pi(q^2) = \frac{1}{2\pi i} \int_{R\to\infty} ds \frac{\Pi(s)}{s - q^2} + \frac{1}{\pi} \int_0^\infty ds \frac{\Delta\Pi(s)}{s - q^2} \ . \tag{2.11}$$

The discontinuity along the positive real axis

$$\Delta\Pi(\omega) = \frac{1}{2i} \lim_{\epsilon\to 0} [\Pi(s + i\epsilon) - \Pi(s - i\epsilon)] \tag{2.12}$$

can by the Schwartz reflection principle $\Pi(s^*) = \Pi^*(s)$ be transformed, and is related to the spectral density

$$\Delta\Pi(s) = \mathrm{Im}\Pi(s) \equiv \rho(s) \ . \tag{2.13}$$

If the integral along the infinite circle vanishes, the dispersion relation

$$\Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\Delta\Pi(s)}{s - q^2} \ , \tag{2.14}$$

is the starting point for the celebrated QCD sum rule. It remains to calculate the correlator, here on the l.h.s., in terms of quarks and gluons. For $q^2 < 0$ this can be performed via Wilson's operator product expansion (OPE) [38]. It is an expansion of nonlocal products of operators A and B

$$A(x)B(y) = \sum_i \tilde{C}_i(x-y)\mathcal{O}_i \tag{2.15}$$

into local operators $\mathcal{O}_i$ accompanied by Wilson coefficients $\tilde{C}_i$ which are singular as $x \to y$. The operators $\mathcal{O}_i$ are sorted such that the degree of singularity increases with index i, and also the operator dimension due to the constant overall mass dimension in this expansion. In momentum space it becomes

$$\int d^4q \ e^{iq(x-y)} A(x)B(y) = \sum_i C_i(q)\mathcal{O}_i \ , \tag{2.16}$$

with $C_i \propto q^{-(i+n)}$, where n is a fixed number depending on the mass dimension of the product $A(x)B(y)$. For time-ordered products, as the correlation function in (2.14), this expansion is equally possible and since ground state expectation values are considered one is naturally lead to a sum

$$\Pi(q^2) = \sum_i C_i(q^2) \langle \mathcal{O}_i \rangle \ . \tag{2.17}$$

The local expectation values $\langle \mathcal{O}_i \rangle = \langle \Psi | \mathcal{O}_i | \Psi \rangle$ are termed QCD condensates. Condensates quantify the complex behavior of the QCD ground state. As universal numbers they occur in many QCD sum rules and are not restricted to a QSR for a specific hadron. Moreover, also qualitative information about symmetry properties of QCD are partially encoded into QCD condensates. The leading term, the coefficient of the identity operator, represents, besides possible renormalization corrections, the perturbative QCD result. The QCD sum rule (2.14) thus has the form

$$\frac{1}{\pi} \int_0^\infty ds \frac{\Delta\Pi(s)}{s - q^2} = \sum_i C_i(q^2) \langle \mathcal{O}_i \rangle \ . \tag{2.18}$$

The hadronic part in integrated form (left hand side) is determined by a set of power corrections in inverse momenta which are weighted by QCD condensates (right hand side). In this argument we have neglected the first integral contribution in Eq. (2.11) assuming sufficient convergence of the integrand as $q^2 \to \infty$. Otherwise the circular integral does not vanish but converges into a finite polynomial [39]. Reversing this, the subtraction of a finite polynomial of suitable degree resolves the problem. Note, that this is equivalently obtained when starting with a dispersion relation for the derivative of $\Pi(q^2)$. There remains the subtlety that the degree of this polynomial and the coefficients had to be found. This can be dealt with using the Borel transformation under which any polynomial vanishes. Such technical aspects are postponed to the more detailed section on in-medium QCD sum rules and to the appendix.

In summary, this motivation shows how spectral properties of hadrons, entering the hadronic integral, are related to characteristics of QCD collected in the condensates. These two sides of the dispersion relation Eq. (2.18) build the QCD sum rule and this principle also holds for any other applications of the method.

2.2 Generalization to Medium

The introduction to QCD sum rules will now be extended to a medium which in the thermodynamic limit is characterized by the baryon density n and the temperature T. In this section especially new aspects in the medium case shall be described. Furthermore, important technical details are given to strengthen the conceptional ideas presented in the previous motivation.

2.2.1 Dispersion Relations

In a thermodynamic system the propagation of a hadron probing the medium characterized by the four-velocity v_μ is described by the Gibbs averaged correlation function (Green's function)

$$\Pi(q,v) = i \int d^4x \, e^{iqx} \langle\langle T \left[\eta(x)\bar{\eta}(0)\right]\rangle\rangle \, . \tag{2.19}$$

We concentrate on the fermion example, in the bosonic case the derivation of the dispersion relations proceeds in the same way.

The Gibbs averaged expectation value of an operator $\hat{\mathcal{O}}$ is defined as

$$\langle\langle \hat{\mathcal{O}} \rangle\rangle = \frac{\mathrm{Tr}(\hat{\mathcal{O}} e^{-\beta(\hat{H}-\mu\hat{N})})}{\mathrm{Tr}(e^{-\beta(\hat{H}-\mu\hat{N})})} \, , \tag{2.20}$$

with the Hamilton operator $\hat{H}$, particle number operator $\hat{N}$, chemical potential μ and $\beta = \frac{1}{k_B T}$ (T is the temperature and k_B the Boltzmann constant). The trace represents the summation over the complete sets of states to any particle numbers N. The Gibbs averaged correlation function possesses poles above and below the real energy axis. The medium generalization of QCD sum rules in [8] resorted to the retarded (advanced) correlation functions

$$\Pi^R(q,v) = \quad i \int d^4x \, e^{iqx} \, \langle\langle \eta(x)\bar{\eta}(0)\rangle\rangle \, \Theta(x_0) \, , \tag{2.21}$$

$$\Pi^A(q,v) = -i \int d^4x \, e^{iqx} \, \langle\langle \bar{\eta}^T(0)\eta^T(x)\rangle\rangle^T \, \Theta(-x_0) \, , \tag{2.22}$$

which are analytic in the upper (lower) half energy plane in Fig. 2.2. However, if one evaluates the correlation functions far off the real axis, $\mathrm{Im}\, q_0 \neq 0$, the distinction between the causal, retarded and advanced correlators has no importance [40]. The contours in Fig. 2.2 avoid the branch cuts close to the real axis, and we choose for the derivation of dispersion relations the case of the time-ordered correlation function. Note that the following argument can be repeated separately for the retarded and advanced correlators.

Since the operator product expansion is valid for large spacelike Euclidean momenta $-Q^2 \equiv q^2 < 0$, as applied in Fig. 2.1, one has to analytically continue the correlation function $\Pi(q_0)$ to imaginary values of q_0 (Wick rotation). Thus if $q_0 = iq_0'$ with $q_0' \in \mathbb{R}$ and sufficiently large $q_0' > |\vec{q}|$ for fixed $\vec{q}$, the condition $q^2 < 0$ is always fulfilled, and there the OPE representation can be used. Assuming the analyticity of the correlation function in the whole complex energy plane except the previously discussed region close to the (physical) real axis one can apply Cauchy's integral formula to relate the operator product expansion in the upper or lower complex plane

$$\Pi(q_0) = \frac{1}{2\pi i} \left(\int_{\mathcal{C}_1} da \frac{\Pi(a)}{a - q_0} + \int_{\mathcal{C}_2} da \frac{\Pi(a)}{a - q_0} \right) \tag{2.23}$$

to the hadronic representation on the real energy axis, see Fig. 2.2. One of the integrals vanishes depending on the sign of the imaginary part of q_0. The integration over the contours $\mathcal{C}_i$ can be

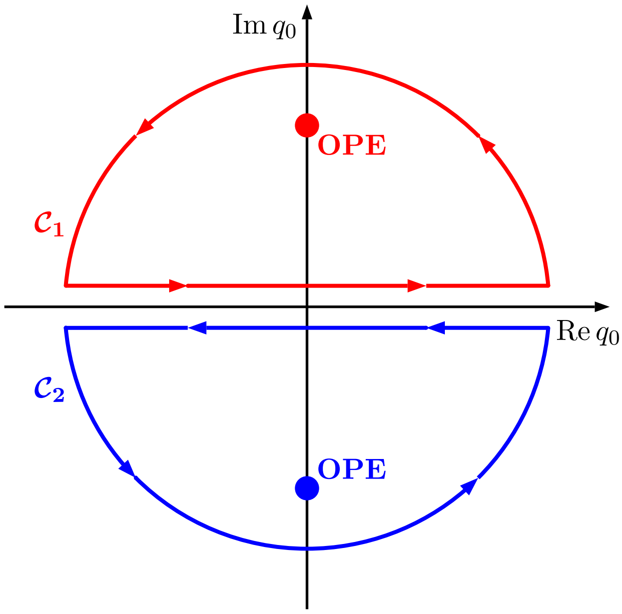

Figure 2.2: The contours $\mathcal{C}_1$ and $\mathcal{C}_2$ in the complex energy plane q_0 applied to derive dispersion relations which relate the operator product expansion (OPE) to the hadronic representation of the correlation function along the real energy axis.

decomposed into an integration along the real axis shifted by $\pm i\epsilon$ $(\epsilon > 0)$ and one along the sector of a circle of radius R

$$\Pi(q_0) = \frac{1}{2\pi i}\int_{-R+i\epsilon}^{+R+i\epsilon} da \frac{\Pi(a)}{a-q_0} + \frac{1}{2\pi i}\int_{\substack{|a|=R \\ \mathrm{Im}\, a>\epsilon}} da \frac{\Pi(a)}{a-q_0}$$
$$+ \frac{1}{2\pi i}\int_{+R-i\epsilon}^{-R-i\epsilon} da \frac{\Pi(a)}{a-q_0} + \frac{1}{2\pi i}\int_{\substack{|a|=R \\ \mathrm{Im}\, a<\epsilon}} da \frac{\Pi(a)}{a-q_0} . \qquad (2.24)$$

In order to get rid of the contribution from the circular arcs one considers the limit $R \to \infty$. However the ultraviolet behavior of the correlation function is not generally given to converge in a way that the specified integral vanishes. For the time being this problem can be circumvented by subtracting out the ultraviolet divergent part of $\Pi(q_0)$ in the form of a polynomial of degree $(n-1)$

$$\bar{\Pi}(q_0) = \Pi(q_0) - \sum_{l=1}^{n} \frac{\Pi^{(l-1)}(0)}{(l-1)!} q_0^{(l-1)} , \qquad (2.25)$$

defined such that n is the number of subtractions or more precisely the number of subtracted terms. The $(l-1)\mathrm{th}$ derivative $\Pi^{(l-1)}$ is with respect to q_0; using $\bar{\Pi}(q_0)$ equivalently means to consider the $n\mathrm{th}$ derivative of $\Pi(q_0)$ of the operator product representation.[1] Combination of the subtractions with Eq. (2.23) or the direct use of Cauchy's formula for the derivative expression yield

$$\bar{\Pi}_{\mathrm{OPE}}(q_0) = \frac{q_0^n}{2\pi i}\left(\int_{\mathcal{C}_1} da \frac{\Pi(a)}{a^n(a-q_0)} + \int_{\mathcal{C}_2} da \frac{\Pi(a)}{a^n(a-q_0)}\right) . \qquad (2.26)$$

[1]For such subtractions $\Pi(q_0)$ must be analytic around $q_0 = 0$ and the contour in Fig. 2.2 could be connected. Rigorous statements about the integral contribution at infinity are given in [39].

Now if a finite n exists which satisfies

$$\int_{|a|=R} da \frac{\Pi(a)}{a^n(a-q_0)} \xrightarrow{R\to\infty} 0\,, \tag{2.27}$$

the contour integration along $\mathcal{C}_1$ in the limit $R \to \infty$ leads to

$$\frac{1}{2\pi i}\int_{\mathcal{C}_1} da \frac{\bar{\Pi}(a)}{a-q_0} = \frac{q_0^n}{2\pi i}\int_{-\infty+i\epsilon}^{+\infty+i\epsilon} da \frac{\Pi(a)}{a^n(a-q_0)} = \frac{q_0^n}{2\pi i}\int_{-\infty}^{+\infty} da \frac{\Pi(a+i\epsilon)}{(a+i\epsilon)^n(a+i\epsilon-q_0)}\,, \tag{2.28}$$

and similarly for the contour $\mathcal{C}_2$

$$\frac{1}{2\pi i}\int_{\mathcal{C}_2} da \frac{\bar{\Pi}(a)}{a-q_0} = \frac{q_0^n}{2\pi i}\int_{+\infty-i\epsilon}^{-\infty-i\epsilon} da \frac{\Pi(a)}{a^n(a-q_0)} = \frac{q_0^n}{2\pi i}\int_{+\infty}^{-\infty} da \frac{\Pi(a-i\epsilon)}{(a-i\epsilon)^n(a-i\epsilon-q_0)}\,, \tag{2.29}$$

where in the last steps the integration variables have been shifted. This leads to the n-times subtracted dispersion relation or QCD sum rule, respectively,

$$\frac{q_0^n}{\pi}\int_{-\infty}^{+\infty} d\omega \frac{\Delta\Pi(\omega)}{\omega^n(\omega-q_0)} = \bar{\Pi}(q_0)\,, \tag{2.30}$$

with the discontinuity defined as

$$\Delta\Pi(\omega) = \frac{1}{2i}\lim_{\epsilon\to 0}\left[\Pi(\omega+i\epsilon) - \Pi(\omega-i\epsilon)\right]. \tag{2.31}$$

If we neglect subtractions, that means $n = 0$ and so $\bar{\Pi}(q_0) = \Pi(q_0)$, then the operator product expansion at $q_0 = iq_0'$ is related to the physical correlation function by

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega \frac{\Delta\Pi(\omega)}{\omega-q_0} = \Pi(q_0), \tag{2.32}$$

which is the celebrated form employed in QCD sum rules. The l.h.s. contains the hadronic spectral function, while the r.h.s. is evaluated, for large Euclidean energies q_0 and fixed momenta $\vec{q}$, by means of the OPE.

In the vacuum case the dispersion relation Eq. (2.18) depends only on the Lorentz scalar q^2, such that the correlation function is Lorentz invariant. The presence of a medium frame with definite momentum interferes with this Lorentz invariant vacuum situation. In fact, transformation in another frame of reference affects the characteristic Lorentz vectors describing the medium as well. Thus a Lorentz invariant decomposition of the correlation function has to incorporate the transformation properties of the medium itself. The medium may therefore be described by its four-velocity v_μ, which in the medium rest frame, we will work in later, simplifies to $v_\mu = (1, \vec{0}\,)$. Any Lorentz invariant structures built from the hadron momentum q_μ, the given velocity v_μ, and the metric tensor $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and the pseudo-tensor $\epsilon_{\mu\nu\kappa\lambda}$ become arguments of the invariant functions, which the correlator $\Pi(q, v)$ is decomposed into. Note that in addition the Dirac structure of the correlator increases the number of such invariants.[2] The decomposition into invariant functions combined with dispersion relations leads in general to a number of coupled QCD sum rule equations as elaborated for the nucleon in Section 3.2.

[2]Concerning conventions on metric, Dirac and Gell-Mann matrices, etc. we follow [41].

2.2.2 Weighted Spectral Moments

The QCD sum rules, e.g. Eq. (2.30) and (2.32), reveal a generic limitation of this approach. The spectral density $\Delta\Pi$ enters the integral on the hadronic side. Thus the operator product expansion only restricts the spectral integral on the left hand side but not the integrand itself. This can be used to constrain unknown parameters of a given hadronic model or to test given spectral functions (see also Subsection 3.1.1).

To identify the vacuum limit in Eq. (2.30) the medium specific parts can be singled out. Especially, new terms which are of odd power in q_0 appear in medium. (Remember that Lorentz invariance prohibits such terms in the vacuum case.) Define the even (”e”) and odd (”o”) contributions

$$\Pi^{\mathrm{e}}(q_0^2) \equiv \frac{1}{2}\left(\Pi(q_0)+\Pi(-q_0)\right) , \tag{2.33a}$$

$$\Pi^{\mathrm{o}}(q_0^2) \equiv \frac{1}{2q_0}\left(\Pi(q_0)-\Pi(-q_0)\right) . \tag{2.33b}$$

This leads to

$$\Pi^{\mathrm{e}}(q_0^2) = \frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega \frac{\Delta\Pi(\omega)}{\omega^n(\omega^2-q_0^2)} \cdot \begin{cases} \omega q_0^n & \text{if } n \text{ is even,} \\ q_0^{n+1} & \text{if } n \text{ is odd,} \end{cases} \tag{2.34a}$$

$$\Pi^{\mathrm{o}}(q_0^2) = \frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega \frac{\Delta\Pi(\omega)}{\omega^n(\omega^2-q_0^2)} \cdot \begin{cases} q_0^n & \text{if } n \text{ is even,} \\ \omega q_0^{n-1} & \text{if } n \text{ is odd.} \end{cases} \tag{2.34b}$$

In order to eliminate the specified subtractions a Borel transform is applied to Eqs. (2.34). The Borel transform[3] with respect to the energy q_0,

$$\mathcal{F}(Q^2) \;\longrightarrow\; \mathcal{F}(\mathcal{M}^2) \equiv \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=\mathcal{M}^2}} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n \mathcal{F}(Q^2) , \tag{2.35}$$

introduces instead as new energy scale the Borel mass $\mathcal{M}$ [42]. The separation of even and odd parts and therewith the dependence on the squared energy allows the contact to the standard Borel transform which was introduced w.r.t. q^2 in vacuum [3]. Some details of this specific inverse Laplace transform and a collection of Borel transforms are supplemented in Appendix A. This leads to the Borel transformed sum rules with the hadronic spectral density on the l.h.s. and the operator product expansion entering the r.h.s.,

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, \omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} = \Pi^{\mathrm{e}}(\mathcal{M}^2) , \tag{2.36a}$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} = \Pi^{\mathrm{o}}(\mathcal{M}^2) , \tag{2.36b}$$

independent of the number of subtractions n (see Appendix A). Therefore the polynomial subtractions above are not relevant for Borel sum rules as the Borel transform delivers the required convergence. Nevertheless can subtractions at another point $q_0 \neq 0$ be used to separate scattering contributions from the spectral density [40].

[3] Here and elsewhere the different arguments denote a different function in spite of the same letter used.

In the sum rules (2.36a) and (2.36b) one is interested only in the lowest hadronic excitation which is isolated by splitting the spectral integral

$$\int_{-\infty}^{+\infty} d\omega \mathcal{F}(\omega) = \underbrace{\int_{-\infty}^{\omega_-} d\omega \mathcal{F}(\omega)}_{\text{"anti-continuum"}} + \underbrace{\int_{\omega_-}^{0} d\omega \mathcal{F}(\omega)}_{\text{"anti-particle"}} + \underbrace{\int_{0}^{\omega_+} d\omega \mathcal{F}(\omega)}_{\text{"particle"}} + \underbrace{\int_{\omega_+}^{+\infty} d\omega \mathcal{F}(\omega)}_{\text{"continuum"}} \quad (2.37)$$

into a particle and an anti-particle contribution, and related continuum contributions as indicated. The final sum rules have than the form

$$\frac{1}{\pi}\int_{\omega_-}^{0} d\omega\, \omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} + \frac{1}{\pi}\int_{0}^{\omega_+} d\omega\, \omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} \quad (2.38a)$$

$$= \Pi^{\rm e}(\mathcal{M}^2) - \frac{1}{\pi}\int_{-\infty}^{\omega_-} d\omega\, \omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} - \frac{1}{\pi}\int_{\omega_+}^{+\infty} d\omega\, \omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} \equiv R_{\rm e}\,,$$

$$\frac{1}{\pi}\int_{\omega_-}^{0} d\omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} + \frac{1}{\pi}\int_{0}^{\omega_+} d\omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} \quad (2.38b)$$

$$= \Pi^{\rm o}(\mathcal{M}^2) - \frac{1}{\pi}\int_{-\infty}^{\omega_-} d\omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} - \frac{1}{\pi}\int_{\omega_+}^{+\infty} d\omega\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2} \equiv R_{\rm o}\,,$$

where the continua are again estimated within the semi-local quark-hadron duality hypothesis by the operator product expansion (right hand sides), extended down to the respective continuum thresholds $\omega_\pm$. The hadronic information is contained in the spectral integrals (left hand sides). The vacuum limit is encoded in the first sum rule. In a particle-antiparticle symmetric situation, for vacuum or neutral mesons in nuclear matter, with an anti-symmetric spectral function, $\Delta\Pi(\omega) = -\Delta\Pi(-\omega)$ one obtains with $\omega_- = -\omega_+$

$$R_{\rm e} = \frac{1}{\pi}\int_{0}^{\omega_+^2} d(\omega^2)\, \Delta\Pi(\omega)\, e^{-\omega^2/\mathcal{M}^2}\,, \quad (2.39a)$$

$$R_{\rm o} = 0\,, \quad (2.39b)$$

in accordance with (2.14), if there the continuum part is separated, variables $\omega^2 \leftrightarrow s = q^2$ are changed and the Borel transform is performed.

For ω and ρ^0 meson e.g. one can define a normalized moment for the Borel transform

$$\overline{m}^2(n, \mathcal{M}^2, s_+) \equiv \frac{\int_0^{s_+} ds\ \Delta\Pi(s,n)\ e^{-s/\mathcal{M}^2}}{\int_0^{s_+} ds\ \Delta\Pi(s,n)\ s^{-1} e^{-s/\mathcal{M}^2}}\,, \quad (2.40)$$

which is determined by the ratio of $R_{\rm e}$ and its logarithmic derivative w.r.t. the squared Borel mass $\mathcal{M}^2$. This quantity is model independent (but suffers from the ad hoc introduced continuum threshold s_+). Its meaning becomes obvious for a pole ansatz with pole mass m, $\Delta\Pi(s) = F\delta(m^2 - s)$, where $\bar{m}(n, \mathcal{M}^2, s_+) = m$ follows.

The introduction of such moments, which can be calculated by means of a QCD sum rule without relying on a specific hadronic model, can partially be extended to arbitrary particle and anti-particle spectral functions. Defining further the moments

$$\bar{E} = \frac{\int_{\omega_-}^0 d\omega \Delta\Pi(\omega)\omega e^{-\omega^2/\mathcal{M}^2}}{\int_{\omega_-}^0 d\omega \Delta\Pi(\omega) e^{-\omega^2/\mathcal{M}^2}} \quad \text{and} \quad E = \frac{\int_0^{\omega_+} d\omega \Delta\Pi(\omega)\omega e^{-\omega^2/\mathcal{M}^2}}{\int_0^{\omega_+} d\omega \Delta\Pi(\omega) e^{-\omega^2/\mathcal{M}^2}}\,, \quad (2.41)$$

the even and odd sum rules can be rephrased to formulate relations which rather depend on ratios than purely on absolute spectral integrals. These integrals are the first moments of the Borel weighted spectral density for the negative and positive energy excitations. In a combined sum rule ansatz, suitable for the nucleon case (Section 3.2),

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega(\omega-\bar{E})\Delta\Pi(\omega)e^{-\omega^2/\mathcal{M}^2} = \Pi^{\rm e}(\mathcal{M}^2) - \bar{E}\Pi^{\rm o}(\mathcal{M}^2)\,, \tag{2.42}$$

the integral over the negative energy contribution seems to be ”eliminated”. (This contribution, however, is still included in the ratio $\bar{E}$.) This delivers the Borel transformed sum rule

$$\begin{aligned} (E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega\Delta\Pi(\omega)e^{-\omega^2/\mathcal{M}^2} &= \Pi^{\rm e}(\mathcal{M}^2) - \frac{1}{\pi}\int_{\omega_+}^{\infty} d\omega\omega\Pi^{\rm e}_{\rm per}(\omega)e^{-\omega^2/\mathcal{M}^2} \\ -\bar{E}\left\{\Pi^{\rm o}(\mathcal{M}^2) - \frac{1}{\pi}\int_{\omega_+}^{\infty} d\omega\Pi^{\rm o}_{\rm per}(\omega)e^{-\omega^2/\mathcal{M}^2}\right\} &+ \frac{1}{\pi}\int_{\omega_-}^{-\omega_+} d\omega\Delta\Pi(\omega)[\omega-\bar{E}]e^{-\omega^2/\mathcal{M}^2}\,. \end{aligned} \tag{2.43}$$

The continuum contributions are rearranged as $\Pi^{\rm e,o}_{\rm per}(\omega) \equiv \Delta\Pi(\omega) \mp \Delta\Pi(-\omega)$. Based on semi-local quark-hadron duality these integrals are extended towards the respective continuum thresholds $\omega_\pm$. Typically only the logarithmic terms in Π provide discontinuities which enter the continuum integrals. To summarize, Eq. (2.43) exhibits the typical structure of QCD sum rules: the hadronic properties on the l.h.s., i.e. the low-lying hadronic spectral function, are thought to be given by the operator product representation of Π including condensates on the r.h.s. The last term on the r.h.s. accounts for asymmetric continuum thresholds, i.e. $\omega_- \neq -\omega_+$, and could be estimated by semi-local quark-hadron duality.

In the generic case, however, the excitation spectrum of particles and anti-particles in a medium is asymmetric, and the dispersion relations (2.38a) and (2.38b) alone seem not to allow separate model-independent statements about the low-lying excitation at positive energy. This appears as a severe limitation of this form of the QCD sum rule approach.

2.2.3 Operator Product Expansion

The dispersion relations in vacuum Eq. (2.14) and in medium Eqs. (2.30) and (2.32) relate integrals of the weighted spectral density of a hadron entering the left hand side of a QCD sum rule to a momentum region on the right hand side, where a treatment of the correlator in terms of QCD parameters is possible. Such parameters are the masses and coupling constants which appear in the classical chromodynamic Lagrangian density

$$\mathcal{L}(x) = \sum_{q=1}^{f} \bar{\psi}_q(x)(i\not{D} - m_q)\psi_q(x) - \frac{1}{4}G^A_{\mu\nu}G^{\mu\nu}_A\,, \tag{2.44}$$

being the foundation of quantum chromodynamics. The covariant derivative $D_\mu = \partial_\mu + igT_A A^A_\mu$, the gluon field strength tensor $G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - gf^{ABC}A^B_\mu A^C_\nu$, and the abbreviations $\not{D} = D_\mu\gamma^\mu$, $G_{\mu\nu} = T_A G^A_{\mu\nu}$ are used in this work. The index q enumerates the flavor degrees of freedom; T_A are color generators, γ_μ Dirac matrices; μ, ν are Lorentz indices; A, B, C color indices; summation which underlies the matrix structures of Dirac and color objects is implied.

For a complete QCD sum rule the correlator has to be calculated by means of QCD in the validity range of large Euclidean space-like momenta $Q^2 = -q^2 > 0$ (compare Fig. 2.1). This condition is consistently met in the medium case for large imaginary energies with fixed three momentum $\vec{q}$. In this so called deep Euclidean space-like region the correlator can be expressed using the operator product expansion (OPE). It allows to expand a non-local product of composite operators in a series of local operators with increasingly divergent coefficient functions for $(x-y) \to 0$ (compare Section 2.1). The expectation values of the local operators lead to the QCD condensates.

Conceptionally, the OPE is organized to separate perturbative from non-perturbative effects, which requires the separation scale μ defining the two regimes. This has to be respected when perturbative loop integrals cover also small momenta $|p| < \mu$. Mass logarithms appearing then contribute to the non-perturbative parts and can be absorbed by the renormalization of condensates. Exemplary, for the D meson the redefinition of such condensates becomes important, see Section 3.3.1.

Qualitatively, the correlators take then, after Fourier transformation, the form

$$\Pi(q,v) = \sum_n C_n \left\langle \mathcal{O}_n \right\rangle \ . \tag{2.45}$$

The weights of the condensates, the Wilson coefficients C_n, can be calculated perturbatively. The OPE can further be improved by the renormalization group equation. Here, the condensates will be fixed by their values at a renormalization point of $\mu = 1$ GeV; $\langle \ldots \rangle = \langle \ldots \rangle_{\mu=1\ \mathrm{GeV}}$ will be understood throughout the thesis. A special role is played by the leading term $C_{\mathbb{1}}\mathbb{1}$, which contains the perturbative result. Based on the order of divergences the next terms are suppressed by higher powers of Q^2 and are therefore named non-perturbative power corrections in [2]. The non-perturbative character is encoded in the fact that condensates arise from a Wick expansion in normal ordered products. These vanish in perturbation theory. Via the non-vanishing condensates the non-perturbative effects are thus added to perturbation theory. The Wilson coefficients themselves are calculable by perturbative techniques as follows. The standard perturbative expansion of a correlation function is given by

$$\Pi^c_{[\mu][\nu]}(q) = i \int d^4x\, e^{iqx} \langle \Psi | T \left[\bar{\eta}_{[\mu]}(x) \eta_{[\nu]}(0) e^{i \int d^4x \mathcal{L}_I(x)} \right] |\Psi\rangle \,, \tag{2.46}$$

where

$$\begin{aligned}\mathcal{L}_I(x) =& - g\bar{\psi}_q(x) T_A A^A_\mu(x) \gamma^\mu \psi_q(x) + g f^{ABC} (\partial_\mu A^A_\nu(x)) A^\mu_B(x) A^\nu_C(x) \\ & - \frac{g^2}{4} f^{ABC} f^{ADE} A^B_\mu(x) A^C_\nu(x) A^\mu_D(x) A^\nu_E(x)\end{aligned} \tag{2.47}$$

is the interaction part of the Lagrangian density (derived from Eq. (2.44) with $\mathcal{L}_I = \mathcal{L} - \mathcal{L}_0$; $\mathcal{L}_0 = \mathcal{L}(g=0)$ is the free Lagrangian density). The superscript c denotes the restriction to connected graphs, $[\mu]$ stands for a set of Lorentz indices for example. The expansion is derived by means of Wick's theorem on the operator basis applied to any expansion parts.

A remark on the generalization to non-vanishing temperatures and/or densities is in order: The expectation value to be calculated becomes the Gibbs averaged correlation function (see Eq. (2.19)). Using a generalized form of Wick's theorem valid for expectation values, the expansion could be derived with the rules of Thermal Field Theory. However, finite temperatures T or chemical potentials μ represent additional new scales in the expansion. In QCD sum rules for a medium it is assumed that for small T and μ (compared to $\lambda_{QCD} \approx 1$ GeV) the Wilson coefficients remain the same, and the only T and μ dependence enters the condensates [43]. Therefore we can proceed the OPE calculation in the same way as in vacuum with two important generalizations: (a) The vacuum expectation values $\langle \mathcal{O} \rangle$ are to be replaced by the Gibbs averages $\langle\langle \ldots \rangle\rangle_{T,\mu}$; (b) The projection onto Lorentz scalar condensates involves further structures in medium, because the occurrence of matter is characterized by a medium specific Lorentz vector, e.g., the velocity of the matter rest frame v_μ.

The interpolating fields η in Eq. (2.46) consist of quark and gluon operators. Performing the Wick expansion for a specific composition of η leaves uncontracted products of quark and gluon operators taken in the QCD ground state. Upon projection onto Lorentz scalars they assemble the QCD condensates. The remaining contracted parts are treated as in perturbation theory using well known propagators. In this way, any operator product expansion (2.45) is technically obtained. Details for OPE calculations required for clarity and completeness are postponed to Appendix B.

2.3 QCD Condensates

The in the operator product expansion formally introduced QCD condensates measure as corrections to perturbation theory the non-perturbative structure of the QCD ground state. The condensates are ground state expectation values of normal ordered products of quark and gluons, projected onto Lorentz scalars for a Lorentz invariant parametrization. These expectation values are further supposed to be color singlets, gauge invariant and invariant with respect to time and parity reversal if we consider cold nuclear matter.

The strength of a power correction term $\propto Q^{-n}$ in the operator product expansion (2.45) is given by the Wilson coefficient. The overall constant mass dimension is set by the interpolating fields. Hence the dimension of the Wilson coefficient is compensated by the mass dimension of the accompanying condensate. Higher power corrections correspond therefore to condensates of higher mass dimension. It is thus reasonable to perform QSR evaluations up to a given mass dimension of condensates. In this section condensates ordered by increasing mass dimension up to dimension 6 are discussed.

Similar condensates appear in OPE's of various problems and can thus be understood like universal parameters of the QCD ground state. They absorb essentially much of the complexity of the theory of QCD. Partially, such expectation values can also be related to symmetries of the theory as follows.

2.3.1 Symmetries of QCD

The classical chromodynamic Lagrangian density $\mathcal{L}(x)$ given in Eq. (2.44), which QCD is based on, exhibits in the limit of vanishing quark masses $m_q = 0$ a set of symmetries, summarized in the chiral symmetry group $U(n_f)_L \times U(n_f)_R$ (e.g. [44, 45]). The independence of the symmetries $U(n_f)_L$ and $U(n_f)_R$ expresses the decoupling of left and right helicity states. In the assumed chiral limit $m_q = 0$, where chirality and helicity coincide, this corresponds to the decoupling of the left and right handed quark fields

$$\psi_L = \frac{1}{2}(1+\gamma_5)\psi \, , \qquad \psi_R = \frac{1}{2}(1-\gamma_5)\psi \, . \tag{2.48}$$

The spinors ψ without flavor index q, compare Eq. (2.44), are n_f-dimensional flavor vectors. As any unitary group can be decomposed into $U(n) = U(1) \times SU(n)$, the chiral symmetry group can be written $U(1)_L \times SU(n_f)_L \times U(1)_R \times SU(n_f)_R$ with the transformations

$$U(1)_L: \quad \psi_L \to \exp(i\theta_L)\psi_L \, , \qquad SU(n_f)_L: \quad \psi_L \to \exp(i\theta_L^a T_a)\psi_L \, , \tag{2.49}$$

$$U(1)_R: \quad \psi_R \to \exp(i\theta_R)\psi_R \, , \qquad SU(n_f)_R: \quad \psi_R \to \exp(i\theta_R^a T_a)\psi_R \, . \tag{2.50}$$

The transformations given in the fundamental (quark) representation are characterized by arbitrary angles $\theta_{L,R}$ or, respectively, sets of angles $\theta^a_{L,R}$ $(a = 1 \ldots (n_f^2 - 1))$.

The (N^2-1) traceless generators $T_a = \frac{1}{2}\lambda_a$ in the standard representation of any special unitary group $SU(N)$ are normalized, $Tr(T_a T_b) = \frac{1}{2}\delta_{ab}$, and obey

$$[T_a, T_b] \equiv T_a T_b - T_b T_a = i f_{abc} T_c \, , \tag{2.51}$$

$$\{T_a, T_b\} \equiv T_a T_b + T_b T_a = \frac{1}{N}\delta_{ab}\mathbb{1} + i d_{abc} T_c \, , \tag{2.52}$$

with the totally anti-symmetric structure constants f_{abc} and the symmetric structure constants d_{abc}.

Simultaneous transformations in the left and right handed sector can be combined into the equivalent form $U(1)_V \times SU(n_f)_V \times U(1)_A \times SU(n_f)_A$, which leads to the common vector

$(V : \alpha^{(a)} = \frac{1}{2}(\theta_L^{(a)} + \theta_R^{(a)}))$ and axial-vector $(A : \beta^{(a)} = \frac{1}{2}(\theta_L^{(a)} - \theta_R^{(a)}))$ transformations

$$
\begin{aligned}
U(1)_V: &\quad \psi \to \exp(i\alpha)\psi && \Rightarrow j_\mu(x) \propto \bar{\psi}\gamma_\mu\psi\ , && (2.53)\\
U(1)_A: &\quad \psi \to \exp(i\beta\gamma_5)\psi && \Rightarrow j_\mu(x) \propto \bar{\psi}\gamma_\mu\gamma_5\psi\ , && (2.54)\\
SU(n_f)_V: &\quad \psi \to \exp(i\alpha_a T^a)\psi && \Rightarrow j_\mu^a(x) \propto \bar{\psi}\gamma_\mu T^a\psi\ , && (2.55)\\
SU(n_f)_A: &\quad \psi \to \exp(i\beta_a T^a\gamma_5)\psi && \Rightarrow j_\mu^a(x) \propto \bar{\psi}\gamma_\mu\gamma_5 T^a\psi\ . && (2.56)
\end{aligned}
$$

The realized symmetries in the massless case imply the given classically conserved Noether currents $\partial^i j_i^{(a)}(x) = 0$.

The $U(1)_V$ symmetry, which yields the conservation of the vector current and thus of the total baryon number, is even satisfied in the case of arbitrarily different flavor quark masses, and is always fulfilled. The classical $U(1)_A$ symmetry, only satisfied in the $m_q = 0$ limit, is special, because the axial vector current is not conserved in the quantized theory due to the QCD axial anomaly. Within the discussion of condensates and symmetries we will especially be concerned with the behavior under $SU(n_f)_V$ and $SU(n_f)_A$ transformations.

In the presence of equal, finite current quark masses for all flavor degrees of freedom, the flavor-vector symmetry $SU(n_f)_V$, often abbreviated flavor symmetry, remains valid meaning the conservation of the flavor-vector current, the isospin current for $n_f = 2$. The symmetry with respect to $SU(n_f)_A$ is immediately violated by finite quark masses, as can be seen from a typical diagonal mass term in the Lagrangian density, which transforms like

$$
\begin{aligned}
SU(n_f)_V: &\quad m\bar{\psi}\psi \to m\bar{\psi}\psi\ , && (2.57)\\
SU(n_f)_A: &\quad m\bar{\psi}\psi \to m\bar{\psi}\gamma_0 \exp(-i\beta^a T_a\gamma_5)\gamma_0 \exp(i\beta^a T_a\gamma_5)\psi \\
&\qquad \approx m\bar{\psi}(1 - 2(\beta^a T_a)^2)\psi\ +\ 2im\beta^a\bar{\psi}\gamma_5 T_a\psi\ +\ \mathcal{O}(\beta^3) && (2.58)
\end{aligned}
$$

(the kinetic and gluonic terms are invariant in any case). Here, an expansion to second order in the angles β^a is performed; m is the scalar mass parameter common for all flavors (a general mass matrix in flavor space cannot be commutated with the $SU(n_f)$ generators).

Isospin symmetry is considered as approximate symmetry in the $n_f = 2$ sector, e.g. the current quark masses $m_u = (3 \pm 1)$ MeV and $m_d = (7 \pm 3)$ MeV (at a renormalization scale $\mu = 1$ GeV [1]), as well as their difference, are small compared to typical hadronic scales of 1 GeV. The isospin symmetry is realized in the hadron spectrum where for example the mass difference of the isospin partners proton and neutron amounts to only about 1 MeV. If also $SU(n_f)_A$ was realized in the physical hadron spectrum the energy states of parity partners, particles of opposite parity, would be degenerate. An excellent example are here the mesons ρ (770) and a_1 (1260). Since the small quark masses are not expected to generate via explicit $SU(n_f)_A$ symmetry breaking such strong splitting effects, another mechanism is considered responsible: The $SU(n_f)_A$ symmetry is said to be spontaneously broken, meaning that among all degenerated vacuum configurations, one ground state is realized. This leads to the existence of $(n_f^2 - 1)$ massless Goldstone bosons. (The explicit breaking by non-zero quark masses assigns a finite mass to the associated Goldstone bosons, which are then called pseudo-Goldstone bosons. In the $n_f = 2$ case, these bosons are identified with the pion triplet π^a having still masses significantly below 1 GeV.) The chiral symmetry group $SU(n_f)_V \times SU(n_f)_A$ is thus broken down to $SU(n_f)_V$.

Nambu-Goldstone Theorem

The violation of a symmetry by the ground state can be measured in ground state expectation values of quark and gluon operators, which were already introduced as QCD condensates. However, not every condensate provides information about broken symmetries. Obviously, purely gluonic expectation values have, for example, no direct relation to the chiral symmetries considered here.

Suitable quantities to measure the degree of spontaneous symmetry breaking in the physically realized ground state are called order parameters, e.g. [46]. Such parameters shall be formally introduced using the Goldstone theorem. Starting point is the Noether theorem which provides a conserved current with each global, continuous symmetry of the action $S = \int d^4x \; \mathcal{L}(x)$. The symmetries (2.53)-(2.56) of the Lagrangian itself are thereby included. These symmetries have to be extended to the quantized theory. The classically conserved quantities, the integrals over the time component of the respective Noether currents

$$Q^{(a)} = \int d^3x \, j_0^{(a)}(x) \,, \tag{2.59}$$

are thus substituted by operators $\hat{Q}^{(a)}$. These are the generators of the transformation group, meaning that their commutators with the field recover the infinitesimal field transformation $\delta\psi = \psi' - \psi$ by

$$i[\hat{Q}^{(a)}, \hat{\psi}] = -\delta\hat{\psi}^{(a)} \,. \tag{2.60}$$

From here on we abandon the operator hat symbols for simplicity. If the charge operator $Q^{(a)}$ represents a conserved quantity, it commutes with the Hamiltonian H, which implies

$$[Q^{(a)}, H] = 0 \;\Rightarrow\; H(Q^{(a)}|s\rangle) = Q^{(a)}H|s\rangle = E_s(Q^{(a)}|s\rangle) \,, \tag{2.61}$$

if $|s\rangle$ is an eigenstate of H to energy E_s: $H|s\rangle = E_s|s\rangle$.

If the ground state $|\Psi\rangle$ is symmetric w.r.t. the transformation generated by $Q^{(a)}$ then especially

$$e^{i\beta^{(a)}Q^{(a)}}|\Psi\rangle = |\Psi\rangle \;\Leftrightarrow\; Q^{(a)}|\Psi\rangle = 0 \,. \tag{2.62}$$

This realization of the symmetry is known as Wigner-Weyl phase.

Elsewise, if the ground state is not invariant under the transformation, one can formally assume $Q^{(a)}|\Psi\rangle \neq 0$.[4] A more rigorous definition of the violation of the symmetry by the ground state is given by the expectation value of the commutator between $Q^{(a)}$ and a local operator $\Phi(x)$

$$\langle\Psi|[Q^{(a)}, \Phi(x)]|\Psi\rangle \neq 0 \,. \tag{2.63}$$

Using the conservation of the underlying current $j_i^{(a)}(x)$, and upon insertion of a complete set of states in the commutator, properties of these states are deduced. This leads to the Nambu-Goldstone theorem, which says that these states are massless excitations with quantum numbers of $j_0^{(a)}(x)$, the Goldstone bosons.[5] The excitations couple besides $j_\mu^{(a)}$ also to the field Φ. This necessarily resides on the non-zero value in Eq. (2.63). Otherwise one falls back to the Wigner-Weyl realization of the symmetry. The quantity in (2.63) thus distinguishes between the Nambu-Goldstone and the Wigner-Weyl mode and qualifies as possible order parameter. Related to the generating property of the charge Eq. (2.60), this order parameter can be written as

$$\langle\Psi|[Q^{(a)}, \Phi(x)]|\Psi\rangle = i\langle\Psi|\delta\Phi^{(a)}|\Psi\rangle \,. \tag{2.64}$$

Now, the field Φ can itself consist of field operators ψ and the abstract commutator can be evaluated using standard commutators and anti-/commutator rules. This eventually leads to QCD condensates identified as potential order parameters for spontaneous symmetry breaking.

[4] This statement is not rigorous, since the existence of the charge integral implies symmetry as in the Wigner-Weyl phase. In the broken phase, the integral is divergent, which is circumvented introducing the commutator of the charge operator and a local operator $\Phi(x)$ [47]. This ensures convergence of the commutator.

[5] Variations of this proof can be found in e.g. [47, 48], for a collection of alternative proofs of Goldstone's theorem cf. [49].

As preliminary summary, the Goldstone theorem shows that dynamical breaking of a symmetry is qualitatively parametrized by order parameters, which behave differently in the symmetric and asymmetric phase under the considered symmetry transformation: In the unbroken phase they vanish, in the broken (or hidden) symmetry case they acquire non-zero values. This distinction is caused only by the ground state itself; the Lagrangian is assumed to possess the considered symmetry in any case.

An Example: The Chiral Condensate

The statements about order parameters have so far not been restricted to specific symmetries. As example the chiral condensate shall be identified as order parameter of spontaneous $SU(n_f)_A$ symmetry breaking in the given framework.

Using the canonical equal time anti-commutators (lower indices a, b denote flavor; α, β are Dirac indices; A, B stand for color)

$$\{\psi_{a\alpha A}(x), \psi^{\dagger}_{b\beta B}(y)\} = \delta_{ab}\delta_{\alpha\beta}\delta_{AB}\delta^{(3)}(x-y)\,, \tag{2.65}$$

$$\{\psi_{a\alpha A}(x), \psi_{b\beta B}(y)\} = \{\psi^{\dagger}_{a\alpha A}(x), \psi^{\dagger}_{b\beta B}(y)\} = 0\,, \tag{2.66}$$

one deduces with $[AB, C] = A\{B, C\} - \{A, C\}B$ the field variations from Eq. (2.60) as

$$[Q^a, \psi(x)] = -\gamma_5 T^a \psi(x)\,, \tag{2.67}$$

$$[Q^a, \psi^{\dagger}(x)] = \psi^{\dagger}(x)\gamma_5 T^a\,, \tag{2.68}$$

with the $SU(n_f)_A$ charge $Q^a = \int d^3x\, \psi^{\dagger}(x)\gamma_5 T^a \psi(x)$. These can also be obtained from the infinitesimal transformation Eq. (2.56).

To identify the chiral condensate we specify now the commutator (2.64) with $\Phi^b \equiv \overline{\psi} D T^b \psi$

$$[Q^a, \overline{\psi} D T^b \psi] = i\delta\Phi^{ab}\,, \tag{2.69}$$

where D is an arbitrary Dirac matrix. With $[A, BC] = [A, B]C + B[A, C]$ the commutator can be traced back to (2.67) and (2.68), and yields

$$i\delta\Phi^{ab} = -\bar{\psi}(\gamma_5 D T^a T^b + D\gamma_5 T^b T^a)\psi\,. \tag{2.70}$$

For $D = \gamma_5$ and $n_f = 2$, i.e. $T^a = \frac{1}{2}\tau^a$ (τ^a are Pauli matrices) one recovers in

$$\langle\Psi|[Q^a, \bar{\psi} D T^b \psi]|\Psi\rangle = -\frac{\delta_{ab}}{2}\langle\Psi|\bar{\psi}\psi|\Psi\rangle \tag{2.71}$$

the chiral condensate $\langle\Psi|\bar{\psi}\psi|\Psi\rangle$ as order parameter in the sense of definition (2.64). Fixing $b = a$, Φ^b directly induces the chiral condensate as order parameter for the spontaneous breaking of the symmetry related to the charge Q^a.

In the chiral limit of vanishing quark masses a non-zero value of the chiral condensate is directly linked to the spontaneous break down of chiral symmetry. However, in the real physical world the small values of the current quark masses contribute a decisive explicit symmetry breaking effect, generating a finite pion mass. This is still small in hadronic scales. Thus caused by the mass terms in the Lagrangian density the corresponding current is only partially conserved. The partial conservation of the axial vector current (PCAC) is utilized to determine the numerical value of the chiral condensate in Section 2.3.2.

Additional Chiral Order Parameters

Identification of further potential order parameters is now possible on the basis of Eq. (2.64) calculating commutators between the charge operator $Q^{(a)}$ and appropriate products of field operators Φ. If gluon operators are included in an ansatz for Φ which already induces a condensate then the expectation value with the gluon field added qualifies also as potential order parameter. The induced possible order parameters may however be excluded by other symmetry demands.

We focus on the spontaneous breaking of $SU(n_f)_A$ from here on. To find further candidates for order parameters with two quark operators we consider a general commutator

$$\langle \Psi | [Q^a, \bar{\psi} X \psi] | \Psi \rangle = -\langle \Psi | \bar{\psi} [\gamma_5 T^a X + X \gamma_5 T^a] \psi | \Psi \rangle \,, \tag{2.72}$$

where X is an arbitrary product of Dirac, flavor and color matrices. If we assume flavor symmetry for the condensates then the flavor part of X has to be chosen to eliminate T^a in order to introduce a unit matrix in flavor space. For parity reversal invariance, X must include a pseudo-tensor (i.e. a γ_5 or $\gamma_5 \gamma_\mu$ structure). Finally, an identity matrix in color space ensures color neutrality of the object. With $X^b = \gamma_5 D T^b \mathbb{1}_{col}$ ($D = \mathbb{1}, \gamma_\mu, \sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu]$) one obtains for any number of flavors n_f

$$\langle \Psi | [Q^a, \bar{\psi} X^b \psi] | \Psi \rangle = -\langle \Psi | \bar{\psi} D \{T^a, T^b\} \psi | \Psi \rangle \,, \tag{2.73}$$

$$= -\frac{\delta_{ab}}{n_f} \langle \Psi | \bar{\psi} D \psi | \Psi \rangle - i d_{abc} \langle \Psi | \bar{\psi} D T_c \psi | \Psi \rangle \,. \tag{2.74}$$

The latter term again vanishes due to flavor symmetry. For $D = \mathbb{1}$, Eq. (2.71) is recovered. The structure for $D = \sigma_{\mu\nu}$ contracted with the gluon field strength tensor $G^{\mu\nu}$ introduces the mixed quark-gluon condensate as order parameter candidate. In case $D = \gamma_\mu$, no Lorentz scalar condensate can be built in vacuum but in medium, see Section 2.3.2. By studying expectation values of commutators of the form

$$\langle \Psi | [Q^a, \Phi] | \Psi \rangle = \langle \Psi | [Q^a, \bar{\psi} X \psi \bar{\psi} Y \psi] | \Psi \rangle \,, \tag{2.75}$$

the list of potential order parameters can be increased by a number of four-quark condensates, as accomplished in Section 2.4.3.

The non-zero order parameters cause the existence of Goldstone bosons and measure a symmetry violation due to the ground state. If these particular condensates vanish, this reflects the symmetry restoration of the ground state, the Wigner-Weyl realization. The vanishing of these condensates in the latter symmetric phase is deeply connected to the influence of other (i.e. not spontaneously broken) symmetries. Such ones can be flavor symmetry or discrete symmetries like parity reversal invariance. We have already applied these to rule out some order parameters initially suggested by the commutator definition.

To point out the contact to other symmetries suppose the $SU(n_f)_A$ axial symmetry were also realized in the Wigner-Weyl form. Then the axial charge Q^a exists. With the invariance of the ground state, the expectation values of operators $\mathcal{O}$ are then equal under $SU(n_f)_A$

$$\langle \Psi | \mathcal{O} | \Psi \rangle = \langle \Psi' | \mathcal{O} | \Psi' \rangle = \langle \Psi | e^{-iQ} \mathcal{O} e^{iQ} | \Psi \rangle = \langle \Psi | \mathcal{O}' | \Psi \rangle \,. \tag{2.76}$$

If $\langle \Psi | \mathcal{O} | \Psi \rangle$ lies in a multiplet together with other expectation values which individually vanish due to ulterior symmetries, e.g. flavor symmetry, then the condensate $\langle \Psi | \mathcal{O} | \Psi \rangle$ itself must vanish. Consider for example the infinitesimal transformation of the bilinears

$$\begin{aligned} SU(n_f)_A : \quad \bar{\psi} D \psi \to & \bar{\psi} \gamma_0 \exp(-i\beta^a T_a \gamma_5) \gamma_0 D \exp(i\beta^a T_a \gamma_5) \psi \\ & \approx \bar{\psi} D (1 - (\beta^a T_a)^2) \psi \; - \bar{\psi} (\beta^a T_a)^2 \gamma_5 D \gamma_5 \psi \\ & + i\beta^a \bar{\psi} T_a [\gamma_5 D + D \gamma_5] \psi \; + \; \mathcal{O}(\beta^3) \end{aligned} \tag{2.77}$$

up to quadratic order in the parameter β^a (compare Eq. (2.58)), which are no invariants for $D = \mathbb{1}, \sigma_{\mu\nu}, \gamma_5$. The term linear in β^a then does not vanish but becomes only zero by the demanded flavor symmetry as the contributions from the projection of $(\beta^a T_a)^2$ proportional to T^b do. Then the condensate is under $SU(n_f)_A$ reproduced:

$$\langle\Psi|\bar{\psi}D\psi|\Psi\rangle = \gamma\langle\Psi|\bar{\psi}D\psi|\Psi\rangle\,, \tag{2.78}$$

besides the factor $\gamma \neq 1$, due to the projection of $(\beta^a T_a)^2$ onto the identity matrix. The equality in this multiplet can than only be satisfied by $\langle\Psi|\bar{\psi}D\psi|\Psi\rangle = 0$. This is again consistent with the vanishing of an order parameter in the Wigner-Weyl phase. For bilinears with Dirac basis elements $D\gamma_5 = -\gamma_5 D$, which is the case for $D = \gamma_\mu$ (the kinetic term) and $D = \gamma_5\gamma_\mu$, invariance can be shown beyond this infinitesimal treatment.

Therefore singlets, being also symmetric w.r.t. all other symmetries, have the advantage that their value is not constrained by other symmetry demands and can be a pure measure for the $SU(n_f)_A$ symmetry. Generally, also multiplets could fulfill these requirements, whereby the condensates would then be degenerate. Note, in the case $n_f = 2$, closed expressions can be given for finite transformations, which evidence these statements (cf. Section 2.4.3).

Finally we comment on the correspondence of chiral symmetry breaking terms and the mixing of left and right helicity contributions. To do so, we return to the decomposition into left and right handed spinors in the Dirac representation (2.48). Therewith, the decomposition of bilinears can be carried out to form two classes, namely,

(i) non-invariant forms

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\,, \tag{2.79}$$

$$\bar{\psi}\sigma_{\mu\nu}\psi = \bar{\psi}_L\sigma_{\mu\nu}\psi_R + \bar{\psi}_R\sigma_{\mu\nu}\psi_L\,, \tag{2.80}$$

$$\bar{\psi}\gamma_5\psi = \bar{\psi}_L\gamma_5\psi_R + \bar{\psi}_R\gamma_5\psi_L\,, \tag{2.81}$$

(ii) invariant forms

$$\bar{\psi}\gamma_\mu\psi = \bar{\psi}_L\gamma_\mu\psi_L + \bar{\psi}_R\gamma_\mu\psi_R\,, \tag{2.82}$$

$$\bar{\psi}\gamma_5\gamma_\mu\psi = \bar{\psi}_L\gamma_5\gamma_\mu\psi_L + \bar{\psi}_R\gamma_5\gamma_\mu\psi_R\,. \tag{2.83}$$

The first two non-invariants correspond to the chiral and the mixed quark-gluon condensates. The prescription left and right is convention, and neither one is distinguished. They should then be of equal size. If the order parameters are non-zero, each addend should be non-zero, too. This is physically interpreted as the mixing of left and right helicity states by the spontaneous symmetry breaking. Originally, this argument is already present with the mass term of the Lagrangian.

The invariant forms contain pairs of merely left or right parts, mixed terms vanish as consequence of $D\gamma_5 = -\gamma_5 D$, which also leads directly to the invariance under the axial transformation $SU(n_f)_A$ in (2.77). An interpretation as intermixing of the left and the right handed parts of the theory is then not apparent, similarly for four-quark condensates, where even both invariant and non-invariant forms admix.

In conclusion, the transition between phases of broken or restored symmetries is signalled by order parameters. Relying on the chiral condensate might be not sufficient if other mechanisms let this condensate vanish or the symmetry is only broken down to a lower symmetry. This suggests, that chiral symmetry restoration should not be linked only to the value of the chiral condensate, but to a set of qualified order parameters. In Section 2.4.3 the role of four-quark structures as possible order parameters is further pursued.

2.3.2 Catalog of QCD Condensates

The QCD condensates enter QCD sum rules as universal parameters and determine hadronic properties. Effects at finite baryon density n or temperature T are described by the change of condensates and the advent of new structures which are absent at vanishing density or temperature. In the QSR approach these changes reflect in modifications of hadrons embedded in a strongly interacting medium. Although, in principle, one might determine the changes of a few condensates from measured modifications of hadrons, the multitude of condensate values has to be evaluated elsewise. Only then the remaining unknowns can be fixed from experiment unambiguously.

QSR in a medium are conceptionally bound to small deviations from the vacuum case, since the operator expansion relies on a power counting of momenta. Scales of the order of these momenta, high temperatures and large chemical potentials, would demand for a complete rearrangement of the expansion. With the restriction to low temperature and density it suffices to expand the condensate changes to lowest order in these parameters. Extrapolations of condensate modifications and QSR results, e.g. to the phase transition boundary in the QCD phase diagram, must therefore be taken with care.

Commonly, medium QCD condensates are evaluated in the approximation of a thermodynamically equilibrated, dilute gas. Cold nuclear matter being the focus of this work is approximated as a non-interacting Fermi gas of nucleons [50]. The density dependence of condensates is governed by spin-averaged and isospin-averaged nucleon matrix elements taken for nucleons at rest, and has yet to be found for particular operators. In the zero temperature limit $T = 0$, the Fermi-Dirac distribution reduces to the step function and the medium modified condensates are given by

$$\langle \mathcal{O} \rangle_n = \langle \mathcal{O} \rangle_0 + \int \frac{d^3k}{2(2\pi)^3 E_k} \langle N(\vec{k})|\mathcal{O}|N(\vec{k})\rangle \Theta(\mu - E_k)\,, \tag{2.84}$$

with the nucleon states normalized as $\langle N(\vec{k})|N(\vec{k}')\rangle = 2E_k(2\pi)^3\delta(\vec{k}-\vec{k}')$, $E_k = \sqrt{M_N^2 + \vec{k}^2}$, and μ is the chemical potential. If the nucleon matrix element is independent of the nucleon momentum the remaining integral equals the number density of nucleons, the baryon density n. Then the condensates

$$\langle \mathcal{O} \rangle_n = \langle \mathcal{O} \rangle_0 + \frac{n}{2m} \langle N|\mathcal{O}|N\rangle\,. \tag{2.85}$$

change linearly as functions of the baryon density.

On the contrary, matter at zero net baryon density but non-vanishing temperature T is approximated as a Bose gas of the lowest lying hadronic excitations, the pions [51]. In the simplest approach again interactions are excluded. The medium corrections to the vacuum condensate values are then

$$\langle \mathcal{O} \rangle_T = \langle \mathcal{O} \rangle_0 + \sum_{a=1}^{3} \int \frac{d^3p}{2(2\pi)^3 E_\pi} \langle \pi^a(\vec{p})|\mathcal{O}|\pi^a(\vec{p})\rangle \frac{1}{e^{E_\pi/k_B T} - 1}\,. \tag{2.86}$$

The pion states with isospin index a and energy $E_\pi = \sqrt{\vec{p}^{\,2} + m_\pi^2}$ are covariantly normalized as $\langle \pi^a(\vec{p})|\pi^b(\vec{p}')\rangle = 2E_\pi(2\pi)^3\delta^{ab}\delta^{(3)}(\vec{p}-\vec{p}')$. If the pion matrix element is independent of the pion momentum and diagonal in isospin, $\langle \pi^a(\vec{p})|\mathcal{O}|\pi^b(\vec{p})\rangle = \delta^{ab}\langle \pi|\mathcal{O}|\pi\rangle$, the integral further reduces to

$$\langle \mathcal{O} \rangle_T = \langle \mathcal{O} \rangle_0 + \frac{T^2}{8} B_1(m_\pi/T) \langle \pi|\mathcal{O}|\pi\rangle\,. \tag{2.87}$$

The integral $B_1(z) = \frac{6}{\pi^2}\int_z^\infty dy \sqrt{y^2 - z^2}\frac{1}{e^y - 1}$ converges to 1 for $m_\pi \ll T$. Especially, in the chiral limit $m_\pi = 0$ the condensates then change proportional to the temperature squared.

Combining both approximations, the effects at low temperatures and small densities are summarized in

$$\langle\mathcal{O}\rangle_{n,T} = \langle\mathcal{O}\rangle_0 + \frac{n}{2M_N}\langle N|\mathcal{O}|N\rangle + \frac{T^2}{8}\langle\pi|\mathcal{O}|\pi\rangle + \dots . \tag{2.88}$$

in the chiral limit. The effect of non-vanishing temperature in the nucleon gas and of a finite pion mass is respected in Figs. 2.3 and 2.4, where the chiral and the gluon condensate are numerically evaluated within the outlined approximations. The inclusion of further massive excitations, like K and η mesons, becomes relevant only for $T > 100$ MeV [51]. Inclusion of nucleon-nucleon interactions as done in [44] does not significantly alter Fig. 2.3 in the low density region. In the following part the unknown matrix elements are discussed for the most important condensates, the chiral condensate and the gluon condensate. Besides this the numerical values of some condensates relevant for our subsequent evaluation of QCD sum rules are collected.

Chiral Condensate

The chiral condensate was in the previous section motivated as order parameter of spontaneous chiral symmetry breaking. The existence of pions as Goldstone bosons in the hadron spectrum means that the chiral condensate as order parameter should acquire a non-zero value. Beyond the chiral limit its value is also affected by explicit symmetry breaking in terms of the (light) current quark masses entering the QCD Lagrangian Eq. (2.44). The axial vector current $j^a_\mu = \bar\psi\gamma_\mu\gamma_5 T^a\psi$ is then not conserved but, by the equations of motion, satisfies

$$\partial^\mu j^a_\mu = i\bar\psi\gamma_5\{M, T^a\}\psi\,, \tag{2.89}$$

with the diagonal mass matrix

$$M = \mathrm{diag}(m_u, m_d) = \frac{1}{2}(m_u + m_d)\mathbb{1} + \frac{1}{2}(m_u - m_d)\tau_3 \tag{2.90}$$

in flavor space for two flavor degrees of freedom $n_f = 2$. One can use $\partial^\mu j^b_\mu(x) \neq 0$ as field $\Phi(x)$ in the commutator of Eq. (2.64)

$$\langle\Psi|[Q^a, \partial^\mu j^b_\mu]|\Psi\rangle = \langle\Psi|[Q^a, i\bar\psi\gamma_5\{M, T^b\}\psi]|\Psi\rangle\,. \tag{2.91}$$

With Eqs. (2.72) and (2.90) this commutator gives

$$\langle\Psi|[Q^a, \partial^\mu j^b_\mu]|\Psi\rangle = -\frac{i}{2}(m_u + m_d)\delta_{ab}\langle\Psi|\bar\psi\psi|\Psi\rangle - \frac{i}{2}(m_u - m_d)\delta_{3b}\langle\Psi|\bar\psi\tau^a\psi|\Psi\rangle\,, \tag{2.92}$$

where the second term on the r.h.s. drops out due to the assumed flavor symmetry. The Goldstone theorem states that the Goldstone bosons, interpreted as pions, couple to the current of the charge Q^a,

$$\langle\Psi|j^b_\mu(x)|\pi^c(q)\rangle = iq_\mu\delta^{bc}f_\pi(q^2)e^{-iqx}\,. \tag{2.93}$$

Here, the pion momentum q_μ is the only possible Lorentz vector to parametrize the Lorentz structure of the matrix element; the exponential is due to translation invariance and the function $f_\pi(q^2)$ is for a pion at rest defined as the pion decay constant $f_\pi = f_\pi(m^2_\pi)$. It can be determined from the weak decay constant of charged pions $\pi^+ \to \mu^+\nu$ (e.g. [52]). Defining the pion field similarly

$$\langle\Psi|\phi^b(x)|\pi^c(q)\rangle = \delta^{bc}e^{-iqx}\,, \tag{2.94}$$

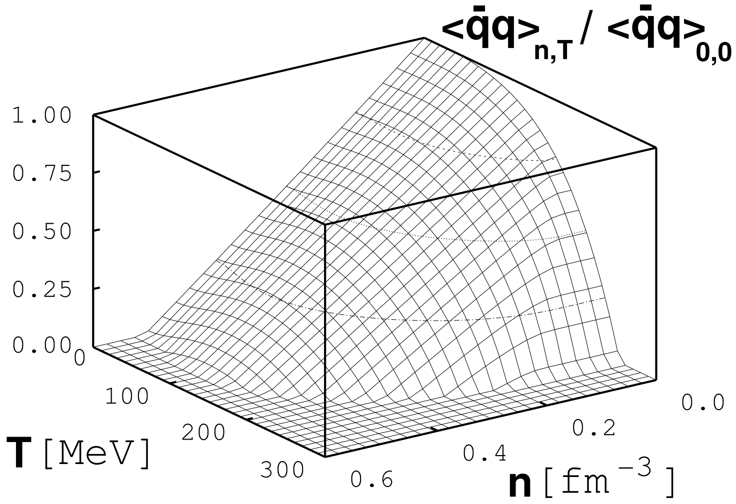

Figure 2.3: The relative changes of the chiral condensate as a function of density and temperature. Due to finite pion masses the temperature effect is negligible for small temperatures T. The slope in direction of the baryon density n is governed by the nucleon sigma term σ_N. The effects of non-vanishing T and n are covered in leading order, i.e. small values of T and n; the displayed range is extended strongly beyond the range of validity to expose the trends when extrapolating towards the confinement-deconfinement phase transition.

one may identify, taking the derivative in Eq. (2.93),

$$\partial^\mu j^b_\mu(x) = m_\pi^2 f_\pi \phi^b(x)\,, \tag{2.95}$$

what is often considered as partial conservation of the axial current (PCAC). For massless pions the axial current is exactly conserved.

Also the charge Q^a can be related to Eq. (2.93) by integration. Insertion of a complete set of covariantly normalized pion states

$$\sum_c \int d^3p \frac{1}{2(2\pi)^3 E_p} |\pi^c(p)\rangle\langle\pi^c(p)| \tag{2.96}$$

into the commutator, l.h.s. of Eq. (2.91), yields for pions at rest $\vec{p} = 0$ at time $t = 0$ the expression

$$\langle\Psi|[Q^a, \partial^\mu j^b_\mu|\Psi\rangle = i\delta^{ab} f_\pi^2 m_\pi^2\,. \tag{2.97}$$

This leads to the Gell-Mann–Oakes–Renner relation (GOR) [53]

$$f_\pi^2 m_\pi^2 = -\frac{m_u + m_d}{2}\langle\bar\psi\psi\rangle\,, \tag{2.98}$$

besides higher corrections in quark masses. We use as abbreviation for the ground state expectation value $\langle\Psi| : \bar\psi\psi : |\Psi\rangle \equiv \langle\bar\psi\psi\rangle$; $|\Psi\rangle$ is the physical QCD ground state, the normal ordering $: \ldots :$ of operators is understood from the proper definition of normal ordered currents etc. and not explicitly written out. Further, it is common to denote for the single flavor condensates due to flavor symmetry $\langle\bar qq\rangle = \langle\bar uu\rangle = \left\langle\bar dd\right\rangle$; thus $\left\langle\bar\psi\psi\right\rangle = 2\left\langle\bar qq\right\rangle$.

The GOR relation with $f_\pi = 92.4$ MeV, $m_\pi = 139.6$ MeV, $m_u = 4$ MeV, $m_d = 7$ MeV [1] yields the vacuum value of the chiral condensate $\langle\bar qq\rangle = -(247\ \mathrm{MeV})^3$. In what follows, the standard value used for all presented QSR results will be $\langle\bar qq\rangle = -(245\ \mathrm{MeV})^3$.

The change of the chiral condensate in cold nuclear matter in Eq. (2.85) is dictated by the nucleon matrix element $\langle N|\bar{q}q|N\rangle$, which defines the nucleon sigma term σ. We use

$$\langle \bar{q}q\rangle = -(0.245\ \mathrm{GeV})^3 + n\frac{\sigma_N}{2m_q}\,, \tag{2.99}$$

with $\sigma_N = 45\ \mathrm{MeV}$ and $m_q = 5.5\ \mathrm{MeV}$.

Gluon Condensate

The gluon condensate is related to the energy density by the trace of the energy-momentum tensor. For zero or infinite quark masses QCD exhibits a classical scale invariance, i.e. dilatation symmetry. On the quantum level this symmetry is broken by the regularization scale. Quark masses break scale invariance explicitly. The divergence of the dilatation current is then determined by the trace of the energy-momentum tensor, the so-called QCD trace anomaly

$$\Theta^\mu_\mu = -\frac{1}{8}(11 - \frac{2}{3}n_f)\frac{\alpha_s}{\pi}G^2 + \sum_q m_q\bar{q}q\,. \tag{2.100}$$

Therefrom the required nucleon matrix element is obtained, the vacuum part is constrained by charmonium QCD sum rules. For the gluon condensate,

$$\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle = -2\left\langle \frac{\alpha_s}{\pi}(\vec{E}^2 - \vec{B}^2)\right\rangle = -2[-0.5(0.33\ \mathrm{GeV})^4 + 0.325\ \mathrm{GeV}n] \tag{2.101}$$

is used here, where the relation to the chromoelectric and chromomagnetic fields $\vec{E}$ and $\vec{B}$ is also displayed; the contraction $G^2 = G^A_{\mu\nu}G^{\mu\nu}_A$ is understood.

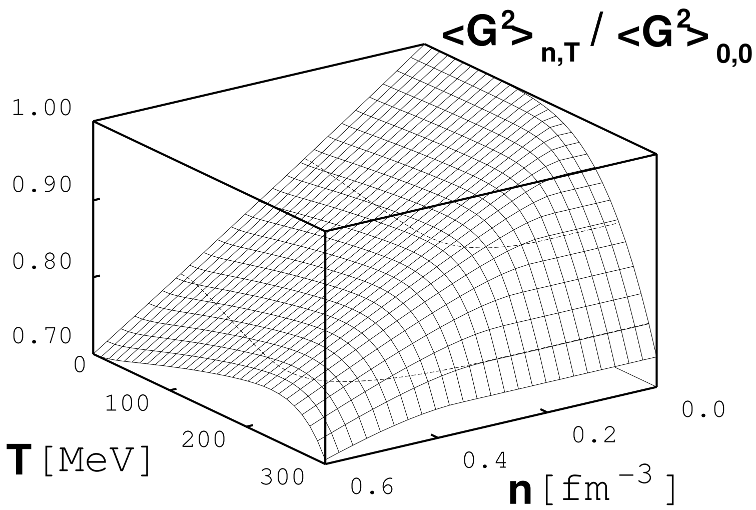

Figure 2.4: The gluon condensate as function of density and temperature normalized to its vacuum value. The impact of density and temperature is significantly weaker than for the chiral condensate, compare Fig. 2.3. The same caution as in Fig. 2.3 applies.

Further Condensates

We compile in increasing mass dimension some further condensates entering the later on presented sum rules. The medium specific condensates are given in the matter rest frame. The vector two-quark condensate (mass dimension 3) is

$$\langle \bar{q} \not{v} q \rangle = \langle \bar{q} \gamma_0 q \rangle = \left\langle q^\dagger q \right\rangle = \frac{3}{2} n \,, \tag{2.102}$$

counting simply the number of quarks in symmetric nuclear matter.

At mass dimension 4 additionally the medium gluon condensate

$$\left\langle \frac{\alpha_s}{\pi} \left[(vG)^2 + (v\tilde{G})^2 \right] \right\rangle = - \left\langle \frac{\alpha_s}{\pi} \left(\vec{E}^2 + \vec{B}^2 \right) \right\rangle = -0.1 \text{ GeV} n \tag{2.103}$$

and a condensate containing a derivative $\left\langle q^\dagger i D_0 q \right\rangle = 0.18 \text{ GeV} n$ arise. Note the abbreviations $G_{\mu\nu} = G^A_{\mu\nu} T_A$ and $\sigma G = \sigma_{\mu\nu} G^{\mu\nu}$.

Mixed quark-gluon condensates of mass dimension 5 are usually parametrized by mass dimension 3 condensates as

$$\langle g_s \bar{q} \sigma G q \rangle = x^2 \langle \bar{q} q \rangle + 3.0 \text{ GeV}^2 n \quad \text{with} \quad x^2 = 0.8 \text{ GeV}^2 \,. \tag{2.104}$$

Necessary are also the mass dimension 5 combinations

$$\langle \bar{q} i D_0 i D_0 q \rangle + \langle g_s \bar{q} \sigma G q \rangle / 8 = 0.3 \text{ GeV}^2 n \,, \tag{2.105}$$

$$\left\langle q^\dagger i D_0 i D_0 q \right\rangle + \left\langle g_s q^\dagger \sigma G q \right\rangle / 12 = (0.176 \text{ GeV})^2 n \,, \tag{2.106}$$

$$\left\langle g_s q^\dagger \sigma G q \right\rangle = -0.33 \text{ GeV}^2 n \,. \tag{2.107}$$

From the discussion in Section 2.3.1, especially Eq. (2.74), we see that besides the purely gluonic terms all condensates above might be related to order parameters. The density dependent parts of the condensates can be expressed through moments of parton distribution functions. If not otherwise stated the numerical values used in this work are those employed and discussed in [54]. At mass dimension 6 triple-gluon condensates and four-quark condensates appear. The latter type is the main focus of this thesis.

2.4 Four-Quark Condensates

Formally, four-quark condensates are QCD ground state expectation values of Hermitian products of four quark operators which are to be Dirac and Lorentz scalars, color singlets and are to be invariant under time and parity reversal. Thereby we restrict ourselves to equilibrated cold nuclear matter[6] but do not impose isospin symmetry from the very beginning in view of further applications, such as the proton-neutron mass difference in asymmetric cold nuclear matter (e.g. [55]). With the following discussion of independent four-quark condensates for arbitrary numbers of flavors we allow for the inclusion of strange quark contributions as well.

Physically, the four-quark condensates quantify the correlated production of two quark-antiquark pairs in the physical vacuum. In contrast to the square of the two-quark condensate, which accounts for uncorrelated production of two of these pairs, the four-quark condensates are a measure of the correlation and thus evidence the complexity of the QCD ground state. Especially, deviations from factorization, the approximation of unknown four-quark condensates in terms of

[6] The catalog can be extended to non-equilibrated systems lifting the demand for time reversal symmetry or to systems at finite temperature and vanishing chemical potential where charge conjugation provides a good symmetry.

the squared chiral condensate justified in the large N_c limit (cf. also [56]), represent effects of these more involved correlations.

In this section the classification of four-quark condensates, in the light quark sector, is performed in some detail. We do not cover other condensates of mass dimension 6 like the triple-gluon condensate or condensates containing higher derivatives.

2.4.1 Projection and Classification

The projections onto Dirac, Lorentz and color structures lead to all possible in-medium four-quark condensates just as for the example of the non-local two-quark expectation value in Appendix B. However, the situation is even simpler since we are only interested in the mass dimension 6 four-quark condensates, so derivatives are not required and all operators in four-quark expectation values are to be taken at $x = 0$.

Using the Clifford bases $O_k \in \{\mathbb{1}, \gamma_\mu, \sigma_{\mu<\nu}, i\gamma_5\gamma_\mu, \gamma_5\}$ and $O^m \in \{\mathbb{1}, \gamma^\mu, \sigma^{\mu<\nu}, i\gamma_5\gamma^\mu, \gamma_5\}$, which fulfill $\mathrm{Tr}\,(O_k O^m) = 4\delta_k^m$, one can project out the Dirac indices of products of four arbitrary quark operators

$$\Big(\underset{\mathrm{e}}{\bar{q}_1}^{a'}\underset{\mathrm{f}}{q_2}^{a}\underset{\mathrm{g}}{\bar{q}_3}^{b'}\underset{\mathrm{h}}{q_4}^{b}\Big) = \frac{1}{16}\sum_{k,l=1}^{16}\left(\bar{q}_1^{a'} O_k q_2^a \bar{q}_3^{b'} O^l q_4^b\right) \underset{\mathrm{f,e}}{O^k}\, \underset{\mathrm{h,g}}{O_l}\,. \tag{2.108}$$

Note, here Dirac indices, if explicitly shown, are attached below the concerned objects. From Eq. (2.108) there are 25 combinatorial Lorentz structures which have to be projected on condensates to obey Lorentz invariance (using the four-velocity v_μ), time/parity reversal and hermiticity. For each of the remaining 5 (10) Lorentz scalars in vacuum (medium) two possible color singlet combinations can be formed using contractions with the unity element and the generators $\lambda^A = 2T^A$ of $SU(N_c = 3)$. Thus one obtains the projection formula

$$\bar{q}_1^{a'} q_2^a \bar{q}_3^{b'} q_4^b = \frac{1}{9}\left(\bar{q}_1 q_2 \bar{q}_3 q_4\right)\mathbb{1}_{aa'}\mathbb{1}_{bb'} + \frac{1}{12}\left(\bar{q}_1\lambda^A q_2 \bar{q}_3 \lambda^A q_4\right)\lambda^B_{aa'}\lambda^B_{bb'}\,. \tag{2.109}$$

Especially, in the calculation of an operator product expansion for baryons the color condensate structures naturally arise from the product $\epsilon_{abc}\epsilon_{a'b'c'}\ \delta^{cc'} = \epsilon_{abc}\epsilon_{a'b'c} = \delta_{aa'}\delta_{bb'} - \delta_{ab'}\delta_{a'b}$ due to the color structure of the baryon interpolating fields. Hence there the four-quark condensates generally appear in linear combinations of color structures in the form

$$\epsilon_{abc}\epsilon_{a'b'c}\ \bar{q}_1^{a'} q_2^a \bar{q}_3^{b'} q_4^b = \frac{2}{3}\left\{\left(\bar{q}_1 q_2 \bar{q}_3 q_4\right) - \frac{3}{4}\left(\bar{q}_1\lambda^A q_2 \bar{q}_3\lambda^A q_4\right)\right\}\,. \tag{2.110}$$

This would imply two condensate structures for each Lorentz scalar term. However, for expectation values with just one flavor (pure flavor four-quark condensates) these structures are not independent. Combining Fierz rearrangement of the Dirac contractions of pure four-quark operators with the rearrangement of the color structures, compare Appendix C.3, one derives the transformation equation

$$\left(\bar{u}O_k\lambda^A u\bar{u}O^l\lambda^A u\right) = -\frac{2}{3}\left(\bar{u}O_k u\bar{u}O^l u\right) - \frac{1}{8}\mathrm{Tr}\left(O_k O_n O^l O^m\right)\left(\bar{u}O_m u\bar{u}O^n u\right)\,, \tag{2.111}$$

which relates the two different color combinations. This transformation can be brought in matrix form $\vec{y} = \hat{A}\vec{x}$ with

$$\vec{y} = \begin{pmatrix} \langle \bar{q}\lambda^A q \bar{q}\lambda^A q \rangle \\ \langle \bar{q}\gamma_\alpha \lambda^A q \bar{q}\gamma^\alpha \lambda^A q \rangle \\ \langle \bar{q}\not{v}\lambda^A q \bar{q}\not{v}\lambda^A q \rangle /v^2 \\ \langle \bar{q}\sigma_{\alpha\beta}\lambda^A q \bar{q}\sigma^{\alpha\beta}\lambda^A q \rangle \\ \langle \bar{q}\sigma_{\alpha\beta}\lambda^A q \bar{q}\sigma^{\gamma\delta}\lambda^A q \rangle g^\alpha_\gamma v^\beta v_\delta /v^2 \\ \langle \bar{q}\gamma_5\gamma_\alpha \lambda^A q \bar{q}\gamma_5\gamma^\alpha \lambda^A q \rangle \\ \langle \bar{q}\gamma_5\not{v}\lambda^A q \bar{q}\gamma_5\not{v}\lambda^A q \rangle /v^2 \\ \langle \bar{q}\gamma_5\lambda^A q \bar{q}\gamma_5\lambda^A q \rangle \\ \langle \bar{q}\not{v}\lambda^A q \bar{q}\lambda^A q \rangle \\ \langle \bar{q}\gamma_5\gamma^\alpha \lambda^A q \bar{q}\sigma^{\beta\gamma}\lambda^A q \rangle i\epsilon_{\alpha\beta\gamma\delta}v^\delta/2 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} \langle \bar{q}q\bar{q}q \rangle \\ \langle \bar{q}\gamma_\alpha q \bar{q}\gamma^\alpha q \rangle \\ \langle \bar{q}\not{v} q \bar{q}\not{v} q \rangle /v^2 \\ \langle \bar{q}\sigma_{\alpha\beta} q \bar{q}\sigma^{\alpha\beta} q \rangle \\ \langle \bar{q}\sigma_{\alpha\beta} q \bar{q}\sigma^{\gamma\delta} q \rangle g^\alpha_\gamma v^\beta v_\delta /v^2 \\ \langle \bar{q}\gamma_5\gamma_\alpha q \bar{q}\gamma_5\gamma^\alpha q \rangle \\ \langle \bar{q}\gamma_5\not{v} q \bar{q}\gamma_5\not{v} q \rangle /v^2 \\ \langle \bar{q}\gamma_5 q \bar{q}\gamma_5 q \rangle \\ \langle \bar{q}\not{v} q \bar{q} q \rangle \\ \langle \bar{q}\gamma_5\gamma^\alpha q \bar{q}\sigma^{\beta\gamma} q \rangle i\epsilon_{\alpha\beta\gamma\delta}v^\delta/2 \end{pmatrix}, \tag{2.112}$$

$$\hat{A} = \begin{pmatrix} -7/6 & -1/2 & 0 & -1/4 & 0 & 1/2 & 0 & -1/2 & 0 & 0 \\ -2 & 1/3 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ -1/2 & 1/2 & -5/3 & -1/4 & 1 & 1/2 & -1 & 1/2 & 0 & 0 \\ -6 & 0 & 0 & 1/3 & 0 & 0 & 0 & -6 & 0 & 0 \\ -3/2 & -1/2 & 2 & 1/4 & -2/3 & 1/2 & -2 & -3/2 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1/3 & 0 & -2 & 0 & 0 \\ 1/2 & 1/2 & -1 & 1/4 & -1 & 1/2 & -5/3 & -1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & -1/4 & 0 & -1/2 & 0 & -7/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5/3 & -i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3i & 1/3 \end{pmatrix}. \tag{2.113}$$

We emphasize that the inverse transformation $\hat{A}^{-1}$ exists. However, structures for baryon sum rules typically are combinations of two color contractions, dictated by Eq. (2.110), which form components of the vector

$$\vec{z} = \frac{2}{3}\left(\vec{x} - \frac{3}{4}\vec{y}\right) = \hat{B}\vec{x} = \frac{2}{3}\left(\hat{A}^{-1} - \frac{3}{4}\mathbb{1}\right)\vec{y}, \quad \hat{B} \equiv \tfrac{2}{3}(\mathbb{1} - \tfrac{3}{4}\hat{A}). \tag{2.114}$$

The matrix $\hat{B}$ has the fivefold eigenvalues 0 and 2, and the corresponding eigenspaces both have dimension 5, especially the kernel of $\hat{B}$ spanned by the eigenvectors to eigenvalue 0. The fact that the kernel contains more than the null vector implies that $\hat{B}$ has no inverse. The transformation of this equation into the basis of eigenvectors yields a new vector $\vec{z}\,'$ where 5 elements are to be zero. Written in components of $\vec{z}$ these relations are

$$z_2 + z_6 = 0\,, \tag{2.115a}$$
$$4z_1 - 2z_2 - z_4 = 0\,, \tag{2.115b}$$
$$2z_1 - z_4 + 2z_8 = 0\,, \tag{2.115c}$$
$$z_1 - z_3 - z_5 + z_7 = 0\,, \tag{2.115d}$$
$$z_9 - iz_{10} = 0\,. \tag{2.115e}$$

The first three conditions occur already in the vacuum set, the latter two constraints are additional in the medium case. Of course, the conditions can be written differently, e.g., the second and third line may be conveniently combined to $z_1 - z_2 - z_8 = 0$ for applications. An alternative derivation of these relations is presented in Appendix C.1.

The relations (2.115) have two important consequences: firstly, they allow to simplify pure flavor four-quark condensates in baryon sum rules; secondly, since Eq. (2.114) can not be inverted, they forbid a direct translation from pure flavor four-quark condensates in baryon sum rules at the order α_s^0 to those which occur e.g. in sum rules for light vector mesons in the order α_s^1.

2.4.2 Factorization and Parametrization of Four-Quark Condensates

Up to now we have introduced all possible four-quark condensates in the light quark sector. These structures will appear in the QCD sum rules for the ω meson and the nucleon. To evaluate the sum rule equations with the focus on particular combinations of four-quark condensates one is faced with the common problem of the poor knowledge of four-quark condensates. Usually assuming the vacuum saturation hypothesis or resorting to the large N_c limit the four-quark condensates are factorized into products of condensates with two quark operators. The factorization of four-quark condensates allows to set the proper units, however, its reliability is a matter of debate. For instance, [57] state that the four-quark condensates in the nucleon sum rule are the expectation value of a chirally invariant operator, while $\langle\bar{q}q\rangle^2$ is not invariant and thus a substitution by the factorized form would be inconsistent with the chiral perturbation theory expression for the nucleon self-energy. The four-quark condensates breaking chiral symmetry might have a meaningful connection to the chiral condensate but for the chirally invariant structures such a closer relation to $\langle\bar{q}q\rangle$ is not clear [58].

Moreover, for nucleon sum rules at finite temperature T (and vanishing chemical potential) it was argued in [59] that the four-quark condensates are T independent quite different from the behavior of $\langle\bar{q}q\rangle^2$ which is why a naive factorization would lead to artificial temperature effects in the nucleon mass.

For numerical purposes it is convenient to correct the values deduced from factorization by factors, denoted as κ, and examine the effect of these correction factors on predictions from QCD sum rules. In this section, the four-quark condensates classified so far in general are spelled out and the parametrization with a set of quantities κ is defined. In doing so one includes a density dependent factor $\kappa(n)$ in the factorized result

$$\langle\bar{q}_{f1}\Gamma_1\mathbb{C}_1 q_{f1}\bar{q}_{f2}\Gamma_2\mathbb{C}_2 q_{f2}\rangle = \kappa(n)\,\langle\bar{q}_{f1}\Gamma_1\mathbb{C}_1 q_{f1}\bar{q}_{f2}\Gamma_2\mathbb{C}_2 q_{f2}\rangle_{\mathrm{fac}}\ , \tag{2.116}$$

where κ and the following parametrization depend on the specific condensate structure. In linear density approximation this product ansatz obtains contributions both from the expansion $\kappa(n) = \kappa^{(0)} + \kappa^{(1)}n$ with $\kappa^{(1)} = \frac{\partial\kappa(0)}{\partial n}$ and from the linearized, factorized four-quark condensate expression $\langle\bar{q}_{f1}\Gamma_1\mathbb{C}_1 q_{f1}\bar{q}_{f2}\Gamma_2\mathbb{C}_2 q_{f2}\rangle_{\mathrm{fac}} = a + bn$. If $\kappa^{(0)} = 1$, then $\kappa^{(1)} = 0$ recovers the usual factorization, which means the four-quark condensate behaves like the product of two two-quark condensates; $\kappa^{(1)} > 0$ represents a stronger density dependence with respect to the factorization and vice versa. Inserting both expansions one can also describe the total density dependence of the condensates by the combination $\kappa^{\mathrm{med}} = \kappa^{(0)} + \frac{a}{b}\kappa^{(1)}$,

$$\langle\bar{q}_{f1}\Gamma_1\mathbb{C}_1 q_{f1}\bar{q}_{f2}\Gamma_2\mathbb{C}_2 q_{f2}\rangle = a\kappa^{(0)} + b\kappa^{\mathrm{med}}n \tag{2.117}$$

such that for $\kappa^{\mathrm{med}} = 0$ the condensate is (in first order) independent of density. For condensates with vanishing a or b in factorization we choose $a = \langle\bar{q}q\rangle_{\mathrm{vac}}^2$ and $b = \langle\bar{q}q\rangle_{\mathrm{vac}}\,\sigma_{\mathrm{N}}/m_q$ as scales to study deviations from zero and denote these instances by $\tilde{\kappa}$.[7] The classification of possible four-quark condensates is collected together with the specific κ parametrization in Tabs. 2.1 and 2.2.

[7] For consistency with earlier publications we use the labels "0" and "vac" in parallel to denote the vacuum limit.

Indices	Full condensate	Parametrized Factorization in Linear Density Approximation
1s	$\langle \bar{u}u\bar{u}u \rangle$	$\frac{11}{12}\left(\kappa_{1\mathrm{s}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{s}}^{\mathrm{med}} n\xi\right)$
1v	$\langle \bar{u}\gamma_\alpha u\bar{u}\gamma^\alpha u \rangle$	$-\frac{1}{3}\left(\kappa_{1\mathrm{v}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{v}}^{\mathrm{med}} n\xi\right)$
1v′	$\langle \bar{u}\not{v} u\bar{u}\not{v} u \rangle /v^2$	$-\frac{1}{12}\left(\frac{1}{4}\kappa_{1\mathrm{v}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{v}'}^{\mathrm{med}} n\xi\right)$
1t	$\left\langle \bar{u}\sigma_{\alpha\beta} u\bar{u}\sigma^{\alpha\beta} u \right\rangle$	$-\left(\kappa_{1\mathrm{t}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{t}}^{\mathrm{med}} n\xi\right)$
1t′	$\left\langle \bar{u}\sigma_{\alpha\beta} u\bar{u}\sigma^{\gamma\delta} u \right\rangle g_\gamma^\alpha v^\beta v_\delta/v^2$	$-\frac{1}{4}\left(\frac{1}{4}\kappa_{1\mathrm{t}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{t}'}^{\mathrm{med}} n\xi\right)$
1a	$\langle \bar{u}\gamma_5\gamma_\alpha u\bar{u}\gamma_5\gamma^\alpha u \rangle$	$\frac{1}{3}\left(\kappa_{1\mathrm{a}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{a}}^{\mathrm{med}} n\xi\right)$
1a′	$\langle \bar{u}\gamma_5\not{v} u\bar{u}\gamma_5\not{v} u \rangle /v^2$	$\frac{1}{12}\left(\frac{1}{4}\kappa_{1\mathrm{a}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{a}'}^{\mathrm{med}} n\xi\right)$
1p	$\langle \bar{u}\gamma_5 u\bar{u}\gamma_5 u \rangle$	$-\frac{1}{12}\left(\kappa_{1\mathrm{p}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{1\mathrm{p}}^{\mathrm{med}} n\xi\right)$
1vs	$\langle \bar{u}\not{v} u\bar{u}u \rangle$	$\tilde{\kappa}_{1\mathrm{vs}}^{\mathrm{med}} n\xi$
1at	$\langle \bar{u}\gamma_5\gamma_\kappa u\bar{u}\sigma_{\lambda\pi} u \rangle \epsilon^{\kappa\lambda\pi\xi} v_\xi$	$\tilde{\kappa}_{1\mathrm{at}}^{\mathrm{med}} n\xi$
2s	$\left\langle \bar{u}\lambda^A u\bar{u}\lambda^A u \right\rangle$	$-\frac{4}{9}\left(\kappa_{2\mathrm{s}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{s}}^{\mathrm{med}} n\xi\right)$
2v	$\left\langle \bar{u}\gamma_\alpha\lambda^A u\bar{u}\gamma^\alpha\lambda^A u \right\rangle$	$-\frac{16}{9}\left(\kappa_{2\mathrm{v}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{v}}^{\mathrm{med}} n\xi\right)$
2v′	$\left\langle \bar{u}\not{v}\lambda^A u\bar{u}\not{v}\lambda^A u \right\rangle /v^2$	$-\frac{4}{9}\left(\frac{1}{4}\kappa_{2\mathrm{v}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{v}'}^{\mathrm{med}} n\xi\right)$
2t	$\left\langle \bar{u}\sigma_{\alpha\beta}\lambda^A u\bar{u}\sigma^{\alpha\beta}\lambda^A u \right\rangle$	$-\frac{16}{3}\left(\kappa_{2\mathrm{t}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{t}}^{\mathrm{med}} n\xi\right)$
2t′	$\left\langle \bar{u}\sigma_{\alpha\beta}\lambda^A u\bar{u}\sigma^{\gamma\delta}\lambda^A u \right\rangle g_\gamma^\alpha v^\beta v_\delta/v^2$	$-\frac{4}{3}\left(\frac{1}{4}\kappa_{2\mathrm{t}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{t}'}^{\mathrm{med}} n\xi\right)$
2a	$\left\langle \bar{u}\gamma_5\gamma_\alpha\lambda^A u\bar{u}\gamma_5\gamma^\alpha\lambda^A u \right\rangle$	$\frac{16}{9}\left(\kappa_{2\mathrm{a}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{a}}^{\mathrm{med}} n\xi\right)$
2a′	$\left\langle \bar{u}\gamma_5\not{v}\lambda^A u\bar{u}\gamma_5\not{v}\lambda^A u \right\rangle /v^2$	$\frac{4}{9}\left(\frac{1}{4}\kappa_{2\mathrm{a}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{a}'}^{\mathrm{med}} n\xi\right)$
2p	$\left\langle \bar{u}\gamma_5\lambda^A u\bar{u}\gamma_5\lambda^A u \right\rangle$	$-\frac{4}{9}\left(\kappa_{2\mathrm{p}}^{\mathrm{vac}} \langle\bar{q}q\rangle_{\mathrm{vac}}^2 + \kappa_{2\mathrm{p}}^{\mathrm{med}} n\xi\right)$
2vs	$\left\langle \bar{u}\not{v}\lambda^A u\bar{u}\lambda^A u \right\rangle$	$\tilde{\kappa}_{2\mathrm{vs}}^{\mathrm{med}} n\xi$
2at	$\left\langle \bar{u}\gamma_5\gamma_\kappa\lambda^A u\bar{u}\sigma_{\lambda\pi}\lambda^A u \right\rangle \epsilon^{\kappa\lambda\pi\xi} v_\xi$	$\tilde{\kappa}_{2\mathrm{at}}^{\mathrm{med}} n\xi$

Table 2.1: Two complete sets (indices 1 and 2) of independent non-flavor-mixing four-quark condensates differing in color structure and their parametrization with κ in strict linear density approximation ($\xi = \langle\bar{q}q\rangle_{\mathrm{vac}}\sigma_{\mathrm{N}}/m_{\mathrm{q}}$). The sets are related by a Fierz transformation. A similar table for flavor d instead u appears for an exhaustive list of four-quark condensates for the two-flavor case $n_f = 2$.

Indices	Full condensate	Parametrized Factorization in Linear Density Approximation
3s	$\left\langle \bar{u}u\bar{d}d \right\rangle$	$\kappa_{3s}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \kappa_{3s}^{\rm med} n\xi$
3v	$\left\langle \bar{u}\gamma_\alpha u\bar{d}\gamma^\alpha d \right\rangle$	$\tilde{\kappa}_{3v}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{3v}^{\rm med} n\xi$
3v′	$\left\langle \bar{u}\not{v}u\bar{d}\not{v}d \right\rangle/v^2$	$\frac{1}{4}\tilde{\kappa}_{3v}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{3v'}^{\rm med} n\xi$
3t	$\left\langle \bar{u}\sigma_{\alpha\beta}u\bar{d}\sigma^{\alpha\beta}d \right\rangle$	$\tilde{\kappa}_{3t}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{3t}^{\rm med} n\xi$
3t′	$\left\langle \bar{u}\sigma_{\alpha\beta}u\bar{d}\sigma^{\gamma\delta}d \right\rangle g^\alpha_\gamma v^\beta v_\delta/v^2$	$\frac{1}{4}\tilde{\kappa}_{3t}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{3t'}^{\rm med} n\xi$
3a	$\left\langle \bar{u}\gamma_5\gamma_\alpha u\bar{d}\gamma_5\gamma^\alpha d \right\rangle$	$\tilde{\kappa}_{3a}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{3a}^{\rm med} n\xi$
3a′	$\left\langle \bar{u}\gamma_5\not{v}u\bar{d}\gamma_5\not{v}d \right\rangle/v^2$	$\frac{1}{4}\tilde{\kappa}_{3a}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{3a'}^{\rm med} n\xi$
3p	$\left\langle \bar{u}\gamma_5 u\bar{d}\gamma_5 d \right\rangle$	$\tilde{\kappa}_{3p}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{3p}^{\rm med} n\xi$
3vs	$\left\langle \bar{u}\not{v}u\bar{d}d \right\rangle$	$\kappa_{3vs}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$
3at	$\left\langle \bar{u}\gamma_5\gamma_\kappa u\bar{d}\sigma_{\lambda\pi}d \right\rangle \epsilon^{\kappa\lambda\pi\xi}v_\xi$	$\kappa_{3at}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$
4s	$\left\langle \bar{u}\lambda^A u\bar{d}\lambda^A d \right\rangle$	$\tilde{\kappa}_{4s}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4s}^{\rm med} n\xi$
4v	$\left\langle \bar{u}\gamma_\alpha\lambda^A u\bar{d}\gamma^\alpha\lambda^A d \right\rangle$	$\tilde{\kappa}_{4v}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4v}^{\rm med} n\xi$
4v′	$\left\langle \bar{u}\not{v}\lambda^A u\bar{d}\not{v}\lambda^A d \right\rangle/v^2$	$\frac{1}{4}\tilde{\kappa}_{4v}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4v'}^{\rm med} n\xi$
4t	$\left\langle \bar{u}\sigma_{\alpha\beta}\lambda^A u\bar{d}\sigma^{\alpha\beta}\lambda^A d \right\rangle$	$\tilde{\kappa}_{4t}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4t}^{\rm med} n\xi$
4t′	$\left\langle \bar{u}\sigma_{\alpha\beta}\lambda^A u\bar{d}\sigma^{\gamma\delta}\lambda^A d \right\rangle g^\alpha_\gamma v^\beta v_\delta/v^2$	$\frac{1}{4}\tilde{\kappa}_{4t}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4t'}^{\rm med} n\xi$
4a	$\left\langle \bar{u}\gamma_5\gamma_\alpha\lambda^A u\bar{d}\gamma_5\gamma^\alpha\lambda^A d \right\rangle$	$\tilde{\kappa}_{4a}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4a}^{\rm med} n\xi$
4a′	$\left\langle \bar{u}\gamma_5\not{v}\lambda^A u\bar{d}\gamma_5\not{v}\lambda^A d \right\rangle/v^2$	$\frac{1}{4}\tilde{\kappa}_{4a}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4a'}^{\rm med} n\xi$
4p	$\left\langle \bar{u}\gamma_5\lambda^A u\bar{d}\gamma_5\lambda^A d \right\rangle$	$\tilde{\kappa}_{4p}^{\rm vac} \langle \bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{4p}^{\rm med} n\xi$
4vs	$\left\langle \bar{u}\not{v}\lambda^A u\bar{d}\lambda^A d \right\rangle$	$\tilde{\kappa}_{4vs}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$
4at	$\left\langle \bar{u}\gamma_5\gamma_\kappa\lambda^A u\bar{d}\sigma_{\lambda\pi}\lambda^A d \right\rangle \epsilon^{\kappa\lambda\pi\xi}v_\xi$	$\kappa_{4at}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$
5vs	$\left\langle \bar{d}\not{v}d\bar{u}u \right\rangle$	$\kappa_{5vs}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$
5at	$\left\langle \bar{d}\gamma_5\gamma_\kappa d\bar{u}\sigma_{\lambda\pi}u \right\rangle \epsilon^{\kappa\lambda\pi\xi}v_\xi$	$\kappa_{5at}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$
6vs	$\left\langle \bar{d}\not{v}\lambda^A d\bar{u}\lambda^A u \right\rangle$	$\tilde{\kappa}_{6vs}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$
6at	$\left\langle \bar{d}\gamma_5\gamma_\kappa\lambda^A d\bar{u}\sigma_{\lambda\pi}\lambda^A u \right\rangle \epsilon^{\kappa\lambda\pi\xi}v_\xi$	$\kappa_{6at}^{\rm med} \langle \bar{q}q\rangle_{\rm vac} 3n/2$

Table 2.2: A complete set of independent flavor-mixing four-quark condensates and their parametrization by κ parameters in strict linear density approximation. Additional parameters (indices 5 and 6) are required for structures which cannot be exchanged.

Non-Flavor Mixing Case

The condensates which contain only one flavor are listed in Tab. 2.1. From the demand for parity and time reversal invariance only 5 (10) Dirac and Lorentz scalar four quark operators remained in vacuum (medium). Further, these structures carry color indices and must be projected on colorless objects for which there are two ways. However, since the same flavors occur, both color combinations can be alternatively rearranged via Fierz transformation. Hence, there are only 5 (10) independent κ parameter sets in the Tab. 2.1, although both color alternatives are listed. The parameter sets with indices $1, 2$ are related by the transformation (2.113).

Flavor Mixing Case

Here the condensates containing two quark operator pairs are distinguished by flavor. The numbering is as for the pure flavor structures. However, the conversion of the two color contractions is not possible due to different flavors. Compared to the non-flavor mixing case the missing exchange symmetry of $\bar{q}q$ contractions due to different flavors allows additional placements of Dirac matrices and thus leads to 4 additional condensate structures in medium (see Tab. 2.2). Therefore, 10 (24) flavor-mixed four quark condensates and thus κ parameter pairs appear in vacuum (medium).

To sum up, there exist in medium $\{$vacuum$\}$ for n_f flavors without flavor symmetry taken into account $2n_f(6n_f-1)$ $\{5n_f^2\}$ independent four-quark condensates being Lorentz invariant expectation values of Hermitian products of four quark operators constrained by time and parity reversal invariance. Symmetry under flavor rotation reduces these numbers to 20 $\{10\}$, respectively. Finally note that these are also the numbers of necessary $\kappa^{\rm med}$ parameters. Since the four-quark condensates in operator product expansions obtained from the medium projections in the limit of vanishing baryon density n should coincide with the vacuum result, this leads by contraction of vacuum and medium projections of four-quark condensates to the relations $\kappa^{\rm vac}_{\rm v',t',a'} = \frac{1}{4}\kappa^{\rm vac}_{\rm v,t,a}$, which have already been included in Tabs. 2.1 and 2.2. Further, Lorentz projections which exist only in medium imply no new $\kappa^{\rm vac}$ parameters and so the number of $\kappa^{\rm med}$ in medium reduces consistently to the number of $\kappa^{\rm vac}$ and four-quark condensates in vacuum.

Density dependence of four-quark condensates from models

It is instructive to derive values for the effective density dependence parameters $\kappa^{\rm med}$. Expectation values of four-quark operators in the nucleon were previously calculated in a perturbative chiral quark model [60] and taken into account in sum rule evaluations for the in-medium nucleon [61]. (Corrections to the factorization of four-quark condensates in nucleon sum rules have also been considered in the framework of the Nambu-Jona-Lasinio model in [62].) Lattice evaluations of four-quark operators in the nucleon are yet restricted to combinations which avoid the mixing with lower dimensional operators on the lattice [63], and provide not yet enough information to constrain the four-quark condensate combinations entering specific QCD sum rules.

The results in [60] can be translated to our κ parameters. However, only such color combinations being significant in baryon sum rules are considered, see left column in Tab. 2.3. We note that the values given in [60] have to be corrected slightly in order to reach full consistency with the Fierz relations (2.115), which are an operator identity and thus must be fulfilled also for expectation values in the nucleon. An optimized minimally corrected set is found by the following procedure: minimize the relative deviation of all separate values compared to values delivered in the parametrization of [60] (this is in the order of 10 %, however with different possible adjustments); from these configurations choose the set with smallest sum of separate deviations (this

Mean Nucleon Matrix Element (to be color contracted with $\epsilon_{abc}\epsilon_{a'b'c'}$)	PCQM model $[\langle \bar{q}q \rangle_{\mathrm{vac}}]$
$\left\langle \bar{u}^{a'} u^a \bar{u}^{b'} u^b \right\rangle_N$	3.993
$\left\langle \bar{u}^{a'} \gamma_\alpha u^a \bar{u}^{b'} \gamma^\alpha u^b \right\rangle_N$	1.977
$\left\langle \bar{u}^{a'} \not{v} u^a \bar{u}^{b'} \not{v} u^b \right\rangle_N / v^2$	0.432
$\left\langle \bar{u}^{a'} \sigma_{\alpha\beta} u^a \bar{u}^{b'} \sigma^{\alpha\beta} u^b \right\rangle_N$	12.024
$\left\langle \bar{u}^{a'} \sigma_{\alpha\beta} u^a \bar{u}^{b'} \sigma^{\alpha\delta} u^b \right\rangle_N v^\beta v_\delta / v^2$	3.045
$\left\langle \bar{u}^{a'} \gamma_5 \gamma_\alpha u^a \bar{u}^{b'} \gamma_5 \gamma^\alpha u^b \right\rangle_N$	-1.980
$\left\langle \bar{u}^{a'} \gamma_5 \not{v} u^a \bar{u}^{b'} \gamma_5 \not{v} u^b \right\rangle_N / v^2$	-0.519
$\left\langle \bar{u}^{a'} \gamma_5 u^a \bar{u}^{b'} \gamma_5 u^b \right\rangle_N$	2.016
$\left\langle \bar{u}^{a'} \not{v} u^a \bar{u}^{b'} u^b \right\rangle_N$	$-$
$\left\langle \bar{u}^{a'} \gamma_5 \gamma_\kappa u^a \bar{u}^{a'} \sigma_{\lambda\pi} u^b \right\rangle_N \epsilon^{\kappa\lambda\pi\xi} v_\xi$	$-$
$\left\langle \bar{u}^{a'} u^a \bar{d}^{b'} d^b \right\rangle_N$	3.19
$\left\langle \bar{u}^{a'} \gamma_\alpha u^a \bar{d}^{b'} \gamma^\alpha d^b \right\rangle_N$	-2.05
$\left\langle \bar{u}^{a'} \not{v} u^a \bar{d}^{b'} \not{v} d^b \right\rangle_N / v^2$	-0.73
$\left\langle \bar{u}^{a'} \sigma_{\alpha\beta} u^a \bar{d}^{b'} \sigma^{\alpha\beta} d^b \right\rangle_N$	3.36
$\left\langle \bar{u}^{a'} \sigma_{\alpha\beta} u^a \bar{d}^{b'} \sigma^{\alpha\delta} d^b \right\rangle_N v^\beta v_\delta / v^2$	1.11
$\left\langle \bar{u}^{a'} \gamma_5 \gamma_\alpha u^a \bar{d}^{b'} \gamma_5 \gamma^\alpha d^b \right\rangle_N$	1.66
$\left\langle \bar{u}^{a'} \gamma_5 \not{v} u^a \bar{d}^{b'} \gamma_5 \not{v} d^b \right\rangle_N / v^2$	0.37
$\left\langle \bar{u}^{a'} \gamma_5 u^a \bar{d}^{b'} \gamma_5 d^b \right\rangle_N$	-0.185
$\left\langle \bar{u}^{a'} \not{v} u^a \bar{d}^{b'} d^b \right\rangle_N$	-0.245
$\left\langle \bar{u}^{a'} \gamma_5 \gamma_\kappa u^a \bar{d}^{b'} \sigma_{\lambda\pi} d^b \right\rangle_N \epsilon^{\kappa\lambda\pi\xi} v_\xi$	$-$

Table 2.3: The combinations arranged as in the vector $\vec{z}$ of four-quark expectation values obtained from the (partially modified) set taken from a perturbative chiral quark model calculation (PCQM) in [60]. Therefrom the characteristic density dependence of four-quark condensates, the value of κ^{med}, is derived. Isospin symmetry $N = \frac{1}{2}(p+n)$ of the nuclear matter ground state is assumed. The values in the pure flavor sector (upper part) are tuned to obey Fierz relations (2.115) on the accuracy level $< 0.01 \, \langle \bar{q}q \rangle_{\mathrm{vac}}$. For three combinations no results are provided in [60] as indicated by ”$-$”.

deviation sum estimates to 40 % and different configurations are close to this value). The results from which the relevant density dependence for our condensate classification is obtained are collected in Tab. 2.3; our slight modifications of values in the original parametrization [60] are documented in Tabs. C.1 and C.2 in Appendix C.2.

The connection to our κ parameters is derived as follows: Generally, in linear density approximation condensates behave like $\langle \Psi |\mathcal{O}| \Psi \rangle = \langle \Psi |\mathcal{O}| \Psi \rangle_0 + n \langle N |\mathcal{O}| N \rangle$. The normalization of the nucleon state differs from that of Eq. (2.85) in order to be comparable to [60, 61]. If one compares our parametrized density dependent part of each four-quark condensate with the evaluation of nucleon matrix elements of four-quark operators in the combinations in Tab. 2.3 one obtains values for linear combinations of κ parameters. The linear combinations refer to the two distinct color alternatives representing, as mentioned above, the typical color combination in baryon sum rules. For further attempts to gain estimates of four-quark condensates we refer the interested reader to [64].

2.4.3 Four-Quark Condensates as Chiral Order Parameters

In Section 2.3.1 the meaning of condensates as order parameters of spontaneously broken symmetries was covered. Chiral symmetry restoration is linked to the limit of a vanishing chiral condensate $\langle \bar{q}q \rangle$. The previously collected four-quark condensates shall now be discussed in this respect.

For this discussion we assume exact isospin symmetry and focus on the two-flavor case. Then the condensates tabulated in Tabs. 2.1 and 2.2 can be projected onto flavor singlet structures, that means the one-dimensional invariant subspaces of $SU(n_f = 2)_V$

$$\left\langle \bar{\psi}_a \Gamma_1 \psi_b \bar{\psi}_c \Gamma_2 \psi_d \right\rangle = \frac{1}{4} \left\langle \bar{\psi} \Gamma_1 \psi \bar{\psi} \Gamma_2 \psi \right\rangle \mathbb{1}_{ba} \mathbb{1}_{dc} + \frac{1}{4} \left\langle \bar{\psi} \tau_a \Gamma_1 \psi \bar{\psi} \tau_a \Gamma_2 \psi \right\rangle \tau^b_{ba} \tau^b_{dc} \, . \tag{2.118}$$

The arbitrary index arrangement of the basis matrices is chosen to simplify the structure of the coefficients. Basis systems of flavor symmetric four-quark condensates, as specification of $\Gamma_{1,2}$, can for example be O_c or O_f as given in Tab. 2.4. There ψ denotes a flavor vector ($\psi = (u, d)^T$ for $n_f = 2$) and the first as well as third (second and fourth) blocks are the vacuum (the in medium additionally appearing) four-quark condensates. This exemplifies the previously claimed numbers of 10 (20) vacuum (medium) four-quark condensates in the flavor symmetric case. The flavor or color matrices containing parts in the lower half of Tab. 2.4 can be (Fierz) reordered to transform the basis systems into each other, see Appendix C.3.

To classify a four-quark condensate as potential order parameter along definition (2.64) the considered four-quark operator has to be identified as commutator of Q^a with a generating field Φ being itself a four-quark structure

$$[Q^a, \Phi] = [Q^a, \bar{\psi} X \psi \bar{\psi} Y \psi] = [Q^a, \bar{\psi} X \psi] \bar{\psi} Y \psi + \bar{\psi} X \psi [Q^a, \bar{\psi} Y \psi] \, . \tag{2.119}$$

The terms X, Y cover Dirac, flavor and color structures. Adopting Eq. (2.72) this reads

$$[Q^a, \bar{\psi} X \psi \bar{\psi} Y \psi] = -\bar{\psi}(\gamma_5 T^a X + X \gamma_5 T^a) \psi \bar{\psi} Y \psi - \bar{\psi} X \psi \bar{\psi}(\gamma_5 T^a Y + Y \gamma_5 T^a) \psi \, , \tag{2.120}$$

a relation symmetric under exchange of $X \leftrightarrow Y$. This reduces the number of possible configurations in the following discussion. The transformation of $SU(n_f)_A$ does not act in color space, and therefore would not change the initially given color combinations in X and Y. It suffices to study the flavor matrices ($\mathbb{1}_f \equiv \mathbb{1}_{2\times 2}$, τ^a) and Dirac structures $D_{x,y}$.

For $X = D_x \mathbb{1}_f$ and $Y = D_y \mathbb{1}_f$ it yields

$$[Q^a, \Phi] = -\frac{1}{2} \bar{\psi} \{\gamma_5, D_x\} \tau^a \psi \bar{\psi} D_y \psi - \frac{1}{2} \bar{\psi} D_x \psi \bar{\psi} \{\gamma_5, D_y\} \tau^a \psi \, , \tag{2.121}$$

Basis O_c	Basis O_f	
$\left\langle\bar{\psi}\psi\bar{\psi}\psi\right\rangle$	$\left\langle\bar{\psi}\psi\bar{\psi}\psi\right\rangle$	(2.135)
$\left\langle\bar{\psi}\gamma_\alpha\psi\bar{\psi}\gamma^\alpha\psi\right\rangle$	$\left\langle\bar{\psi}\gamma_\alpha\psi\bar{\psi}\gamma^\alpha\psi\right\rangle$	
$\left\langle\bar{\psi}\sigma_{\alpha\beta}\psi\bar{\psi}\sigma^{\alpha\beta}\psi\right\rangle$	$\left\langle\bar{\psi}\sigma_{\alpha\beta}\psi\bar{\psi}\sigma^{\alpha\beta}\psi\right\rangle$	(2.140)
$\left\langle\bar{\psi}\gamma_5\gamma_\alpha\psi\bar{\psi}\gamma_5\gamma^\alpha\psi\right\rangle$	$\left\langle\bar{\psi}\gamma_5\gamma_\alpha\psi\bar{\psi}\gamma_5\gamma^\alpha\psi\right\rangle$	
$\left\langle\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi\right\rangle$	$\left\langle\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi\right\rangle$	(2.134)
$\left\langle\bar{\psi}\not{v}\psi\bar{\psi}\not{v}\psi\right\rangle/v^2$	$\left\langle\bar{\psi}\not{v}\psi\bar{\psi}\not{v}\psi\right\rangle/v^2$	
$\left\langle\bar{\psi}\sigma_{\alpha\beta}\psi\bar{\psi}\sigma^{\gamma\delta}\psi\right\rangle g^\alpha_\gamma v^\beta v_\delta/v^2$	$\left\langle\bar{\psi}\sigma_{\alpha\beta}\psi\bar{\psi}\sigma^{\gamma\delta}\psi\right\rangle g^\alpha_\gamma v^\beta v_\delta/v^2$	(2.140)
$\left\langle\bar{\psi}\gamma_5\not{v}\psi\bar{\psi}\gamma_5\not{v}\psi\right\rangle/v^2$	$\left\langle\bar{\psi}\gamma_5\not{v}\psi\bar{\psi}\gamma_5\not{v}\psi\right\rangle/v^2$	
$\left\langle\bar{\psi}\not{v}\psi\bar{\psi}\psi\right\rangle$	$\left\langle\bar{\psi}\not{v}\psi\bar{\psi}\psi\right\rangle$	(2.136)
$\left\langle\bar{\psi}\gamma_5\gamma_\kappa\psi\bar{\psi}\sigma_{\lambda\pi}\psi\right\rangle\epsilon^{\kappa\lambda\pi\xi}v_\xi$	$\left\langle\bar{\psi}\gamma_5\gamma_\kappa\psi\bar{\psi}\sigma_{\lambda\pi}\psi\right\rangle\epsilon^{\kappa\lambda\pi\xi}v_\xi$	(2.139)
$\left\langle\bar{\psi}\lambda^A\psi\bar{\psi}\lambda^A\psi\right\rangle$	$\left\langle\bar{\psi}\tau_a\psi\bar{\psi}\tau_a\psi\right\rangle$	(2.134)
$\left\langle\bar{\psi}\gamma_\alpha\lambda^A\psi\bar{\psi}\gamma^\alpha\lambda^A\psi\right\rangle$	$\left\langle\bar{\psi}\gamma_\alpha\tau_a\psi\bar{\psi}\gamma^\alpha\tau_a\psi\right\rangle$	(2.127)
$\left\langle\bar{\psi}\sigma_{\alpha\beta}\lambda^A\psi\bar{\psi}\sigma^{\alpha\beta}\lambda^A\psi\right\rangle$	$\left\langle\bar{\psi}\sigma_{\alpha\beta}\tau_a\psi\bar{\psi}\sigma^{\alpha\beta}\tau_a\psi\right\rangle$	(2.140)
$\left\langle\bar{\psi}\gamma_5\gamma_\alpha\lambda^A\psi\bar{\psi}\gamma_5\gamma^\alpha\lambda^A\psi\right\rangle$	$\left\langle\bar{\psi}\gamma_5\gamma_\alpha\tau_a\psi\bar{\psi}\gamma_5\gamma^\alpha\tau_a\psi\right\rangle$	(2.127)
$\left\langle\bar{\psi}\gamma_5\lambda^A\psi\bar{\psi}\gamma_5\lambda^A\psi\right\rangle$	$\left\langle\bar{\psi}\gamma_5\tau_a\psi\bar{\psi}\gamma_5\tau_a\psi\right\rangle$	(2.135)
$\left\langle\bar{\psi}\not{v}\lambda^A\psi\bar{\psi}\not{v}\lambda^A\psi\right\rangle/v^2$	$\left\langle\bar{\psi}\not{v}\tau_a\psi\bar{\psi}\not{v}\tau_a\psi\right\rangle/v^2$	(2.127)
$\left\langle\bar{\psi}\sigma_{\alpha\beta}\lambda^A\psi\bar{\psi}\sigma^{\gamma\delta}\lambda^A\psi\right\rangle g^\alpha_\gamma v^\beta v_\delta/v^2$	$\left\langle\bar{\psi}\sigma_{\alpha\beta}\tau_a\psi\bar{\psi}\sigma^{\gamma\delta}\tau_a\psi\right\rangle g^\alpha_\gamma v^\beta v_\delta/v^2$	(2.140)
$\left\langle\bar{\psi}\gamma_5\not{v}\lambda^A\psi\bar{\psi}\gamma_5\not{v}\lambda^A\psi\right\rangle/v^2$	$\left\langle\bar{\psi}\gamma_5\not{v}\tau_a\psi\bar{\psi}\gamma_5\not{v}\tau_a\psi\right\rangle/v^2$	(2.127)
$\left\langle\bar{\psi}\not{v}\lambda^A\psi\bar{\psi}\lambda^A\psi\right\rangle$	$\left\langle\bar{\psi}\not{v}\tau_a\psi\bar{\psi}\tau_a\psi\right\rangle$	(2.128), (2.137)
$\left\langle\bar{\psi}\gamma_5\gamma_\kappa\lambda^A\psi\bar{\psi}\sigma_{\lambda\pi}\lambda^A\psi\right\rangle\epsilon^{\kappa\lambda\pi\xi}v_\xi$	$\left\langle\bar{\psi}\gamma_5\gamma_\kappa\tau_a\psi\bar{\psi}\sigma_{\lambda\pi}\tau_a\psi\right\rangle\epsilon^{\kappa\lambda\pi\xi}v_\xi$	(2.129), (2.138)

Table 2.4: Two complete basis sets of four-quark condensates when exact flavor symmetry is assumed. The second and fourth blocks are only present in medium. The bases differ in the lower half, where either color matrices λ^A (O_c) or flavor matrices τ_a (O_f) are used. The last column states in which order parameter combination a condensate in the basis O_f appears.

only containing flavor triplets and thus not suitable to be identified with a four-quark condensate in Tab. 2.4 regardless of the Dirac structures in X and Y.

Then assume $X = D_x \tau^b$ and $Y = D_y \tau^c$

$$
\begin{aligned}
[Q^a, \Phi^{bc}] = &- \frac{1}{2}\bar{\psi}(\gamma_5 D_x \tau^a \tau^b + D_x \gamma_5 \tau^b \tau^a)\psi\bar{\psi} D_y \tau^c \psi \\
&- \frac{1}{2}\bar{\psi} D_x \tau^b \psi \bar{\psi}(\gamma_5 D_y \tau^a \tau^c + D_y \gamma_5 \tau^c \tau^a)\psi \,.
\end{aligned} \tag{2.122}
$$

With $\tau_a \tau_b = i\epsilon_{abc}\tau_c + \delta_{ab}\mathbb{1}$ one obtains

$$
\begin{aligned}
[Q^a, \Phi^{bc}] = &- \frac{1}{2}\bar{\psi}(i\epsilon_{abd}\tau^d[\gamma_5, D_x] + \delta_{ab}\{\gamma_5, D_x\})\psi\bar{\psi} D_y \tau^c \psi \\
&- \frac{1}{2}\bar{\psi} D_x \tau^b \psi \bar{\psi}(i\epsilon_{acd}\tau^d[\gamma_5, D_y] + \delta_{ac}\{\gamma_5, D_y\})\psi \,,
\end{aligned} \tag{2.123}
$$

where the only remaining relevant terms are the contributions to flavor singlets

$$
\begin{aligned}
\langle\Psi|[Q^a, \Phi^{bc}]|\Psi\rangle = &- \frac{i}{2}\epsilon_{abd}\langle\Psi|\bar{\psi}\tau^d[\gamma_5, D_x]\psi\bar{\psi} D_y \tau^c \psi|\Psi\rangle \\
&- \frac{i}{2}\epsilon_{acd}\langle\Psi|\bar{\psi} D_x \tau^b \psi\bar{\psi}\tau^d[\gamma_5, D_y]\psi|\Psi\rangle \,.
\end{aligned} \tag{2.124}
$$

Upon flavor singlet projection with Eq. (2.118) this leads to

$$
\begin{aligned}
\langle\Psi|[Q^a, \Phi^{bc}]|\Psi\rangle = &- \frac{i}{2}\epsilon_{abc}\left(\langle\Psi|\bar{\psi}\tau^e[\gamma_5, D_x]\psi\bar{\psi} D_y \tau^e \psi|\Psi\rangle\right. \\
&\left. -\langle\Psi|\bar{\psi} D_x \tau^e \psi\bar{\psi}\tau^e[\gamma_5, D_y]\psi|\Psi\rangle\right) \,,
\end{aligned} \tag{2.125}
$$

or using as ansatz for Φ the contraction with ϵ_{abc}

$$
\begin{aligned}
\langle\Psi|[Q^a, \epsilon_{abc}\Phi^{bc}]|\Psi\rangle = &- 3i\left(\langle\Psi|\bar{\psi}\tau^e[\gamma_5, D_x]\psi\bar{\psi} D_y \tau^e \psi|\Psi\rangle\right. \\
&\left. -\langle\Psi|\bar{\psi} D_x \tau^e \psi\bar{\psi}\tau^e[\gamma_5, D_y]\psi|\Psi\rangle\right) \,.
\end{aligned} \tag{2.126}
$$

Specifications of the Dirac structures determine four-quark condensates as potential order parameters. Non-zero commutators arise for $D = \gamma_\mu, \gamma_5\gamma_\mu$. Therewith the following four-quark structures lead, performing Lorentz projection and symmetry consideration along the line of Section 2.4.1, to order parameters in the sense of definition (2.64):

$$
\begin{aligned}
&D_x = \gamma_5\gamma_\mu \,, \quad && D_y = \gamma_\nu \quad && \longrightarrow \langle\Psi|\bar{\psi}\gamma_\mu\tau_e\psi\bar{\psi}\gamma_\nu\tau_e\psi|\Psi\rangle \\
& && && \quad - \langle\Psi|\bar{\psi}\gamma_5\gamma_\mu\tau_e\psi\bar{\psi}\gamma_5\gamma_\nu\tau_e\psi|\Psi\rangle \,, && (2.127) \\
&D_x = \gamma_5\gamma_\mu \,, \quad && D_y = \mathbb{1} \quad && \longrightarrow \langle\Psi|\bar{\psi}\gamma_\mu\tau_e\psi\bar{\psi}\tau_e\psi|\Psi\rangle \,, && (2.128) \\
&D_x = \gamma_\mu \,, \quad && D_y = \sigma_{\kappa\lambda} \quad && \longrightarrow \langle\Psi|\bar{\psi}\gamma_5\gamma_\mu\tau_e\psi\bar{\psi}\sigma_{\kappa\lambda}\tau_e\psi|\Psi\rangle \,. && (2.129)
\end{aligned}
$$

It is crucial to recognize that the order parameter candidate in (2.127) is a combination of two condensates, both terms individually cannot be generated from a commutator. The other two potential order parameters (2.128) and (2.129) correspond to condensates that are specific to the medium scenario but vanish in vacuum.

The third possibility to specify the ansatz of Φ is realized by $X = D_x\tau^b$ and $Y = D_y\mathbb{1}_f$

$$
[Q^a, \Phi^b] = -\frac{1}{2}\bar{\psi}(\gamma_5 D_x \tau^a \tau^b + D_x \gamma_5 \tau^b \tau^a)\psi\bar{\psi} D_y \psi - \frac{1}{2}\bar{\psi} D_x \tau^b \psi\bar{\psi}(\gamma_5 D_y \tau^a + D_y \gamma_5 \tau^a)\psi \,, \tag{2.130}
$$

which yields

$$
[Q^a, \Phi^b] = -\frac{1}{2}\bar{\psi}(i\epsilon_{abd}\tau^d[\gamma_5, D_x] + \delta_{ab}\{\gamma_5, D_x\})\psi\bar{\psi} D_y \psi - \frac{1}{2}\bar{\psi} D_x \tau^b \psi\bar{\psi}\{\gamma_5, D_y\}\tau^a \psi \,. \tag{2.131}
$$

Flavor symmetry demands

$$\langle\Psi|[Q^a,\Phi^b]|\Psi\rangle = -\frac{1}{2}\delta_{ab}\left(\langle\Psi|\bar\psi\{\gamma_5,D_x\}\psi\bar\psi D_y\psi|\Psi\rangle\right.$$
$$\left.+\ \langle\Psi|\bar\psi D_x\tau_e\psi\bar\psi\{\gamma_5,D_y\}\tau_e\psi|\Psi\rangle\right)\ , \tag{2.132}$$

such that four-quark condensates are derived from the contraction

$$\langle\Psi|[Q^a,\Phi^a]|\Psi\rangle = -\left(\langle\Psi|\bar\psi\{\gamma_5,D_x\}\psi\bar\psi D_y\psi|\Psi\rangle + \langle\Psi|\bar\psi D_x\tau_e\psi\bar\psi\{\gamma_5,D_y\}\tau_e\psi|\Psi\rangle\right)\ . \tag{2.133}$$

Analog to the previous case one finds order parameters from

$$D_x = \mathbb{1}\,,\qquad D_y=\gamma_5 \longrightarrow \langle\Psi|\bar\psi\gamma_5\psi\bar\psi\gamma_5\psi|\Psi\rangle + \langle\Psi|\bar\psi\tau_e\psi\bar\psi\tau_e\psi|\Psi\rangle\,, \tag{2.134}$$
$$D_x = \gamma_5\,,\qquad D_y=\mathbb{1} \longrightarrow \langle\Psi|\bar\psi\psi\bar\psi\psi|\Psi\rangle + \langle\Psi|\bar\psi\gamma_5\tau_e\psi\bar\psi\gamma_5\tau_e\psi|\Psi\rangle\,, \tag{2.135}$$
$$D_x = \gamma_5\,,\qquad D_y=\gamma_\mu \longrightarrow \langle\Psi|\bar\psi\psi\bar\psi\gamma_\mu\psi|\Psi\rangle\,, \tag{2.136}$$
$$D_x = \gamma_\mu\,,\qquad D_y=\gamma_5 \longrightarrow \langle\Psi|\bar\psi\gamma_\mu\tau_e\psi\bar\psi\tau_e\psi|\Psi\rangle\,, \tag{2.137}$$
$$D_x = \gamma_5\gamma_\mu\,,\qquad D_y=\sigma_{\kappa\lambda} \longrightarrow \langle\Psi|\bar\psi\gamma_5\gamma_\mu\tau_e\psi\bar\psi\gamma_5\sigma_{\kappa\lambda}\tau_e\psi|\Psi\rangle\,, \tag{2.138}$$
$$D_x = \sigma_{\kappa\lambda}\,,\qquad D_y=\gamma_5\gamma_\mu \longrightarrow \langle\Psi|\bar\psi\gamma_5\sigma_{\kappa\lambda}\psi\bar\psi\gamma_5\gamma_\mu\psi|\Psi\rangle\,, \tag{2.139}$$
$$D_x = \sigma_{\mu\nu}\,,\qquad D_y=\sigma_{\kappa\lambda} \longrightarrow \langle\Psi|\bar\psi\gamma_5\sigma_{\mu\nu}\psi\bar\psi\sigma_{\kappa\lambda}\psi|\Psi\rangle$$
$$+\ \langle\Psi|\bar\psi\sigma_{\mu\nu}\tau_e\psi\bar\psi\gamma_5\sigma_{\kappa\lambda}\tau_e\psi|\Psi\rangle\,. \tag{2.140}$$

Again one obtains linear combinations, this time mixing the two different flavor contractions, Eqs. (2.134), (2.135) and (2.140). By contraction with the epsilon pseudo-tensor the latter can be traced back the tensor structures listed in Tab. 2.4. The table also allocates in the last column the identified order parameter candidates. The pairs Eqs. (2.136), (2.137) and Eqs. (2.138), (2.139) differ by the flavor structure and are absent in vacuum.

These possible order parameters could also determine the spontaneous symmetry breaking of the axial vector symmetry. Note that it remains an open issue how the Goldstone bosons, coupling to the fields Φ used in the ansatz Eq. (2.119), are to be identified in a unique way with the hadronic spectrum.

The four-quark condensates qualified as possible order parameters in the two flavor case can similarly be found for scenarios with arbitrary numbers of flavors generalizing the products of Pauli matrices with Eqs. (2.51) and (2.52) for $SU(N)$ groups. For the color contractions, which have not been specified in the ansatz, both color singlet structures, for example in Eq. (2.127) (the color unit matrix $\mathbb{1}_c$ explicitly shown)

$$\langle\Psi|\bar\psi\mathbb{1}_c\gamma_\mu\tau_e\psi\bar\psi\mathbb{1}_c\gamma_\nu\tau_e\psi|\Psi\rangle - \langle\Psi|\bar\psi\mathbb{1}_c\gamma_5\gamma_\mu\tau_e\psi\bar\psi\mathbb{1}_c\gamma_5\gamma_\nu\tau_e\psi|\Psi\rangle\,, \tag{2.141}$$
$$\langle\Psi|\bar\psi\lambda^A\gamma_\mu\tau_e\psi\bar\psi\lambda^A\gamma_\nu\tau_e\psi|\Psi\rangle - \langle\Psi|\bar\psi\lambda^A\gamma_5\gamma_\mu\tau_e\psi\bar\psi\lambda^A\gamma_5\gamma_\nu\tau_e\psi|\Psi\rangle\,, \tag{2.142}$$

are allowed. The transformation of combination (2.142) to the basis sets O_c and O_f is given in Appendix C.3.

As discussed in Section 2.3.1 in the Wigner-Weyl phase the vanishing of some order parameters is strongly correlated to symmetries distinct from $SU(n_f)_A$. In the two flavor case this can be verified beyond infinitesimal transformations in closed form. For this we concentrate on the elements of O_f in Tab. 2.4 in the Wigner-Weyl scenario.

The vector and axial vector structures $\left\langle\bar\psi\gamma_\alpha\psi\bar\psi\gamma_\beta\psi\right\rangle$ and $\left\langle\bar\psi\gamma_5\gamma_\alpha\psi\bar\psi\gamma_5\gamma_\beta\psi\right\rangle$ are invariant under the transformation $\psi\rightarrow\exp\left(i\beta_aT_a\gamma_5\right)\psi$ in Section 2.3.1. The $SU(n_f)_A$ transformation in infinitesimal form

$$
\begin{aligned}
&\bar{\psi}X\psi\bar{\psi}Y\psi \longrightarrow \\
&\bar{\psi}\gamma_0 \exp(-i\beta^a T_a\gamma_5)\gamma_0 X \exp(i\beta^a T_a\gamma_5)\psi\bar{\psi}\gamma_0 \exp(-i\beta^a T_a\gamma_5)\gamma_0 Y \exp(i\beta^a T_a\gamma_5)\psi \\
&\approx \bar{\psi}X\psi\bar{\psi}Y\psi \,+\, i\beta^a(\bar{\psi}[\gamma_5 X + X\gamma_5]T_a\psi\bar{\psi}Y\psi + \bar{\psi}X\psi\bar{\psi}[\gamma_5 Y + Y\gamma_5]T_a\psi) \\
&+ \bar{\psi}(i\beta^a T_a)[\gamma_5 X + X\gamma_5]\psi\bar{\psi}(i\beta^a T_a)[\gamma_5 Y + Y\gamma_5]\psi \\
&+ \bar{\psi}(i\beta^a T_a)^2 X\psi\bar{\psi}Y\psi + \bar{\psi}(i\beta^a T_a)^2\gamma_5 X\gamma_5\psi\bar{\psi}Y\psi \\
&+ \bar{\psi}X\psi\bar{\psi}(i\beta^a T_a)^2 Y\psi + \bar{\psi}X\psi\bar{\psi}(i\beta^a T_a)^2\gamma_5 Y\gamma_5\psi \,+\, \mathcal{O}(\beta^3)
\end{aligned}
\tag{2.143}
$$

reveals that only if X, Y are γ_μ and/or $\gamma_5\gamma_\mu$ the four-quark structure is invariant, the remaining structures are no $SU(n_f)_A$ invariants. It remains questionable whether further suitable linear combinations possess the $SU(n_f)_A$ symmetry, which would describe another singlet. In the two-flavor scenario the $SU(2)_A$ transformations

$$
\psi \longrightarrow \exp\left(i\beta_a\tau_a\gamma_5\right)\psi \tag{2.144}
$$

of the given elements of O_f are now calculated in non-infinitesimal form. Using the properties of the Pauli-Matrices τ_a and $\gamma_5^2 = \mathbb{1}$, the exponential is evaluated and the transformation of the flavor vectors becomes

$$
\psi \longrightarrow \quad \left(\mathbb{1}\cos\beta + i\gamma_5\frac{\vec{\beta}\vec{\tau}}{\beta}\sin\beta\right)\psi \,, \tag{2.145}
$$

$$
\bar{\psi} \longrightarrow \bar{\psi}\left(\mathbb{1}\cos\beta + i\gamma_5\frac{\vec{\beta}\vec{\tau}}{\beta}\sin\beta\right) \quad , \tag{2.146}
$$

with $\beta = |\vec{\beta}|$. As verification and prerequisite for the discussion of four-quark condensates consider the bilinear terms with an arbitrary Dirac structure D

$$
\bar{\psi}D\psi \longrightarrow \begin{cases} \bar{\psi}D\psi & \text{if } D\gamma_5 = -\gamma_5 D \,, \\ \bar{\psi}D\psi\cos(2\beta) + i\bar{\psi}D\gamma_5\dfrac{\vec{\beta}\vec{\tau}}{\beta}\psi\sin(2\beta) & \text{if } D\gamma_5 = \gamma_5 D \quad . \end{cases} \tag{2.147}
$$

The last term vanishes if flavor symmetry is demanded. Especially, the chiral condensate transforms as $\langle\bar{\psi}\psi\rangle \rightarrow \cos(2\beta)\langle\bar{\psi}\psi\rangle$, which demonstrates the fate of an order parameter in the Wigner-Weyl phase. Only a vanishing $\langle\bar{\psi}\psi\rangle$ ensures symmetry with respect to $SU(2)_A$. Note that the exact transformation reveals that the infinitesimal approximation in first order of $\vec{\beta}$ would lead to an invariant term (compare Eq. (2.58)). Similarly one finds for

$$
\bar{\psi}D\tau_c\psi \longrightarrow \begin{cases} \bar{\psi}D\tau_c\psi\cos(2\beta) - \dfrac{\beta_a}{\beta}\epsilon_{cab}\bar{\psi}D\tau_b\gamma_5\psi\sin(2\beta) + \dfrac{2\beta_c}{\beta}\bar{\psi}D\dfrac{(\vec{\beta}\vec{\tau})}{\beta}\psi\sin^2\beta & \\ & \text{if } D\gamma_5 = -\gamma_5 D \,, \\ \bar{\psi}D\tau_c\psi + \dfrac{i\beta_c}{\beta}\bar{\psi}D\gamma_5\psi\sin(2\beta) - \dfrac{2\beta_c}{\beta}\bar{\psi}D\dfrac{\vec{\beta}\vec{\tau}}{\beta}\psi\sin^2\beta & \text{if } D\gamma_5 = \gamma_5 D \quad . \end{cases} \tag{2.148}
$$

With this compilation of formulas the elements of O_f with any Dirac structures can be transformed. Hence, the four-quark condensates behave under $SU(2)_A$ like

$$\left\langle \bar{\psi} X \psi \bar{\psi} Y \psi \right\rangle \longrightarrow$$
$$\begin{cases} \left\langle \bar{\psi} X \psi \bar{\psi} Y \psi \right\rangle & \text{if } X\gamma_5 = -\gamma_5 X \,,\, Y\gamma_5 = -\gamma_5 Y \,, \\ \cos(2\beta) \left\langle \bar{\psi} X \psi \bar{\psi} Y \psi \right\rangle + \text{fac.} & \text{if } X\gamma_5 = -\gamma_5 X \,,\, Y\gamma_5 = +\gamma_5 Y \,, \\ \cos(2\beta) \left\langle \bar{\psi} X \psi \bar{\psi} Y \psi \right\rangle + \text{fac.} & \text{if } X\gamma_5 = +\gamma_5 X \,,\, Y\gamma_5 = -\gamma_5 Y \,, \\ \cos^2(2\beta) \left\langle \bar{\psi} X \psi \bar{\psi} Y \psi \right\rangle - \sin^2(2\beta) \left\langle \bar{\psi} X \gamma_5 \vec{\tau} \psi \bar{\psi} Y \gamma_5 \vec{\tau} \psi \right\rangle + \text{fac.} & \\ & \text{if } X\gamma_5 = +\gamma_5 X \,,\, Y\gamma_5 = +\gamma_5 Y \,, \end{cases} \tag{2.149}$$

and

$$\left\langle \bar{\psi} X \vec{\tau} \psi \bar{\psi} Y \vec{\tau} \psi \right\rangle \longrightarrow$$
$$\begin{cases} \left\langle \bar{\psi} X \vec{\tau} \psi \bar{\psi} Y \vec{\tau} \psi \right\rangle + \text{fac.} & \text{if } X\gamma_5 = -\gamma_5 X \,,\, Y\gamma_5 = -\gamma_5 Y \,, \\ \cos(2\beta) \left\langle \bar{\psi} X \vec{\tau} \psi \bar{\psi} Y \vec{\tau} \psi \right\rangle + \text{fac.} & \text{if } X\gamma_5 = -\gamma_5 X \,,\, Y\gamma_5 = +\gamma_5 Y \,, \\ \cos(2\beta) \left\langle \bar{\psi} X \vec{\tau} \psi \bar{\psi} Y \vec{\tau} \psi \right\rangle + \text{fac.} & \text{if } X\gamma_5 = +\gamma_5 X \,,\, Y\gamma_5 = -\gamma_5 Y \,, \\ \cos^2(2\beta) \left\langle \bar{\psi} X \vec{\tau} \psi \bar{\psi} Y \vec{\tau} \psi \right\rangle - \sin^2(2\beta) \left\langle \bar{\psi} X \gamma_5 \psi \bar{\psi} Y \gamma_5 \psi \right\rangle + \text{fac.} & \\ & \text{if } X\gamma_5 = +\gamma_5 X \,,\, Y\gamma_5 = +\gamma_5 Y \,. \end{cases} \tag{2.150}$$

We have not written out the flavor asymmetric contributions ”fac.”, which are absent under the applied flavor symmetry. Note that, for example, condensates with $X = Y = \mathbb{1}$ mix with those containing $X = Y = \gamma_5$. It is possible to derive a linear combination of these condensates which transforms into itself. However, such a behavior is accidental for specific values of $\vec{\beta}$ und does not imply an invariant subspace of $SU(2)_A$, since therefore invariance must be realized for any arbitrary value of the continuous parameters $\vec{\beta}$.

In conclusion, there exist no additional pure $SU(n_f)_A$ singlets and thus the vanishing of all four-quark condensates in the given basis derived as possible order parameters of chiral symmetry breaking is related to flavor symmetry. Only the vector and axial vector structures without Pauli matrices are invariant, but these have not been realized as commutator and are in this sense no potential order parameters. The other condensates appear partially only in order parameter combinations. In any QSR applications it is required to test whether the occurring four-quark condensates can be candidates for order parameters. This cannot be deduced from a calculation in the factorization approach, where the chiral condensate enters in any way. One observes, e.g. in Tab. 2.1, that even the $SU(2)_A$ invariant condensates are expressed by the chiral condensate. It should be emphasized that a statement on chiral symmetry restoration from such terms would be misleading.

QCD Sum Rules for Parity Partners

The previous discussion of order parameters was based on the transformation behavior of the condensates. A much stronger connection to spontaneous chiral symmetry breaking would start on the observable hadronic side of a QCD sum rule. Indeed, an alternative to the study of chiral symmetry restoration in the QCD phase diagram via modifications of individual hadron properties is to consider parity partners in the hadronic spectrum.

In this context the difference between vector and axial-vector correlators has been investigated [65]. If both vector and axial-vector symmetry $SU(n_f)_{V,A}$ are realized in the ground state, this is equivalent to independent symmetries in the left and the right handed sector. Therefrom the equality of the expectation values $\langle\Psi|V^a_\mu(x)V^b_\nu(y)|\Psi\rangle = \langle\Psi|A^a_\mu(x)A^b_\nu(y)|\Psi\rangle$ of the vector current $V^a_\mu = \bar\psi\gamma_\mu T^a\psi$ and the axial-vector current $A^a_\mu = \bar\psi\gamma_5\gamma_\mu T^a\psi$ follows [66].

Especially, the thereby described parity partners ρ ($J^P = 1^-$) and a_1 ($J^P = 1^+$) would become degenerate when chiral symmetry was realized in the Wigner-Weyl phase. Their currents

$$j^V_\mu = \frac{1}{2}(\bar u\gamma_\mu u - \bar d\gamma_\mu d)\,, \tag{2.151}$$

$$j^A_\mu = \frac{1}{2}(\bar u\gamma_\mu\gamma_5 u - \bar d\gamma_\mu\gamma_5 d) \tag{2.152}$$

mix under $SU(n_f)_A$ transformations. The study of the difference of their spectral functions (commonly abbreviated as $V-A$ difference) has the advantage that many chirality independent terms vanish. This concerns especially the operator product expansion in a QCD sum rule formulation. The perturbative contributions are the same for the V and A correlator in the chiral limit of vanishing quark masses; also gluon condensate contributions cancel in the $V-A$ difference. This results in the sum rules

$$\int_0^\infty \frac{ds}{s}\,[\rho_V(s)-\rho_A(s)] = f_\pi^2\,, \tag{2.153}$$

$$\int_0^\infty ds\,[\rho_V(s)-\rho_A(s)] = 0\,, \tag{2.154}$$

$$\int_0^\infty ds\,s\,[\rho_V(s)-\rho_A(s)] = -2\pi\alpha_s\left\langle\mathcal{O}^\mu_\mu\right\rangle\,, \tag{2.155}$$

where ρ_V (ρ_A) denote the spectral densities in the vector (axial vector) channel. The first two equations are the famous Weinberg sum rules [65]. The third sum rule was added in [67]. There, also a generalization to medium is given, where the transverse and longitudinal contributions to the spectral densities $\rho^{T,L}_{V,A}$ have to be distinguished. The third sum rule (2.155) becomes then

$$\int_0^\infty d\omega\,\omega^3\,[2\Delta\rho^T(\omega,|\vec p\,|)+\Delta\rho^L(\omega,|\vec p\,|)] = -2\pi\alpha_s[\left\langle\mathcal{O}^\mu_\mu + 2\mathcal{O}^{00}\right\rangle]\,, \tag{2.156}$$

with $\Delta\rho^{T,L} = \rho^{T,L}_V - \rho^{T,L}_A$. The decisive condensates contributing to the QCD sum rule for $V-A$ spectral functions are the four-quark condensates [67]

$$\langle O^{\mu\nu}\rangle = \left\langle(\bar u_L\gamma^\mu\lambda^A u_L - \bar d_L\gamma^\mu\lambda^A d_L)(\bar u_R\gamma^\nu\lambda^A u_R - \bar d_R\gamma^\mu\lambda^A d_R)\right\rangle\,, \tag{2.157}$$

following from the difference of the operator product expansions of the currents (2.151) and (2.152). In vacuum the important condensates have the form

$$\langle O_{V-A}\rangle \equiv \left\langle\mathcal{O}^\mu_\mu\right\rangle = \left\langle(\bar u\gamma_\mu\gamma_5\lambda^A u - \bar d\gamma_\mu\gamma_5\lambda^A d)^2\right\rangle - \left\langle(\bar u\gamma_\mu\lambda^A u - \bar d\gamma_\mu\lambda^A d)^2\right\rangle\,, \tag{2.158}$$

which becomes, with flavor symmetry imposed,

$$\langle O_{V-A}\rangle = \left\langle\bar\psi\gamma_\mu\tau_a\lambda^A\psi\bar\psi\gamma^\mu\tau_a\lambda^A\psi\right\rangle - \left\langle\bar\psi\gamma_\mu\gamma_5\tau_a\lambda^A\psi\bar\psi\gamma^\mu\gamma_5\tau_a\lambda^A\psi\right\rangle\,. \tag{2.159}$$

It is remarkable that the combination entering $\langle O_{V-A}\rangle$ can be identified with the order parameter derived in Eq. (2.142). The concept of order parameters for spontaneous chiral symmetry breaking built in an abstract way for the condensates in Section 2.3.1 is related by the QSR to the meaning of chiral symmetry in the hadronic spectrum. The importance of the four-quark condensates $\langle O_{V-A}\rangle$

is thus made obvious. The contribution $\langle \mathcal{O}^{00} \rangle$, being the rest frame limit of $\langle \mathcal{O}^{\mu\nu} \rangle v_\mu v_\nu$, obviously plays an analog role in medium. In [58] these four-quark condensates are used to estimate the transition line of chiral symmetry restoration in the T-μ-plane. The degeneracy of ρ and a^1 meson admittedly not yet restricts the fate of the spectral functions, since only their difference would decrease approaching the chirally restored phase. Even the nature of ρ as quark-antiquark state and the a_1 as meson-molecule may be different [68].

So far $V - A$ data have been provided only for vacuum by ALEPH [69] and OPAL [70], where the semi-hadronic τ decays into $\pi\pi$ and $\pi\pi\pi$ are analyzed. It has been attempted to constrain not only the running coupling α_s from the individual vector and axial-vector spectral functions [71] but also the condensates of dimension 6, 8, and even dimension 10 and 12 [72–74]. Such analyses rely on the choice of suitable moments or otherwise improved weighting of the sum rules. The vacuum determination of the four-quark condensates in Eq. (2.158) lead to values, which agree with the factorization hypothesis (vacuum saturation) [75]. Nevertheless, this statements holds only for the combination $\langle O_{V-A} \rangle$ in vacuum. It cannot be excluded that cancellation effects occur and the factorization of individual four-quark condensates fails. From the conceptional point of view, also ambiguities in the factorization prescription, e.g. for terms of order $1/N_c^2$ in dimension 8 condensates [72], question the validity of factorization for $N_c = 3$ colors.

In this chapter all prerequisites for building QCD sum rules have been provided. These sum rules basically relate, via dispersion relations, an integral over a hadronic spectral function to QCD condensates which enter through an operator product expansion. The condensates are expectation values which are only partially known. Especially, the four-quark condensates are not well determined. The condensates are abstract numbers universal to all QSR applications. They might also measure symmetry effects in QCD. We elaborated on the spontaneous chiral symmetry breaking and showed how order parameters could be defined. The last application to parity partners joined all the covered aspects: How does symmetry realize in the hadron spectrum, how is this related to quantities of QCD, the condensates, and when is such a condensate a qualified order parameter?

3 Analysis of QCD Sum Rules

QCD sum rules are now specified for the examples ω meson, nucleon and D meson. We utilize Borel transformed sum rules in this chapter. Their quantitative evaluation is discussed. The focus is here on the effects at non-vanishing nuclear densities. The vacuum limits are reproduced along the way. Numerical evaluations reveal for the light quark sector significant correlations between the density dependence of four-quark condensates and changes of spectral functions quantified in moments thereof, e.g. in hadron masses. The case of the D meson is distinct from that. The mass of heavy quarks cannot be neglected in the OPE calculation; this new scale influences the relative weights of condensates in the sum rule and thus reshuffles their impact on moments of spectral functions.

3.1 Light Vector Mesons: ω Meson

The neutral vector mesons, the iso-scalar ω meson and the iso-vector ρ^0 meson, have been investigated within the approach of QCD sum rules already in the pioneering works [4], including the mixing of these mesons [76]. They are described by the currents

$$j_\mu^{(V)} = \frac{1}{2}\left(\bar{u}\gamma_\mu u \pm \bar{d}\gamma_\mu d\right) , \tag{3.1}$$

where the upper (lower) sign denotes the $V = \omega$ (ρ) vector meson. QCD sum rules for vector mesons in medium are well documented, cf. [77] for our notation and references therein. The vector meson is considered at rest $Q^2 \equiv -q^2 = -q_0^2$ in the nuclear matter frame of reference. Then, instead of studying the transverse and longitudinal components of the correlator

$$\Pi_{\mu\nu}(q,v) = i\int d^4x\, e^{iqx}\langle\langle T\left[j_\mu^V(x) j_\nu^V(0)\right]\rangle\rangle\,, \tag{3.2}$$

if suffices to deal with the contracted part $\Pi = \Pi_\mu^\mu/(-3q_0^2)$.

3.1.1 Hadronic Models

The quantity which will be determined by the QCD sum rule is the ratio of moments, Eq. (2.40),

$$\overline{m}_V^2(n,\mathcal{M}^2,s_+) = \frac{\int_0^{s_V} ds\ \Delta\Pi(s,n)\ e^{-s/\mathcal{M}^2}}{\int_0^{s_V} ds\ \Delta\Pi(s,n)\ s^{-1}e^{-s/\mathcal{M}^2}}\,. \tag{3.3}$$

In the pole + continuum ansatz this turned out to be just the squared pole mass. In medium the sum rule relates changes of this quantity to changes of condensates. Whereas on the OPE side the uncertainties lie in the values of condensates, the hadronic description of changes in $\overline{m}_V$ should be taken with care. Based on the conceptional restriction that only integrals of spectral densities can be determined, it was early recognized that numerous changes in the form of a spectral function can for example cause a dropping of $\overline{m}_V$. Such reasons can be width effects or multiple peak shapes, cf. [78–82] for discussions w.r.t. the ρ meson or [82–84] for the ω meson.

The interpretation of QSR results allows to restrict hadronic models. In case of the pole ansatz, the pole mass can be exactly determined. For any model containing more parameters the sum rule correlates these parameter. To elucidate this aspect consider as model for the spectral function a Breit-Wigner distribution

$$\Delta\Pi(s) = \frac{1}{\pi}\frac{\sqrt{s}\Gamma(s)}{(s-m_0^2)^2 + s\Gamma^2(s)}\,, \tag{3.4}$$

where m_0 is the center of the Breit-Wigner curve, $\Gamma(s)$ the (energy dependent) width. In the limit $\Gamma(s) = \Gamma_0 \to 0$ the ansatz reduces to the pole limit $\Delta\Pi(s) \to \delta(s-m_0^2)$. For the ω meson the correlation between central mass m_0 and a constant width Γ_0 is exhibited in Fig. 3.1. The regions shaded in specific colors correspond to constant moments $\overline{m}_V$, while the band denotes the impact of different threshold parameters s_V in the definition of $\overline{m}_V$. This figure reveals that passing to a numerically reduced moment $\overline{m}_V$ is possible by either dropping the mass center m_0 or increasing the width Γ_0, or a combination of both effects, due to the asymmetry factor $e^{-s/\mathcal{M}^2}$ in Eq. (3.3). The axis $\Gamma_0 = 0$ recovers the pole ansatz. Note, since mass and width effects can compensate

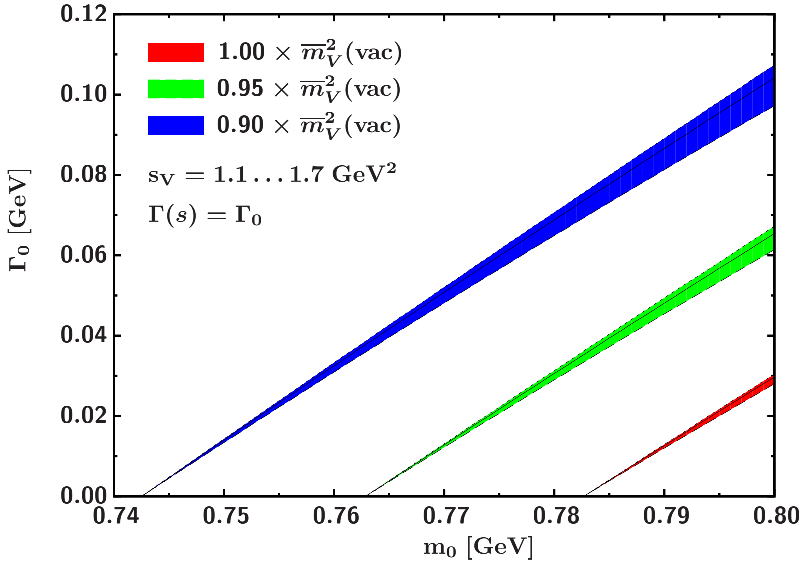

Figure 3.1: Correlation of fixed width Γ_0 and pole mass parameter m_0 under the assumption of constant moments (3.3) for the ω meson. Three different values for $\overline{m}_V \equiv \overline{m}_\omega$ are compared (red, green, blue) and the possible explanations for a decrease of $\overline{m}_\omega$ can be read of (see text). The error bands are due to different values of the threshold s_0 (short-dashed line: $s_0 = 1.7$ GeV^2, continuous line: $s_0 = 1.4$ GeV^2, long-dashed line: $s_0 = 1.1$ GeV^2).

each other, even an increase of m_0 with significant broadening would explain a change of $\overline{m}_V$. The situation of decreasing width but smaller pole mass seems not realistic, especially for a narrow resonance like the ω meson. Modelling the width as in [79],

$$\Gamma(s) = \Gamma_0\sqrt{\frac{1-m_\pi^2/s}{1-m_\pi^2/m_0^2}}\Theta(s-m_\pi^2)\,, \tag{3.5}$$

with the pion mass m_π, does not alter these qualitative statements. However, then the slopes in Fig. 3.1 increase. These results are obtained for a fixed Borel mass $\mathcal{M}^2 = 1$ GeV^2. As test for the impact of this parameter, the moments, averaged over the range of applied Borel masses, have been adjusted to be constant. This treatment further increases the error bands and even leads in combination with the energy dependent width (3.5) to overlapping bands. This exemplifies the restrictions laid upon the interpretation of the ratio (3.3), compare also [84].

Experimentally, significant broadening effects have been reported for the ω meson by the CB-TAPS/ELSA collaboration [21]. For the ρ meson the CLAS collaboration reported no significant mass shift but some broadening [22, 23]. In the following we will dwell on the consequences for the condensates under the hypothesis that the ratio Eq. (3.3) does not increase in a nuclear medium. This adds some details to the main results already summarized in [25]. There a major motivation was the observed lowering of the ω decay strength inside a nuclear medium [19]. Although these experimental results are still under debate (for instance the authors of [85] argue that they are not conclusive due to a particular choice of background separation), our hypothesis would be equally well supported by considerable broadening effects. Fig. 3.1 showed that also an increase of width alone drives the general moment $\overline{m}_V$ to smaller values. The expectation that this ratio is not increasing in a medium is therefore a reasonable assumption which alone leads to the consequences for condensates as published in [25] and complemented hereafter. Note that the vector meson is considered at rest which stands against a quantitative comparison with experimental results where the mesons have non-zero three-momentum. Quantitative statements about the moment require the treatment of final state interaction etc. We restrict ourselves to the qualitative hypothesis, that $\overline{m}_\omega$ not increases with increasing baryon density.

3.1.2 QCD Sum Rule

For completeness the essential steps towards the analyzable sum rule equation are collected. The dispersion relation is improved by two subtractions

$$\frac{\Pi^{(V)}(Q^2,n)}{Q^2} = \frac{\Pi^{(V)}(0,n)}{Q^2} + \Pi^{(V)\prime}(0) + \frac{Q^2}{\pi}\int_0^\infty ds \frac{\Delta\Pi^{(V)}(s,n)}{s^2(s+Q^2)} \,. \tag{3.6}$$

Any polynomials vanish under Borel transformation (see Section 2.2.2), however, dividing by Q^2 the subtraction effect is retained also in the Borel sum rule

$$\mathcal{B}_{\mathcal{M}^2}\left(\frac{\Pi^{(V)}(Q^2,n)}{Q^2}\right) = \Pi^{(V)}(0,n) - \frac{1}{\pi}\int_0^\infty ds \frac{\Delta\Pi^{(V)}(s,n)}{s} e^{-s/M^2} \,. \tag{3.7}$$

The l.h.s. is given by the operator product expansion organized in terms of mass dimension d and twist τ [77]

$$\Pi^{(V)}(Q^2,n) = \Pi^{(V)}_{\text{scalar}} + \Pi^{(V)}_{d=4,\tau=2} + \Pi^{(V)}_{d=6,\tau=2} + \Pi^{(V)}_{d=6,\tau=4} + \dots \,, \tag{3.8}$$

which contains the Wilson coefficients and QCD condensates in

$$\begin{aligned}
\Pi^{(V)}_{\text{scalar}} = &-\frac{1}{8\pi^2}\left(1+\frac{3\alpha_s}{4\pi}C_F\right)Q^2\ln\frac{Q^2}{\mu^2} - \frac{3}{8\pi^2}(m_u^2+m_d^2) \\
&+\frac{1}{2}\left(1+\frac{\alpha_s}{4\pi}C_F\right)\frac{1}{Q^2}\langle\Psi|m_u\bar{u}u+m_d\bar{d}d|\Psi\rangle + \frac{1}{24}\frac{1}{Q^2}\langle\Psi|\frac{\alpha_s}{\pi}G^2|\Psi\rangle \\
&-\frac{\pi}{2}\alpha_s\frac{1}{Q^4}\langle\Psi|\left(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{u}\gamma^\mu\gamma_5\lambda^a u + \bar{d}\gamma_\mu\gamma_5\lambda^a d\bar{d}\gamma^\mu\gamma_5\lambda^a d\right)|\Psi\rangle \\
&\mp\pi\alpha_s\frac{1}{Q^4}\langle\Psi|\left(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{d}\gamma^\mu\gamma_5\lambda^a d\right)|\Psi\rangle \\
&-\frac{\pi}{9}\alpha_s\frac{1}{Q^4}\langle\Psi|\left(\bar{u}\gamma_\mu\lambda^a u\bar{u}\gamma^\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d\bar{d}\gamma^\mu\lambda^a d\right)|\Psi\rangle \\
&-\frac{2\pi}{9}\alpha_s\frac{1}{Q^4}\langle\Psi|\left(\bar{u}\gamma_\mu\lambda^a u\bar{d}\gamma^\mu\lambda^a d\right)|\Psi\rangle
\end{aligned}$$

$$+ g_s \frac{1}{12}\frac{1}{Q^6}\left(m_u^2\langle\Psi|m_u\bar{u}\sigma_{\mu\nu}G^{\mu\nu}u|\Psi\rangle + m_d^2\langle\Psi|m_d\bar{d}\sigma_{\mu\nu}G^{\mu\nu}d|\Psi\rangle\right), \tag{3.9a}$$

$$\Pi^{(V)}_{d=4,\tau=2} = \frac{\alpha_s}{2\pi}n_f\frac{1}{Q^4}q^\mu q^\nu\langle\Psi|\hat{S}\hat{T}\left(G_\mu{}^\alpha G_{\alpha\nu}\right)|\Psi\rangle - \left(\frac{2}{3}-\frac{5\alpha_s}{18\pi}C_F\right)i\frac{1}{Q^4}q^\mu q^\nu\langle\Psi|\hat{S}\hat{T}\left(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d\right)|\Psi\rangle, \tag{3.9b}$$

$$\Pi^{(V)}_{d=6,\tau=2} = -\frac{41\alpha_s}{27\pi}n_f\frac{1}{Q^8}q^\mu q^\nu q^\lambda q^\sigma\langle\Psi|\hat{S}\hat{T}\left(G_\mu{}^\rho D_\nu D_\lambda G_{\rho\sigma}\right)|\Psi\rangle + \left(\frac{8}{3}+\frac{67\alpha_s}{30\pi}C_F\right)i\frac{1}{Q^8}q^\mu q^\nu q^\lambda q^\sigma\times \langle\Psi|\hat{S}\hat{T}\left(\bar{u}\gamma_\mu D_\nu D_\lambda D_\sigma u + \bar{d}\gamma_\mu D_\nu D_\lambda D_\sigma d\right)|\Psi\rangle, \tag{3.9c}$$

$$\begin{aligned}\Pi^{(V)}_{d=6,\tau=4} = &\mp\frac{1}{3}\frac{1}{Q^6}q^\mu q^\nu\langle\Psi|g_s^2\hat{S}\hat{T}\left(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{d}\gamma_\nu\gamma_5\lambda^a d\right)|\Psi\rangle\\
&-\frac{1}{6}\frac{1}{Q^6}q^\mu q^\nu\langle\Psi|g_s^2\hat{S}\hat{T}\left(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{u}\gamma_\nu\gamma_5\lambda^a u + \bar{d}\gamma_\mu\gamma_5\lambda^a d\bar{d}\gamma_\nu\gamma_5\lambda^a d\right)|\Psi\rangle\\
&-\frac{1}{24}\frac{1}{Q^6}q^\mu q^\nu\langle\Psi|g_s^2\hat{S}\hat{T}\left(\bar{u}\gamma_\mu\lambda^a u\left(\bar{u}\gamma_\nu\lambda^a u + \bar{d}\gamma_\nu\lambda^a d\right)\right)|\Psi\rangle\\
&-\frac{1}{24}\frac{1}{Q^6}q^\mu q^\nu\langle\Psi|g_s^2\hat{S}\hat{T}\left(\bar{d}\gamma_\mu\lambda^a d\left(\bar{u}\gamma_\nu\lambda^a u + \bar{d}\gamma_\nu\lambda^a d\right)\right)|\Psi\rangle\\
&-\frac{5}{12}\frac{1}{Q^6}q^\mu q^\nu\langle\Psi|g_s^2\hat{S}\hat{T}\left(\bar{u}\left[D_\mu,\tilde{G}_{\nu\alpha}\right]_+\gamma^\alpha\gamma_5 u + \bar{d}\left[D_\mu,\tilde{G}_{\nu\alpha}\right]_+\gamma^\alpha\gamma_5 d\right)|\Psi\rangle\\
&-\frac{7}{3}\frac{1}{Q^6}q^\mu q^\nu\langle\Psi|g_s^2\hat{S}\hat{T}\left(m_u\bar{u}D_\mu D_\nu u + m_d\bar{d}D_\mu D_\nu d\right)|\Psi\rangle.\end{aligned} \tag{3.9d}$$

The operators $\hat{S}$, $\hat{T}$ project on symmetric and traceless terms w.r.t. Lorentz indices. For the Borel transformed OPE side on the left of Eq. (3.7) one finds then

$$\mathcal{B}_{\mathcal{M}^2}\left(\frac{\Pi^{(V)}(Q^2,n)}{Q^2}\right) = c_0 M^2 + \sum_{i=1}^{\infty}\frac{c_i}{(i-1)!M^{2(i-1)}}, \tag{3.10}$$

with the coefficient functions given by

$$c_0 = \frac{1}{8\pi^2}\left(1+\frac{3\alpha_s}{4\pi}C_F\right), \tag{3.11a}$$

$$c_1 = -\frac{3}{8\pi^2}(m_u^2+m_d^2), \tag{3.11b}$$

$$c_2 = \frac{1}{2}\left(1+\frac{\alpha_s}{4\pi}C_F\right)\left(m_u\langle\bar{u}u\rangle_{\rm vac} + m_d\langle\bar{d}d\rangle_{\rm vac} + \sigma_N n\right) + \frac{1}{24}\left[\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rm vac} - \frac{8}{9}M_N^0 n\right] + \left(\frac{1}{4}-\frac{5\alpha_s}{48\pi}C_F\right)A_2^{(u+d)}M_N n - \frac{3}{16}n_f\frac{\alpha_s}{\pi}A_2^G M_N n, \tag{3.11c}$$

$$c_3 = -\frac{112}{81}\pi\alpha_s\left[\kappa_\omega^{\rm vac}\langle\bar{q}q\rangle_{\rm vac}^2 + \kappa_\omega^{\rm med}\frac{\sigma_N\langle\bar{q}q\rangle_{\rm vac}}{m_q}n\right] - \left(\frac{5}{12}+\frac{67\alpha_s}{192\pi}C_F\right)A_4^{(u+d)}M_N^3 n + \frac{205\alpha_s}{864\pi}n_f A_4^G M_N^3 n + \frac{1}{4}M_N n\left(\frac{3}{8}K_u^2 + \frac{3}{2}K_u^1 + \frac{15}{16}K_u^g\right) - \frac{7}{144}\sigma_N M_N^2 n, \tag{3.11d}$$

where we adopted the notation and values given for the condensates from [77]. Especially, some further moments of parton distribution functions are used to determine the density dependence of condensates. The used values in this notation are $C_F = \frac{4}{3}$, $n_f = 3$, $\alpha_s = 0.38$, $m_u = 0.004$ GeV, $m_d = 0.007$ GeV, $m_q = 0.0055$ GeV, $M_N^0 = 0.77$ GeV, $A_2^{(u+d)} = 1.02$, $A_2^g = 0.83$, $M_N =$

0.939 GeV, $Q_0 = 0.15$ GeV, $f_\pi = 0.093$ GeV, $A_4^{(u+d)} = 0.12$, $A_4^g = 0.04$, $K_u^2 = 0.11$ $K_u^1 = -0.112$, $K_u^g = -0.3$, and the saturation density $n_0 = 0.15\ \mathrm{fm}^{-3}$.

The evaluation is based on the final Borel sum rule

$$\Pi^{(V)}(0,n) - \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi^{(V)}(s,n)}{s} e^{-s/M^2} = c_0 M^2 + \sum_{i=1}^{\infty} \frac{c_i}{(i-1)!M^{2(i-1)}} \,. \tag{3.12}$$

The term $\Pi^{(V)}(0,n)$ accounts for the Landau damping effect and represents the meson-nucleon forward scattering amplitude being $9n/4M_N$ ($n/4M_N$) for the ω (ρ) meson. Combining this sum rule with its derivative one obtains the expression for the ratio of generalized moments

$$\overline{m}_\omega^2(n,\mathcal{M}^2,s_V) = \frac{c_0\mathcal{M}^2\left[1-\left(1+\frac{s_V}{\mathcal{M}^2}\right)e^{-s_V/\mathcal{M}^2}\right] - \frac{c_2}{\mathcal{M}^2} - \frac{c_3}{\mathcal{M}^4} - \frac{c_4}{2\mathcal{M}^6}}{c_0\left[1-e^{-s_V/\mathcal{M}^2}\right] + \frac{c_1}{\mathcal{M}^2} + \frac{c_2}{\mathcal{M}^4} + \frac{c_3}{2\mathcal{M}^6} + \frac{c_4}{6\mathcal{M}^8} - \frac{\Pi^{(V)}(0,n)}{\mathcal{M}^2}} \,. \tag{3.13}$$

In this form the QCD sum rule equation will be evaluated below.

3.1.3 Constraints on Four-Quark Condensates

In the following we concentrate on the four-quark condensates entering Eq. (3.9a) for the ω meson. The combined four-quark condensates there

$$\begin{aligned}
&\frac{1}{2}\left\langle \bar{u}\gamma_5\gamma_\mu\lambda^A u\bar{u}\gamma_5\gamma^\mu\lambda^A u\right\rangle + \frac{1}{2}\left\langle \bar{d}\gamma_5\gamma_\mu\lambda^A d\bar{d}\gamma_5\gamma^\mu\lambda^A d\right\rangle + \left\langle \bar{u}\gamma_5\gamma_\mu\lambda^A u\bar{d}\gamma_5\gamma^\mu\lambda^A d\right\rangle \\
&+\frac{2}{9}\left\langle \bar{u}\gamma_\mu\lambda^A u\bar{d}\gamma^\mu\lambda^A d\right\rangle + \frac{1}{9}\left\langle \bar{u}\gamma_\mu\lambda^A u\bar{u}\gamma^\mu\lambda^A u\right\rangle + \frac{1}{9}\left\langle \bar{d}\gamma_\mu\lambda^A d\bar{d}\gamma^\mu\lambda^A d\right\rangle \\
&\quad = \frac{112}{81}\left(\kappa_\omega^{\mathrm{vac}}\left\langle\bar{q}q\right\rangle_{\mathrm{vac}}^2 + \kappa_\omega^{\mathrm{med}}\left\langle\bar{q}q\right\rangle_{\mathrm{vac}}\frac{\sigma_{\mathrm{N}}}{m_{\mathrm{q}}}n\right)
\end{aligned} \tag{3.14}$$

are already present in vacuum. Note that in this case the additional, medium specific, four-quark condensates in Eq. (3.9d) are directly linked to the parametrization $K_u^{1,2}$ of twist-4 matrix elements in the nucleon [77, 86], and are not modified here. This does not suffice to determine all four-quark condensates in the ω sum rule. Using for the investigated four-quark condensates in Eq. (3.14) the individual parametrizations from Tabs. 2.1 and 2.2 with the slightly different definition of $\tilde{\kappa}_{4a,v}$ from [25], the relations

$$\kappa_\omega^{\mathrm{vac}} = -\frac{2}{7}\kappa_{2\mathrm{v}}^{\mathrm{vac}} + \frac{9}{7}\kappa_{2\mathrm{a}}^{\mathrm{vac}} + \frac{Q_0^2}{\pi^2 f_\pi^2}\left(\frac{9}{28}\tilde{\kappa}_{4\mathrm{a}}^{\mathrm{vac}} - \frac{1}{14}\tilde{\kappa}_{4\mathrm{v}}^{\mathrm{vac}}\right) , \tag{3.15}$$

$$\kappa_\omega^{\mathrm{med}} = -\frac{2}{7}\kappa_{2\mathrm{v}}^{\mathrm{med}} + \frac{9}{7}\kappa_{2\mathrm{a}}^{\mathrm{med}} + \frac{Q_0^2}{\pi^2 f_\pi^2}\left(\frac{9}{28}\tilde{\kappa}_{4\mathrm{a}}^{\mathrm{med}} - \frac{1}{14}\tilde{\kappa}_{4\mathrm{v}}^{\mathrm{med}}\right) \tag{3.16}$$

elucidate the important point that only statements about certain linear combinations of four-quark condensates can be derived when comparing to hadronic models. This accounts especially for the strength of the density dependence of the examined four-quark condensates.

The dependence of the finite density behavior of the moment $\overline{m}_\omega$ on the value of $\kappa_\omega^{\mathrm{med}}$ is understood qualitatively as follows. Expand all functions $f(n)$ according to

$$f(n) = f^{(0)} + f^{(1)}\frac{n}{n_0} \quad \text{with} \quad f^{(0)} = f(0) \quad \text{and} \quad f^{(1)} = n_0\frac{\partial}{\partial n}f(0) \,, \tag{3.17}$$

$$\overline{m}_\omega^2(n,\mathcal{M}^2,s_V) = R + \Delta\frac{n}{n_0} \,, \tag{3.18}$$

$$c_i = c_i^{(0)} + c_i^{(1)} \frac{n}{n_0}\,, \tag{3.19}$$

$$s_\omega = s_\omega^{(0)} + s_\omega^{(1)} \frac{n}{n_0}\,, \tag{3.20}$$

which gives

$$R = \frac{1}{N}\left(c_0^{(0)} \mathcal{M}^2 \left[1 - \left(1 + \frac{s_\omega^{(0)}}{\mathcal{M}^2} \right) e^{-s_\omega^{(0)}/\mathcal{M}^2} \right] - \frac{c_2^{(0)}}{\mathcal{M}^2} - \frac{c_3^{(0)}}{\mathcal{M}^4} - \frac{c_4^{(0)}}{2\mathcal{M}^6} \right)\,, \tag{3.21}$$

$$\begin{aligned} \Delta = \frac{1}{N\mathcal{M}^2}\Bigg\{ \left[\frac{9R}{4M_N} n_0 + c_0^{(0)} e^{-s_\omega^{(0)}/\mathcal{M}^2} s_\omega^{(1)} (s_\omega^{(0)} - R) - c_2^{(1)} \left(1 + \frac{R}{\mathcal{M}^2} \right) \right] \\ - \frac{c_3^{(1)}}{\mathcal{M}^2} \left(1 + \frac{R}{2\mathcal{M}^2} \right) - \frac{c_4^{(1)}}{2\mathcal{M}^4} \left(1 + \frac{R}{3\mathcal{M}^2} \right) \Bigg\}\,, \end{aligned} \tag{3.22}$$

$$N = c_0^{(0)} \left[1 - e^{-s_\omega^{(0)}/\mathcal{M}^2} \right] + \frac{c_1^{(0)}}{\mathcal{M}^2} + \frac{c_2^{(0)}}{\mathcal{M}^4} + \frac{c_3^{(0)}}{2\mathcal{M}^6} + \frac{c_4^{(0)}}{6\mathcal{M}^8}\,. \tag{3.23}$$

From the above set of parameters one finds the values of $c_i^{(0)}$ and $c_i^{(1)}$, where especially

$$\begin{aligned} c_2^{(1)} = & n_0 \Bigg\{ \frac{1}{2}\left(1 + \frac{\alpha_s}{4\pi} C_F \right) \sigma_N - \frac{1}{27} M_N^0 \\ & + \left(\frac{1}{4} - \frac{5\alpha_s}{48\pi} C_F \right) A_2^{(u+d)} M_N - \frac{3}{16} n_f \frac{\alpha_s}{\pi} A_2^G M_N \Bigg\}\,, \end{aligned} \tag{3.24}$$

$$\begin{aligned} c_3^{(1)} = & n_0 \Bigg\{ -\frac{112}{81} \pi \alpha_s \kappa_\omega^{\mathrm{med}} \frac{\sigma_N \langle \bar{q} q \rangle_{\mathrm{vac}}}{m_q} - \left(\frac{5}{12} + \frac{67\alpha_s}{192\pi} C_F \right) A_4^{(u+d)} M_N^3 \\ & + \frac{205\alpha_s}{864\pi} n_f A_4^G M_N^3 + \frac{1}{4} M_N \left(\frac{3}{8} K_u^2 + \frac{3}{2} K_u^1 + \frac{15}{16} K_u^g \right) - \frac{7}{144} \sigma_N M_N^2 \Bigg\}\,. \end{aligned} \tag{3.25}$$

The sign of Δ dictates whether the measure $\overline{m}_\omega$ decreases or increases in medium. Since the values of N are positive for relevant parameters, the qualitative behavior is given by the sign of the expression in brackets in Eq. (3.22). Assuming $s_\omega^{(0)} = 1.4$ GeV^2, $s_\omega^{(1)} = -0.15$ GeV^2, the vacuum reference value $R = 0.612$ GeV^2 and $c_4 = 0$, the sign change is driven by $\kappa_\omega^{\mathrm{med}}$ as

$$\mathcal{M}^2 = 0.4 \text{ GeV}^2 : \Delta \propto (1.86 - \kappa_\omega^{\mathrm{med}})\,, \tag{3.26}$$

$$\mathcal{M}^2 = 1.0 \text{ GeV}^2 : \Delta \propto (3.96 - \kappa_\omega^{\mathrm{med}})\,, \tag{3.27}$$

$$\mathcal{M}^2 = 2.0 \text{ GeV}^2 : \Delta \propto (5.32 - \kappa_\omega^{\mathrm{med}})\,. \tag{3.28}$$

In [25] the case $\mathcal{M}^2 = 1$ GeV^2 was representative for the numerical evaluation of the sum rule explained below. The other cases also denote that the critical situation, an approximately constant moment $\overline{m}_\omega$ is related to $\kappa_\omega^{\mathrm{med}} > 1$, i.e. the four-quark condensates change stronger than expected in the factorization picture.

Beyond this estimate the sum rule equation (3.13) is evaluated by numerical means. Therefore the dependence of $\overline{m}_\omega$ on the Borel mass $\mathcal{M}^2$ has to be reduced. Technically, the threshold parameter s_ω is optimized to produce a curve of maximum flatness within a certain range of Borel masses. This range, the so-called Borel window, has to contain physically reasonable values of the Borel mass. Its borders are here determined by standard criteria: the upper limit is set to restrict the continuum contribution to the initial hadronic side to at most 50 %; the lower Borel limit is determined such that the highest dimensional condensates contribute less than 10 % to the operator product expansion. These criteria are well established for the vector meson sum rules, however, they cannot be taken as general rules. Since the asymptotic behavior of the OPE in the non-perturbative sector is not understood, higher condensates could still spoil the sum rule. Also extending local quark-hadron duality down to the threshold s_ω might not describe the continuum appropriately. For compatibility we follow in the analysis these techniques well proven in the literature. The QSR considered here is fairly robust w.r.t. different evaluation approaches. We have checked that the method applied to the nucleon, compare Section 3.2, successfully recovers similar results.

In the vacuum limit the working Borel window is $\mathcal{M} = 0.8 \dots 2.7$ GeV, changing slightly in medium. The undetermined four-quark condensates are in vacuum adjusted to reproduce the vacuum ω mass with $\kappa_\omega^{\rm vac} = 2.94$. The density dependence of the four-quark condensates is then adjusted to remain at the same moment $\overline{m}_\omega$ at a baryon density $n > 0$. This depends of course on the density where the moment is calculated for comparison: The critical values of $\kappa_\omega^{\rm med}$ for a constant moment are 4.25 at $n = 0.01 n_0$, 4.06 at $n = 0.1 n_0$, 3.79 at $n = 0.6 n_0$, and 3.57 at $n = n_0$. Fig. 3.2 shows the squared moment $\overline{m}_\omega^2$ for this particular adjustments of $\kappa_\omega^{\rm med}$, it exhibits also the general impact of variations in $\kappa_\omega^{\rm med}$: $\overline{m}_\omega$ increases when the density dependence of the combination of four-quark condensates (3.14) is reduced. In all of the following discussions the choice of a reference density of 10 percent saturation density is made.

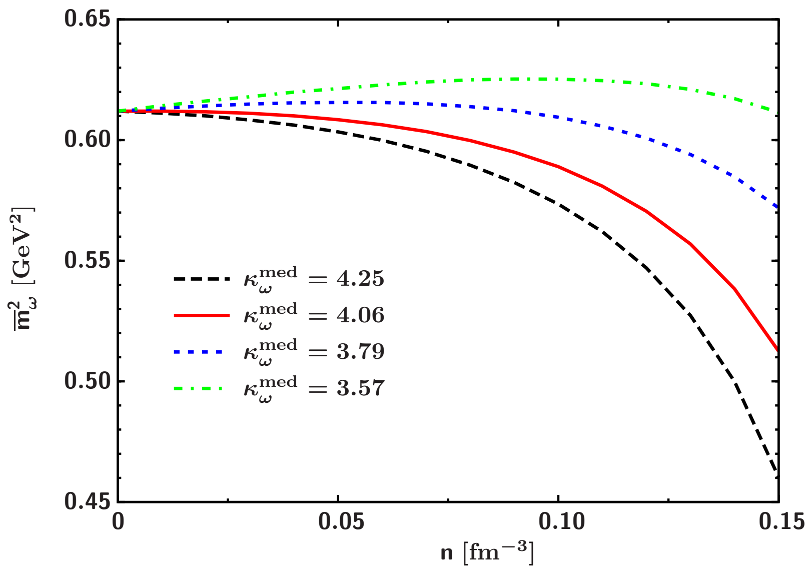

Figure 3.2: Adjustment of $\kappa_\omega^{\rm med}$ with $c_4 = 0$ to the case of a non-changed value of $\overline{m}_\omega^2$ with respect to different reference points in density. Increasing density dependence $\kappa_\omega^{\rm med}$ of the combined four-quark condensate suppresses the moment $\overline{m}_\omega$ at non-vanishing baryon density n.

We also estimated the impact of mass dimension 8 condensates entering c_4. The influence of c_4 is correlated to $\kappa_\omega^{\rm med}$. Variation of $c_4^{(1)}$ from $0 \dots 10^{-5}$ GeV^8 yields the following condition when $\overline{m}_\omega^2$ is to give the same value in vacuum and at our selected reference density or is even reduced,

$$\kappa_\omega^{\rm med} > 4.05 - 2.80 \cdot 10^3 \ {\rm GeV}^{-8} c_4^{(1)} \quad (c_4^{(0)} = 0)\,. \tag{3.29}$$

The influence of different contributions of $c_4^{(0)}$ is exemplified in Tab. 3.1. Modifications in the vacuum sum rule require readjustments to the vacuum mass with $\kappa_\omega^{\rm vac}$. The critical parameters $\kappa_\omega^{\rm med}$ are not significantly altered.

$c_4^{(0)}$	$\kappa_\omega^{\rm vac}$	$\kappa_\omega^{\rm med}(n = 0.1\, n_0)$
$-1.0 \cdot 10^{-3}$	1.14	3.81
$-5.0 \cdot 10^{-4}$	2.05	4.36
0.0	2.94	4.06
$5.0 \cdot 10^{-4}$	3.80	3.75
$1.0 \cdot 10^{-3}$	4.64	3.48

Table 3.1: The impact of constant dimension 8 condensates on the adjustment of $\kappa_\omega^{\rm vac}$ and $\kappa_\omega^{\rm med}$. A non-vanishing term $c_4^{(0)}$ requires a readjustment of the vacuum limit. Also the critical density dependence $\kappa_\omega^{\rm med}$ to ensure a constant moment is modified.

Fig. 3.3 collects the essential results: The moment $\overline{m}_\omega^2$ is calculated as a function of the baryon density n adjusted such that it remains approximately constant at least at small densities. The critical value therefore is $\kappa_\omega^{\rm med} \propto 4$ meaning that four-quark condensates change stronger with density compared to the factorization hypothesis.

Supplementary, we study the influence of a variation of several input parameters in the QCD sum rule: For a variation of the vacuum initial moment (vacuum mass) R by $\pm 10\%$ one obtains in the linearized sum rule Eq. (3.18) the values of Δ given in Tab. 3.2.

$\mathcal{M}^2$	$(a) R = 0.5508$ GeV^2	$(b) R = 0.612$ GeV^2	$(c) R = 0.6732$ GeV^2
0.4 GeV^2	$(1.76 - \kappa_\omega^{\rm med}) 0.247/n_0$	$(1.86 - \kappa_\omega^{\rm med}) 0.311/n_0$	$(1.95 - \kappa_\omega^{\rm med}) 0.425/n_0$
1.0 GeV^2	$(3.39 - \kappa_\omega^{\rm med}) 0.027/n_0$	$(3.96 - \kappa_\omega^{\rm med}) 0.028/n_0$	$(4.50 - \kappa_\omega^{\rm med}) 0.029/n_0$
2.0 GeV^2	$(3.63 - \kappa_\omega^{\rm med}) 0.009/n_0$	$(5.32 - \kappa_\omega^{\rm med}) 0.009/n_0$	$(6.97 - \kappa_\omega^{\rm med}) 0.009/n_0$

Table 3.2: The value of the analytical estimate Δ in units of GeV^2 for 10% variations of the vacuum limit $R = \overline{m}_\omega(n = 0)$ using $s_\omega = 1.4$ GeV^2 $-$ $0.15 n/n_0$ GeV^2 and $c_4 = 0$. In all cases a critical $\kappa_\omega^{\rm med} > 1$ is found.

If the complete numerical analysis of the QSR equation (3.13) is carried out with the same modifications, then the new constraints for a decreasing mass at our chosen reference density of $0.1\, n_0$ are

(a) $R = 0.5508 \text{ GeV}^2 : \kappa_\omega^{\rm vac} = 2.35, \kappa_\omega^{\rm med} > 3.37 - 2.99 \cdot 10^3 \text{ GeV}^{-8} n_0 c_4^{(1)}$,

(b) $R = 0.6120 \text{ GeV}^2 : \kappa_\omega^{\rm vac} = 2.94, \kappa_\omega^{\rm med} > 4.05 - 2.80 \cdot 10^3 \text{ GeV}^{-8} n_0 c_4^{(1)}$,

(c) $R = 0.6732 \text{ GeV}^2 : \kappa_\omega^{\rm vac} = 3.63, \kappa_\omega^{\rm med} > 4.76 - 2.43 \cdot 10^3 \text{ GeV}^{-8} n_0 c_4^{(1)}$.

Again $c_4 = c_4^{(0)} + c_4^{(1)} n$ with $c_4^{(0)} = 0$ has been used.

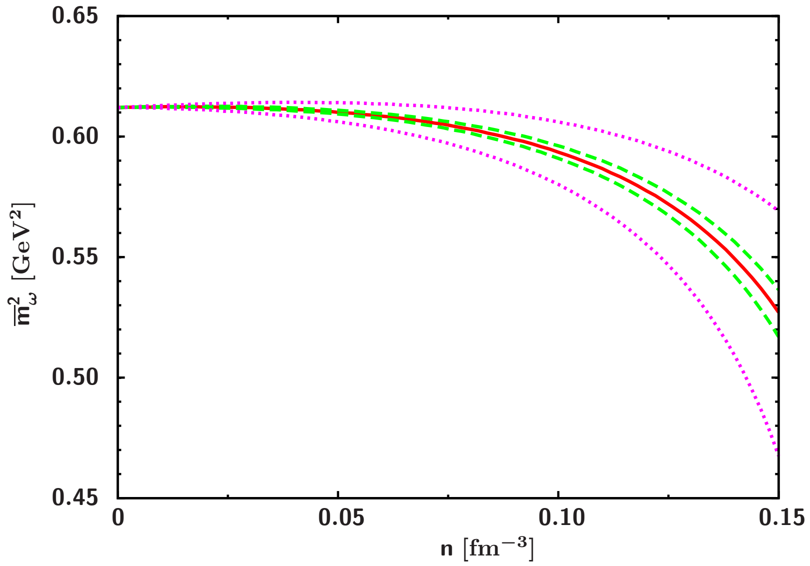

Figure 3.3: The mass parameter $\overline{m}^2_\omega$ in Eq. (3.3), averaged within the Borel window, as a function of the baryon density for $\kappa_N = 4$ and $c_4 = 0$ (solid curve). Note that the parameter $\overline{m}^2_\omega$ coincides only in zero-width approximation with the ω pole mass squared; in general it is a normalized moment of $\Delta\Pi^\omega$ to be calculated from data or models. The sum rule Eq. (3.13) is evaluated as described in the text with appropriately adjusted $\kappa^{\rm vac}_\omega$. Inclusion of $c_4^{(0)} = \mathcal{O}(\pm 10^{-3})$ GeV^8 requires a readjustment of $\kappa^{\rm vac}_\omega$ in the range $1 \cdots 5$ to $\overline{m}^{(0)2}_\omega$. A simultaneous change of $\kappa^{\rm med}_\omega$ in the order of 20 % is needed to recover the same density dependence as given by the solid curve at small values of n. The effect of a $c_4^{(1)}$ term is exhibited, too ($c_4^{(1)} = \pm 10^{-5} n_0^{-1}$ GeV^8: dashed curves, $c_4^{(1)} = \pm 5 \times 10^{-5} n_0^{-1}$ GeV^8: dotted curves; the upper (lower) curves are for negative (positive) signs).

Finally, the uncertainties emerging from other input parameters are investigated. A conservative estimate with uncorrelated $\pm 10\%$ deviations of R, α_s, m_u, m_d, $\langle \bar{q} q \rangle_{\rm vac}$, σ_N, $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rm vac}$, M_N^0, $A_2^{(u+d)}$, A_2^g, Q_0, $A_4^{(u+d)}$, A_4^g, K_u^2, K_u^1, K_u^g, $s^{(0)}$, $s^{(1)}$, with 0.4 GeV$^2 < \mathcal{M}^2 < 1.6$ GeV^2, $2.24 < \kappa^{\rm vac}_\omega < 3.64$, yields for the ω meson

$$\Delta = (a - \kappa^{\rm med}_\omega) \frac{b}{n_0} \; \mathrm{GeV}^2 \, , \tag{3.30}$$

where the uncertainties for the standard values $a = 3.96$ and $b = 0.0283$ are $1.24 < a < 9.5$, $0.00842 < b < 1.14$. This shows how robust the conclusion $\kappa^{\rm med}_\omega > 1$ would be.

For the ρ meson the numerically large Landau damping term pushes up the weighted strength [87]. Indeed, the term Δ takes for the ρ meson the values $a = -1.07$ and $b = 0.0283$, with uncertainties $-6.79 < a < 0.656$, $0.00842 < b < 1.14$. The negative value of a signals that for any positive $\kappa^{\rm med}_\rho$ the ratio $\overline{m}^2_\rho$ will always be decreasing. Note, that the underlying combination of four-quark condensates slightly differs from that in the ω sum rule.

In an analysis of finite energy sum rules for the ρ meson [88], the values of four-quark condensates enter a third sum rule, where for consistency reasons a strong violation of factorization of four-quark condensates is supported, although the authors use finite energy sum rules to avoid the impact of these dimension 6 condensates. In the vacuum limit, comparable to the ω case, the reported value $\kappa_\rho^{\mathrm{vac}} \gtrsim 4.5$ supports our findings. In the medium case strong deviations from factorization are indicated there for the ρ meson. The evaluation of finite energy sum rules for the ω meson [84] supports our findings.

Certainly, the errors assigned to the critical $\kappa_\omega^{\mathrm{med}}$ do not allow to determine the density dependence exactly. Nevertheless, the main insight is, that comparisons of hadronic information with condensates approve serious doubts about the factorization of the four-quark condensates.

Coming back to chiral symmetry, the condensate combination in Eq. (3.14) reads in terms of (two-component) flavor vectors

$$\begin{aligned} &\frac{1}{2}\left\langle \bar{u}\gamma_5\gamma_\mu\lambda^A u\bar{u}\gamma_5\gamma^\mu\lambda^A u\right\rangle + \frac{1}{2}\left\langle \bar{d}\gamma_5\gamma_\mu\lambda^A d\bar{d}\gamma_5\gamma^\mu\lambda^A d\right\rangle + \left\langle \bar{u}\gamma_5\gamma_\mu\lambda^A u\bar{d}\gamma_5\gamma^\mu\lambda^A d\right\rangle \\ &+\frac{2}{9}\left\langle \bar{u}\gamma_\mu\lambda^A u\bar{d}\gamma^\mu\lambda^A d\right\rangle + \frac{1}{9}\left\langle \bar{u}\gamma_\mu\lambda^A u\bar{u}\gamma^\mu\lambda^A u\right\rangle + \frac{1}{9}\left\langle \bar{d}\gamma_\mu\lambda^A d\bar{d}\gamma^\mu\lambda^A d\right\rangle \\ &\qquad = \frac{1}{2}\left\langle \bar{\psi}\gamma_5\gamma_\mu\lambda^A\psi\bar{\psi}\gamma_5\gamma^\mu\lambda^A\psi\right\rangle + \frac{1}{9}\left\langle \bar{\psi}\gamma_\mu\lambda^A\psi\bar{\psi}\gamma^\mu\lambda^A\psi\right\rangle . \end{aligned} \tag{3.31}$$

This combination could not be directly constructed by a generating operator Φ in Section 2.4.3. Under the restriction to the applied definition of potential order parameters, spontaneous breaking of chiral symmetry can therewith not be related to the specific four-quark condensate combination entering the QCD sum rule for the ω meson. However, the change of these four-quark condensate combination would still signal the complicated modifications of strongly interacting matter in the vicinity of a nuclear medium.

3.2 Light-Quark Baryons: Nucleon

The nucleon is of fundamental interest as it represents as carrier of mass the hard core of visible matter in the universe and thus an important source for gravitation. Our investigations presented are to be considered in line with previous QCD sum rule investigations [42, 54, 89, 90] for nucleons inside cold nuclear matter, which are also discussed in [91] and continuously explored in [55, 61, 92–97]. For the nucleon an important dependence of self-energies on four-quark condensates was found. Comparisons with results of chiral effective field theory [98], where nucleon self-energies show strong cancellation effects (i.e. they change with the same magnitude but have opposite signs) suggest that the relevant four-quark condensates should be weakly density dependent [42]. The aim is to specify these condensates and their impact on nucleon self-energies. Chiral effective field theory provides self-energies that our QSR results will be compared to. Parts of the presented material can be also found in [27].

Basis of the QSR for the nucleon is again a correlator, this time defined as in Eq. (2.19),

$$\Pi(q,v) = i \int d^4x \, e^{iqx} \langle\langle T\left[\eta(x)\bar{\eta}(0)\right]\rangle\rangle \,, \tag{3.32}$$

with the nucleon current η. Following Ioffe [13] one can write down two independent interpolating fields representing the nucleon with the corresponding quantum numbers $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$, $\epsilon^{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu d_c$ and $\epsilon^{abc}[u_a^T C\sigma_{\mu\nu} u_b]\gamma_5\sigma^{\mu\nu} d_c$, when restricting to fields that contain no derivatives and couple to spin $\frac{1}{2}$ only.[1] Extended forms of the nucleon current include derivatives [99, 100], or make use of tensor interpolating fields [101, 102] (also used to extrapolate the vacuum nucleon mass via QCD sum rules [103] to (unphysical) larger values obtained on the lattice, comparable to similar efforts within chiral perturbation theory [104]). The complications in nucleon sum rules can further be dealt with when taking into consideration the coupling of positive and negative parity states to the nucleon interpolating field [105].

In this work, our structures are always written for the proton; by exchanging u and d the neutron is obtained. Even the neutron-proton mass difference has been analyzed in this framework [106]. For the case of exact flavor symmetry $SU(3)_V$ a compilation of baryon interpolating fields and their chiral representations is given in [107].

As interpolating field for the nucleon a Fierz rearranged and thus simplified linear combination is widely used [42]

$$\eta(x) = 2\epsilon^{abc}\left\{t[u_a^T(x)C\gamma_5 d_b(x)]u_c(x) + [u_a^T(x)Cd_b(x)]\gamma_5 u_c(x)\right\} \,, \tag{3.33}$$

which in the above basis reads

$$\tilde{\eta}(x) = \frac{1}{2}\epsilon^{abc}\left\{(1-t)[u_a^T(x)C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x) + (1+t)[u_a^T(x)C\sigma_{\mu\nu} u_b(x)]\gamma_5\sigma^{\mu\nu} d_c(x)\right\} \,. \tag{3.34}$$

Both currents, η and $\tilde{\eta}$, are related by Fierz transformations whereby in such a straightforward calculation the remaining difference vanishes for symmetry reasons (analog to the exclusion of Dirac structures in [13] when constructing all possible nucleon fields). The consequence of these two equivalent representations (3.33) and (3.34) is that two different forms of the OPE arise. On the level of four-quark condensates the identity is not obvious, but is understood with the constraints on pure flavor four-quark condensates, Eqs. (2.115), derived in Section 2.4.1. Our subsequent equations will be given for the ansatz (3.33) with arbitrary mixing parameter t. In

[1]Note, the second term can be rewritten with the identity $\gamma_5\sigma^{\alpha\beta} = \frac{i}{2}\epsilon^{\alpha\beta\mu\nu}\sigma_{\mu\nu}$. The charge conjugation matrix is defined as $C = i\gamma_0\gamma_2$.

nucleon sum rule calculations the particular choice of the field with $t = -1$, the so-called Ioffe interpolating field, is preferred for reasons of applicability of the method and numerical stability of the evaluation procedure (cf. also [108] for a discussion of an optimal nucleon interpolating field; another choice of t would emphasize the negative-parity state in the sum rule [109]).

3.2.1 QCD Sum Rule Equations

The OPE for $\Pi(x)$ is the important step towards the sum rule formulation. Lorentz invariance and the requested symmetry with respect to time/parity reversal allow the decomposition of the correlator into invariant functions

$$\Pi(q) = \Pi_s(q^2, qv) + \Pi_q(q^2, qv)\not{q} + \Pi_v(q^2, qv)\not{v} \,, \tag{3.35}$$

where v again is the four-velocity vector of the medium. The three invariant functions, which accordingly yield three sum rule equations, can be projected out by appropriate Dirac traces

$$\Pi_s(q^2, qv) = \frac{1}{4}\mathrm{Tr}\left(\Pi(q)\right) \,, \tag{3.36a}$$

$$\Pi_q(q^2, qv) = \frac{1}{4[q^2v^2 - (qv)^2]}\left\{v^2\mathrm{Tr}\left(\not{q}\Pi(q)\right) - (qv)\mathrm{Tr}\left(\not{v}\Pi(q)\right)\right\} \,, \tag{3.36b}$$

$$\Pi_v(q^2, qv) = \frac{1}{4[q^2v^2 - (qv)^2]}\left\{q^2\mathrm{Tr}\left(\not{v}\Pi(q)\right) - (qv)\mathrm{Tr}\left(\not{q}\Pi(q)\right)\right\} \,, \tag{3.36c}$$

and are furthermore decomposed into even (e) and odd (o) parts w.r.t. qv

$$\Pi_i(q^2, qv) = \Pi_i^{\mathrm{e}}(q^2, (qv)^2) + (qv)\Pi_i^{\mathrm{o}}(q^2, (qv)^2) \,. \tag{3.37}$$

For the nucleon interpolating field (3.33), this leads to

$$\begin{aligned}\Pi_s^{\mathrm{e}}(q^2, (qv)^2) = &+ \frac{c_1}{16\pi^2}q^2\ln(-q^2)\langle\bar{q}q\rangle + \frac{3c_2}{16\pi^2}\ln(-q^2)\langle\bar{q}g_s(\sigma G)q\rangle \\ &+ \frac{2c_3}{3\pi^2v^2}\frac{(qv)^2}{q^2}\left(\langle\bar{q}(viD)^2q/v^2\rangle + \frac{1}{8}\langle\bar{q}g_s(\sigma G)q\rangle\right) \,,\end{aligned} \tag{3.38a}$$

$$\Pi_s^{\mathrm{o}}(q^2, (qv)^2) = -\frac{1}{3v^2}\frac{1}{q^2}\left\{c_1\langle\bar{q}q\rangle\langle\bar{q}\not{v}q\rangle\right\}_{\mathrm{eff}}^1 \,, \tag{3.38b}$$

$$\begin{aligned}\Pi_q^{\mathrm{e}}(q^2, (qv)^2) = &- \frac{c_4}{512\pi^4}q^4\ln(-q^2) - \frac{c_4}{256\pi^2}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \\ &+ \frac{c_4}{72\pi^2v^2}\left(5\ln(-q^2) - \frac{8(qv)^2}{q^2v^2}\right)\langle\bar{q}\not{v}(viD)q\rangle \\ &- \frac{c_4}{1152\pi^2v^2}\left(\ln(-q^2) - \frac{4(qv)^2}{q^2v^2}\right)\left\langle\frac{\alpha_s}{\pi}[(vG)^2 + (v\tilde{G})^2]\right\rangle \\ &- \frac{1}{6}\frac{1}{q^2}\left\{c_1\langle\bar{q}q\rangle^2 + \frac{c_4}{v^2}\langle\bar{q}\not{v}q\rangle^2\right\}_{\mathrm{eff}}^q \,,\end{aligned} \tag{3.38c}$$

$$\begin{aligned}\Pi_q^{\mathrm{o}}(q^2, (qv)^2) = &+ \frac{c_4}{24\pi^2v^2}\ln(-q^2)\langle\bar{q}\not{v}q\rangle + \frac{c_5}{72\pi^2v^2}\frac{1}{q^2}\langle\bar{q}g_s\not{v}(\sigma G)q\rangle \\ &- \frac{c_4}{12\pi^2v^2}\frac{1}{q^2}\left(1 + \frac{2(qv)^2}{q^2v^2}\right)\left(\langle\bar{q}\not{v}(viD)^2q/v^2\rangle + \frac{1}{12}\langle\bar{q}g_s\not{v}(\sigma G)q\rangle\right) \,,\end{aligned} \tag{3.38d}$$

$$\begin{aligned}\Pi_v^{\mathrm{e}}(q^2, (qv)^2) = &+ \frac{c_4}{12\pi^2v^2}q^2\ln(-q^2)\langle\bar{q}\not{v}q\rangle - \frac{c_5}{48\pi^2v^2}\ln(-q^2)\langle\bar{q}g_s\not{v}(\sigma G)q\rangle \\ &+ \frac{c_4}{2\pi^2v^4}\frac{(qv)^2}{q^2}\left(\langle\bar{q}\not{v}(viD)^2q/v^2\rangle + \frac{1}{12}\langle\bar{q}g_s\not{v}(\sigma G)q\rangle\right) \,,\end{aligned} \tag{3.38e}$$

$$\Pi_v^{\mathrm{o}}(q^2,(qv)^2) = +\frac{c_4}{288\pi^2 v^4}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}[(vG)^2+(v\tilde{G})^2]\right\rangle \\ -\frac{5c_4}{18\pi^2 v^4}\ln(-q^2)\langle\bar{q}\not{v}(viD)q\rangle - \frac{1}{3v^2}\frac{1}{q^2}\left\{\frac{c_4}{v^2}\langle\bar{q}\not{v}q\rangle^2\right\}^v_{\mathrm{eff}}, \tag{3.38f}$$

where the $c_i(t)$'s, being polynomials of the mixing parameter t, are

$$c_1 = 7t^2 - 2t - 5\,, \tag{3.39a}$$
$$c_2 = -t^2 + 1\,, \tag{3.39b}$$
$$c_3 = 2t^2 - t - 1\,, \tag{3.39c}$$
$$c_4 = 5t^2 + 2t + 5\,, \tag{3.39d}$$
$$c_5 = 7t^2 + 10t + 7\,. \tag{3.39e}$$

Numerical values for condensates are collected in Section 2.3.2.

In Eqs. (3.38) the contributions from four-quark condensates are written as the usual factorized result denoted by $\{\ldots\}^{1,q,v}_{\mathrm{eff}}$; full expressions which replace and overcome this simplification are the focus of Section 3.2.2 (see especially Eqs. (3.50) below). Note that, in contrast to the OPE for light vector mesons Eqs. (3.8) and (3.9), four-quark condensates enter already without a factor α_s (the strong coupling), and the chiral condensate $\langle\bar{q}q\rangle$ does not appear in a renormalization invariant combination with the quark mass.

We utilize the weighted sum rule in the form of Eq. (2.43) repeated here as basis for our investigations

$$(E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega\Delta\Pi(\omega)e^{-\omega^2/\mathcal{M}^2} = \Pi^{\mathrm{e}}(\mathcal{M}^2) - \frac{1}{\pi}\int_{\omega_+}^{\infty} d\omega\omega\Pi^{\mathrm{e}}_{\mathrm{per}}(\omega)e^{-\omega^2/\mathcal{M}^2} \\ -\bar{E}\left\{\Pi^{\mathrm{o}}(\mathcal{M}^2) - \frac{1}{\pi}\int_{\omega_+}^{\infty} d\omega\Pi^{\mathrm{o}}_{\mathrm{per}}(\omega)e^{-\omega^2/\mathcal{M}^2}\right\} + \frac{1}{\pi}\int_{\omega_-}^{-\omega_+} d\omega\Delta\Pi(\omega)[\omega-\bar{E}]e^{-\omega^2/\mathcal{M}^2}\,. \tag{3.40}$$

In general, the Dirac structure of $\Delta\Pi$ would require definitions E_i, $\bar{E}_i$ to account for the distinct invariant functions $(i = s, q, v)$ of the decomposition (3.35). In the case considered here we assume that these weighted moments coincide with $E_{s,q,v} = E$ (analogously $\bar{E}$). Also for the threshold parameters $\omega_\pm$ we use common values for the s, q, v parts. For shortness, in the shown Borel transformed equations the decomposed terms are symbolically rearranged to full Dirac structures.

It should be emphasized that the applied sum rule Eq. (3.40) is for a certain, weighted moment of a part of the nucleon spectral function. Without further assumptions, local properties of $\Delta\Pi(\omega)$ cannot be deduced. Note also that in this form the anti-nucleon enters inevitably the sum rule. The reasoning behind the choice of the combined sum rule, Eq. (2.42), with the moments E and $\bar{E}$, Eqs. (2.41), is that in mean field approximation, where self-energy contributions in the propagator are real and energy-momentum independent (cf. also [42]), the pole contribution of the nucleon propagator $G(q) = (\not{q} - M_N - \Sigma)^{-1}$ can be written as

$$G(q) = \frac{1}{1-\Sigma_q}\frac{\not{q} + M_N^* - \not{v}\Sigma_v}{(q_0 - E_+)(q_0 - E_-)}\,. \tag{3.41}$$

Pauli corrections to positive-energy baryons and propagation of holes in the Fermi sea give rise to an additional piece $G_{\mathrm{D}}(q) \propto \Theta(|\vec{q_F}| - |\vec{q}\,|)$ [110] vanishing for nucleon momenta $\vec{q}$ above the Fermi surface $|\vec{q_F}|$ considered here. The self-energy Σ is decomposed into invariant structures $\Sigma = \tilde{\Sigma}_s + \Sigma_q\not{q} + \tilde{\Sigma}_v\not{v}$ [111] (for mean field $\Sigma_q = 0$), where one introduces scalar $\Sigma_s = M_N^* - M_N$ and vector self-energies Σ_v, which are related to the decomposition as [42]

$$M_N^* = \frac{M_N + \tilde{\Sigma}_s}{1-\Sigma_q}\,, \quad \Sigma_v = \frac{\tilde{\Sigma}_v}{1-\Sigma_q}\,. \tag{3.42}$$

In the rest frame of nuclear matter the energy of the nucleon is E_+, correspondingly E_- that of the antinucleon excitation, where

$$E_\pm = \Sigma_v \pm \sqrt{\vec{q}^{\,2} + M_N^{*2}}\,. \tag{3.43}$$

Since the sum rule explicitly depends on the nucleon momentum, however, the self-energy Σ as well as invariant structures Σ_i and derived quantities acquire now a momentum dependence and become functions of the Lorentz invariants q^2, qv and v^2, extending mean field theory towards the relativistic Hartree-Fock approximation [110]. Eq. (3.41) is giving rise to a discontinuity $\Delta G(q_0) = \frac{1}{2i}\lim_{\epsilon\to 0}(G(q_0+i\epsilon) - G(q_0 - i\epsilon))$ with a simple pole structure

$$\Delta G(q_0) = \frac{\pi}{1-\Sigma_q}\frac{\not{q} + M_N^* - \not{v}\Sigma_v}{E_+ - E_-}\left(\delta(q_0 - E_-) - \delta(q_0 - E_+)\right)\,, \tag{3.44}$$

where the general expression, Eq. (2.41), identifies the moment $\bar{E}$ with the anti-nucleon pole energy E_- for all 3 Dirac structures (analogously, E is identified with E_+). Then the l.h.s. of the sum rule (3.40) reads

$$(E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi(\omega) e^{-\omega^2/\mathcal{M}^2} = -\frac{\lambda_N^2}{1-\Sigma_q}(\not{q} + M_N^* - \not{v}\Sigma_v)e^{-E_+^2/\mathcal{M}^2}\,. \tag{3.45}$$

Here, λ_N enters through the transition from the correlator in terms of quarks (3.32) to the nucleon propagator (3.41), compare Eq. (2.1)-(2.3), and can be combined into an effective coupling $\lambda_N^{*2} = \frac{\lambda_N^2}{(1-\Sigma_q)}$. More general one can interpret Eq. (3.45) as parametrization of the l.h.s. of (3.40), where integrated information of $\Delta\Pi$ is mapped onto the quantities M_N^*, Σ_v, λ_N^{*2} and $E_\pm$, which are subject of our further analysis, by virtue of Eqs. (3.36)

$$-\lambda_N^{*2} M_N^* e^{-E_+^2/\mathcal{M}^2} = (E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_s(\omega) e^{-\omega^2/\mathcal{M}^2}\,, \tag{3.46a}$$

$$-\lambda_N^{*2} e^{-E_+^2/\mathcal{M}^2} = (E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_q(\omega) e^{-\omega^2/\mathcal{M}^2}\,, \tag{3.46b}$$

$$\lambda_N^{*2}\Sigma_v e^{-E_+^2/\mathcal{M}^2} = (E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_v(\omega) e^{-\omega^2/\mathcal{M}^2}\,. \tag{3.46c}$$

Due to the supposed pole structure in (3.41) the self-energy components are related to $E_\pm$ (or more general to E and $\bar{E}$) and the relations from distinct Dirac structures are coupled equations. The given general spectral integrals however not yet relate the unknown quantities, so that our numerical results presented here are not completely independent of the given nucleon propagator ansatz. These relations highlight also the dependence on the Borel mass $\mathcal{M}$ which determines how the spectral density is weighted in the general spectral integrals on the right hand sides.

In [112], it has been pointed out, that Π also contains chiral logarithms, e.g. $\overset{\circ}{m}{}_\pi^2 \log \overset{\circ}{m}{}_\pi^2$, which, however, do not appear in the chiral perturbation theory expression for M_N. It was argued [113, 114] that low-lying continuum like πN excitations around M_N cancel such unwanted pieces. In this respect, the parameters M_N^*, Σ_q, Σ_v in Eqs. (3.46) are hardly to be identified with pure nucleon pole characteristics, but should be considered as measure of integrated strength of nucleon like excitations in a given interval. Moreover, many hadronic models point to a quite distributed strength or even multi-peak structures (e.g. [115]). The importance of an explicit inclusion of scattering contributions in the interval $0\ldots\omega_+$ has been demonstrated in [59, 116, 117] for finite temperature effects on the in-medium nucleon. In vacuum QCD sum rules for baryons, e.g. the nucleon, improvement of the continuum treatment is achieved by the inclusion of negative-parity states, which are equally described by a given correlation function as the corresponding positive-parity states [105, 109, 118–120]. Resorting to integrated strength distributions avoids these problems, but loses the tight relation to simple pole parameters.

Eq. (3.40) is the sum rule we are going to evaluate with respect to the above motivated identifications. Inserting the decomposition (3.35) with Eqs. (3.36)-(3.38) we arrive at the three coupled sum rule equations

$$\lambda_N^{*2} M_N^* e^{-(E_+^2-\vec{q}^{\,2})/\mathcal{M}^2} = A_1\mathcal{M}^4 + A_2\mathcal{M}^2 + A_3\,, \tag{3.47a}$$

$$\lambda_N^{*2} e^{-(E_+^2-\vec{q}^{\,2})/\mathcal{M}^2} = B_0\mathcal{M}^6 + B_2\mathcal{M}^2 + B_3 + B_4/\mathcal{M}^2\,, \tag{3.47b}$$

$$\lambda_N^{*2} \Sigma_v e^{-(E_+^2-\vec{q}^{\,2})/\mathcal{M}^2} = C_1\mathcal{M}^4 + C_2\mathcal{M}^2 + C_3\,, \tag{3.47c}$$

with coefficients

$$A_1 = -\frac{c_1}{16\pi^2} E_1 \langle \bar{q}q \rangle\ , \tag{3.48a}$$

$$A_2 = -\frac{3c_2}{16\pi^2} E_0 \langle g_s\bar{q}\sigma Gq \rangle\ , \tag{3.48b}$$

$$A_3 = -\frac{2c_3}{3\pi^2}\vec{q}^{\,2}\left(\langle \bar{q}iD_0iD_0q\rangle + \frac{1}{8}\langle g_s\bar{q}\sigma Gq\rangle\right) - \frac{1}{3}E_-\left\{c_1\langle\bar{q}q\rangle\langle\bar{q}\not{v}q\rangle\right\}^1_{\text{eff}}\ , \tag{3.48c}$$

$$B_0 = \frac{c_4}{256\pi^4}E_2\,, \tag{3.48d}$$

$$B_2 = \frac{c_4E_0}{24\pi^2}E_-\left\langle q^\dagger q\right\rangle - \frac{5c_4E_0}{72\pi^2}\left\langle q^\dagger iD_0q\right\rangle + \frac{c_4E_0}{256\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{c_4E_0}{1152\pi^2}\left\langle\frac{\alpha_s}{\pi}[(vG)^2+(v\tilde{G})^2]\right\rangle\ , \tag{3.48e}$$

$$B_3 = \frac{c_4\vec{q}^{\,2}}{9\pi^2}\left\langle q^\dagger iD_0q\right\rangle - \frac{c_4\vec{q}^{\,2}}{288\pi^2}\left\langle\frac{\alpha_s}{\pi}[(vG)^2+(v\tilde{G})^2]\right\rangle + \frac{c_5E_-}{72\pi^2}\left\langle g_sq^\dagger\sigma Gq\right\rangle - \frac{c_4}{4}E_-\left(\left\langle q^\dagger iD_0iD_0q\right\rangle + \frac{1}{12}\left\langle g_sq^\dagger\sigma Gq\right\rangle\right) + \frac{1}{6}\left\{c_1\langle\bar{q}q\rangle^2 + \frac{c_4}{v^2}\langle\bar{q}\not{v}q\rangle^2\right\}^q_{\text{eff}}\ , \tag{3.48f}$$

$$B_4 = \frac{c_4}{6\pi^2}\vec{q}^{\,2}\left(\left\langle q^\dagger iD_0iD_0q\right\rangle + \frac{1}{12}\left\langle g_sq^\dagger\sigma Gq\right\rangle\right)\ , \tag{3.48g}$$

$$C_1 = \frac{c_4}{12\pi^2}E_1\left\langle q^\dagger q\right\rangle\ , \tag{3.48h}$$

$$C_2 = \frac{5c_4}{18\pi^2}E_0E_-\left\langle q^\dagger iD_0q\right\rangle - \frac{c_4E_0}{288\pi^2}E_-\left\langle\frac{\alpha_s}{\pi}[(vG)^2+(v\tilde{G})^2]\right\rangle - \frac{c_5E_0}{48\pi^2}\left\langle g_sq^\dagger\sigma Gq\right\rangle\ , \tag{3.48i}$$

$$C_3 = \frac{c_4}{2\pi^2}\vec{q}^{\,2}\left(\left\langle q^\dagger iD_0iD_0q\right\rangle + \frac{1}{12}\left\langle g_sq^\dagger\sigma Gq\right\rangle\right) + \frac{1}{3}E_-\left\{\frac{c_4}{v^2}\langle\bar{q}\not{v}q\rangle^2\right\}^v_{\text{eff}}\ , \tag{3.48j}$$

and factors E_j emerging from continuum contributions, with the definition $s_0 = \omega_+^2 - \vec{q}^{\,2}$,

$$E_0 = 1 - e^{-s_0/M^2}\ , \tag{3.49a}$$

$$E_1 = 1 - \left(1 + \frac{s_0}{M^2}\right)e^{-s_0/M^2}\ , \tag{3.49b}$$

$$E_2 = 1 - \left(1 + \frac{s_0}{M^2} + \frac{s_0^2}{2M^4}\right)e^{-s_0/M^2}\ , \tag{3.49c}$$

and the asymmetric continuum threshold integral in Eq. (3.40) neglected. The list (3.48) is exhaustive for all condensates up to mass dimension 5 in the limit of vanishing quark masses.

Note that nucleon QCD sum rules in the vacuum case are included: Thereby only Eqs. (3.47a) and (3.47b) remain, the condensates entering the r.h.s. of Eq. (3.47c) vanish at zero density as well as the vector-self energy Σ_v on the left hand side. Also the pole energies are simplified to $E_+ = -E_-$. Care should be taken with condensate projections. They change qualitatively when the impact of the medium, accounted for by projection onto v, is absent.

3.2.2 Impact of Four-Quark Condensates

We are especially concerned with the impact of four-quark condensates on the nucleon self-energies. The full expressions for the four-quark condensates in the order α_s^0, abbreviated in Eqs. (3.38) and (3.48) so far symbolically, are

$$\begin{aligned}\{c_1 \langle\bar{q}q\rangle \langle\bar{q}\not{v}q\rangle\}^1_{\text{eff}} = \frac{3}{2}\epsilon_{abc}\epsilon_{a'b'c}\Big(&-2c_2 \left\langle \bar{u}^{a'}\not{v}u^a\bar{u}^{b'}u^b\right\rangle + c_6 \left\langle \bar{u}^{a'}\not{v}u^a\bar{d}^{b'}d^b\right\rangle \\ &- 3c_2 \left\langle \bar{u}^{a'}u^a\bar{d}^{b'}\not{v}d^b\right\rangle + c_7 \left\langle \bar{u}^{a'}\gamma_5\gamma_\kappa u^a\bar{d}^{b'}\sigma_{\lambda\pi}d^b\epsilon^{\kappa\lambda\pi\xi}v_\xi\right\rangle\Big)\ ,\end{aligned} \tag{3.50a}$$

$$\begin{aligned}\left\{c_1 \langle\bar{q}q\rangle^2 + \frac{c_4}{v^2}\langle\bar{q}\not{v}q\rangle^2\right\}^q_{\text{eff}} = \epsilon_{abc}\epsilon_{a'b'c}\Big(&2c_9 \left\langle \bar{u}^{a'}\gamma_\tau u^a\bar{u}^{b'}\gamma^\tau u^b\right\rangle - 2c_9 \left\langle \bar{u}^{a'}\not{v}u^a\bar{u}^{b'}\not{v}u^b/v^2\right\rangle \\ &+ 4t \left\langle \bar{u}^{a'}\gamma_5\gamma_\tau u^a\bar{u}^{b'}\gamma_5\gamma^\tau u^b\right\rangle - 4t \left\langle \bar{u}^{a'}\gamma_5\not{v}u^a\bar{u}^{b'}\gamma_5\not{v}u^b/v^2\right\rangle \\ &- 9c_2 \left\langle \bar{u}^{a'}u^a\bar{d}^{b'}d^b\right\rangle + \frac{9}{2}c_2 \left\langle \bar{u}^{a'}\sigma_{\kappa\lambda}u^a\bar{d}^{b'}\sigma^{\kappa\lambda}d^b\right\rangle \\ &- 9c_2 \left\langle \bar{u}^{a'}\gamma_5 u^a\bar{d}^{b'}\gamma_5 d^b\right\rangle \\ &+ c_{10} \left\langle \bar{u}^{a'}\gamma_\tau u^a\bar{d}^{b'}\gamma^\tau d^b\right\rangle - 2c_9 \left\langle \bar{u}^{a'}\not{v}u^a\bar{d}^{b'}\not{v}d^b/v^2\right\rangle \\ &+ c_8 \left\langle \bar{u}^{a'}\gamma_5\gamma_\tau u^a\bar{d}^{b'}\gamma_5\gamma^\tau d^b\right\rangle - 4t \left\langle \bar{u}^{a'}\gamma_5\not{v}u^a\bar{d}^{b'}\gamma_5\not{v}d^b/v^2\right\rangle\Big)\ ,\end{aligned} \tag{3.50b}$$

$$\begin{aligned}\left\{\frac{c_4}{v^2}\langle\bar{q}\not{v}q\rangle^2\right\}^v_{\text{eff}} = \epsilon_{abc}\epsilon_{a'b'c}\Big(&-c_9 \left\langle \bar{u}^{a'}\gamma_\tau u^a\bar{u}^{b'}\gamma^\tau u^b\right\rangle + 4c_9 \left\langle \bar{u}^{a'}\not{v}u^a\bar{u}^{b'}\not{v}u^b/v^2\right\rangle \\ &- 2t \left\langle \bar{u}^{a'}\gamma_5\gamma_\tau u^a\bar{u}^{b'}\gamma_5\gamma^\tau u^b\right\rangle + 8t \left\langle \bar{u}^{a'}\gamma_5\not{v}u^a\bar{u}^{b'}\gamma_5\not{v}u^b/v^2\right\rangle \\ &- c_9 \left\langle \bar{u}^{a'}\gamma_\tau u^a\bar{d}^{b'}\gamma^\tau d^b\right\rangle + 4c_9 \left\langle \bar{u}^{a'}\not{v}u^a\bar{d}^{b'}\not{v}d^b/v^2\right\rangle \\ &-2t \left\langle \bar{u}^{a'}\gamma_5\gamma_\tau u^a\bar{d}^{b'}\gamma_5\gamma^\tau d^b\right\rangle + 8t \left\langle \bar{u}^{a'}\gamma_5\not{v}u^a\bar{d}^{b'}\gamma_5\not{v}d^b/v^2\right\rangle\Big)\ .\end{aligned} \tag{3.50c}$$

Here, additional polynomials which express the mixing of interpolating fields are

$$c_6 = t^2 - 2t + 1, \tag{3.51a}$$
$$c_7 = t^2 - t, \tag{3.51b}$$
$$c_8 = 9t^2 + 10t + 9, \tag{3.51c}$$
$$c_9 = t^2 + 1, \tag{3.51d}$$
$$c_{10} = 11t^2 + 6t + 11. \tag{3.51e}$$

These expressions extend the non-factorized four-quark condensates for the nucleon in vacuum listed in [33, 59].

Parametrization

Similar to the case of the ω meson the individual four-quark condensates can be parametrized according to Tabs. 2.1 and 2.2. However, numerically only a statement about the obtained linear combinations of four-quark condensates can be made. Therefore we work with effective parametrizations for the combinations (3.50). The contraction of tensors in color space, $\epsilon_{abc}\epsilon_{a'b'c}$, also exhibits the linear combination of two independent color structures of four-quark condensates, compare Eq. (2.110). In the sum rule analysis this is collected in the parameters $\kappa_{\rm s}^{\rm med}$, $\kappa_{\rm q}^{\rm med}$, $\tilde{\kappa}_{\rm v}^{\rm med}$ describing the density dependence as

$$\left\{c_1 \langle\bar{q}q\rangle \langle\bar{q}\not{v}q\rangle\right\}^1_{\rm eff} = c_1 \left(\kappa_{\rm s}^{\rm med} \langle\bar{q}q\rangle_{\rm vac} \frac{3}{2} n\right) , \tag{3.52a}$$

$$\left\{c_1 \langle\bar{q}q\rangle^2 + \frac{c_4}{v^2} \langle\bar{q}\not{v}q\rangle^2\right\}^q_{\rm eff} = c_1 \left(\kappa_{\rm q}^{\rm vac} \langle\bar{q}q\rangle^2_{\rm vac} + \kappa_{\rm q}^{\rm med} \langle\bar{q}q\rangle_{\rm vac} \frac{\sigma_{\rm N}}{m_{\rm q}} n\right) , \tag{3.52b}$$

$$\left\{\frac{c_4}{v^2} \langle\bar{q}\not{v}q\rangle^2\right\}^v_{\rm eff} = c_4 \left(\tilde{\kappa}_{\rm v}^{\rm med} \langle\bar{q}q\rangle_{\rm vac} \frac{\sigma_{\rm N}}{m_{\rm q}} n\right) , \tag{3.52c}$$

and have to be specified for a mixing parameter t. Here, we restrict the discussion to the limit of the Ioffe interpolating field, $t = -1$. Note again, the $\kappa^{\rm med}$ values are effective combinations representing the density dependence of the respective condensate lists (3.50) and thus e.g. negative $\kappa^{\rm med}$, a four-quark condensate behavior contrary to the factorization assumption, comprise cancellation effects within these condensate combinations. The sum rule is only sensitive to these effective combinations and can thus only reveal information on the behavior of these specific linear combinations of four-quark condensates.

The density dependence of four-quark condensates specific to this sum rule was calculated within a perturbative chiral quark model (PCQM) [60] and further analyzed in [61]. With our translation in Tab. 2.3 these values can be used to calculate the respective effective parameters $\kappa^{\rm med}$ (apart from the term $\propto \left\langle \bar{u}^{a'}\gamma_5\gamma_\kappa u^a \bar{d}^{b'}\sigma_{\lambda\pi} d^b\right\rangle_N \epsilon^{\kappa\lambda\pi\xi} v_\xi$ not considered in [60], which we had to neglect in the determination of $\kappa_{\rm s}^{\rm med}$)

$$\kappa_{\rm s}^{\rm med} = -0.25\,, \quad \kappa_{\rm q}^{\rm med} = -0.10\,, \quad \tilde{\kappa}_{\rm v}^{\rm med} = -0.03\,. \tag{3.53}$$

Interestingly, individual $\kappa^{\rm med}$ parameters are not small compared to these effective numbers indicating significant cancellation effects in the density dependent parts of combined four-quark condensates. Moreover, for pure flavor four-quark condensates the ambiguity due to Fierz relations between condensates does not allow to prefer a specific four-quark condensate in the sum rule. This equivalence of certain four-quark condensate combinations has to be respected, especially when such matrix elements are derived independently, as for example in [60]. Finally, notice some difference to the OPE part stated in equations (87)-(89) of [61] for the whole combination of the density dependent four-quark condensate contribution. Our equivalent OPE calculation utilizing the same nucleon four-quark expectation values (encoded in $\kappa_{\rm s,q}^{\rm med}$, $\tilde{\kappa}_{\rm v}^{\rm med}$ as above) yields

$$\Pi_{4q} = \left(0.49\frac{(qp)}{M_N}\mathbb{1} + 0.52\not{q} + 0.57\frac{(qp)}{M_N^2}\not{v}\right) \frac{\langle\bar{q}q\rangle}{q^2} n\,, \tag{3.54}$$

as defined in [61] with $p = M_N v$.

Approximations

The QCD sum rule evaluation for light vector mesons in Section 3.1 was carried out on the basis of an equation for a generalized moment optimized for maximum flatness w.r.t. the Borel window. This, however, includes derivative sum rules and seems not to be appropriate in the case of fermions where the condensates are distributed over coupled sum rule equations for several invariant functions due to the Dirac structure. This disentanglement of condensates likewise impedes the application of convergence rules for the operator product expansion to specify a working Borel window. Despite of this, equations for the self-energies can be formed dividing Eqs. (3.47a) and (3.47c) by (3.47b) thus arriving at a generalization of Ioffe's formula [13] for the nucleon vacuum mass. Approximated forms incorporating only lowest dimension condensates are sometimes used as estimates for in-medium nucleon self-energies [121, 122],

$$\Sigma_v = \frac{64\pi^2}{3\mathcal{M}^2}\left\langle q^\dagger q\right\rangle = 0.36 \text{ GeV}\frac{n}{n_0}\,, \tag{3.55}$$

$$\Sigma_s = -M_N - \frac{8\pi^2}{\mathcal{M}^2}\left\langle \bar{q}q\right\rangle = -0.37 \text{ GeV}\frac{n}{n_0}\,, \tag{3.56}$$

at $\mathcal{M}^2 = 1\ \text{GeV}^2$. Within chiral effective field theory such a direct dependence of the reduction of the in-medium nucleon mass on the change of the chiral condensate, obtained in the same framework, seems to be ruled out [123].

Although to be confirmed by dedicated sum rule analysis, it is instructive to understand the impact of four-quark condensates at finite density from naive decoupled self-energy equations linearized in density. For fixed Borel mass $\mathcal{M}^2 = 1\ \text{GeV}^2$, threshold $s_0 = 2.5\ \text{GeV}^2$ and condensates listed in Section 2.3.2, the self-energies become independent when a constant $E_- = -M_N$ is assumed; with $\kappa_{\text{q}}^{\text{vac}}$ adjusted to yield the vacuum nucleon mass the self-energies are estimated as

$$\Sigma_v = (0.16 + 1.22\tilde{\kappa}_{\text{v}}^{\text{med}})\ \text{GeV}\frac{n}{n_0}\,, \tag{3.57}$$

$$\Sigma_s = -(0.32 + 0.11\kappa_{\text{s}}^{\text{med}} - 0.31\kappa_{\text{q}}^{\text{med}})\ \text{GeV}\frac{n}{n_0}\,. \tag{3.58}$$

Indeed at small values of the Fermi momentum k_F the impact of $\kappa_{\text{s}}^{\text{med}}$, $\kappa_{\text{q}}^{\text{med}}$ and $\tilde{\kappa}_{\text{v}}^{\text{med}}$ is as follows: The vector self-energy Σ_v only depends on $\tilde{\kappa}_{\text{v}}^{\text{med}}$, the scalar self-energy Σ_s is effected by $\kappa_{\text{s}}^{\text{med}}$ and $\kappa_{\text{q}}^{\text{med}}$, whereby a negative $\kappa_{\text{s}}^{\text{med}}$ works equivalent to a positive value for $\kappa_{\text{q}}^{\text{med}}$ and vice versa. Comparable effects in Σ_s point out that a characteristic value of $\kappa_{\text{s}}^{\text{med}}$ is three times the corresponding absolute value of $\kappa_{\text{q}}^{\text{med}}$. Whereas this qualitative estimate from Eqs. (3.57) and (3.58) is in line with the numerical analysis below for small densities $n < 0.7n_0$ corresponding to Fermi momenta $k_F = (3\pi^2 n/2)^{1/3} < 1.2\ \text{fm}^{-1}$, the limit of constant four-quark condensates deviates from the widely excepted picture of cancelling vector and scalar self-energies which can be traced back to competing effects of higher order condensates. Since even in the small density limit for constant four-quark condensates the estimated ratio $\Sigma_v/\Sigma_s \sim \frac{1}{2}$ cannot be confirmed numerically, these estimates cannot substitute a numerical sum rule evaluation.

Numerical Analysis

In order to investigate numerically the importance of the three combinations of four-quark condensates entering the sum rule equations (3.47) at finite baryon density we perform an evaluation for fixed continuum threshold parameter $s_0 = 2.5\ \text{GeV}^2$ in a fixed Borel window $\mathcal{M}^2 = 0.8\ldots1.4\ \text{GeV}^2$. Since we are especially interested in medium modifications we use all sum rule equations although chiral-odd sum rule equations have been identified more reliable in the vacuum case [124] (however note that instanton contributions might change the relevance of particular sum rule equations [125, 126]).

Technically we follow the method used in [89, 127]: Eqs. (3.47) are divided to obtain equal left hand sides $L \equiv \lambda_N^{*2} e^{-(E_+^2 - \vec{q}^{\,2})/\mathcal{M}^2}$. For an exact solution the four extracted terms, L and the right hand sides $R_{s,q,v}$, had to be equal. However, this equality cannot be guaranteed in an extended range of Borel masses, the Borel window. Therefore one proceeds to minimize the difference of these four different terms in a given Borel window. A logarithmic deviation measure

$$\Delta(M_N^*, \Sigma_v, \lambda_N^*, s_0; \mathcal{M}^2) = \ln \frac{\max(L, R_s, R_q, R_v)}{\min(L, R_s, R_q, R_v)}, \tag{3.59}$$

averaged over the Borel window, is therefore optimized as function of M_N^*, Σ_v, and λ_N^* using established numerical, multi-dimensional optimization routines. Similar results are obtained if one uses for example as quadratic deviation measure the sum $\sum_{i=s,q,v}(L - R_i)^2/(L + R_i)^2$ over the equations (3.47). Casually, such optimization methods yield in contrast to mass equations additional information about the coupling, here λ_N^*, between the physical hadron state and its interpolation field.

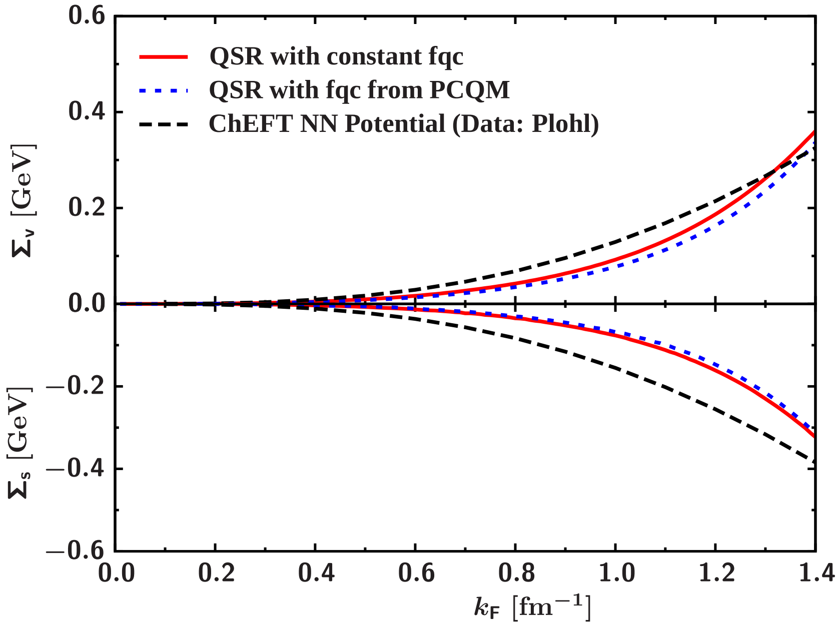

Figure 3.4: Nucleon vector and scalar self-energies as functions of the nucleon Fermi momentum $k_F = (3\pi^2 n/2)^{1/3}$. The sum rule result for constant four-quark condensates (QSR with constant fqc: $\kappa_{\rm s}^{\rm med} = \kappa_{\rm q}^{\rm med} = \tilde{\kappa}_{\rm v}^{\rm med} = 0$, solid curve) is compared to an evaluation with density dependent four-quark condensates as given in Eqs. (3.53) (QSR with fqc from PCQM, dotted curves). The latter choice causes only minor differences in Σ_v and Σ_s, for the scalar self-energy also because of competing impact of $\kappa_{\rm s}^{\rm med}$ and $\kappa_{\rm q}^{\rm med}$. The self-energies from chiral effective field theory [26] (ChEFT, dashed curves) are shown as well but should be used as comparison only at small densities.

The results of these numerical evaluations for a nucleon on the Fermi surface $|\vec{q}_F| = k_F$ are summarized in Figs. 3.4-3.9. Fig. 3.4 shows the scalar and vector self-energies of the nucleon as a function of the Fermi momentum. The situation with four-quark condensate combinations (3.52a)-(3.52c) kept constant at their vacuum value (i.e. $\kappa_{\rm s}^{\rm med} = \kappa_{\rm q}^{\rm med} = \tilde{\kappa}_{\rm v}^{\rm med} = 0$) is compared to the QCD sum rule evaluation with κ parameters from Eqs. (3.53). The results have the same qualitative behavior as self-energies determined from chiral effective field theory with realistic NN potentials [26, 98].

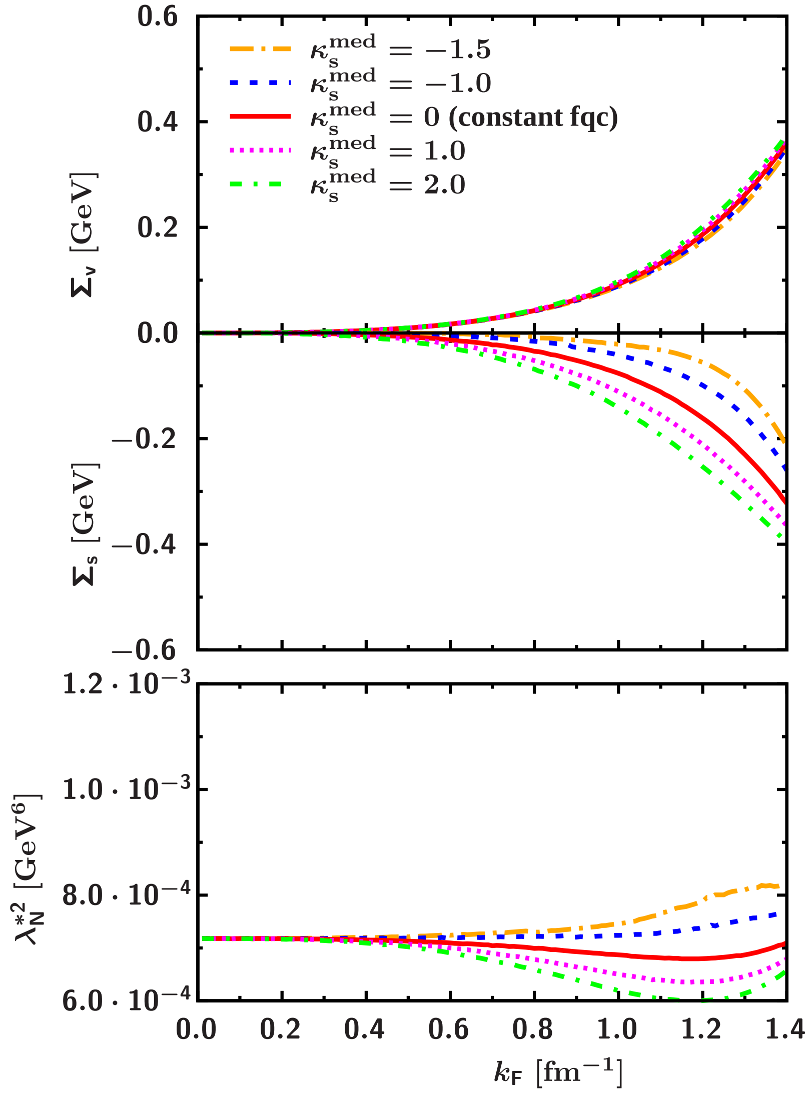

Figure 3.5: The variation of nucleon self-energies (upper panel) and the effective coupling (lower panel) for different assumptions of the density dependence of the four-quark condensates in Eq. (3.47a) parametrized by κ_s^{med}; other four-quark condensate combinations are held constant ($\kappa_q^{med} = \tilde{\kappa}_v^{med} = 0$).

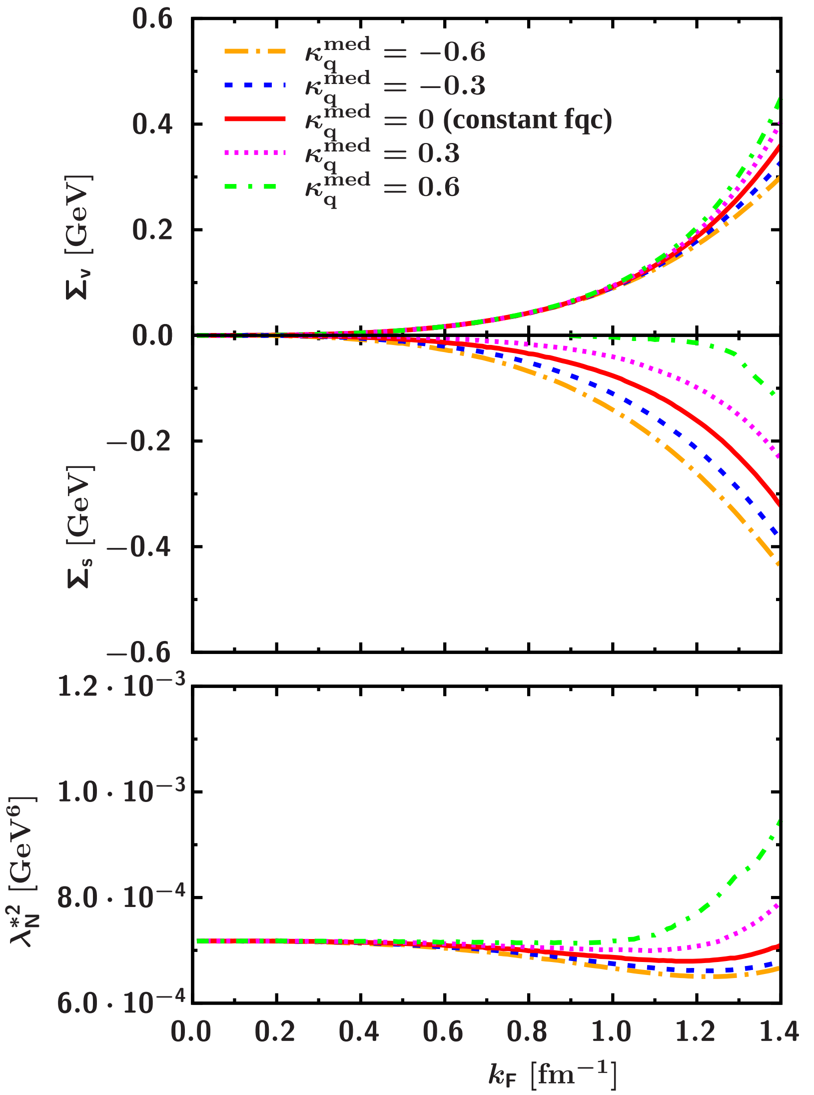

Figure 3.6: The same as Fig. 3.5 but for a variation of the parameter κ_q^{med} for the four-quark condensates in Eq. (3.47b), $\kappa_s^{\mathrm{med}} = \tilde{\kappa}_v^{\mathrm{med}} = 0$.

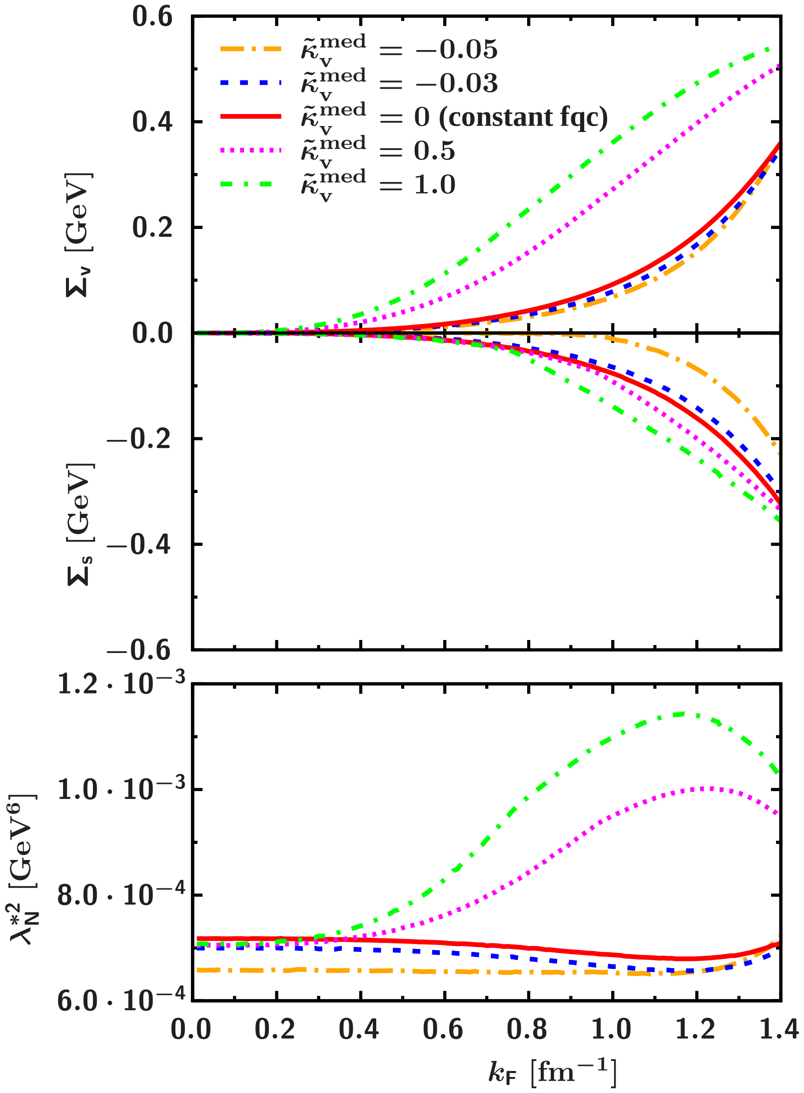

Figure 3.7: The same as Fig. 3.5 but for a variation of the parameter $\tilde{\kappa}_{\rm v}^{\rm med}$ for the four-quark condensates in Eq. (3.47c), $\kappa_{\rm s}^{\rm med} = \kappa_{\rm q}^{\rm med} = 0$.

Figs. 3.5, 3.6 and 3.7 exhibit the impact of the 3 different four-quark condensate combinations: The vector self-energy is, in agreement with Eq. (3.57), mainly determined by $\kappa_{\rm v}^{\rm med}$ especially for small densities (for positive values of $\kappa_{\rm v}^{\rm med}$ even the qualitative form of the vector self-energy changes), $\kappa_{\rm q}^{\rm med}$ has only small impact, and $\kappa_{\rm s}^{\rm med}$ does not effect $\Sigma_{\rm v}$ at all. The scalar self-energy, in contrast, is influenced by all 3 combinations, whereby the change of $\kappa_{\rm v}^{\rm med}$ is only visible for Fermi momenta $k_F > 0.8~{\rm fm}^{-1}$ as also suggested by Eq. (3.58). Figs. 3.5 and 3.6 also reveal the opposed impact of $\kappa_{\rm s}^{\rm med}$ versus $\kappa_{\rm q}^{\rm med}$.

A variation of s_0 is not crucial (see Fig. 3.8). The inclusion of anomalous dimension factors in the sum rule equations as in [89, 90] leads to a reduction of Σ_v in the order of 20% but causes only minor changes in Σ_s. Thereby the naive choice of the anomalous dimension from the factorized form of the four-quark condensates leaves space for improvement since it is known that four-quark condensates mix under renormalization [128]. Our analysis concentrates on the impact of four-

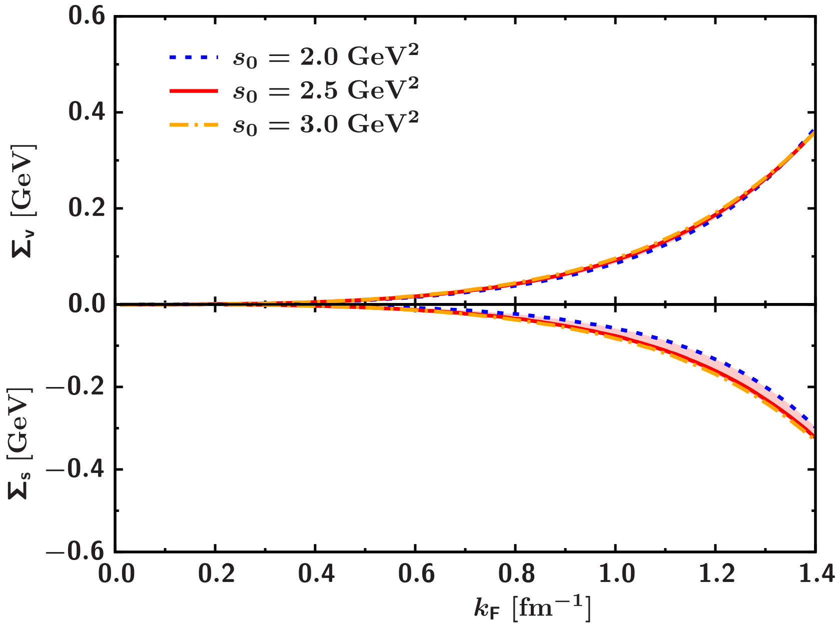

Figure 3.8: The impact of different threshold parameters s_0 on the nucleon self-energies for the case of constant four-quark condensates, i.e. for $\kappa_{\mathrm{s}}^{\mathrm{med}} = \kappa_{\mathrm{q}}^{\mathrm{med}} = \tilde{\kappa}_{\mathrm{v}}^{\mathrm{med}} = 0$.

quark condensates, but also the variation of the density dependence of further condensates can change the result. For example, a large change of the density behavior of the genuine chiral condensate, as determined by the σ_N term, by factor 2 $\{0.5\}$ leads to 8 % decrease $\{4$ % increase$\}$ in the effective mass parameter M_N^* at $k_F \sim 0.8\ \mathrm{fm}^{-1}$, while Σ_v is less sensitive. Correspondingly, the effective coupling λ_N^{*2} is reduced by 10 % $\{$enhanced by 5 %$\}$.

An improved weakly attractive cancellation pattern between Σ_s (attraction) and Σ_v (repulsion), and thus agreement with chiral effective field theory [26], can be achieved for a parameter set $\kappa_{\mathrm{s}}^{\mathrm{med}} = 1.2$, $\kappa_{\mathrm{q}}^{\mathrm{med}} = -0.4$, $\kappa_{\mathrm{v}}^{\mathrm{med}} = 0.1$ (see Fig. 3.9). However, such a fit would allow larger values of $\kappa_{\mathrm{s}}^{\mathrm{med}}$ compensated by a larger magnitude of the negative value of $\kappa_{\mathrm{q}}^{\mathrm{med}}$ and vice versa. Note that in both ways the factorization limit $\kappa_{\mathrm{s,q}}^{\mathrm{med}} = 1$ is violated by one or the other four-quark condensate combination. Such optimized κ parameters, adjusted to results of [26], deviate noticeably from those in Eqs. (3.53) deduced from [60]. To be comparable to the ω case in Fig. 3.3, the same results are again exhibited in Fig. 3.10, now as function of the density. Slight deviations between the QSR results and values from chiral effective field theory arise already for densities $n > n_0/3$.

Quantities characterizing the energy of an excitation with nucleon quantum numbers are M_N^* and E_+, introduced in Eq. (3.41). Since Σ_s is negative, M_N^* drops continuously with increasing density achieving a value of about 540 MeV at nuclear saturation density (corresponding to $k_F \sim 1.35\ \mathrm{fm}^{-1}$) if extrapolated from the optimized fit in Fig. 3.9. The energy E_+ barely changes as function of k_F.

The behavior of the effective coupling parameter as function of the Fermi momentum is also investigated in Figs. 3.5-3.7. The maximum impact of $\kappa_{\mathrm{s}}^{\mathrm{med}}$ $\{\kappa_{\mathrm{q}}^{\mathrm{med}}\}$ on λ_N^{*2} is 6 % $\{3$ %$\}$ at $k_F \sim 0.8\ \mathrm{fm}^{-1}$. In the extreme case, $\tilde{\kappa}_{\mathrm{v}}^{\mathrm{med}} = 1$ leads to a 40 % increase of λ_N^{*2}. The variation

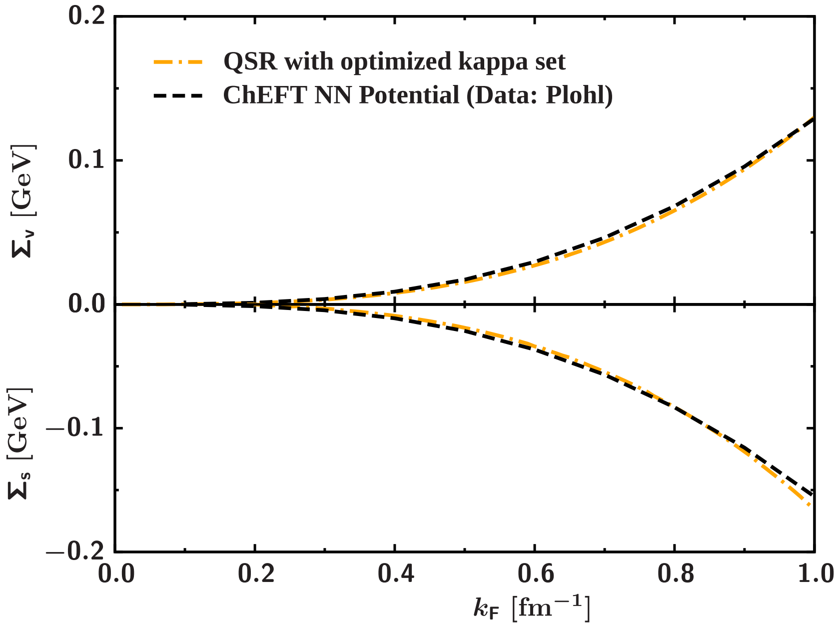

Figure 3.9: QCD sum rule evaluations of nucleon self-energies with the parameter set $\kappa_s^{\mathrm{med}} = 1.2$, $\kappa_q^{\mathrm{med}} = -0.4$, $\tilde{\kappa}_v^{\mathrm{med}} = 0.1$ (dash-dotted curves) compare to chiral effective field theory [26] with realistic NN forces as input.

of this coupling as a function of k_F is in the order of 10 % in the optimized scenario of Fig. 3.9. Generally, specific assumptions on the four-quark condensates can cause a decrease or an increase as well. This alternation of λ_N^{*2} has already been pointed out in [89], whereby their assumptions yield even a ± 20 % change at nuclear density compared to the vacuum limit (cf. also [101]). The vacuum limit of the calculated λ_N^{*2} agrees with the existing range of values (see [108] for a compilation of results for the coupling strength of the nucleon excitation to the interpolating field in vacuum).

These investigations show that nucleon self-energies are subject to numerous four-quark condensates. Originating from the color structure of baryons the four-quark condensates entering here cannot be translated to those in the ω sum rule. Although a few constraints on their density dependence could be derived, their significance for spontaneous break-down of chiral symmetry remains an open issue. One of the elementary limitations is, like for the ω meson, that the four-quark condensate combinations entering QCD sum rules for a specific hadron cannot be directly linked to order parameters in the sense of definition (2.64).

For instance, in vacuum nucleon QCD sum rules the four-quark condensate combination (the vacuum limit of Eq. (3.50b) with isospin symmetry being applied; ψ is the flavor vector) can be divided into a part being invariant under $SU(n_f)_A$ in the Wigner-Weyl phase

$$\begin{aligned}&\left[2(2t^2+t+2)\left\langle\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi\right\rangle+(3t^2+4t+3)\left\langle\bar{\psi}\gamma_5\gamma_\mu\psi\bar{\psi}\gamma_5\gamma^\mu\psi\right\rangle\right]\\&-\frac{3}{4}\left[2(2t^2+t+2)\left\langle\bar{\psi}\gamma_\mu\lambda^A\psi\bar{\psi}\gamma^\mu\lambda^A\psi\right\rangle+(3t^2+4t+3)\left\langle\bar{\psi}\gamma_5\gamma_\mu\lambda^A\psi\bar{\psi}\gamma_5\gamma^\mu\lambda^A\psi\right\rangle\right]\,,\end{aligned} \tag{3.60}$$

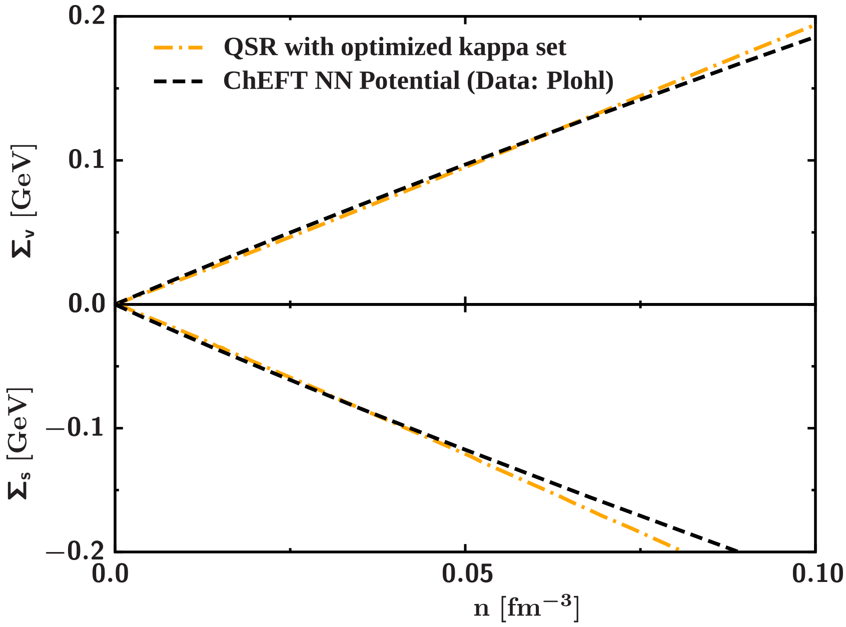

Figure 3.10: Nucleon vector and scalar self-energies as a function of the baryon density n at $T = 0$ for the special choice of $\kappa_{\rm s}^{\rm med}$, $\kappa_{\rm q}^{\rm med}$, $\tilde{\kappa}_{\rm v}^{\rm med}$ used in Fig. 3.9. In this view the deviations from a linear change of the self energies can be read off. This figure is to be compared to Fig. 3.3 for the ω meson.

and a part which breaks this symmetry (pointed out in factorized form already in [109])

$$\left[3(t^2-1)\left(\langle\bar\psi\psi\bar\psi\psi\rangle + \langle\bar\psi\gamma_5\psi\bar\psi\gamma_5\psi\rangle - \frac{1}{2}\langle\bar\psi\sigma_{\mu\nu}\psi\bar\psi\sigma^{\mu\nu}\psi\rangle\right)\right]$$
$$-\frac{3}{4}\left[3(t^2-1)\left(\langle\bar\psi\lambda^A\psi\bar\psi\lambda^A\psi\rangle + \langle\bar\psi\gamma_5\lambda^A\psi\bar\psi\gamma_5\lambda^A\psi\rangle - \frac{1}{2}\langle\bar\psi\sigma_{\mu\nu}\lambda^A\psi\bar\psi\sigma^{\mu\nu}\lambda^A\psi\rangle\right)\right]. \tag{3.61}$$

In the preferred case $t = -1$ only the first part, the invariant contribution (3.60), survives. Would this first part be non-zero, then one could try to quantify the spontaneous breaking of the $SU(n_f)_A$ symmetry in the Nambu-Goldstone phase. However, the first combination (3.60) contains condensates that could not be directly constructed from a correlator like (2.64), cf. Tab. 2.4. The four-quark condensates in the nucleon sum rule thus appear to mix with non-order parameters. These cannot measure pure symmetry effects but are object to other modifications of the QCD ground state at non-vanishing nuclear density as well. In the nucleon sum rule such conclusions are impeded by the interplay of the different linear combinations of four-quark condensates.

A strict statement requires to reorganize the basis of four-quark condensates w.r.t. the possible representations by the commutator in Eq. (2.64) including flavor symmetry and Fierz relations. Note that these results would still be confined to the two flavor case. The opposed numerical impact of two combinations here would even not allow a precise statement from the comparison to (otherwise provided) self-energies. Finally, there are arguments that doubt that condensate changes in medium are pure symmetry restoration effects. The change of the chiral condensate, for example, might partially originate from virtual low-momentum pions and thus could not clearly signal partial restoration of chiral symmetry in matter [114].

Additional insight into the change of four-quark condensates could be acquired from other hadronic channels, as the generalization of further baryon sum rules in vacuum [127, 129, 130] to the medium case, e.g. for the Δ [131, 132]. What their role as order parameters of spontaneous chiral symmetry breaking is concerned, the study of chiral partners is a promising alternative for undisturbed statements on measures for chiral symmetry breaking. An analogy to the difference between vector and axial-vector correlators in the baryon sector is challenging.

The impact of four-quark condensates on hadronic quantities was studied for the ω meson and the nucleon as function of the nuclear density. The main results of these numerical studies are exhibited in Figs. 3.3 and 3.10. In both cases a specific density dependence of (different) four quark condensate combinations had to be assumed. Whereas for the ω example this density dependence turned out stronger than the factorization hypothesis would suggest, it is not uniquely determined for the nucleon case. Here three combinations enter but the results also disprove the factorization limit for four-quark condensates.

3.3 Pseudoscalar Heavy-Light Quark Mesons: D Meson

The description of properties of charmed mesons is increasingly important in view of the upcoming experimental prospects at FAIR with its dedicated possibilities to study the in-medium effects of, especially, D mesons. Due to the quantum numbers one could expect in-medium modifications of excitation strength similar to effects reported for kaons [133, 134], a down shift of K^- (D^+, D^0) and some increase for K^+ (D^-, $\bar{D}^0$).

The constitution of the pseudoscalar open charm mesons, D^+ ($\bar{d}c$), D^- ($\bar{c}d$), D^0 ($\bar{u}c$), $\bar{D}^0$ ($\bar{c}u$), reveals as new physical scale the mass m_c of the charm quark. Whereas in our previous examples the masses of the light quarks $m_{u,d}$ were negligible compared to λ_{QCD}, this is no more the case for $m_c \approx 1.5$ GeV [1]. For example, it was very early noted, that the D meson QCD sum rule is entered by the combination $m_c \langle \bar{q}q \rangle$ [5], i.e., the charm mass m_c acts as magnifier of the chiral condensate. For light vector mesons its impact was numerically small due to the small light quark mass appearing there. This naive argument not yet suggests a measurable chiral symmetry order parameter, since it might be shadowed by the influence of further condensates. Nevertheless it is a strong motivation to ask for the conceptional new aspects for such heavy-light quark systems. Such aspects based on the new mass scale are the topic of Section 3.3.1. In a simple-minded classical picture with one nearly static quark it would be not surprising that the mass of a heavy meson, e.g. $m_{D^\pm} = 1.87$ GeV, is driven by the large current quark mass of its constituent. Some numerical results of a specific QCD sum rule for the D meson, given in Section 3.3.2, are collected in Section 3.3.3.

The QCD sum rule method is applied to the Lorentz and Dirac scalar correlation function

$$\Pi(q,v) = i \int d^4x \, e^{iqx} \langle\langle T \left[j(x) j^\dagger(0) \right] \rangle\rangle \,, \tag{3.62}$$

with the pseudoscalar interpolating fields

$$j_{D^+}(x) = i\bar{d}(x)\gamma_5 c(x) \,, \tag{3.63a}$$
$$j_{D^-}(x) = i\bar{c}(x)\gamma_5 d(x) \,, \tag{3.63b}$$
$$j_{D^0}(x) = i\bar{u}(x)\gamma_5 c(x) \,, \tag{3.63c}$$
$$j_{\bar{D}^0}(x) = i\bar{c}(x)\gamma_5 u(x) \,. \tag{3.63d}$$

QCD sum rules for D mesons in vacuum are documented, e.g. in [135–137], however, generalizations to sum rules in a nuclear medium are rarely published [138–140]. A complete re-evaluation of the in-medium D meson QCD sum rule, in particular the complete OPE side up to mass dimension 5 for products of quark masses and condensates, is presented in [141], which we will rely on.

3.3.1 Operator Product Expansion for Heavy-Light Quark Systems

The evaluation of the sum rule for the $D^\pm$ mesons involves the renormalization of non-perturbative condensates as follows. In an OPE for light quark systems the small quark mass contributions can be included as minor corrections, technically in the quark propagator (B.1). If masses cannot be neglected one is confronted with an increasing number of terms. These are significantly reduced in the limit $m_q \to 0$ for the light quark, here the d quark. Problematically, not all OPE parts converge in this limit due to terms $\sim 1/m_q$ or mass logarithms $\sim \ln m_q$. They arise from (perturbative) loop integrations which cover the full range of loop momenta. With the operator product expansion a conceptional separation of scales into non-perturbative (small momenta: condensates) and perturbative physics (large momenta: Wilson coefficients) is performed. However, perturbative calculation of Wilson coefficients results in another non-perturbative contribution. The divergent parts for $m_q \to 0$ in the OPE calculation are of non-perturbative origin and thus should be covered by the appropriately defined non-perturbative condensates (usually called "non-normal ordered" condensates [6])

$$\langle \bar{q}\,\hat{O}\,(\partial_\mu - igA_\mu)\,q\rangle = \langle :\bar{q}\,\hat{O}\,(\partial_\mu - igA_\mu)\,q:\rangle - i\int d^4p\,\left\langle Tr\left[\hat{O}\left(-ip_\mu - i\tilde{A}_\mu\right)S^{\rm per}(p)\right]\right\rangle , \tag{3.64}$$

which quantify the non-perturbative regime [142]. The integral denotes the perturbatively calculated contribution [143]. The operator $\hat{O}$ is a function of the covariant derivative and contains general Dirac structures, A_μ is the gluon gauge field in fixed-point gauge and $\tilde{A}_\mu$ its Fourier transform; $:\cdots:$ denotes again normal ordering, and $S^{\rm per}$ is the quark propagator in the gluonic background field. For technical aspects we refer to Appendix B and [141, 144].

Introducing these renormalized (physical) condensates cancels mass logarithms in OPE calculations [145] but also causes a mixing between different types of condensates, e.g., the chiral condensate mixes with the gluon condensate (cf. also [6])

$$\langle\bar{q}q\rangle = \langle:\bar{q}q:\rangle + \frac{3}{4\pi^2}m_q^3\left(\ln\frac{\mu^2}{m_q^2}+1\right) - \frac{1}{12m_q}\left\langle:\frac{\alpha_s}{\pi}G^2:\right\rangle + \ldots \quad , \tag{3.65}$$

which relates in the heavy quark sector the heavy quark condensate (e.g. $\langle\bar{c}c\rangle$ for the charm condensate) to gluon condensates [143, 146]

$$m_c\langle\bar{c}c\rangle = -\frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{1}{m_c^2}\frac{1}{1440\pi^2}\langle g_s^3G^3\rangle + \ldots \quad . \tag{3.66}$$

This is also utilized in the QCD sum rule for the J/ψ meson entered by the combination $m_c\langle\bar{c}c\rangle$, which one expects to exhibit the weak density dependence of the gluon condensate, as shown in Fig. 2.4. In fact, the evaluation of the QCD sum rule for J/ψ shows only a tiny change of the in-medium mass [147].

In finite density QCD sum rules for the D meson, medium-specific condensates appear and introduce, via their mixing, further gluon condensates into the OPE. The even and odd parts of the operator product expansion, compare definition (2.33), of the in-medium D meson in the limit $m_d \to 0$ in the nuclear matter rest frame are [141]

$$\begin{aligned}\Pi^e_{D^+}(q_0^2) &= c_0(q_0^2) + \langle\bar{d}d\rangle\frac{m_c}{q_0^2-m_c^2} - \langle\bar{d}g_s\sigma Gd\rangle\frac{1}{2}\left(\frac{m_c^3}{(q_0^2-m_c^2)^3}+\frac{m_c}{(q_0^2-m_c^2)^2}\right)\\ &+\left\langle\frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2}-\frac{G^2}{4}\right)\right\rangle\left(\frac{7}{18}+\frac{1}{3}\ln\frac{\mu^2}{m_c^2}+\frac{2}{3}\ln\left(-\frac{m_c^2}{q_0^2-m_c^2}\right)\right)\left(\frac{m_c^2}{(q_0^2-m_c^2)^2}+\frac{1}{q_0^2-m_c^2}\right)\\ &-\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{1}{12}\frac{1}{q_0^2-m_c^2} + \left\langle d^\dagger iD_0d\right\rangle 2\left(\frac{m_c^2}{(q_0^2-m_c^2)^2}+\frac{1}{q_0^2-m_c^2}\right)\\ &-\left[\frac{1}{3}\left\langle\bar{d}D_0^2d\right\rangle - \frac{1}{24}\left\langle\bar{d}g_s\sigma Gd\right\rangle\right]12\left(\frac{m_c^3}{(q_0^2-m_c^2)^3}+\frac{m_c}{(q_0^2-m_c^2)^2}\right) ,\end{aligned} \tag{3.67a}$$

$$\begin{aligned}\Pi^o_{D^+}(q_0^2) &= -\left\langle d^\dagger d\right\rangle\frac{1}{q_0^2-m_c^2} - \left\langle d^\dagger g_s\sigma Gd\right\rangle\frac{1}{(q_0^2-m_c^2)^2}\\ &+\left\langle d^\dagger D_0^2d\right\rangle 4\left(\frac{m_c^2}{(q_0^2-m_c^2)^3}+\frac{1}{(q_0^2-m_c^2)^2}\right) ,\end{aligned} \tag{3.67b}$$

where the condensates are properly renormalized according to Eq. (3.64) in first order α_s; c_0 is the perturbative term.

3.3.2 QCD Sum Rules for the D Meson

The operator product expansion Eq. (3.67) is via the dispersion relation related to the spectral integral, cf. Section 2.2. Model independent statements require the definition of suitably combined moments of the hadron spectral density like for the ω meson in Eq. (2.40). The intrinsic description of both particle and anti-particle by the correlator (3.62) complicates such general formulation. This was already realized in the nucleon case. To visualize the impact of condensates and the significance of QCD sum rules for the D meson one can still dwell on the pole + continuum ansatz. Certainly, one cannot claim predictions for the shape of the spectral density in a nuclear medium, as they are provided with many-body approaches, e.g. [148, 149]. Here QCD sum rules can at best offer a constraint to test such a given hadronic model if the relevant condensates are under control.

Nevertheless, in order to investigate the impact of individual condensates we apply the following pole ansatz for $D^\pm$ (D^0 and $\bar{D}^0$ are equivalently described, we restrict ourselves to the first case). Inserting a complete set of hadronic states, compare Eq. (2.8), where only the D^+ and D^- remain, the spectral density is motivated as

$$\Delta\Pi(q,v) = \pi\left(F_+\delta(q_0 - m_+) - F_-\delta(q_0 + m_-)\right) , \tag{3.68}$$

where $F_\pm$ are independent couplings between hadron and interpolating fields, $m_\pm$ the pole masses of $D^\pm$. We have set the hadron momentum $\vec{q} = 0$ and the momentum of the matter ground state $\vec{p}_\Psi = 0$. The effective pole masses, incorporating the ground state energy of matter E_Ψ like $m_\pm = M_\pm - E_\Psi$, can be related to medium-dependent self energies $\Sigma_\pm$ introduced in the propagators (e.g. [149])

$$\Pi_{D^\pm}(q,v) = \frac{1}{q^2 - M_\pm^2 - \Sigma_\pm} . \tag{3.69}$$

The even and odd Borel transformed sum rules (2.38) in the ansatz (3.68) with the OPE given in Eqs. (3.67) are

$$\begin{aligned} m_+F_+e^{-m_+^2/\mathcal{M}^2} + m_-F_-e^{-m_-^2/\mathcal{M}^2} &= \frac{1}{\pi}\int_{m_c^2}^{s_0} ds\ e^{-s/\mathcal{M}^2}\mathrm{Im}\Pi_{per}(s) \\ &+ e^{-m_c^2/\mathcal{M}^2}\left(-m_c\left\langle\bar{d}d\right\rangle + \frac{1}{2}\left(\frac{m_c^3}{2\mathcal{M}^4} - \frac{m_c}{\mathcal{M}^2}\right)\left\langle\bar{d}g_s\sigma Gd\right\rangle + \frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right. \\ &+ \left[\left(\frac{7}{18} + \frac{1}{3}\ln\frac{\mu^2 m_c^2}{\mathcal{M}^4} - \frac{2\gamma_E}{3}\right)\left(\frac{m_c^2}{\mathcal{M}^2} - 1\right) - \frac{2m_c^2}{3\mathcal{M}^2}\right]\left\langle\frac{\alpha_s}{\pi}\left[\frac{(vG)^2}{v^2} - \frac{G^2}{4}\right]\right\rangle \\ &\left.+ 2\left(\frac{m_c^2}{\mathcal{M}^2} - 1\right)\left\langle d^\dagger iD_0 d\right\rangle + 4\left(\frac{m_c^3}{2\mathcal{M}^4} - \frac{m_c}{\mathcal{M}^2}\right)\left[\left\langle\bar{d}D_0^2 d\right\rangle - \frac{1}{8}\left\langle\bar{d}g_s\sigma Gd\right\rangle\right]\right) , \end{aligned} \tag{3.70a}$$

$$\begin{aligned} &F_+e^{-m_+^2/\mathcal{M}^2} - F_-e^{-m_-^2/\mathcal{M}^2} = \\ &\quad e^{-m_c^2/\mathcal{M}^2}\left(\left\langle d^\dagger d\right\rangle - 4\left(\frac{m_c^2}{2\mathcal{M}^4} - \frac{1}{\mathcal{M}^2}\right)\left\langle d^\dagger D_0^2 d\right\rangle - \frac{1}{\mathcal{M}^2}\left\langle d^\dagger g_s\sigma Gd\right\rangle\right) . \end{aligned} \tag{3.70b}$$

The continuum contribution, approximated in the semi-local quark-hadron duality hypothesis, is brought to the r.h.s. in the even part and elucidates in the upper bound s_0 of the finite integration over the perturbative term

$$\begin{aligned} \mathrm{Im}\Pi_{per}(s) = \frac{3}{8\pi}\frac{(s-m_c^2)^2}{s}&\left\{1 + \frac{4\alpha_s}{3\pi}\left[\frac{9}{4} + \mathrm{Li}_2\left(\frac{m_c^2}{s}\right) + \ln\left(\frac{s}{m_c^2}\right)\ln\left(\frac{s}{s-m_c^2}\right)\right.\right. \\ &\left.\left.+\frac{3}{2}\ln\left(\frac{m_c^2}{s-m_c^2}\right) + \ln\left(\frac{s}{s-m_c^2}\right) + \frac{m_c^2}{s}\ln\left(\frac{s-m_c^2}{m_c^2}\right) + \frac{m_c^2}{s-m_c^2}\ln\left(\frac{s}{m_c^2}\right)\right]\right\} , \end{aligned} \tag{3.71}$$

with the Spence function $\mathrm{Li}_2 = -\int_0^x dt\, t^{-1}\ln(1-t)$; γ_E is the Euler constant. The relation $\left\langle \frac{\alpha_s}{\pi}\left[\frac{(vG)^2}{v^2} - \frac{G^2}{4}\right]\right\rangle = \frac{1}{2}\left\langle \frac{\alpha_s}{\pi}\left[\frac{(vG)^2}{v^2} + \frac{(v\tilde{G})^2}{v^2}\right]\right\rangle$ holds; for the renormalization scale $\mu \approx m_c$ is used. The medium-specific mixed quark-gluon condensate $\langle d^\dagger g_s \sigma G d\rangle$, even its sign, is not well determined. A value of $+0.33\ \mathrm{GeV}^2 n$ is used in this section, cf. [150], in contrast to Eq. (2.107). The condensate $\langle q^\dagger D_0^2 q\rangle$ is then derived from Eq. (2.106).

3.3.3 QCD Sum Rules for the D Meson

The sign structure on the l.h.s. of Eqs. (3.70) suggests to introduce instead of the masses $m_\pm$ the centroid $m = \frac{1}{2}(m_+ + m_-)$ of this hadron doublet and its splitting $\Delta m = \frac{1}{2}(m_+ - m_-)$, i.e.

$$m_+ = m + \Delta m\,, \tag{3.72a}$$
$$m_- = m - \Delta m\,. \tag{3.72b}$$

Indeed if one assumes constant $F = F_\pm$ and performs an expansion in first order of Δm (with $\Delta m \ll \overline{m}$ and $\Delta m \ll \mathcal{M}$) one obtains for the hadronic side on the left of the sum rules (3.70)

$$m_+ F_+ e^{-m_+^2/\mathcal{M}^2} + m_- F_- e^{-m_-^2/\mathcal{M}^2} \approx 2Fm e^{-m^2/\mathcal{M}^2}\,, \tag{3.73a}$$
$$F_+ e^{-m_+^2/\mathcal{M}^2} - F_- e^{-m_-^2/\mathcal{M}^2} \approx -\frac{4Fm\Delta m}{\mathcal{M}^2} e^{-m^2/\mathcal{M}^2}\,. \tag{3.73b}$$

Eqs. (3.73) indicate that at sufficiently small densities the odd part of the OPE dominates the mass splitting. Strictly, it is still coupled via m to the even part of the OPE. For a typical Borel mass $\mathcal{M} = 1$ GeV the weight of the condensates in Eq. (3.70b) is approximately

$$\left\langle d^\dagger d\right\rangle : \left\langle d^\dagger D_0^2 d\right\rangle : \left\langle d^\dagger g_s \sigma G d\right\rangle \propto 4 : 0.005 : -1\,. \tag{3.74}$$

The large impact of the exactly given positive value of $\langle d^\dagger d\rangle$ explains, due to the negative sign in Eq. (3.73b), that the splitting takes only negative values, $\Delta m < 0$. Since quite restrictive, changes of $F_\pm$ and m are neglected in Eq. (3.73b), this gives only a rough estimate for the impact of condensates, to be compared with the numerical results of the QCD sum rule evaluation.

In order to reach numerical results, the even and odd sum rules (3.70) are added (subtracted) with a factor m_+ (m_-) for the second Eq. (3.70b). In this new set the couplings are disentangled, and by dividing each of these combination by its derivative with respect to the inverse squared Borel mass $\mathcal{M}^2$, the terms $F_\pm$ are eliminated in the end. (This is analog to the ω case.) One obtains two final sum rules, one for m_+ and one for m_-. However, due to the previous weighting by $m_\pm$ this is still a system of coupled equations. Subsequently, the Borel curves for $m_\pm$ or m, Δm can be analyzed. A number of possible criteria were used to actually determine quantitative values for the pole masses from such Borel curves. In [139] a region around a minimum of a Borel curve is considered, flatness in a Borel window was demanded in [137].

If one looks for the medium changes one can diminish the technical aspects of the evaluation. For the Borel curve of m it was observed that these curves intersect for different densities in the applied Borel window when the threshold s_0 is varied [141]. This questions the stability of predictions for the mass centroid, and could explain the different findings for the change of m at nuclear saturation density from almost no change [151] to an approximate down shift of 50 MeV [139].

The situation is much better for the behavior of the splitting Δm at non-vanishing densities. Its Borel curve is less deformed but clearly shifted [141]. The authors of [138] claimed a $D^\pm$ splitting of about 50 MeV at nuclear saturation density, a similar value was given by [151]. Latest investigations suggest that these are rather lower bounds for the absolute value of Δm, and point to a value of $2\Delta m \approx -60$ MeV [150]. The different QSR evaluations agree on the sign of Δm meaning the D^+ is lowered with respect to the D^-; $m_{D^-} > m_{D^+}$. This is consistent with the kaon analogy in the introduction to this section.

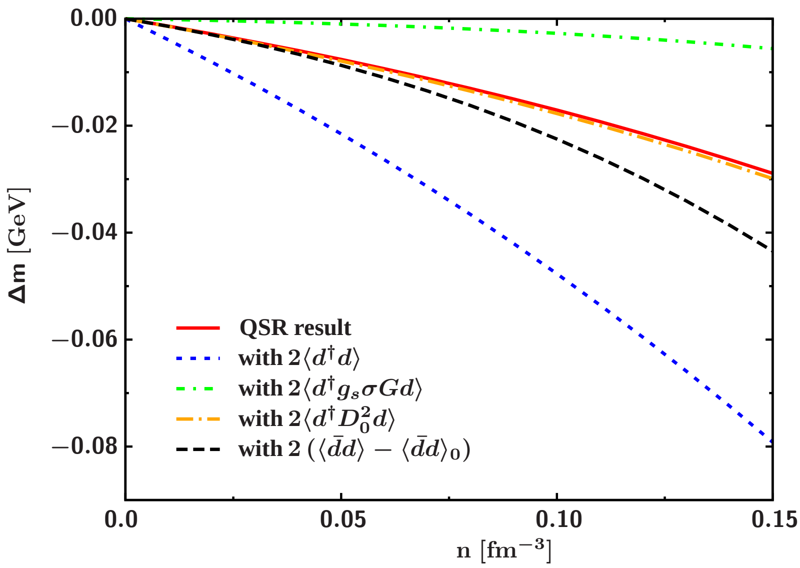

Figure 3.11: The mass splitting $\Delta m = \frac{1}{2}(m_{D^+} - m_{D^-})$ in a pole mass ansatz as a function of the baryon density n at zero temperature. The relevant condensates in the odd part of the OPE (Eq. (3.67b)) and the density dependence of the chiral condensate are varied by factors 2 to highlight the impact of these condensates on the mass splitting (data with courtesy of T. Hilger; obtained with sophisticated threshold criteria [150]).

The result of a numerical sum rule evaluation, selecting particle and anti-particle thresholds to adjust the $m_\pm$ Borel minima at the same Borel mass [150], are shown in Fig. 3.11. The splitting is displayed as a function of the nuclear matter density and, starting from degeneracy of $D^\pm$ in vacuum, reaches a value of $2\Delta m \approx -60$ MeV at saturation density. From the variation of condensates shown there one deduces the largest impact for $\langle d^\dagger d\rangle$, as approximated in Eq. (3.74), followed by the condensate $\langle d^\dagger g_s \sigma G d\rangle$. The choice of this medium-specific mixed quark-gluon condensate as in Eq. (2.107) would amplify the splitting. Although the directions of changes in the splitting are in agreement, compare Fig. 3.11 and Eq. (3.74), the relative modifications deviate from the approximation. The particular choice of the Borel parameter leads in this evaluation procedure to this advanced impact of the condensates. The variation of the density dependence of the chiral condensate also effects the splitting. With respect to the slopes in Fig. 3.11, the modification is relatively pronounced at saturation density. This effect appears in higher order of the density n.

One should be aware that these results are bound to the pole ansatz. The pole parameters can be generalized to combinations of moments of spectral functions, such that the results could be interpreted in view of complicated spectral densities. The transition to B meson sum rules is straightforward and leads to somewhat higher amounts of the splitting [150]. For D_s mesons the intermediate value of the neither light nor heavy strange quark impedes a final conclusion. There the operator product expansions become, as for kaons, conceptionally even more advanced.

4 Summary

QCD sum rules have been applied to the examples of the ω meson, the nucleon, and the D meson embedded in cold nuclear matter. The method is universal since finally only condensates determine the spectral integral of any considered hadron. It is thus conceptionally close to fundamental parameters of quantum chromodynamics. This is highlighted by the relation of condensates to fundamental symmetries of the theory. Especially, chiral symmetry, supposed to be spontaneously broken in the hadronic phase, represents a fundamental concept of hadron physics. Quantities which could measure the degree of this symmetry breaking, order parameters, are thus desired. Most prominently the chiral condensate is considered as such a quantity. The operator product expansion, however, includes numerous other condensates, and to find a clear correspondence between spectral integral and just one significant condensate is difficult. In analogy to an abstract introduction of the chiral condensate as candidate for an order parameter we have discussed other condensates. A number of potential order parameters have been deduced, in particular specific combinations of four-quark condensates.

Four-quark condensates have a strong impact on spectral QCD sum rules for light vector mesons like the ω meson or for the nucleon. Unfortunately, four-quark condensates and their density dependencies are poorly known. One possible solution is to consider a large set of hadronic observables and to try to constrain these parameters characterizing the QCD vacuum. In order to accomplish a systematic approach, we derived a complete catalog of independent four-quark condensates for equilibrated symmetric or asymmetric nuclear matter. While the number of such condensates is fairly large already in the light quark sector, we point out that only special combinations enter the QCD sum rules. The combinations which appear in the ω meson and nucleon sum rule could not be identified as potential order parameters. More promising for investigations of chiral symmetry are differences of sum rules for chiral partners. This was exemplified for the chiral partners ρ meson and a_1 meson. Indeed, the four-quark condensate combination entering there could also be derived from an abstract definition of possible order parameters.

In case of the ω meson a ratio of spectral moments was analyzed. In a pole ansatz this moment can be identified with the pole mass. In general, changes of this moment can be understood by other deformations of the spectral density, e.g. broadening effects. In this sum rule the chiral condensate is suppressed by the light quark mass. Contrary, the impact of four-quark condensates is quite important. Motivated by experimental indications one might expect that this moment does not increase at non-vanishing nuclear densities. Therefrom it was deduced that the relevant combination of four-quark condensates should have a strong density dependence, compared to the factorization hypothesis which approximates four-quark condensates by the squared chiral condensate. This finding suggests that the factorization approximation is questionable at non-vanishing densities.

For the nucleon QCD sum rule three different combinations of four-quark condensates were identified. The knowledge of these combinations (even the individual condensates entering) is not sufficient to convert them into the combination being specific for the spectral QCD sum rule for light vector mesons. This was traced back to the different color decompositions of color neutral interpolating fields for mesons and baryons. In analyzing the set of independent four-quark condensates we found also identities which must be fulfilled in a consistent treatment. Model calculations of four-quark condensates seem not to fulfill automatically these constraints.

On the level of an exploratory study we showed the impact of the three combinations of four-quark condensates on the vector and scalar self-energies of the nucleon. In cold nuclear matter at sufficiently low densities the density dependence of only one effective four-quark condensate combination is found to be important for the vector self-energy and the other two combinations dominate the scalar self-energy. Bearing in mind that the nucleon self-energy pieces are not proved to represent observables, one is tempted to try an adjustment of these parameters to advanced nuclear matter calculations. While the overall pattern agrees fairly well (i.e. large and opposite scalar and vector self-energies) we can reproduce also the fine details on a quantitative level at low densities. Keeping the four-quark condensates constant at vacuum values or giving them a density dependence as suggested by a perturbative chiral quark model induces some quantitative modifications which may be considered as estimator of systematic uncertainties related to the four-quark sector. Furthermore, the special use of sum rules and interpolating current and details of the numerical evaluation procedure may prevent QCD sum rules for the nucleon as a precision tool. The knowledge of this situation may be of relevance for approaches to the nuclear many-body problem which utilize chiral dynamics and condensate-related features of the mean field.

In the example of the D meson the heavy quark mass as new scale changes the impact of condensates compared to the situation for the ω meson. Robust statements in a pole ansatz are only possible for the splitting of the D^+ and D^- mesons. Qualitatively the QCD sum rule predicts the lowering of D^+ with respect to D^-. Deduced quantitative pole mass differences are mainly subject to condensates in the odd part of the operator product expansion, especially to a medium-specific mixed quark-gluon condensate. The chiral condensate enters only the even part of the operator product expansion and affects the splitting for densities approaching nuclear saturation density.

These investigations covered a broad range of conceptional aspects in QCD sum rule evaluations: The nucleon sum rule extends the case of the neutral ω meson due to Dirac structures. Coupled sum rule equations arise from the entanglement of particle and anti-particle spectral contributions. Otherwise, for the D meson as heavy-light quark system with a new physically relevant scale in the operator product expansion, the calculation is complicated by the proper treatment of quark masses and mass divergences.

The dispersion integrals underlying the method do not allow predictions for the spectral shapes. One depends on the choice of a hadronic ansatz or an otherwise obtained spectral density. Nevertheless, the spectral sum rules represent a fundamental constraint on hadronic models, and consequences for condensates may be derived utilizing predictions for the spectral density. In case of the neutral ω meson even a statement about a specific ratio of weighted moments of the spectral density was formulated in a model independent manner. The restrictions on reliable numerical predictions, due to the choice of suitable Borel masses and threshold assumptions, might be reduced when referring to medium changes. Certainly, difference sum rules, as for chiral partners, could again be a very promising alternative. Finally, we remind that our study is restricted to cold nuclear matter. The extension towards finite temperature deserves separate investigations.

In summary, we discussed the QCD sum rules for hadrons in an ambient nuclear medium. The sum rules allowed a direct relation of hadron properties to QCD condensates which change at non-vanishing nuclear densities. For the ω meson and the nucleon, four-quark condensates determine to a large extent the density dependence of these hadrons composed of light quarks. The four-quark condensates entering could not explicitly be identified as possible order parameters for spontaneous chiral symmetry breaking. In contrast, in the heavy-light quark sector, exemplified by D mesons, the impact of the chiral condensate is noticeable.

QCD condensates measure the properties of the ground state. Changes in these universal parameters at non-vanishing nuclear densities, although not always related to symmetry breaking, signal the complicated modifications in the physical ground state of the strong interaction.

A Borel Transforms

The Borel transformation is used in QCD sum rules since the advent of the method [3]. It can be defined as in Eq. (2.35)

$$\mathcal{F}(Q^2) \;\longrightarrow\; \mathcal{F}(\mathcal{M}^2) \equiv \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=\mathcal{M}^2}} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n \mathcal{F}(Q^2)\,, \tag{A.1}$$

where $Q^2 = -q_0^2$ for medium QCD sum rules or $Q^2 = -q^2$ in the vacuum case, respectively. Alternative definitions, for example in [6] (equivalent to the form in [3]),

$$\mathcal{F}(Q^2) \;\longrightarrow\; \tilde{\mathcal{F}}(\mathcal{M}^2) \equiv \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=\mathcal{M}^2}} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^{n+1} \mathcal{F}(Q^2)\,, \tag{A.2}$$

slightly differ in absolute factors or index offsets, and are related to Eq. (A.1), in this case by

$$\mathcal{F}(\mathcal{M}^2) = \mathcal{M}^2\tilde{\mathcal{F}}(\mathcal{M}^2)\,. \tag{A.3}$$

In any application of a Borel transform to a sum rule equation such overall factors cancel. The Borel summation is a suitable tool to deal with divergent series, and therefore it is adequate to be applied to the operator product expansion. Taking derivatives of arbitrary order diminishes polynomials of any degree. For the sum rules of the ω and the nucleon the required Borel transforms are

$$\mathcal{F}(Q^2) = (Q^2)^k \quad (k=0,1,2,\dots) \qquad \longrightarrow\; \mathcal{F}(\mathcal{M}^2) = 0\,, \tag{A.4}$$

$$\mathcal{F}(Q^2) = \frac{1}{(Q^2)^k} \quad (k=1,2,3,\dots) \qquad \longrightarrow\; \mathcal{F}(\mathcal{M}^2) = \frac{1}{(k-1)!}\frac{1}{(\mathcal{M}^2)^{k-1}}\,, \tag{A.5}$$

$$\mathcal{F}(Q^2) = (Q^2)^k \ln Q^2 \quad (k=0,1,2,\dots) \qquad \longrightarrow\; \mathcal{F}(\mathcal{M}^2) = (-1)^{k+1}k!(\mathcal{M}^2)^{k+1}\,. \tag{A.6}$$

The given transforms can directly be obtained taking the derivatives in definition (A.1).

Mathematically, the operator of the Borel transform can be identified with the inversion of the Laplace transform

$$\mathcal{L}[f(t)] = \mathcal{F}(p) = \int_0^{+\infty} e^{-pt} f(t)\,dt\,. \tag{A.7}$$

This may be utilized to find Borel transforms of complicated functions if one can identify the initially given function $\mathcal{F}$ as Laplace transform of f. Eventually the numerous properties of Laplace transforms can be applied therefore. As example consider the function $f(t) = t^{k-1}/(k-1)!$ and its Laplace transform $\mathcal{F}(p) = 1/p^k$. Upon the substitutions $p = Q^2$ and $t = 1/\mathcal{M}^2$ the Borel transform already given in Eq. (A.5) can be read off.

The rule for Laplace transforms $\mathcal{L}[e^{-at}f(t)](p) = \mathcal{L}[f(t)](p+a)$ delivers the Borel transform required for the hadronic side of sum rules

$$\mathcal{F}(Q^2) = \frac{1}{(Q^2+a)^k} \quad (k=1,2,3,\dots) \quad \longrightarrow\; \mathcal{F}(\mathcal{M}^2) = \frac{1}{(k-1)!}\frac{1}{(\mathcal{M}^2)^{k-1}}\,e^{-a/\mathcal{M}^2}\,. \tag{A.8}$$

By iteration of

$$\frac{Q^{2m}}{Q^2+\omega^2} = Q^{2(m-1)} - \frac{Q^{2(m-1)}\omega^2}{Q^2+\omega^2}\,, \tag{A.9}$$

and with Eqs. (A.4), (A.8) the Borel transform for Eqs. (2.34)

$$\mathcal{F}(Q^2) = \frac{(Q^2)^l}{(Q^2+\omega^2)^k} \quad (k=1,2,3,\ldots) \quad \longrightarrow \; \mathcal{F}(\mathcal{M}^2) = \frac{1}{(k-1)!}\frac{(-\omega^2)^l}{(\mathcal{M}^2)^{k-1}}\, e^{-\omega^2/\mathcal{M}^2} \tag{A.10}$$

is obtained.

In more advanced problems the advantage of this approach over the derivative rule (A.1) becomes obvious. In QCD sum rules for the D meson, for example, also the function

$$\mathcal{F}(Q^2) = \frac{1}{(Q^2)^k}\ln Q^2 \quad (k=1,2,3,\ldots) \tag{A.11}$$

appears in the operator product expansion. With $p = Q^2$ this is the tabulated Laplace transform of

$$f(t) = \frac{t^{k-1}}{(k-1)!}\left(\sum_{n=1}^{k-1}\frac{1}{n} - \gamma_E - \ln t\right)\,, \tag{A.12}$$

and the required Borel transform is readily found as

$$\mathcal{F}(\mathcal{M}^2) = \frac{1}{(k-1)!}\frac{1}{(\mathcal{M}^2)^{k-1}}\left(\ln\mathcal{M}^2 - \gamma_E + \sum_{n=1}^{k-1}\frac{1}{n}\right)\,. \tag{A.13}$$

The Euler constant is given by $\gamma_E = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \ln n\right)$. For additional Borel transforms, specific to further QCD sum rules, confer [90, 152].

B Operator Product Expansion Techniques

For completeness and to clarify some technical details we recollect important steps of an OPE calculation for light quark systems. The expressions can than be expanded in powers of the small quark masses, which simplifies the calculation for, e.g., the nucleon and the ω meson. A convenient way to obtain the OPE series is to calculate the Wilson coefficients in an external weak gluon field [144]. In the background field formalism, the correlation function (2.3) is expanded according to Wick's theorem $\Pi(x) = \Pi_{\rm per}(x) + \Pi_{\rm 2q}(x) + \Pi_{\rm 4q}(x) + \dots$, where the full contractions are collected in the perturbative term $\Pi_{\rm per}$ and further terms $\Pi_{\rm 2q,4q,\dots}$ denote the number of non-contracted quark operators. The latter terms give rise to non-local condensates containing the indicated number of quark operators. The use of Wick's theorem naturally introduces the normal ordering of operators $\left\langle \Psi \middle| : \hat{A}_1 \cdots \hat{A}_n : \middle| \Psi \right\rangle \equiv \left\langle \hat{A}_1 \cdots \hat{A}_n \right\rangle$, which will be assumed in all expectation values formed out of products of field operators.

Under the presence of the gluon background field the quark propagator S^q which appears in the terms in $\Pi(x)$ is modified, following from the solution of the Dirac equation in an external field in the Fock-Schwinger gauge for the gluon field. The corrections to the free quark operator appear in an expansion in the coupling $g_s = \sqrt{4\pi\alpha_s}$

$$S^q_{ab}(x) = \langle \Psi \left| T \left[q_a(x) \bar{q}_b(0) \right] \right| \Psi \rangle = \frac{i}{2\pi^2} \frac{\not{x}}{x^4} \delta_{ab} + \frac{i g_s}{8\pi^2} \tilde{G}^A_{\mu\nu}(0) T^A_{ab} \frac{x^\mu}{x^2} \gamma^\nu \gamma_5 + \dots \,, \tag{B.1}$$

with the dual gluon field strength tensor $\tilde{G}^A_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\kappa\lambda} G^{\kappa\lambda A}$ and color matrices T^A_{ab}, valid for massless quarks and inclusion of pure gluon condensates up to mass dimension 4.

The Fock-Schwinger gauge is determined by $(x - x_0)_\mu A^\mu(x) = 0$, and usually one chooses $x_0 = 0$. It allows to express partial derivatives of fields easily by covariant derivatives which matters when expanding non-local products of such operators. In general, results are gauge invariant, however technically fixing this gauge has enormous advantages in calculations of Wilson coefficients. Let us remark, that although the term $\Pi_{\rm 2q}$ initially contains two uncontracted quark field operators, the expansion of the non-local expectation value into local condensates together with weak gluon fields resulting from modified quark propagators and the use of the equations of motion would induce further four-quark condensates at the order α_s.

The use of the quark propagator (B.1) leads to gluon insertions in the expectation values in Π and thus to condensates of higher mass dimension. To obtain the condensates the expectation values are projected onto all possible Dirac, Lorentz and color scalars obeying symmetry w.r.t. time and parity reversal. This introduces all possible condensates up to the considered dimension, and having inserted the projections for the specific correlation function offers also the corresponding Wilson coefficients and therefore the OPE [54].

For example, the non-local diquark expectation value can be projected on color and Dirac structures

$$\langle q_{a\alpha}(x) \bar{q}_{b\beta}(0) \rangle = -\frac{\delta_{ab}}{12} \sum_\Gamma \epsilon_\Gamma \left\langle \bar{q}(0) \Gamma q(x) \right\rangle \Gamma_{\alpha\beta} \,, \tag{B.2}$$

where elements of the Clifford algebra $\Gamma \in \{\mathbb{1}, \gamma_\mu, \sigma_{\mu\nu}, i\gamma_5\gamma_\mu, \gamma_5\}$, are contracted over Lorentz indices, $\epsilon_\Gamma = \frac{1}{2}$ for $\Gamma = \sigma_{\mu\nu}$ and $\epsilon_\Gamma = 1$ otherwise. A Taylor expansion of the quark operator at $x = 0$ in the Fock-Schwinger gauge

$$q(x) = q(0) + x^\mu D_\mu q(0) + \frac{1}{2} x^\mu x^\nu D_\mu D_\nu q(0) + \dots \tag{B.3}$$

leads to additional Lorentz structures, such that the local expansion of the non-local diquark term (B.2) up to mass dimension 5 in the expectation values taken at $x = 0$ yields

$$\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle = -\frac{\delta_{ab}}{12} \sum_\Gamma \epsilon_\Gamma \Gamma_{\alpha\beta} \left(\langle \bar{q}\Gamma q\rangle + x^\mu \langle \bar{q}\Gamma D_\mu q\rangle + \frac{1}{2} x^\mu x^\nu \langle \bar{q}\Gamma D_\mu D_\nu q\rangle \right). \tag{B.4}$$

However, matrix elements $\langle \bar{q}(0)\Gamma q(x)\rangle$ with $\Gamma \in \{\sigma_{\mu\nu}, i\gamma_5\gamma_\mu, \gamma_5\}$ do not contribute due to the demand of time and parity reversal invariance and the multiplication with the symmetric Taylor expansion in x. Condensates with field derivatives can be transformed whereby a couple of manipulations using the equations of motion

$$(i\not{D} - m)q = 0\,, \qquad \bar{q}(i\overleftarrow{\not{D}} + m) = 0\,, \qquad D_\mu^{AB} G_B^{\mu\nu} = g_s \sum_f \bar{q}\gamma^\nu T^A q\,, \tag{B.5}$$

and the representation of the gluon tensor $G_{\mu\nu} = T_A G_{\mu\nu}^A$

$$G_{\mu\nu} = \frac{i}{g_s}[D_\mu, D_\nu], \quad \text{and thus} \quad \frac{1}{2} g_s \sigma G + \not{D}\not{D} = D^2, \qquad D_\mu = \frac{1}{2}\left(\gamma_\mu \not{D} + \not{D}\gamma_\mu\right), \tag{B.6}$$

are exploited to yield simplifications in condensate projections. Terms which contain factors of the small quark mass are neglected in these considerations.

Similar projections can be performed for structures which include gluonic parts from the propagator (B.1) and lead to gluon condensates in $\Pi_{\mathrm{per}}(x)$ and are also carried out to find the linear combinations of four-quark condensates in Π_{4q}. Following this sketched line of manipulations, one arrives for example at Eqs. (3.38) for the nucleon. The OPE for the ω meson is deduced similarly but using also the next order α_s in Eq. (2.46).

To substantiate a complete presentation we list the required projections of Lorentz structures in increasing mass-dimension of the implied condensates. The condensates are rewritten in canonical forms using the equations of motion and identities as well as translational invariance of the nuclear matter ground state; especially we write out the exact forms for finite quark masses m_q, which usually are neglected. Combinations which are not listed are zero by the assumption of time and/or parity reversal invariance or vanish due to symmetry reasons. The projections are those quoted in [54]. Note that here the quark operator and the quark mass m_q are not restricted to the light quark sector. If the mass cannot be neglected then these formula highlight that in counting condensate dimensions the mass factors should be respected.

Condensates of mass-dimension 3:

$$\langle \bar{q}q\rangle\,, \tag{B.7}$$

$$\langle \bar{q}\gamma_\mu q\rangle = \frac{1}{v^2} \langle \bar{q}\not{v} q\rangle\, v_\mu\,. \tag{B.8}$$

Condensates of mass-dimension 4:

$$\langle \bar{q} D_\mu q \rangle = \frac{1}{v^2} \langle \bar{q}(vD)q \rangle \, v_\mu = -\frac{im_q}{v^2} \langle \bar{q}\not{v} q \rangle \, v_\mu \, , \tag{B.9}$$

$$\begin{aligned} \langle \bar{q}\gamma_\mu D_\nu q \rangle &= -\frac{1}{3} \left\langle \bar{q}\not{v}(vD)q/v^2 \right\rangle \left(g_{\mu\nu} - \frac{4v_\mu v_\nu}{v^2} \right) + \frac{1}{3} \langle \bar{q}\not{D} q \rangle \left(g_{\mu\nu} - \frac{v_\mu v_\nu}{v^2} \right) \\ &= -\frac{1}{3} \left\langle \bar{q}\not{v}(vD)q/v^2 \right\rangle \left(g_{\mu\nu} - \frac{4v_\mu v_\nu}{v^2} \right) - \frac{im_q}{3} \langle \bar{q} q \rangle \left(g_{\mu\nu} - \frac{v_\mu v_\nu}{v^2} \right) . \end{aligned} \tag{B.10}$$

Condensates of mass-dimension 5:

$$\begin{aligned} \langle \bar{q} D_\mu D_\nu q \rangle &= -\frac{1}{3} \left\langle \bar{q}(vD)^2 q/v^2 \right\rangle \left(g_{\mu\nu} - \frac{4v_\mu v_\nu}{v^2} \right) + \frac{1}{3} \left\langle \bar{q} D^2 q \right\rangle \left(g_{\mu\nu} - \frac{v_\mu v_\nu}{v^2} \right) \\ &= -\frac{1}{3} \left\langle \bar{q}(vD)^2 q/v^2 \right\rangle \left(g_{\mu\nu} - \frac{4v_\mu v_\nu}{v^2} \right) + \frac{1}{6} \langle \bar{q} g\sigma G q \rangle \left(g_{\mu\nu} - \frac{v_\mu v_\nu}{v^2} \right) \\ &\quad - \frac{m_q^2}{3} \langle \bar{q} q \rangle \left(g_{\mu\nu} - \frac{v_\mu v_\nu}{v^2} \right) , \end{aligned} \tag{B.11}$$

$$\begin{aligned} \langle \bar{q}\gamma_\mu D_\nu D_\alpha q \rangle &= \frac{1}{v^4} \langle \bar{q}\not{v}(vD)^2 q \rangle \left(\frac{2v_\mu v_\nu v_\alpha}{v^2} - \frac{1}{3}[v_\mu g_{\nu\alpha} + v_\nu g_{\mu\alpha} + v_\alpha g_{\mu\nu}] \right) \\ &\quad + \frac{1}{3v^2} \left\langle \bar{q}\not{v} D^2 q \right\rangle \left(v_\mu g_{\nu\alpha} - \frac{v_\mu v_\nu v_\alpha}{v^2} \right) \\ &\quad + \frac{1}{3v^2} \langle \bar{q}(vD)\not{D} q \rangle \left(v_\nu g_{\mu\alpha} - \frac{v_\mu v_\nu v_\alpha}{v^2} \right) \\ &\quad + \frac{1}{3v^2} \langle \bar{q}\not{D}(vD) q \rangle \left(v_\alpha g_{\mu\nu} - \frac{v_\mu v_\nu v_\alpha}{v^2} \right) \\ &= \frac{1}{v^4} \langle \bar{q}\not{v}(vD)^2 q \rangle \left(\frac{2v_\mu v_\nu v_\alpha}{v^2} - \frac{1}{3}[v_\mu g_{\nu\alpha} + v_\nu g_{\mu\alpha} + v_\alpha g_{\mu\nu}] \right) \\ &\quad - \frac{1}{6v^2} \langle \bar{q}\not{v} g\sigma G q \rangle \left(\frac{v_\mu v_\nu v_\alpha}{v^2} - v_\mu g_{\nu\alpha} \right) + \frac{m_q^2}{3v^2} \langle \bar{q}\not{v} q \rangle \left(\frac{v_\mu v_\nu v_\alpha}{v^2} - v_\mu g_{\nu\alpha} \right) \\ &\quad + \frac{im_q}{3v^2} \langle \bar{q}(vD)q \rangle \left(\frac{2v_\mu v_\nu v_\alpha}{v^2} - v_\nu g_{\mu\alpha} - v_\alpha g_{\mu\nu} \right) , \end{aligned} \tag{B.12}$$

$$\begin{aligned} \langle \bar{q}\gamma_5\gamma_\mu D_\nu D_\alpha q \rangle &= -\frac{1}{6v^2} \left\langle \bar{q}\gamma_5 \epsilon^{\kappa\lambda\pi\xi} v_\xi \gamma_\kappa D_\lambda D_\pi q \right\rangle \epsilon_{\mu\nu\alpha\beta} v^\beta \\ &= -\frac{1}{6v^2} \epsilon_{\mu\nu\alpha\beta} v^\beta \left(\frac{i}{2} \langle \bar{q}\not{v} g\sigma G q \rangle + im_q^2 \langle \bar{q}\not{v} q \rangle + im_q^2 \langle \bar{q}(vD)q \rangle \right) , \end{aligned} \tag{B.13}$$

$$\begin{aligned}
\langle\bar{q}\sigma_{\mu\nu}D_\alpha D_\beta q\rangle &= \frac{1}{6}\left\langle\bar{q}\sigma_{\kappa\lambda}D^\kappa D^\lambda q\right\rangle\Big(g_{\mu\alpha}g_{\nu\beta}-g_{\nu\alpha}g_{\mu\beta}\\
&\qquad -g_{\mu\alpha}\frac{v_\nu v_\beta}{v^2}+g_{\nu\alpha}\frac{v_\mu v_\beta}{v^2}+g_{\mu\beta}\frac{v_\nu v_\alpha}{v^2}-g_{\nu\beta}\frac{v_\mu v_\alpha}{v^2}\Big)\\
&-\frac{1}{6v^2}\langle\bar{q}\sigma_{\mu\nu}v^\nu D^\mu(vD)q\rangle\Big(g_{\mu\alpha}g_{\nu\beta}-g_{\nu\alpha}g_{\mu\beta}\\
&\qquad -3g_{\mu\alpha}\frac{v_\nu v_\beta}{v^2}+3g_{\nu\alpha}\frac{v_\mu v_\beta}{v^2}+g_{\mu\beta}\frac{v_\nu v_\alpha}{v^2}-g_{\nu\beta}\frac{v_\mu v_\alpha}{v^2}\Big)\\
&+\frac{1}{6v^2}\langle\bar{q}\sigma_{\mu\nu}v^\nu(vD)D^\mu q\rangle\Big(g_{\mu\alpha}g_{\nu\beta}-g_{\nu\alpha}g_{\mu\beta}\\
&\qquad -g_{\mu\alpha}\frac{v_\nu v_\beta}{v^2}+g_{\nu\alpha}\frac{v_\mu v_\beta}{v^2}+3g_{\mu\beta}\frac{v_\nu v_\alpha}{v^2}-3g_{\nu\beta}\frac{v_\mu v_\alpha}{v^2}\Big)\\
&=-\frac{i}{12}\langle\bar{q}g\sigma Gq\rangle\Big(g_{\mu\alpha}g_{\nu\beta}-g_{\nu\alpha}g_{\mu\beta}\\
&\qquad -g_{\mu\alpha}\frac{v_\nu v_\beta}{v^2}+g_{\nu\alpha}\frac{v_\mu v_\beta}{v^2}+g_{\mu\beta}\frac{v_\nu v_\alpha}{v^2}-g_{\nu\beta}\frac{v_\mu v_\alpha}{v^2}\Big)\\
&+\frac{i}{3v^2}\Big(\left\langle\bar{q}(vD)^2q\right\rangle+m_q\langle\bar{q}\not{v}(vD)q\rangle\Big)\Big(g_{\mu\alpha}g_{\nu\beta}-g_{\nu\alpha}g_{\mu\beta}\\
&\qquad -2g_{\mu\alpha}\frac{v_\nu v_\beta}{v^2}+2g_{\nu\alpha}\frac{v_\mu v_\beta}{v^2}+2g_{\mu\beta}\frac{v_\nu v_\alpha}{v^2}-2g_{\nu\beta}\frac{v_\mu v_\alpha}{v^2}\Big)\ ,
\end{aligned} \tag{B.14}$$

$$\langle\bar{q}g\gamma_5\gamma_\alpha G_{\mu\nu}q\rangle=-\frac{1}{6v^2}\left\langle\bar{q}g\gamma_5\gamma_\kappa G_{\lambda\pi}\epsilon^{\kappa\lambda\pi\xi}v_\xi q\right\rangle\epsilon_{\alpha\mu\nu\sigma}v^\sigma=\frac{1}{6v^2}\langle\bar{q}\not{v}g\sigma Gq\rangle\epsilon_{\alpha\mu\nu\sigma}v^\sigma\ , \tag{B.15}$$

$$\begin{aligned}
\langle\bar{q}g\sigma_{\alpha\beta}G_{\mu\nu}q\rangle &= \frac{1}{6}\left\langle\bar{q}g\sigma_{\kappa\lambda}G^{\kappa\lambda}q\right\rangle\bigg(g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}\\
&\qquad -g_{\alpha\mu}\frac{v_\beta v_\nu}{v^2}+g_{\alpha\nu}\frac{v_\beta v_\mu}{v^2}-g_{\beta\nu}\frac{v_\alpha v_\mu}{v^2}+g_{\beta\mu}\frac{v_\alpha v_\nu}{v^2}\bigg)\\
&-\frac{1}{3v^2}\langle\bar{q}g\sigma_{\kappa\lambda}G_{\pi\xi}q\rangle g^{\kappa\pi}v^\lambda v^\xi\bigg(g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}\\
&\qquad -2g_{\alpha\mu}\frac{v_\beta v_\nu}{v^2}+2g_{\alpha\nu}\frac{v_\beta v_\mu}{v^2}-2g_{\beta\nu}\frac{v_\alpha v_\mu}{v^2}+2g_{\beta\mu}\frac{v_\alpha v_\nu}{v^2}\bigg)\\
&=\frac{1}{6}\langle\bar{q}g\sigma Gq\rangle\bigg(g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}\\
&\qquad -g_{\alpha\mu}\frac{v_\beta v_\nu}{v^2}+g_{\alpha\nu}\frac{v_\beta v_\mu}{v^2}-g_{\beta\nu}\frac{v_\alpha v_\mu}{v^2}+g_{\beta\mu}\frac{v_\alpha v_\nu}{v^2}\bigg)\\
&-\frac{2}{3v^2}\bigg(\left\langle\bar{q}(vD)^2q\right\rangle+im_q\langle\bar{q}\not{v}(vD)q\rangle\bigg)\\
&\bigg(g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}-2g_{\alpha\mu}\frac{v_\beta v_\nu}{v^2}+2g_{\alpha\nu}\frac{v_\beta v_\mu}{v^2}-2g_{\beta\nu}\frac{v_\alpha v_\mu}{v^2}+2g_{\beta\mu}\frac{v_\alpha v_\nu}{v^2}\bigg)\ .
\end{aligned} \tag{B.16}$$

C Addendum: Four-Quark Condensates

Technical details and additional aspects to Section 2.4 concerning four-quark condensates are supplemented here.

C.1 Alternative Derivation of Pure-Flavor Four-Quark Condensate Interrelations

The constraints between two different color structures of pure-flavor four-quark condensates have been presented in Section 2.4.1 by analyzing the specific color structure transformation. For the typical baryon color combination of four-quark condensates, the conversion matrix $\hat{B}$ (2.114) was derived with the decisive property that it cannot be inverted. In algebraic terms, the underlying system of linear equations is linearly dependent. This gave rise to the Fierz relations (2.115). If one is only interested in these relations, another direct way of derivation exists. Thereby one considers the "zero identity"

$$\epsilon^{abc}\epsilon^{a'b'c'}\ \underset{\mathrm{e}}{\bar{q}^{a'}}\underset{\mathrm{f}}{q^a}\underset{\mathrm{g}}{\bar{q}^{b'}}\underset{\mathrm{h}}{q^b}\ \underset{\mathrm{e,g}}{(\Gamma C)}\ \underset{\mathrm{f,h}}{(C\tilde{\Gamma})} = 0 \quad \text{if} \quad (\Gamma C)^T = -(\Gamma C) \quad \text{or} \quad (C\tilde{\Gamma})^T = -(C\tilde{\Gamma}), \tag{C.1}$$

which can be seen by a rearrangement of the product and renaming of indices (this is the analog discussion as for the choice of possible interpolating fields for the nucleon). Fierz transformation of this relations yields the basic formula

$$\epsilon^{abc}\epsilon^{a'b'c'}\ \bar{q}^{a'} O_m q^a \bar{q}^{b'} O^n q^b\ \mathrm{Tr}\left(\tilde{\Gamma} O_n \Gamma C O^{mT} C\right) = 0\,, \tag{C.2}$$

which gives, with insertion of allowed Γ and $\tilde{\Gamma}$, all possible constraints on the color combinations in the sense of the vector $\vec{z}$ in (2.114). From the non-vanishing possibilities we list only combinations relevant for four-quark condensates and contract them to achieve relations between components of $\vec{z}$:

$$\begin{aligned}
\Gamma = \mathbb{1}, \tilde{\Gamma} = \mathbb{1} \quad &\Longrightarrow \quad 0 = -2z_1 + 2z_2 + z_4 + 2z_6 - 2z_8\,,\\
\Gamma = \gamma_5, \tilde{\Gamma} = \gamma_5 \quad &\Longrightarrow \quad 0 = -2z_1 - 2z_2 + z_4 - 2z_6 - 2z_8\,,\\
\Gamma = i\gamma_5\gamma^\alpha, \tilde{\Gamma} = i\gamma_5\gamma_\beta \quad &\Longrightarrow \quad \begin{cases} 0 = -2z_1 + z_2 - z_6 + 2z_8\,,\\ 0 = -2z_1 + 2z_2 - 4z_3 + z_4 - 4z_5 - 2z_6 + 4z_7 + 2z_8\,,\end{cases}\\
\Gamma = i\gamma_5\gamma^\alpha, \tilde{\Gamma} = \gamma_5 \quad &\Longrightarrow \quad 0 = iz_9 + z_{10}\,.
\end{aligned} \tag{C.3}$$

This set of constraints is equivalent to (2.115) in Section 2.4.1.

C.2 Four-Quark Expectation Values in the Nucleon

Supplementary to Tab. 2.3 we collect in Tabs. C.1 and C.2 the underlying coefficients to be understood in connection with the work of Drukarev et al. [60].

Expectation value	Parameters in [60] $N=p$	$N=n$	Minimal Modification $N=p$	$N=n$	Mean Value $N=\frac{p+n}{2}$
$U_N^{S,uu}$	3.94	4.05	3.939	4.047	3.993
$a_N^{V,uu}$	0.52	0.51	0.520	0.510	0.515
$b_N^{V,uu}$	-0.13	-0.02	-0.143	-0.023	-0.083
$a_N^{T,uu}$	0.98	1.02	0.968	1.009	0.989
$b_N^{T,uu}$	0.05	< 0.01	0.045	0.007	0.026
$a_N^{A,uu}$	-0.45	-0.50	-0.471	-0.502	-0.487
$b_N^{A,uu}$	-0.06	-0.01	-0.054	-0.009	-0.032
$U_N^{P,uu}$	1.91	1.96	2.002	2.030	2.016

Table C.1: Coefficients of pure flavor nucleon four-quark expectation values (in units of $\langle \bar{q}q \rangle_{\rm vac} = (-0.245\ {\rm GeV})^3$) as determined in [60] in the terminology introduced there and modified values from a fine-tuned parameter set which fulfill the constraints (2.115). The parameters $\kappa_{\rm s,q}^{\rm med}$ and $\tilde{\kappa}_{\rm v}^{\rm med}$ are finally derived from the right column which shows the result for isospin symmetric baryonic matter.

C.3 Basis Transformations

In this section the transformation laws between the flavor symmetric four-quark condensate basis systems O_c and O_f in Tab. 2.4, derived from multiple application of Fierz transformations, are given. It allows to proceed the discussion of possible order parameters in the basis system O_c and their interrelations. This substantiates also the relation between different color structures of four-quark condensates, Eq. (2.111), which is based on the same technique. But here we generalize it to two flavor degrees of freedom.

We require the general property of the N-dimensional fundamental generators T^a for the special unitary group $SU(N)$ (normalized as $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$)

$$T^a_{ij} T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\right) , \tag{C.4}$$

derived from a completeness relation in matrix space; $a = 1\ldots(N^2-1)$ is summed over and the lower indices running over $1\ldots N$ express the matrix structure .

In color space, $N = N_C = 3$, $T^A = \frac{\lambda^A}{2}$ (λ^a are the Gell-Mann matrices) this reads

$$\lambda^A_{ij}\lambda^A_{kl} = 2\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) . \tag{C.5}$$

Also the equivalent relation in flavor space is now needed, $N = n_f = 2$, $T^a = \frac{\tau^a}{2}$ (τ^a are the Pauli matrices)

$$\tau^a_{ij}\tau^a_{kl} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} . \tag{C.6}$$

Expectation value	Parameters in [60] $N=p$	$N=n$	Mean Value $N=\frac{p+n}{2}$
$U_N^{S,ud}$	3.19	3.19	3.19
$a_N^{V,ud}$	-0.44	-0.44	-0.44
$b_N^{V,ud}$	-0.29	-0.29	-0.29
$a_N^{T,ud}$	0.19	0.19	0.19
$b_N^{T,ud}$	0.18	0.18	0.18
$a_N^{A,ud}$	0.43	0.43	0.43
$b_N^{A,ud}$	-0.06	-0.06	-0.06
$U_N^{P,ud}$	-0.20	-0.17	-0.185
$U_N^{VS,ud}$	-0.28	-0.21	-0.245

Table C.2: As Tab. C.1 but for coefficients of nucleon four-quark expectation values parametrizing mixed flavor structures as determined in [60] and the mean values used to calculate medium strength parameters $\kappa^{\rm med}$ in isospin symmetric matter. The modifications referring to pure-flavor four-quark condensates are not needed here.

Together with the Dirac projection, Eq. (2.108), all requisites are provided to reorganize the contraction of four flavor vectors in the four-quark condensates (Fierz transformation). Starting from $\langle\bar\psi D^x C^x F^x\psi\bar\psi D^y C^y F^y\psi\rangle$, by exchanging the order of the flavor vectors ψ the given index reordering is subsequently pursued for the Dirac (D), color (C) and flavor structures (say $F^x=\tau^e$, for example). Choosing for $D^{x,y}$ the basis elements of the Dirac algebra, see Section 2.4.1, one obtains

$$\left\langle\bar\psi O^k\tau^e\psi\bar\psi O^l\tau^e\psi\right\rangle = -\left\langle\bar\psi O^k\psi\bar\psi O^l\psi\right\rangle \\ -\frac{1}{8}\mathrm{Tr}(O^kO_nO^lO_m)\left[\frac{1}{3}\left\langle\bar\psi O^m\psi\bar\psi O^n\psi\right\rangle+\frac{1}{2}\left\langle\bar\psi O^m\lambda^a\psi\bar\psi O^n\lambda^A\psi\right\rangle\right]. \tag{C.7}$$

The effort to calculate $\mathrm{Tr}(O^kO_nO^lO_m)$ can be circumvented when resorting to the vector formulation (2.112) which is only a way to organize all relevant Dirac structures. Hence, by comparison with Eq. (2.111), the transformation is traced back to the matrix $\hat A$, Eq. (2.113), symbolically

$$\langle\vec\tau\rangle=\frac{1}{3}\left(\hat A-\frac{7}{3}\right)\left\langle\vec{\mathbb{1}}\right\rangle+\frac{1}{2}\left(\hat A+\frac{2}{3}\right)\left\langle\vec{\lambda^A}\right\rangle \tag{C.8}$$

where the formal vectors collect the Dirac structures $D^{x,y}$ of the respective condensates in

$$\langle\vec\tau\rangle\leftrightarrow\langle\bar\psi D^x\tau^e\psi\bar\psi D^y\tau^e\psi\rangle\,, \tag{C.9a}$$

$$\left\langle\vec{\mathbb{1}}\right\rangle\leftrightarrow\langle\bar\psi D^x\psi\bar\psi D^y\psi\rangle\,, \tag{C.9b}$$

$$\left\langle\vec{\lambda^A}\right\rangle\leftrightarrow\langle\bar\psi D^x\lambda^A\psi\bar\psi D^y\lambda^A\psi\rangle\,, \tag{C.9c}$$

$$\left\langle\vec{\tau^e\lambda^A}\right\rangle\leftrightarrow\langle\bar\psi D^x\tau^e\lambda^A\psi\bar\psi D^y\tau^e\lambda^A\psi\rangle\,. \tag{C.9d}$$

Similarly, relations including $\left\langle\bar\psi O^k\tau^e\lambda^A\psi\bar\psi O^l\tau^e\lambda^A\psi\right\rangle$ can be derived. Note, that the transformations in general will depend on n_f and N_c.

D List of Acronyms

QCD	Quantum Chromodynamics
QSR	QCD Sum Rules
OPE	Operator Product Expansion
PCAC	Partial Conservation of the Axial vector Current
GOR	Gell-Mann–Oakes–Renner
PCQM	Perturbative Chiral Quark Model
ChEFT	Chiral Effective Field Theory
fqc	four-quark condensates
CERN	European laboratory for particle physics (French acronym)
SPS	Super Proton Synchrotron
KEK	High Energy Accelerator Research Organization (Japanese acronym)
GSI	Gesellschaft für Schwerionenforschung
HADES	High Acceptance Dielectron Spectrometer
CB-TAPS	Crystal-Barrel - Two Arms Photon Spectrometer
JLAB	Thomas Jefferson Lab National Accelerator Facility
CLAS	CEBAF Large Acceptance Spectrometer
CEBAF	Continuous Electron Beam Accelerator Facility
FAIR	Facility for Antiproton and Ion Research
CBM	Compressed Baryonic Matter
PANDA	$\bar{p}$ Annihilations at Darmstadt

Acknowledgment

I thank Prof. Dr. Burkhard Kämpfer for giving me the opportunity to investigate this topic and his expert guidance during my work at the Forschungszentrum Dresden-Rossendorf. He brought my interest to QCD sum rules and medium modifications, and continuously supported me since my diploma thesis. Thanks are also due to Prof. Dr. Rüdiger Schmidt and the whole Institute for Theoretical Physics at the TU Dresden providing a comfortable host over the whole period of time. I want to thank also Prof. Dr. Gerhard Soff († 2004) who initially accepted me as PhD student at the institute.

The instructive discussions with Prof. Kämpfer, Dr. Sven Zschocke and Dipl. Phys. Thomas Hilger about QCD sum rules are greatfully acknowledged. Sven Zschocke helped me to start working on the topic, and I could base on his investigations. Thomas Hilger checked some of the relations concerning four-quark condensates. Further, he pointed out important aspects in the sum rule approach with his many questions, conceptionally and technically, especially in context with his educated calculations for the D meson.

I also thank PD Dr. Stefan Leupold. His instructive comments during my talks and our discussion on four-quark condensates were clarifying to me. I acknowledge Dipl. Phys. Oliver Plohl and Prof. Dr. Christian Fuchs who provided for comparison data of nucleon self energies beyond their published results.

I am thankful to the division of hadron physics in the Institute of Radiation Physics at the Forschungszentrum Dresden-Rossendorf for the excellent and flexible conditions of work. The pleaseant atmosphere inside the local theory group hosted at the TU Dresden was always motivating and friendly, characterized by physical but also non-physical discussions. For this I am especially thankful to Marcus Bluhm, Henry Schade, Daniel Seipt, Robert Schulze, Thomas Hilger and Oliver Zacharias.

Finally, I am grateful to my sister and my parents for moral support in difficult situations during this period of time. I wish to thank especially my girlfriend Andrea and my son Maximilian for giving me so much pleasure. They stimulated and encouraged me many times.

Bibliography

[1] W. M. Yao et al. (Particle Data Group), J. Phys. **G33**, 1 (2006).

[2] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Rev. Lett. **42**, 297 (1979).

[3] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).

[4] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 448 (1979).

[5] L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rept. **127**, 1 (1985).

[6] S. Narison, *QCD as a Theory of Hadrons* (Cambridge University Press, 2004).

[7] P. Colangelo and A. Khodjamirian, in *Handbook of QCD, Ioffe Festschrift*, edited by M. A. Shifman (World Scientific, 2001).

[8] A. I. Bochkarev and M. E. Shaposhnikov, Nucl. Phys. **B268**, 220 (1986).

[9] T. Hatsuda and S. H. Lee, Phys. Rev. **C46**, 34 (1992).

[10] R. J. Furnstahl, T. Hatsuda, and S. H. Lee, Phys. Rev. **D42**, 1744 (1990).

[11] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991).

[12] G. E. Brown and M. Rho, Phys. Rept. **363**, 85 (2002).

[13] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981), erratum: Nucl. Phys. **B191**, 591 (1981).

[14] G. Agakishiev et al. (CERES), Phys. Rev. Lett. **75**, 1272 (1995).

[15] R. Arnaldi et al. (NA60), Phys. Rev. Lett. **96**, 162302 (2006).

[16] S. Damjanovic et al. (NA60), Nucl. Phys. **A783**, 327 (2007).

[17] M. Naruki et al., Phys. Rev. Lett. **96**, 092301 (2006).

[18] I. Frohlich et al. (HADES), Eur. Phys. J. **A31**, 831 (2007).

[19] D. Trnka et al. (CBELSA/TAPS), Phys. Rev. Lett. **94**, 192303 (2005).

[20] D. P. Weygand, C. Djalali, R. Nasseripour, and M. Wood (CLAS), Int. J. Mod. Phys. **A22**, 380 (2007).

[21] M. Kotulla et al. (CBELSA/TAPS), Phys. Rev. Lett. **100**, 192302 (2008).

[22] R. Nasseripour et al. (CLAS), Phys. Rev. Lett. **99**, 262302 (2007).

[23] M. H. Wood et al. (CLAS), Phys. Rev. **C78**, 015201 (2008).

[24] U. Mosel (2008), `arXiv:0802.0786 [hep-ph]`.

[25] R. Thomas, S. Zschocke, and B. Kämpfer, Phys. Rev. Lett. **95**, 232301 (2005).

[26] O. Plohl and C. Fuchs, Phys. Rev. **C74**, 034325 (2006).

[27] R. Thomas, T. Hilger, and B. Kämpfer, Nucl. Phys. **A795**, 19 (2007).

[28] V. Friese, Nucl. Phys. **A774**, 377 (2006).

[29] K. T. Brinkmann for the PANDA collaboration, Nucl. Phys. **A790**, 75 (2007).

[30] R. Thomas, T. Hilger, and B. Kämpfer, Prog. Part. Nucl. Phys. **61**, 297 (2008).

[31] R. Thomas, T. Hilger, S. Zschocke, and B. Kämpfer, AIP Conf. Proc. **892**, 274 (2007).

[32] R. Thomas, S. Zschocke, T. Hilger, and B. Kämpfer, Proceedings of the 44th International Winter Meeting on Nuclear Physics, Bormio, Italy pp. 61–67 (2006).

[33] R. Thomas, K. Gallmeister, S. Zschocke, and B. Kämpfer, Acta Phys. Hung. **A27**, 35 (2006).

[34] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. **B213**, 109 (1983).

[35] V. A. Nesterenko and A. V. Radyushkin, Phys. Lett. **B115**, 410 (1982).

[36] J. Sugiyama, T. Doi, and M. Oka, Phys. Lett. **B581**, 167 (2004).

[37] R. D. Matheus, F. S. Navarra, M. Nielsen, and R. Rodrigues da Silva, Phys. Rev. **D76**, 056005 (2007).

[38] K. G. Wilson, Phys. Rev. **179**, 1499 (1969).

[39] M. Sugawara and A. Kanazawa, Phys. Rev. **123**, 1895 (1961).

[40] W. Florkowski and W. Broniowski, Nucl. Phys. **A651**, 397 (1999).

[41] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, 1980).

[42] R. J. Furnstahl, D. K. Griegel, and T. D. Cohen, Phys. Rev. **C46**, 1507 (1992).

[43] T. Hatsuda, Y. Koike, and S.-H. Lee, Phys. Rev. **D47**, 1225 (1993).

[44] D. Vretenar and W. Weise, Lect. Notes Phys. **641**, 65 (2004).

[45] A. Hosaka and H. Toki, *Quarks, Baryons and Chiral Symmetry* (World Scientific, 2001).

[46] C. P. Burgess, Phys. Rept. **330**, 193 (2000).

[47] T. Kugo, *Eichtheorie* (Springer, 1997).

[48] U. Mosel, *Fields, Symmetries, and Quarks* (McGraw-Hill, 1989).

[49] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

[50] E. G. Drukarev and E. M. Levin, Prog. Part. Nucl. Phys. **27**, 77 (1991).

[51] T. Hatsuda, Y. Koike, and S.-H. Lee, Nucl. Phys. **B394**, 221 (1993).

[52] F. J. Ynduráin, *The Theory of Quark and Gluon Interactions* (Springer, 1999).

[53] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[54] X.-m. Jin, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. **C47**, 2882 (1993).

[55] E. G. Drukarev, M. G. Ryskin, and V. A. Sadovnikova, Phys. Rev. **C70**, 065206 (2004).

[56] S. Leupold, Phys. Lett. **B616**, 203 (2005).

[57] M. C. Birse and B. Krippa, Phys. Lett. **B381**, 397 (1996).

[58] S. Leupold, J. Phys. **G32**, 2199 (2006).

[59] Y. Koike, Phys. Rev. **D48**, 2313 (1993).

[60] E. G. Drukarev, M. G. Ryskin, V. A. Sadovnikova, V. E. Lyubovitskij, T. Gutsche, and A. Faessler, Phys. Rev. **D68**, 054021 (2003).

[61] E. G. Drukarev, M. G. Ryskin, V. A. Sadovnikova, T. Gutsche, and A. Faessler, Phys. Rev. **C69**, 065210 (2004).

[62] L. S. Celenza, C. M. Shakin, W.-D. Sun, and J. Szweda, Phys. Rev. **C51**, 937 (1995).

[63] M. Gockeler, R. Horsley, B. Klaus, D. Pleiter, P. E. L. Rakow, S. Schaefer, A. Schäfer, and G. Schierholz, Nucl. Phys. **B623**, 287 (2002).

[64] S. Zschocke, B. Kämpfer, and G. Plunien, Phys. Rev. **D72**, 014005 (2005).

[65] S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).

[66] J. Gasser and H. Leutwyler, Phys. Rept. **87**, 77 (1982).

[67] J. I. Kapusta and E. V. Shuryak, Phys. Rev. **D49**, 4694 (1994).

[68] S. Leupold and M. Wagner (2008), `0807.2389`.

[69] R. Barate et al. (ALEPH), Eur. Phys. J. **C4**, 409 (1998).

[70] K. Ackerstaff et al. (OPAL), Eur. Phys. J. **C7**, 571 (1999).

[71] M. Davier, S. Descotes-Genon, A. Hocker, B. Malaescu, and Z. Zhang, Eur. Phys. J. **C56**, 305 (2008).

[72] B. L. Ioffe and K. N. Zyablyuk, Nucl. Phys. **A687**, 437 (2001).

[73] K. N. Zyablyuk, Eur. Phys. J. **C38**, 215 (2004).

[74] A. A. Almasy, K. Schilcher, and H. Spiesberger, Eur. Phys. J. **C55**, 237 (2008).

[75] J. Bordes, C. A. Dominguez, J. Penarrocha, and K. Schilcher, JHEP **02**, 037 (2006).

[76] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 519 (1979).

[77] S. Zschocke and B. Kämpfer, Phys. Rev. **C70**, 035207 (2004).

[78] A. K. Das, V. S. Mathur, and P. Panigrahi, Phys. Rev. **D35**, 2178 (1987).

[79] S. Leupold, W. Peters, and U. Mosel, Nucl. Phys. **A628**, 311 (1998).

[80] S. Leupold, Phys. Rev. **C64**, 015202 (2001).

[81] S. Leupold and M. Post, Nucl. Phys. **A747**, 425 (2005).

[82] E. Santini et al., Phys. Rev. **C78**, 034910 (2008).

[83] B. Kämpfer and S. Zschocke, Prog. Part. Nucl. Phys. **53**, 317 (2004).

[84] B. Steinmüller and S. Leupold, Nucl. Phys. **A778**, 195 (2006).

[85] E. Oset et al., Prog. Part. Nucl. Phys. **61**, 260 (2008).

[86] S. Choi, T. Hatsuda, Y. Koike, and S. H. Lee, Phys. Lett. **B312**, 351 (1993).

[87] S. Zschocke, O. P. Pavlenko, and B. Kämpfer, Eur. Phys. J. **A15**, 529 (2002).

[88] Y. Kwon, M. Procura, and W. Weise (2008), `0803.3262`.

[89] X.-m. Jin, M. Nielsen, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. **C49**, 464 (1994).

[90] T. D. Cohen, R. J. Furnstahl, D. K. Griegel, and X.-m. Jin, Prog. Part. Nucl. Phys. **35**, 221 (1995).

[91] E. M. Henley and J. Pasupathy, Nucl. Phys. **A556**, 467 (1993).

[92] E. G. Drukarev and E. M. Levin, Nucl. Phys. **A511**, 679 (1990).

[93] E. G. Drukarev and M. G. Ryskin, Nucl. Phys. **A578**, 333 (1994).

[94] E. G. Drukarev, M. G. Ryskin, and V. A. Sadovnikova, Prog. Part. Nucl. Phys. **47**, 73 (2001).

[95] E. G. Drukarev, Prog. Part. Nucl. Phys. **50**, 659 (2003).

[96] V. A. Sadovnikova, E. G. Drukarev, and M. G. Ryskin, Phys. Rev. **D72**, 114015 (2005).

[97] V. A. Sadovnikova, E. G. Drukarev, and M. G. Ryskin (2006), `nucl-th/0612045`.

[98] T. Gross-Boelting, C. Fuchs, and A. Faessler, Nucl. Phys. **A648**, 105 (1999).

[99] V. M. Braun, P. Gornicki, L. Mankiewicz, and A. Schäfer, Phys. Lett. **B302**, 291 (1993).

[100] E. Stein, P. Gornicki, L. Mankiewicz, A. Schäfer, and W. Greiner, Phys. Lett. **B343**, 369 (1995).

[101] R. J. Furnstahl, X.-m. Jin, and D. B. Leinweber, Phys. Lett. **B387**, 253 (1996).

[102] D. B. Leinweber, Annals Phys. **254**, 328 (1997).

[103] B. Langwallner, Diploma thesis, TU München (2005).

[104] J. A. McGovern and M. C. Birse, Phys. Rev. **D74**, 097501 (2006).

[105] Y. Kondo, O. Morimatsu, and T. Nishikawa, Nucl. Phys. **A764**, 303 (2006).

[106] K.-C. Yang, W. Y. P. Hwang, E. M. Henley, and L. S. Kisslinger, Phys. Rev. **D47**, 3001 (1993).

[107] H.-X. Chen, V. Dmitrasinovic, A. Hosaka, K. Nagata, and S.-L. Zhu (2008), `0806.1997`.

[108] D. B. Leinweber, Phys. Rev. **D51**, 6383 (1995).

[109] D. Jido, N. Kodama, and M. Oka, Phys. Rev. **D54**, 4532 (1996).

[110] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[111] J. J. Rusnak and R. J. Furnstahl, Z. Phys. **A352**, 345 (1995).

[112] D. K. Griegel and T. D. Cohen, Phys. Lett. **B333**, 27 (1994).

[113] S. H. Lee, S. Choe, T. D. Cohen, and D. K. Griegel, Phys. Lett. **B348**, 263 (1995).

[114] M. C. Birse, Phys. Rev. **C53**, 2048 (1996).

[115] M. Post, S. Leupold, and U. Mosel, Nucl. Phys. **A741**, 81 (2004).

[116] S. Mallik and S. Sarkar, Phys. Rev. **D65**, 016002 (2002).

[117] C. Adami and I. Zahed, Phys. Rev. **D45**, 4312 (1992).

[118] D. Jido and M. Oka (1996), `hep-ph/9611322`.

[119] M. Oka, D. Jido, and A. Hosaka, Nucl. Phys. **A629**, 156c (1998).

[120] Y. Kondo, O. Morimatsu, T. Nishikawa, and Y. Kanada-En'yo, Phys. Rev. **D75**, 034010 (2007).

[121] P. Finelli, N. Kaiser, D. Vretenar, and W. Weise, Eur. Phys. J. **A17**, 573 (2003).

[122] P. Finelli, N. Kaiser, D. Vretenar, and W. Weise, Nucl. Phys. **A735**, 449 (2004).

[123] O. Plohl and C. Fuchs, Nucl. Phys. **A798**, 75 (2008).

[124] X.-m. Jin and J. Tang, Phys. Rev. **D56**, 515 (1997).

[125] A. E. Dorokhov and N. I. Kochelev, Z. Phys. **C46**, 281 (1990).

[126] H. Forkel and M. K. Banerjee, Phys. Rev. Lett. **71**, 484 (1993).

[127] D. B. Leinweber, Ann. Phys. **198**, 203 (1990).

[128] M. Jamin and M. Kremer, Nucl. Phys. **B277**, 349 (1986).

[129] F. X. Lee and X.-y. Liu, Phys. Rev. **D66**, 014014 (2002).

[130] F. X. Lee, Nucl. Phys. **A791**, 352 (2007).

[131] X.-m. Jin, Phys. Rev. **C51**, 2260 (1995).

[132] M. B. Johnson and L. S. Kisslinger, Phys. Rev. **C52**, 1022 (1995).

[133] W. Scheinast et al. (KaoS), Phys. Rev. Lett. **96**, 072301 (2006).

[134] L. Tolos, D. Cabrera, and A. Ramos (2008), `0807.2947`.

[135] T. M. Aliev and V. L. Eletsky, Sov. J. Nucl. Phys. **38**, 936 (1983).

[136] S. Narison, Phys. Lett. **B520**, 115 (2001).

[137] A. Hayashigaki and K. Terasaki (2004), `hep-ph/0411285`.

[138] P. Morath, W. Weise, and S. H. Lee, Proceedings of the 17th Autumn School on QCD: Perturbative of Nonperturbative?, Lisbon, Portugal, p. 425 (1999).

[139] A. Hayashigaki, Phys. Lett. **B487**, 96 (2000).

[140] P. Morath, Ph.D. dissertation, TU München (2001).

[141] T. Hilger, Diploma thesis, TU Dresden (2008).

[142] V. P. Spiridonov and K. G. Chetyrkin, Sov. J. Nucl. Phys. **47**, 522 (1988).

[143] A. G. Grozin, Int. J. Mod. Phys. **A10**, 3497 (1995).

[144] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Fortschr. Phys. **32**, 585 (1984).

[145] M. Jamin and M. Münz, Z. Phys. **C60**, 569 (1993).

[146] D. J. Broadhurst and S. C. Generalis, Phys. Lett. **B165**, 175 (1985).

[147] F. Klingl, S.-s. Kim, S. H. Lee, P. Morath, and W. Weise, Phys. Rev. Lett. **82**, 3396 (1999), erratum: Phys. Rev. Lett. **83**, 4224 (1999).

[148] M. F. M. Lutz and C. L. Korpa, Phys. Lett. **B633**, 43 (2006).

[149] L. Tolos, A. Ramos, and T. Mizutani, Phys. Rev. **C77**, 015207 (2008).

[150] T. Hilger, R. Thomas, and B. Kämpfer (2008), `0809.4996`.

[151] S. Zschocke, private communication (2005).

[152] L. S. Kisslinger, D. Parno, and S. Riordan (2008), `0805.1943`.

Assertion / Versicherung

I declare, that I have written this submitted thesis on my own without inadmissible help of others and without using further not named help; all statements taken from the work of others either directly or indirectly are explicitly referenced. The work in this or similar form has not been submitted for any other qualification to an examination board neither nationally nor internationally.

This thesis was prepared in Dresden under the scientific guidance of Prof. Dr. B. Kämpfer.

Hiermit versichere ich, dass ich die vorliegende Arbeit ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt habe; die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind als solche kenntlich gemacht. Die Arbeit wurde bisher weder im Inland noch im Ausland in gleicher oder ähnlicher Form einer anderen Prüfungsbehörde vorgelegt.

Diese Dissertation wurde in Dresden unter der wissenschaftlichen Betreuung von Prof. Dr. B. Kämpfer angefertigt.

Dresden, 07. Oktober 2008