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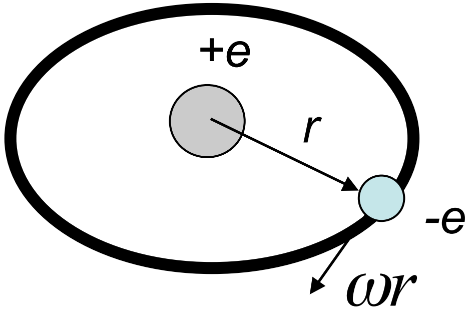
THz coherent control of hydrogenic atoms in silicon for quantum computing applications

Ben Murdin
University of Surrey

EPSRC

COMPASSS

Bohr model of the hydrogen atom



- Consider motion in a circle with attractive Coulomb force

$$m_e r\omega^2 = \frac{e^2}{4\pi\varepsilon r^2} \qquad (1)$$

- With quantised angular momentum

$$m_e r^2\omega = n\hbar \qquad (2)$$

Solve for radius:

$$r_n = \frac{4\pi\varepsilon\ \hbar^2}{e^2}\frac{\hbar^2}{m_e}n^2 = a_0 n^2$$

Bohr radius r_1=a_0=0.53Å

Solve for energy:

$$E_n = E_{KE} + E_{PE}$$

$$= \frac{1}{2}m_e r^2\omega^2 - \frac{e^2}{4\pi\varepsilon r}$$

$$= -\frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon}\right)^2\frac{m_e}{\hbar^2}\frac{1}{n^2} = -E_R\frac{1}{n^2}$$

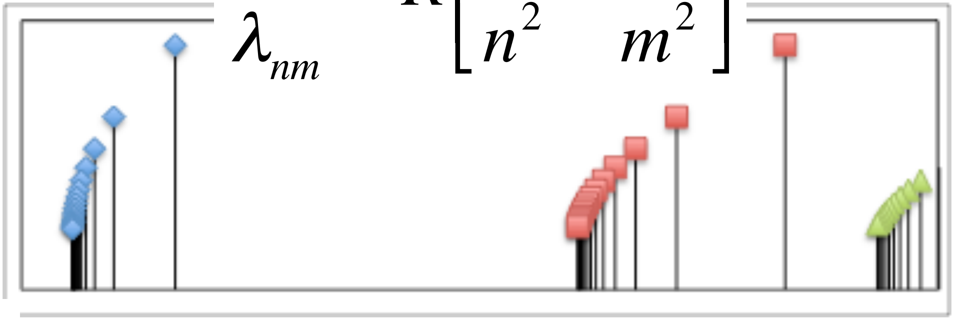
Rydberg energy E_R=13.6eV

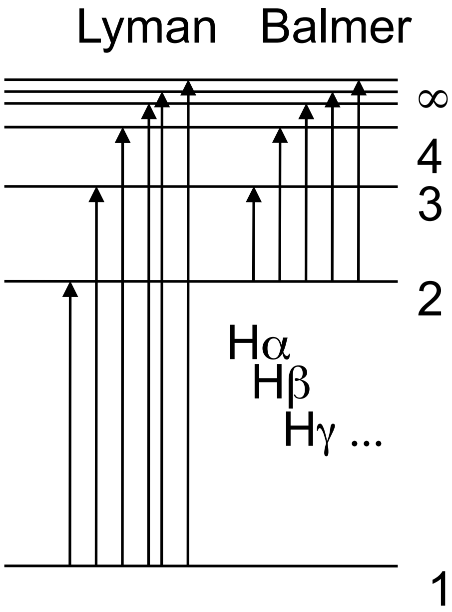
Hα line: E_3-E_2=$^5/_{36}$13.6eV$\equiv$ 656nm

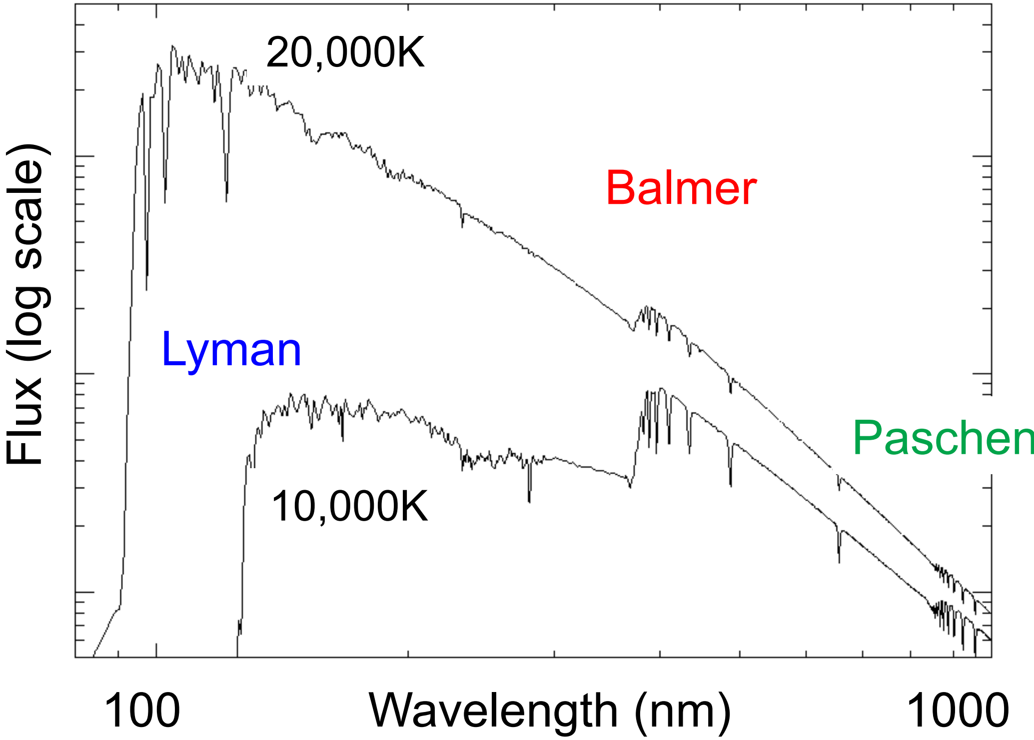
Rydberg spectrum of hydrogen

- Hydrogen absorption spectrum seen superimposed on the emission from a very hot black body (a star)

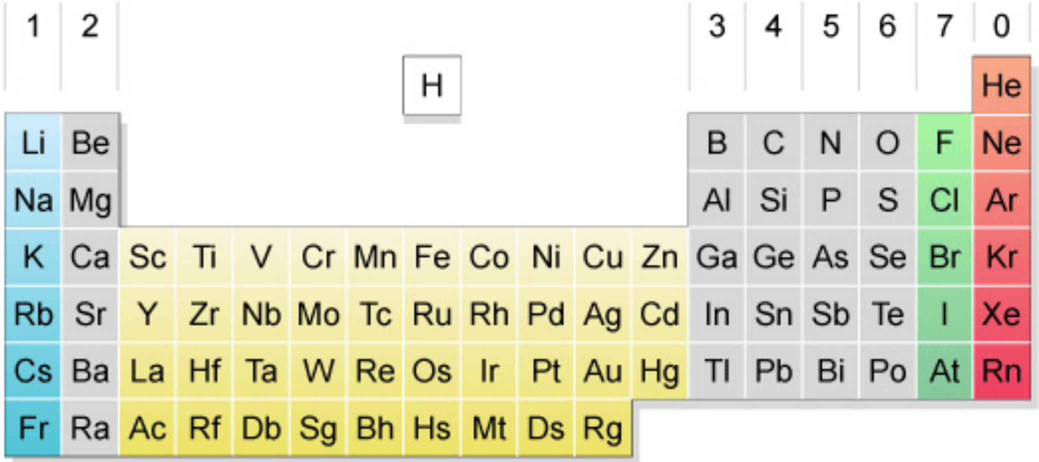
$$\frac{1}{\lambda_{nm}} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$



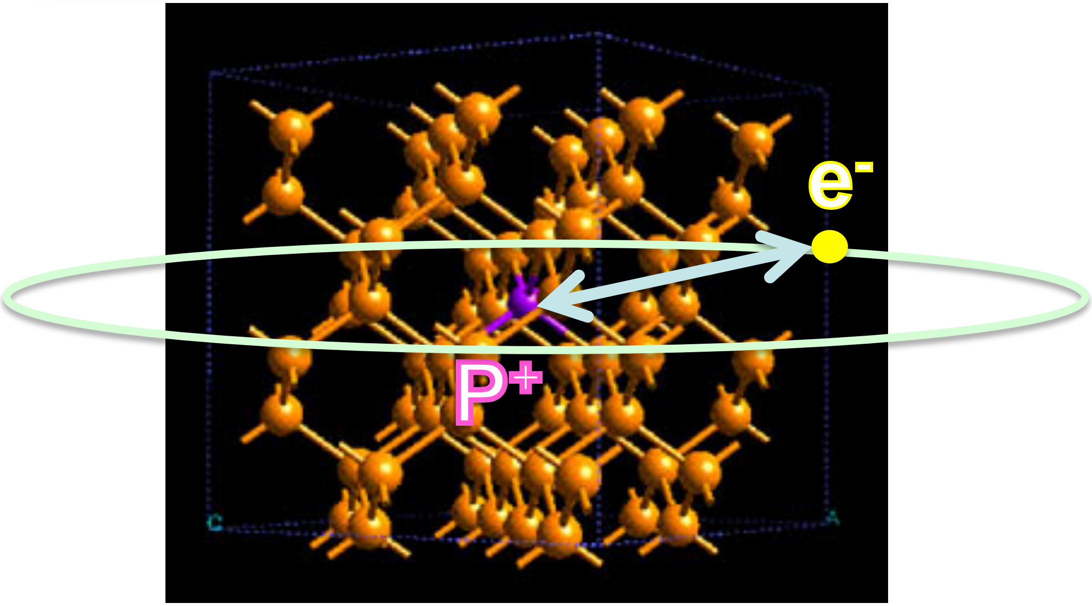




A hydrogen-like atom in a silicon chip: The Group 5 impurity



1	2											3	4	5	6	7	0
							H										He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg							



- P looks like silicon with
  - an extra +ve charge in the ion
  - an extra electron orbiting
- The electron-ion attraction is screened by ε_r
- The mass is reduced by m^*

Scaling from hydrogen to donor

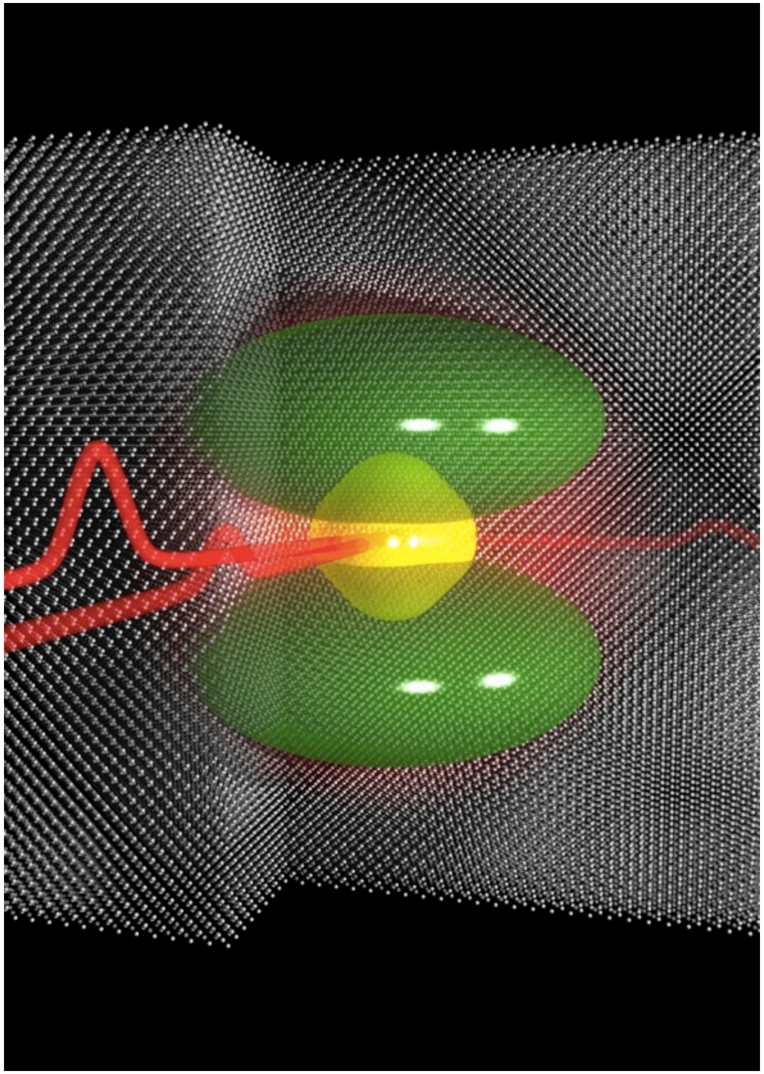
- Binding energy:

$$E_R = \frac{1}{2}\left(\frac{e^2}{4\pi\hbar}\right)^2 \frac{m_e}{\varepsilon^2}$$

- Bohr radius

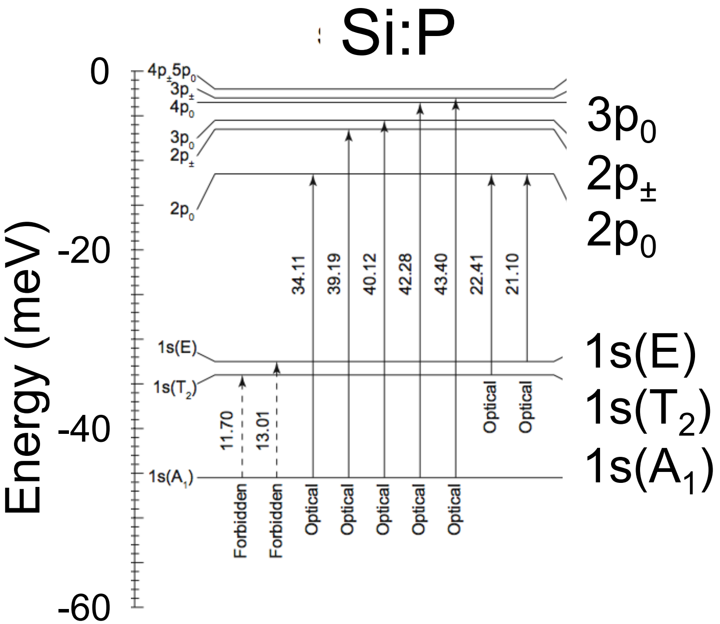
$$a_0 = \frac{4\pi\hbar^2}{e^2}\frac{\varepsilon}{m_e}$$

	H	Si:P
ε_r	1	11.4
m_e	1	0.19
E_R	13.6 eV	0.020 eV
a_0	0.056 nm	3.2 nm

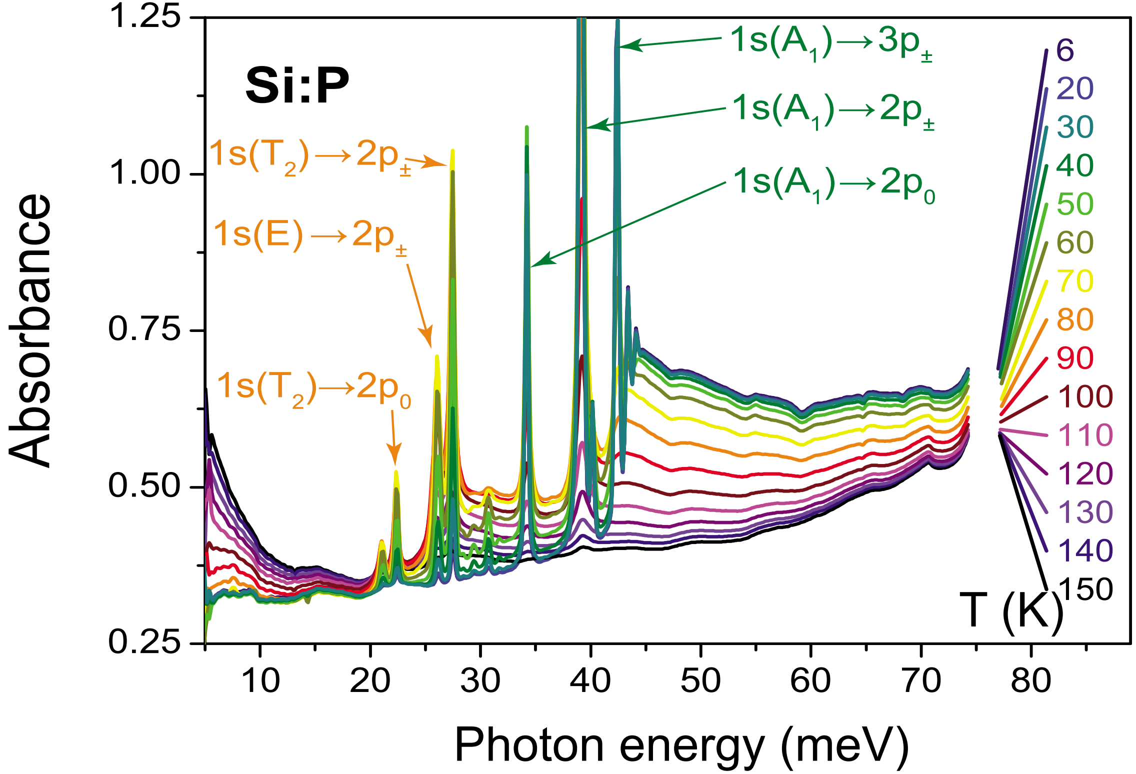


20meV~160cm^{-1}~60μm~5THz

Hydrogen like spectrum in Si:P



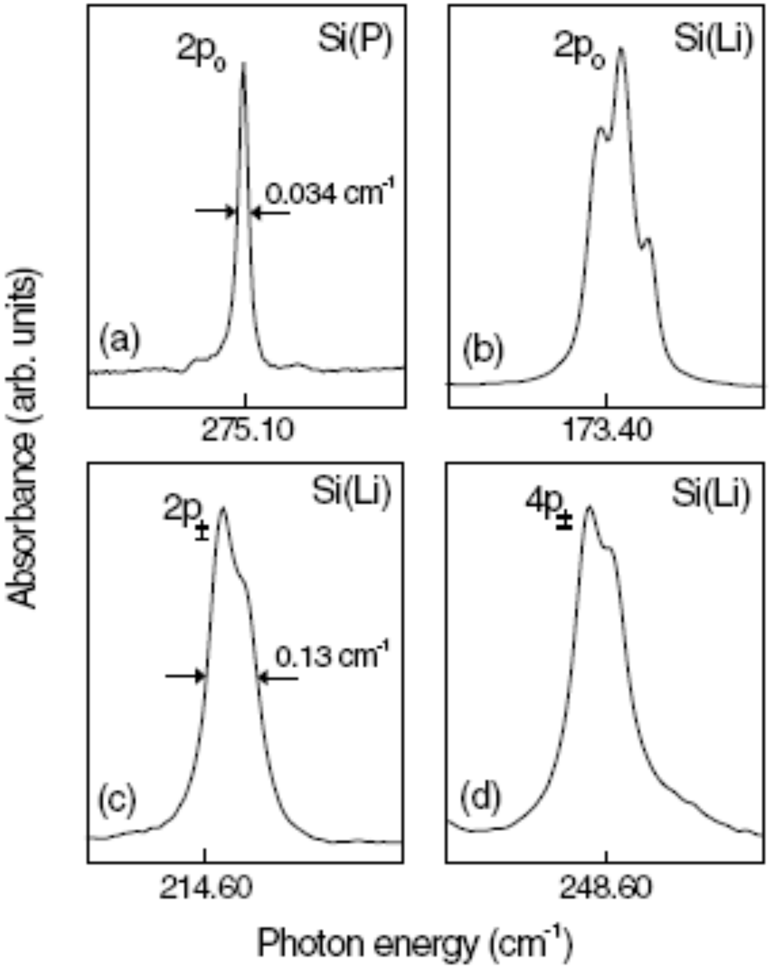
SA Lynch et al, Phys. Rev. B (2010)



- Some qualitative differences between hydrogen and donor
  - States split by cubic crystal symmetry (e.g. $2p_0$ and $2p_\pm$)
  - s-states shifted down in energy by “quantum defect” (a.k.a. “chemical shift” or “central cell correction”) due to the P^+ ion not being point-like

Group V donors in silicon

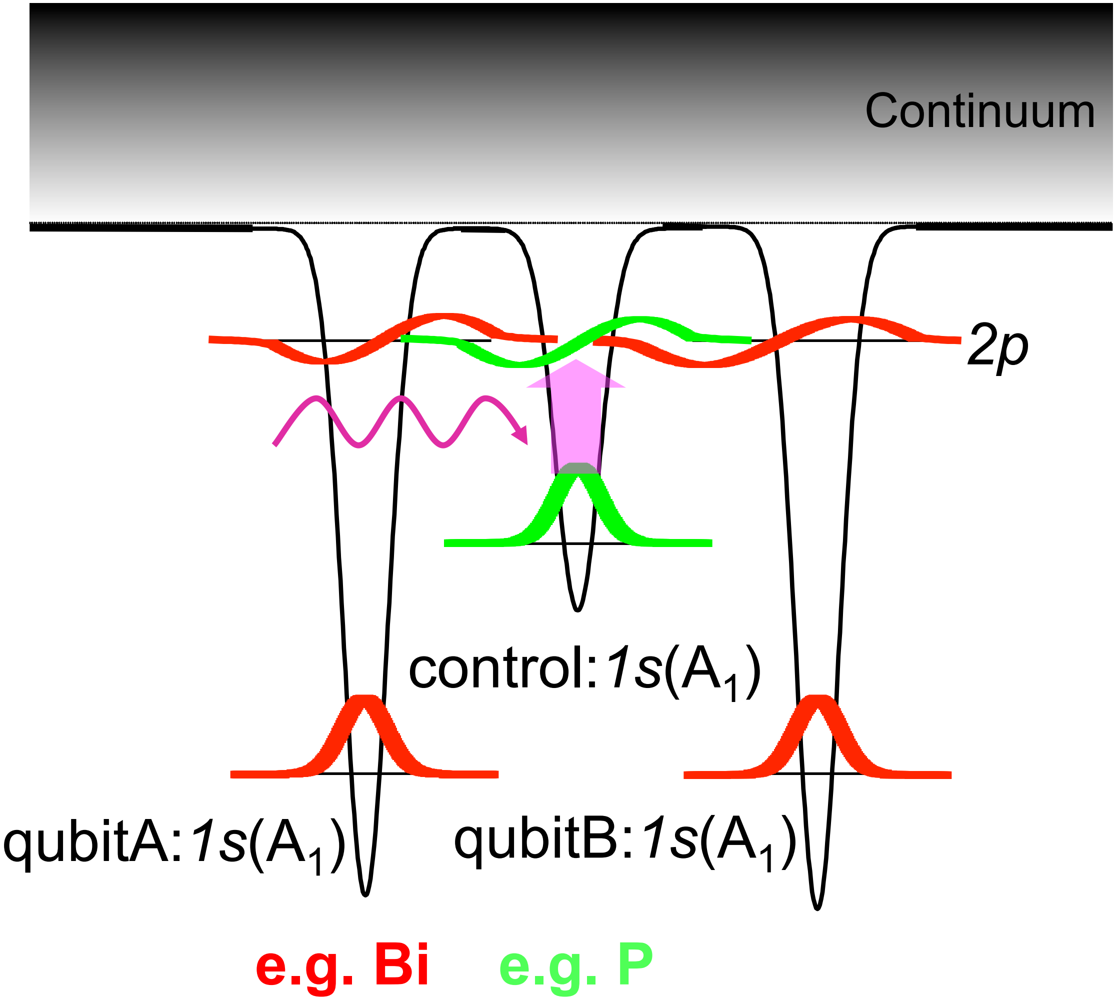
- Very sharp lines: for isotopically pure ^{28}Si:P $h\Delta\nu \sim 4\mu eV \sim 0.03 cm^{-1} \sim 1GHz$ [$1/\Delta\nu \sim 152ps$]
- [c.f. typical commercial (natural) silicon – 250μeV, or single quantum dots – 50μeV]
- for 'quantum control' - need to know how much of this inhomogeneous broadening, and how much is lifetime broadening



Cardona et al. PRL 90 (2003)

Qubit gate control scheme

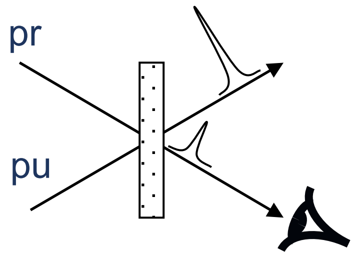
- **Qubits:** spin of deep, well isolated impurity states with very long (msec) dephasing times – they remain in 1s state
- **Entanglement/ control/gating:** pumping different impurity species with overlapping excited state

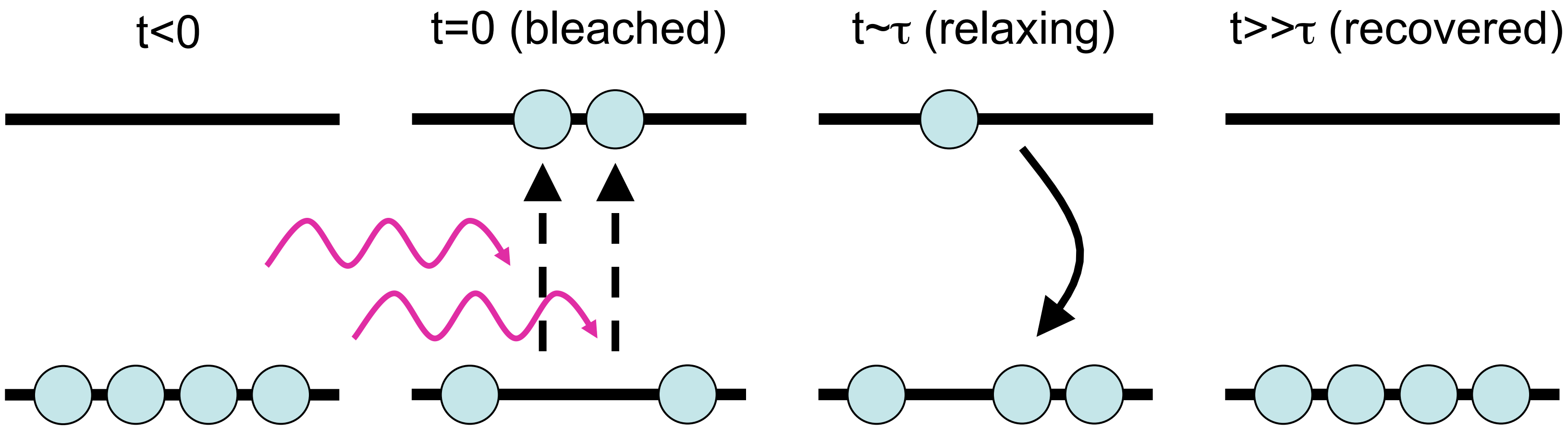


A. M. Stoneham *et al*, *J. Phys. C, 15, L447, 2003.*

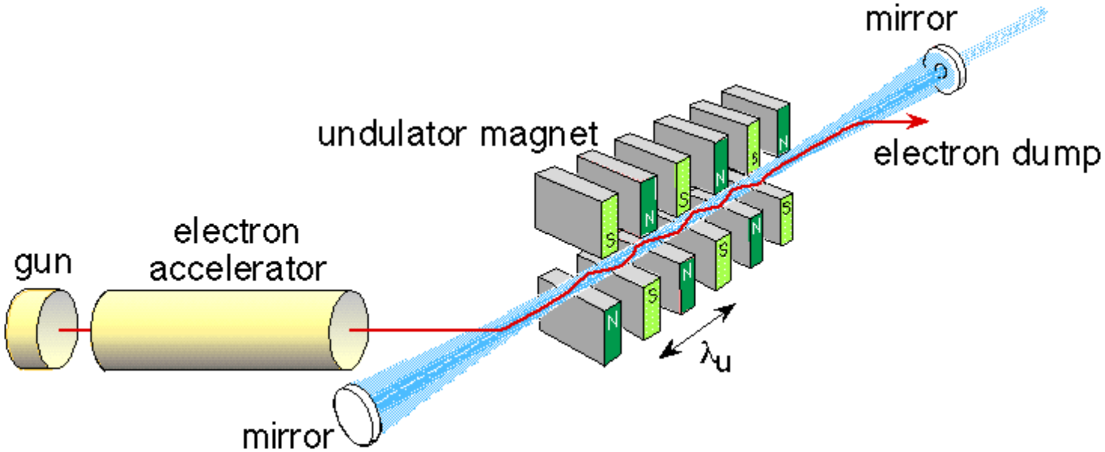
Incoherent dynamics experiment: pump-probe

- A strong pump pulse bleaches the transmission by exciting ~50% of the oscillators
- A weak probe measures the transmission recover as a function of time delay

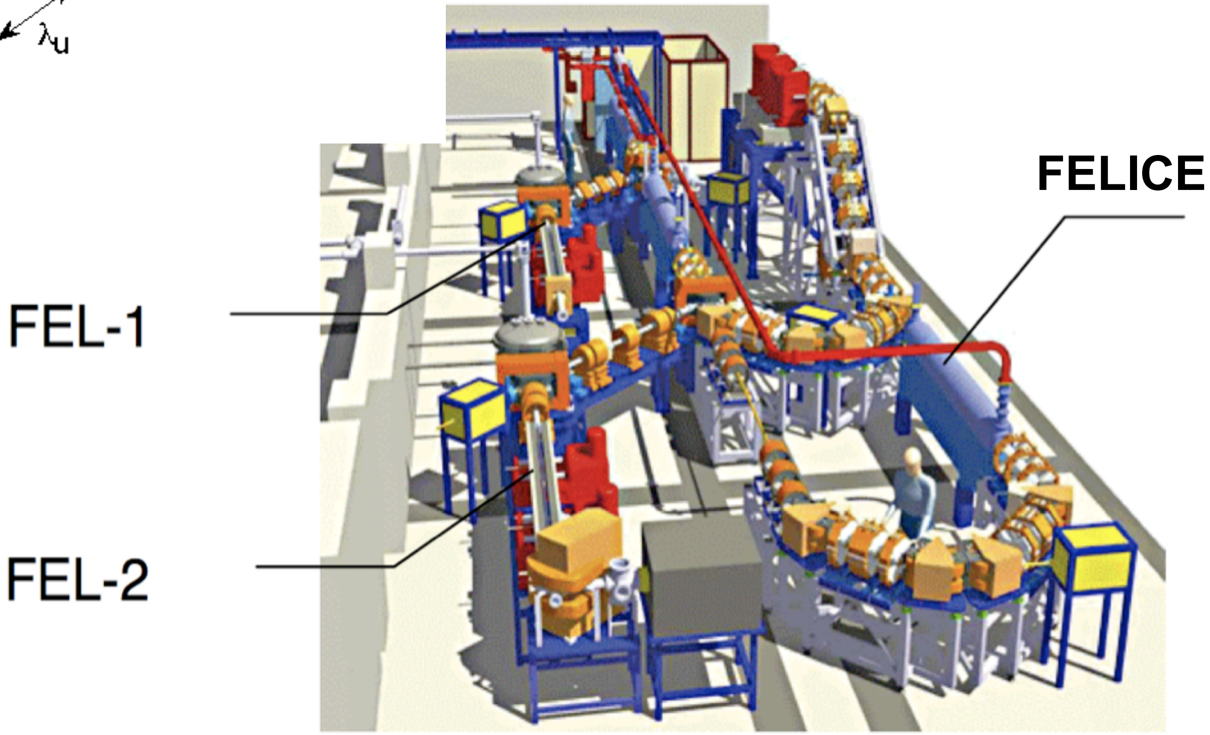




FELIX, the Free-Electron Laser for Infrared eXperiments (Utrecht)

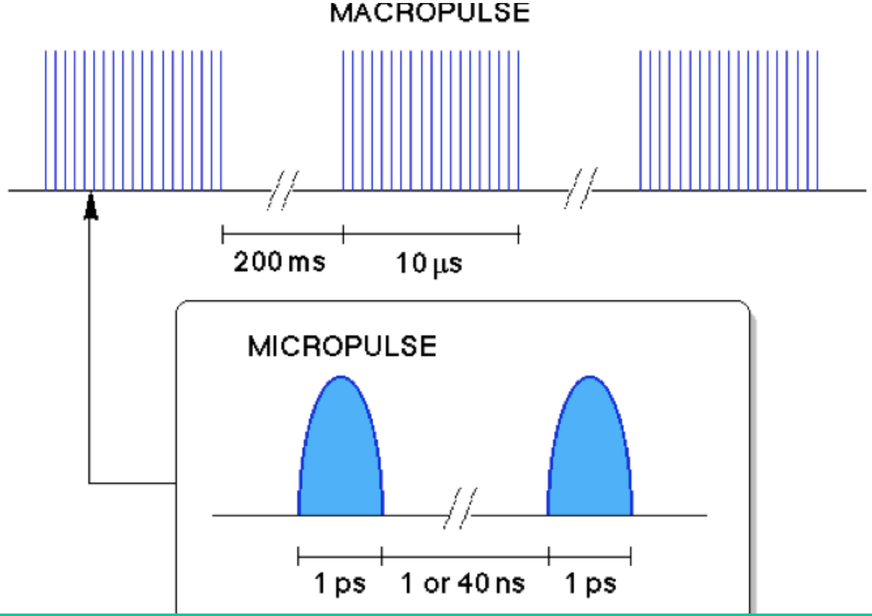


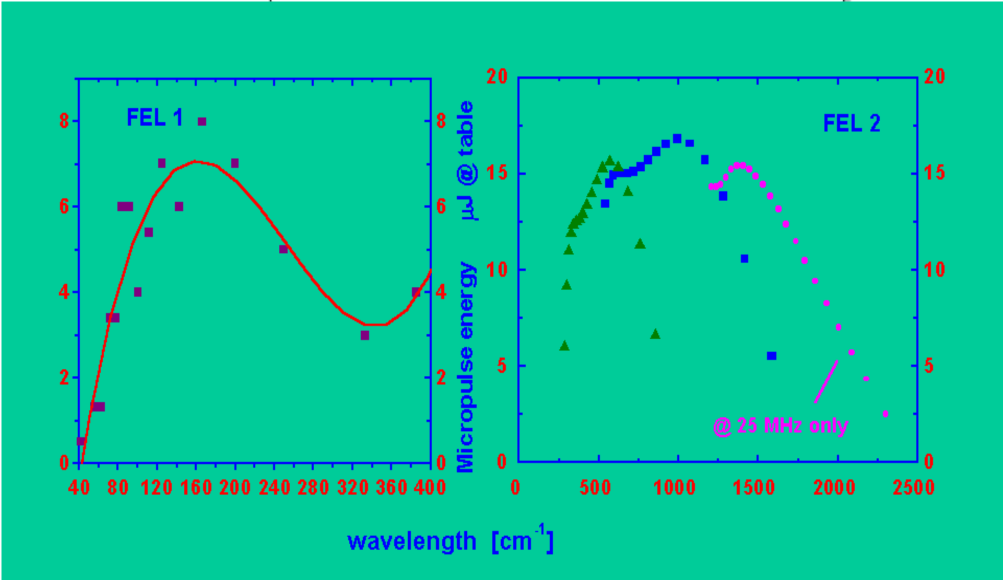
$$n\,\lambda_s = \frac{\lambda_u\,(\,1 + K^2\,)}{2\,\gamma^{\,2}}$$



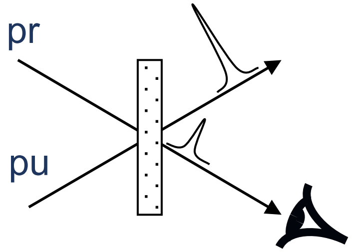
FELIX light output

- Picosecond pulses
  - Ideal for short time dynamics
- MW power
  - Ideal for bleaching
- λ~4-250μm
  - Ideal for silicon impurities at 20-40μm
- Repetition time is long enough that each pulse starts afresh

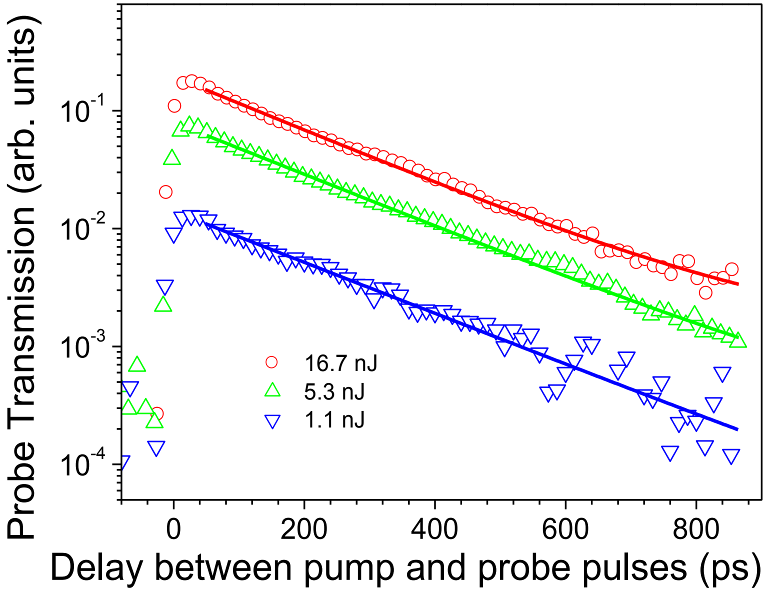




Pump-probe expt for lifetime



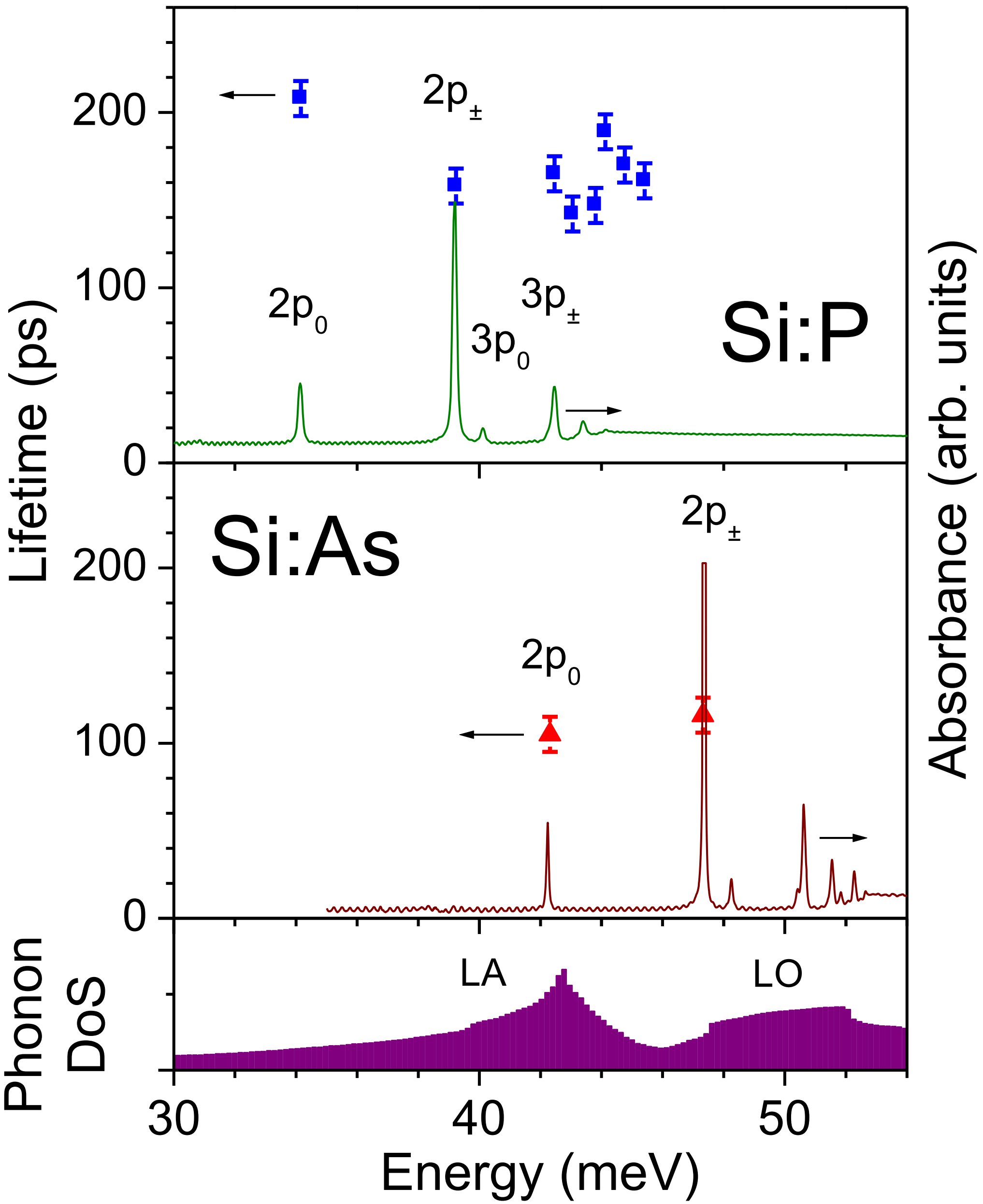
- Si:P
- Lifetimes near 200 ps for 10THz oscillations:
  - $Q = 200\text{x}10^{-12}/10^{-13} = 2000$ for Rydberg states in Si
- Implications
  - Ion trap physics in silicon
  - possibilities for impurity lasers
- Inverse exchange interactions in P-Bi scheme $\hbar/J \sim 10\text{ps} << 200\text{ ps}$
  - Lifetime long enough to make "cube-root of unity" gate, maybe not for "C-NOT" gate



NQ Vinh *et al, Proc Nat Acad Sci USA* **105,** 10649 (2008)

Spectral dependence

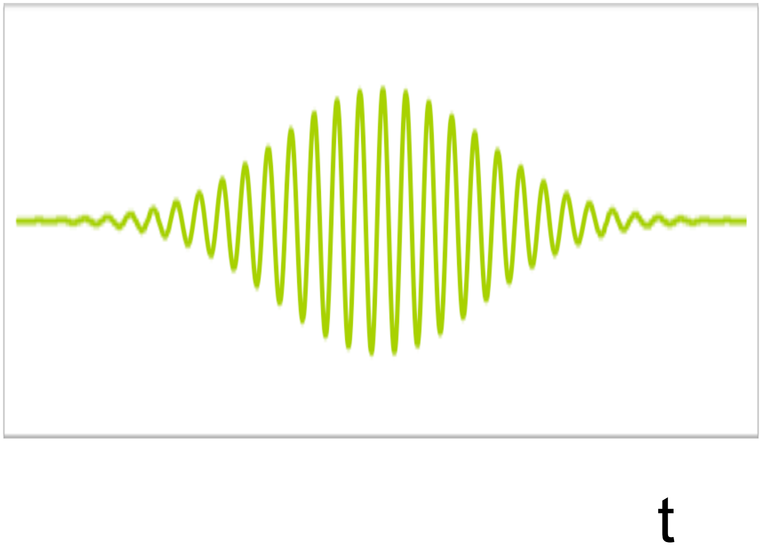
- Measured:
  - Si:P, Si:Bi exhaustively,
  - Si:Sb, Si:As some data
- Lifetimes follow phonon density of states
- No signal when off-resonance
- Bi atoms resonant with optical phonons show evidence of intermediate state trapping

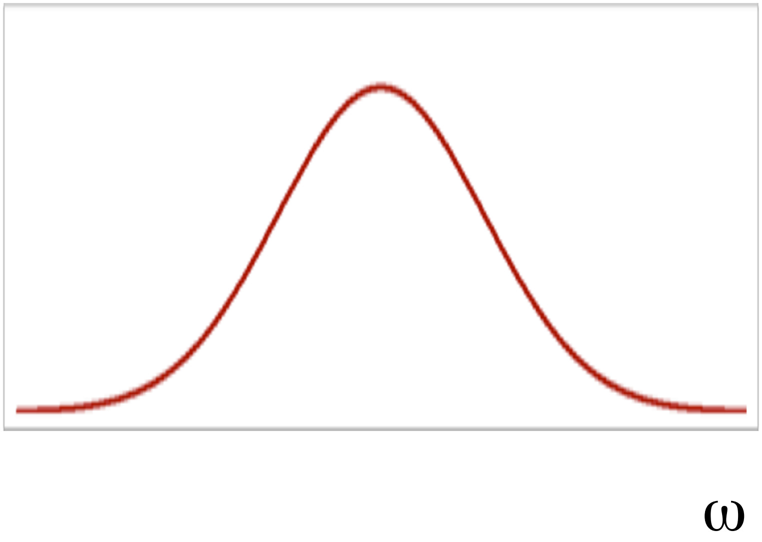


NQ Vinh *et al, Proc Nat Acad Sci USA* **105,** 10649 (2008)

FEL as a coherent source

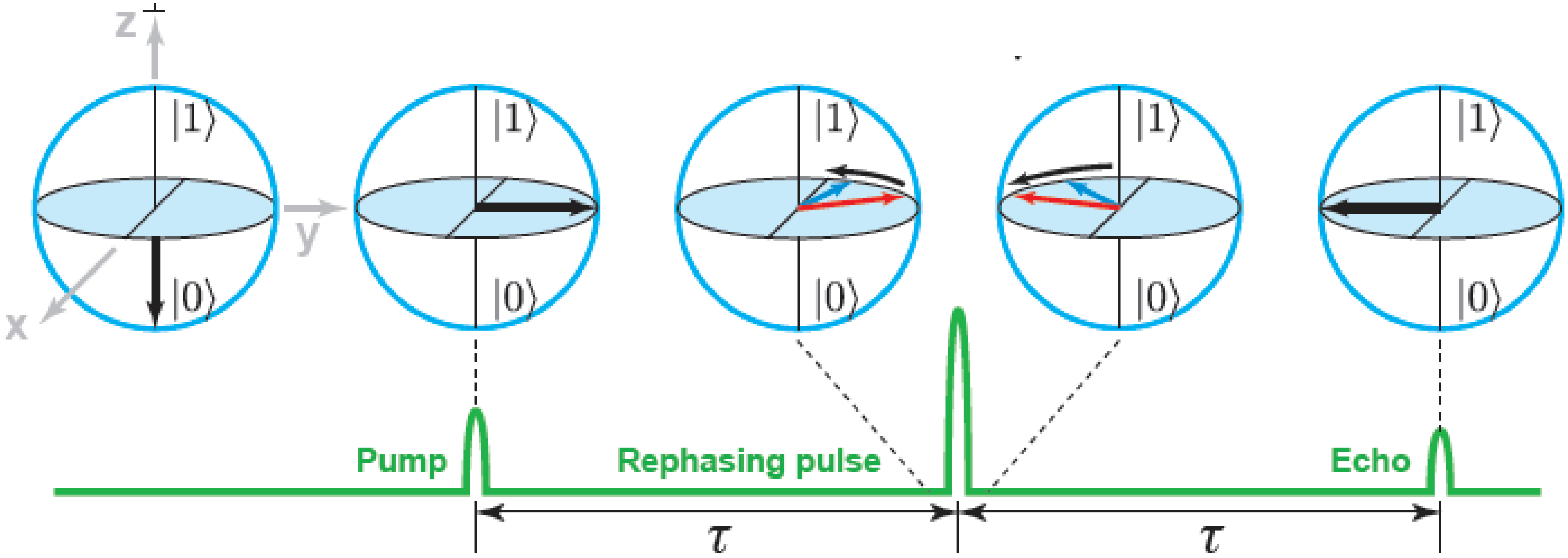
- So far the experiments have not required the coherence of the light, as pump-probe experiments can be incoherent





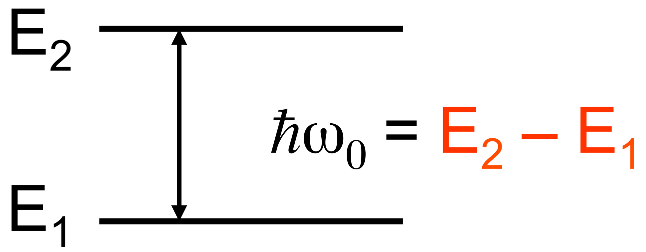
Coherent dynamics: The Hahn Echo

- The photon echo is analogous to the spin echo of ESR/ NMR
- For an ensemble of oscillators with inhomogeneity in the natural frequency, a π-pulse can reverse the loss of phase



The time dependent Schrodinger equation

- Assume we have a two-level atom in the dark



$$\hbar\omega_0 = E_2 - E_1$$

- Now add a strong light field
- The Rabi frequency

$$\Omega_R = F_0\mu_{12}/\hbar$$

- The dipole matrix element

$$\mu_{12} = \int \varphi_1^* e x \varphi_2 d^3\mathbf{r}$$

$$\hat{H}_0\psi_i = i\hbar\frac{\partial\psi_i}{\partial t} = E_i\psi_i \qquad i=1,2$$

$$\psi_i(\mathbf{r},t) = \phi_i(\mathbf{r})\exp(-iE_i t/\hbar)$$

$$\hat{H} = \hat{H}_0 + e\hat{x}F_0\cos\omega t$$

$$\Psi = \sum_{i=1,2} c_i(t)\psi_i(\mathbf{r},t)$$

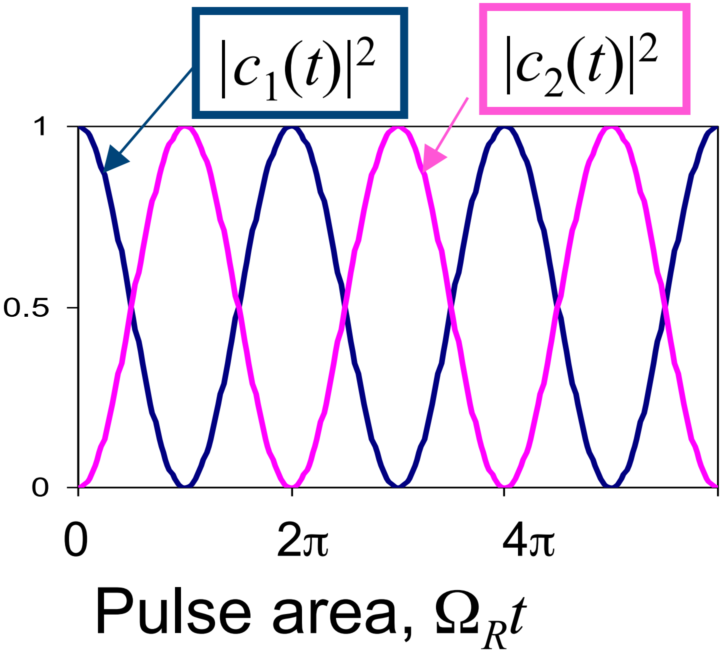
$$\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\vdots$$

$$\dot{c}_1(t) = \frac{i}{2}\Omega_R c_2(t) \qquad \dot{c}_2(t) = \frac{i}{2}\Omega_R c_1(t)$$

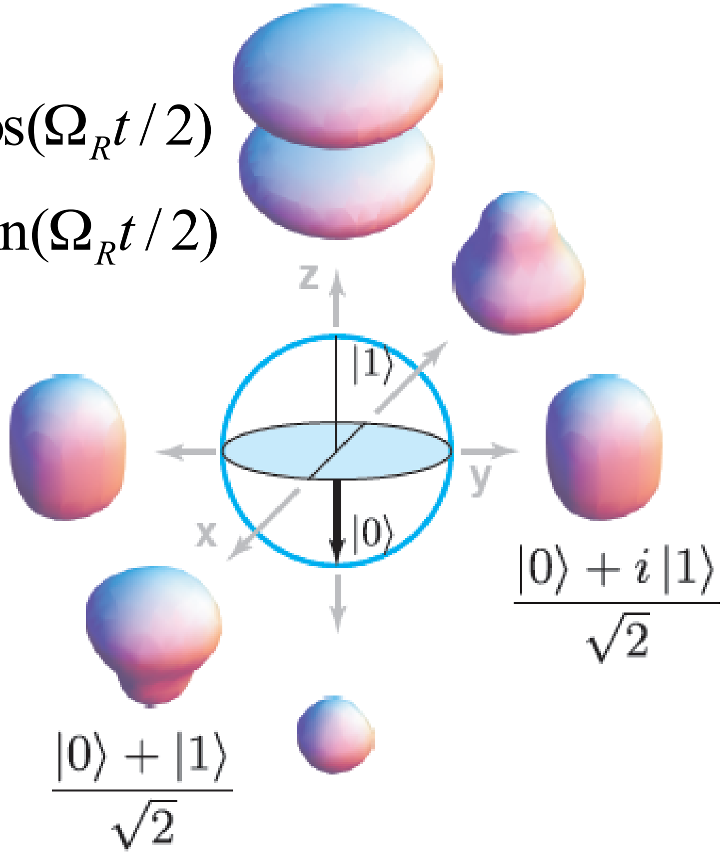
$$c_1(t) = \cos(\Omega_R t/2) \qquad c_2(t) = i\sin(\Omega_R t/2)$$

Rabi flopping in rubidium

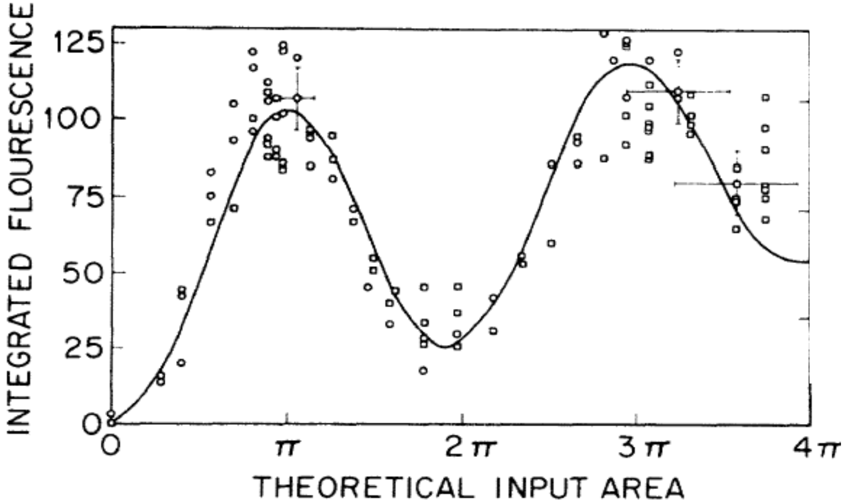


$$c_1(t) = \cos(\Omega_R t/2)$$

$$c_2(t) = i\sin(\Omega_R t/2)$$



The Bloch Sphere



H.M. Gibbs,
Phys. Rev. Lett. 29, 495 (1972),
Phys. Rev. A8, 446 (1973)

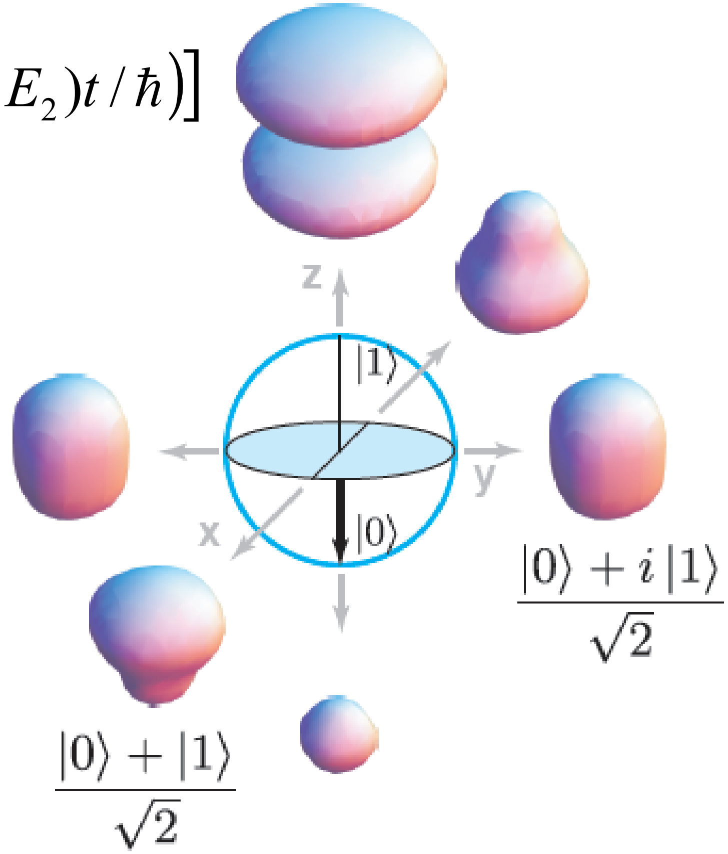
Time-evolution of superpositions

- 50:50 superposition, switch off light

$$\Psi(\mathbf{r},t)=\frac{1}{\sqrt{2}}\left[\phi_1(\mathbf{r})\exp(-iE_1t/\hbar)+\phi_2(\mathbf{r})\exp(-iE_2t/\hbar)\right]$$

$$=\frac{1}{\sqrt{2}}\exp(-iE_1t/\hbar)\left[\phi_1(\mathbf{r})+\phi_2(\mathbf{r})\exp(i(E_1-E_2)t/\hbar)\right]$$

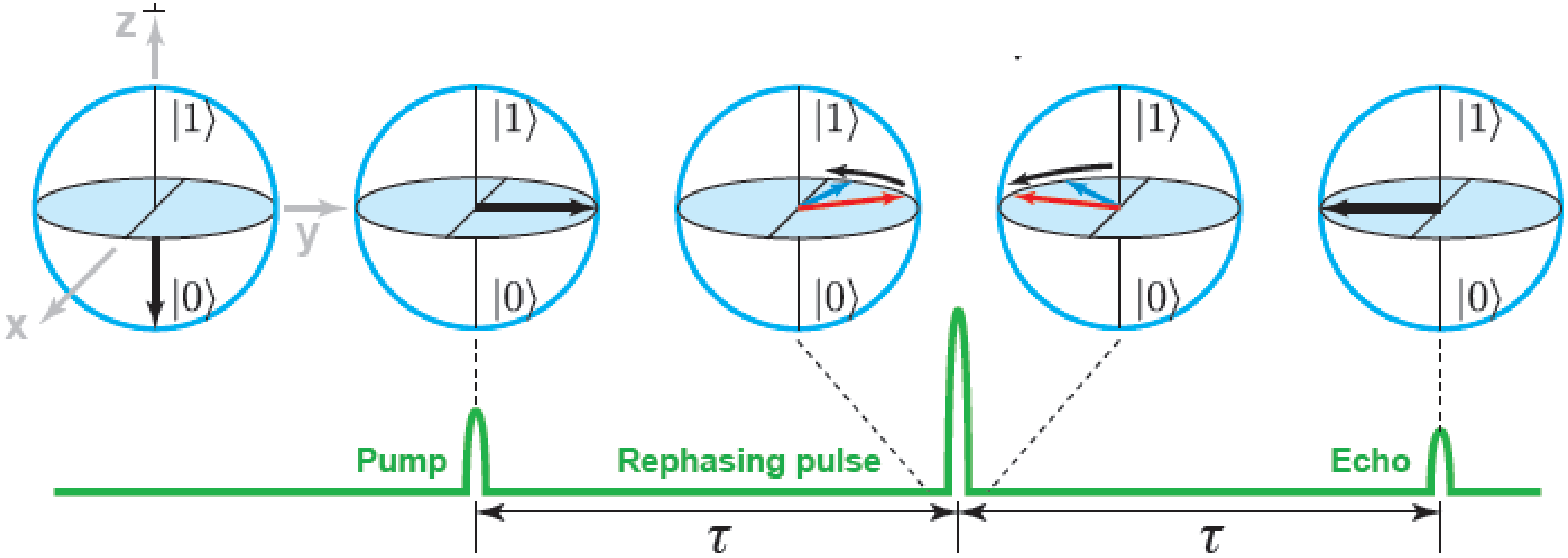
- The relative phase of the superposition oscillates with the natural frequency
  - On Bloch sphere vector precesses around equator
  - The expectation position oscillates up and down
  - The atom radiates



The Bloch Sphere

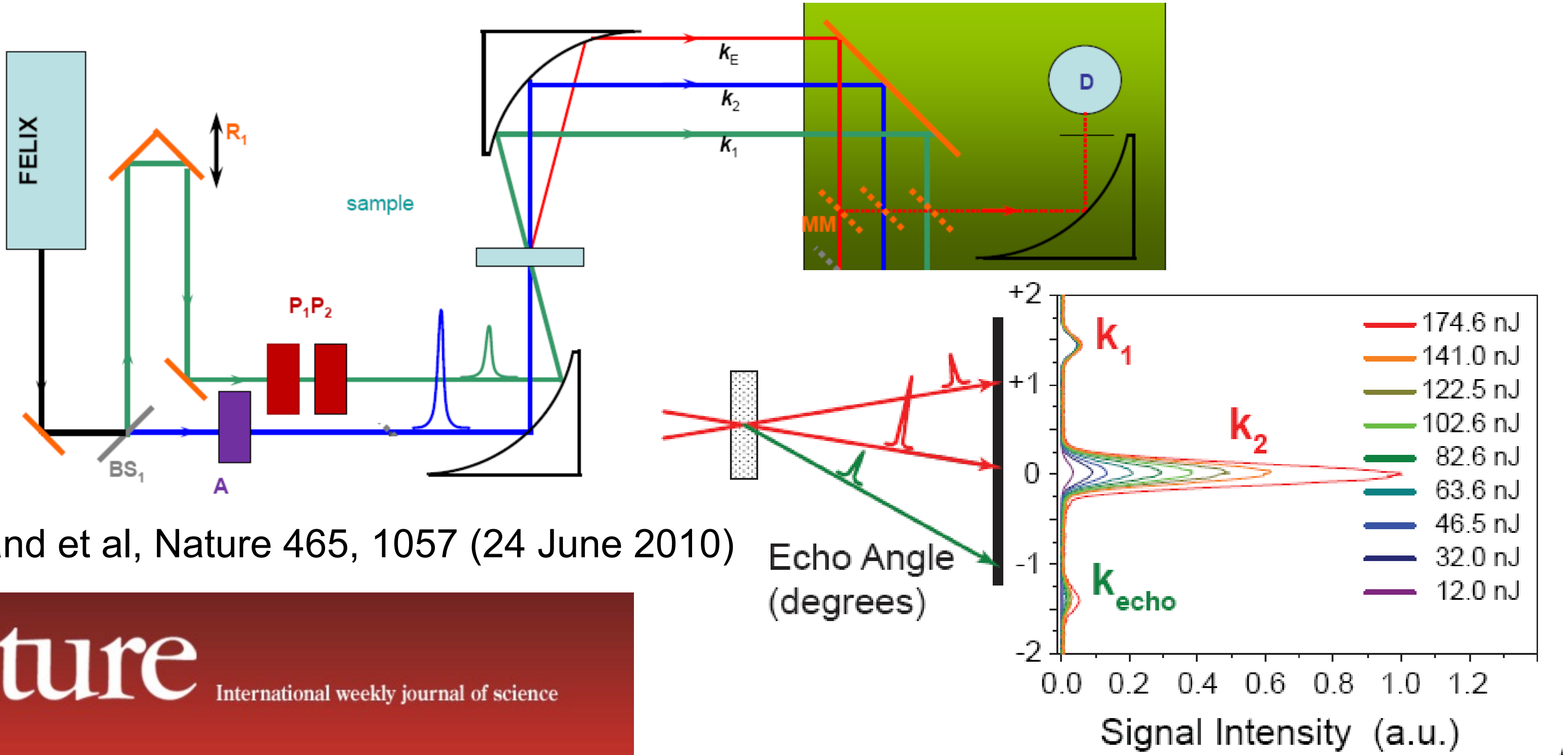
The Hahn Echo

- For an ensemble of oscillators with inhomogeneity in the natural frequency, a π-pulse can reverse the loss of phase



Hahn echo in Si:P

- At optical frequencies with directed laser beams there is spatial interference between the coherence and the light

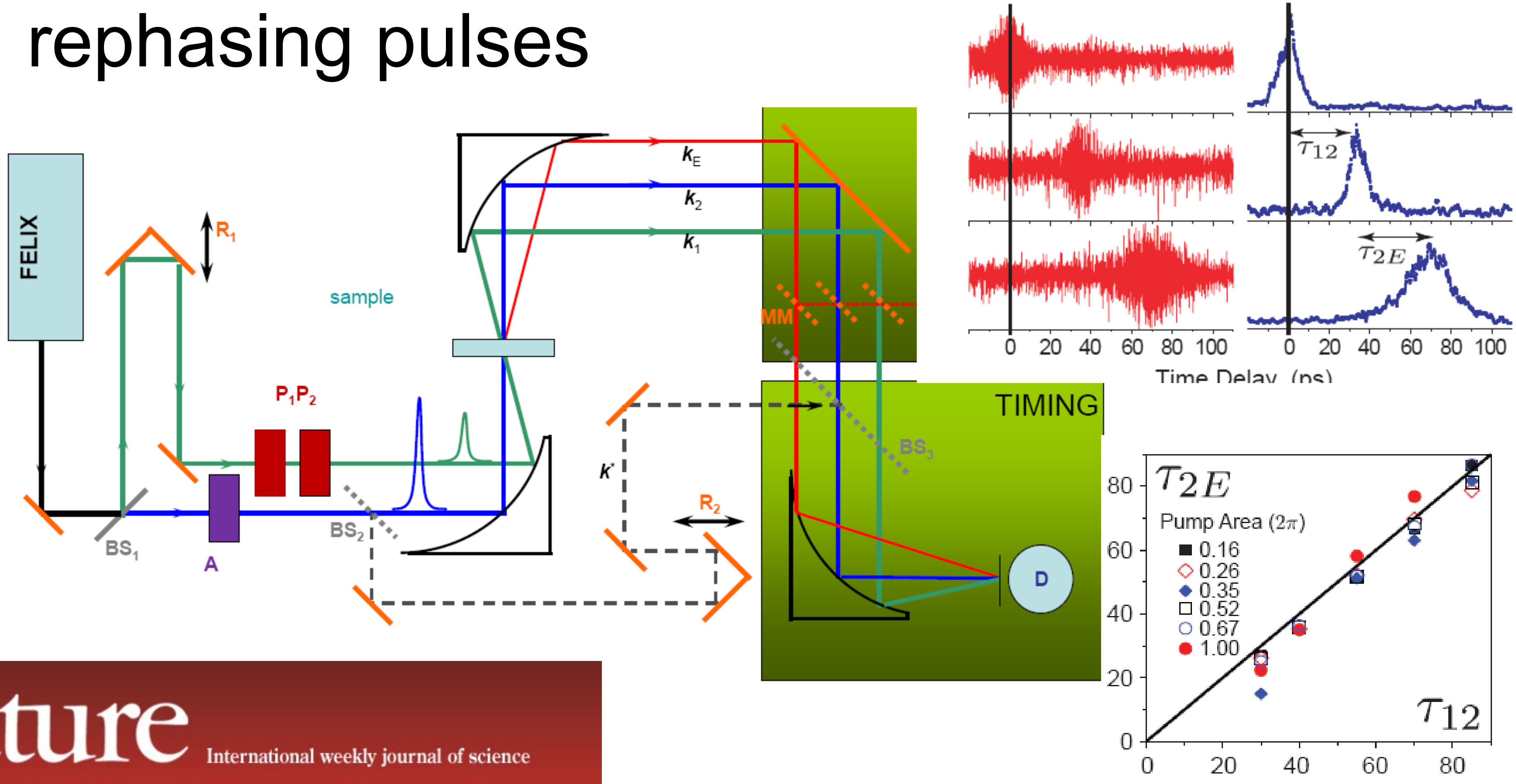


Greenland et al, Nature 465, 1057 (24 June 2010)

nature International weekly journal of science

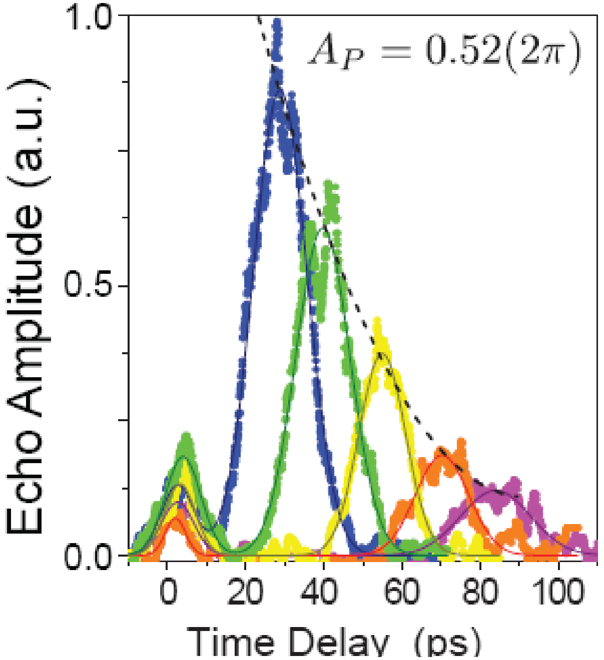
Hahn echo in Si:P

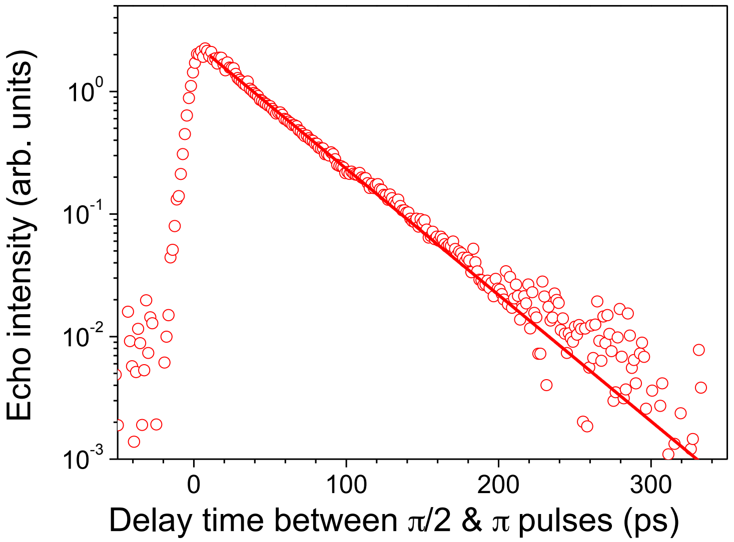
- The arrival time of the echo is controlled by the delay between the pump and rephasing pulses



nature International weekly journal of science

Photon echoes for excited state T_2

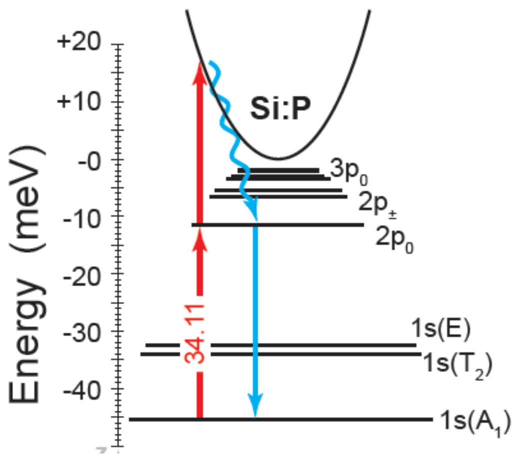


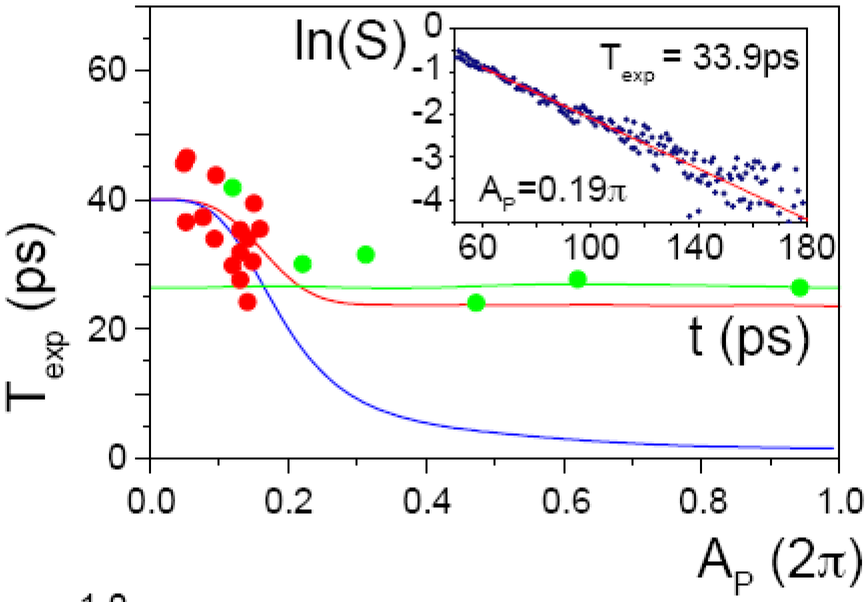


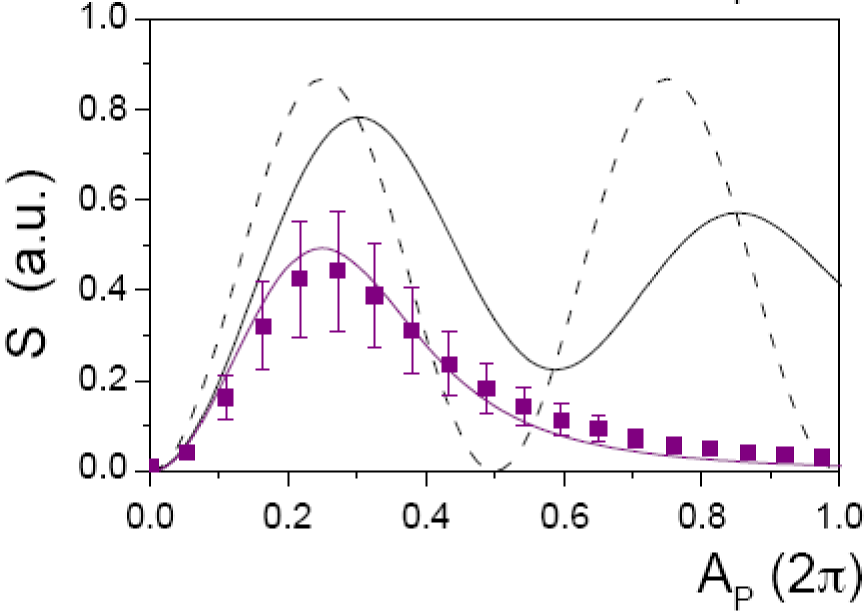
- Measured T_2 200ps ~T_1 from pump-probe in natural Si:P
- Also similar to inverse linewidth of isotopically purified ^{28}Si:P from Cardona et al (1/$\Delta\nu$= 152ps, c.f. 31ps for natural Si:P)

Intensity dependence

- The dephasing time reduces with increasing intensity due to photo-ionisation from the excited state
- The beam profile smoothes out the Rabi oscillations

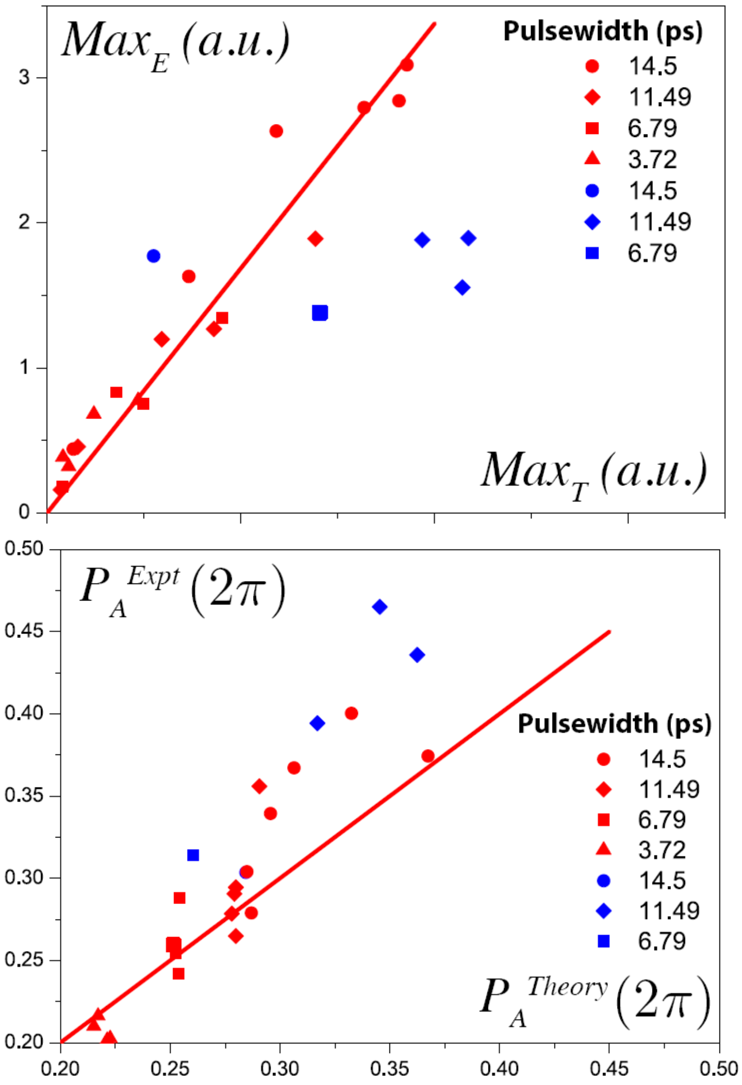






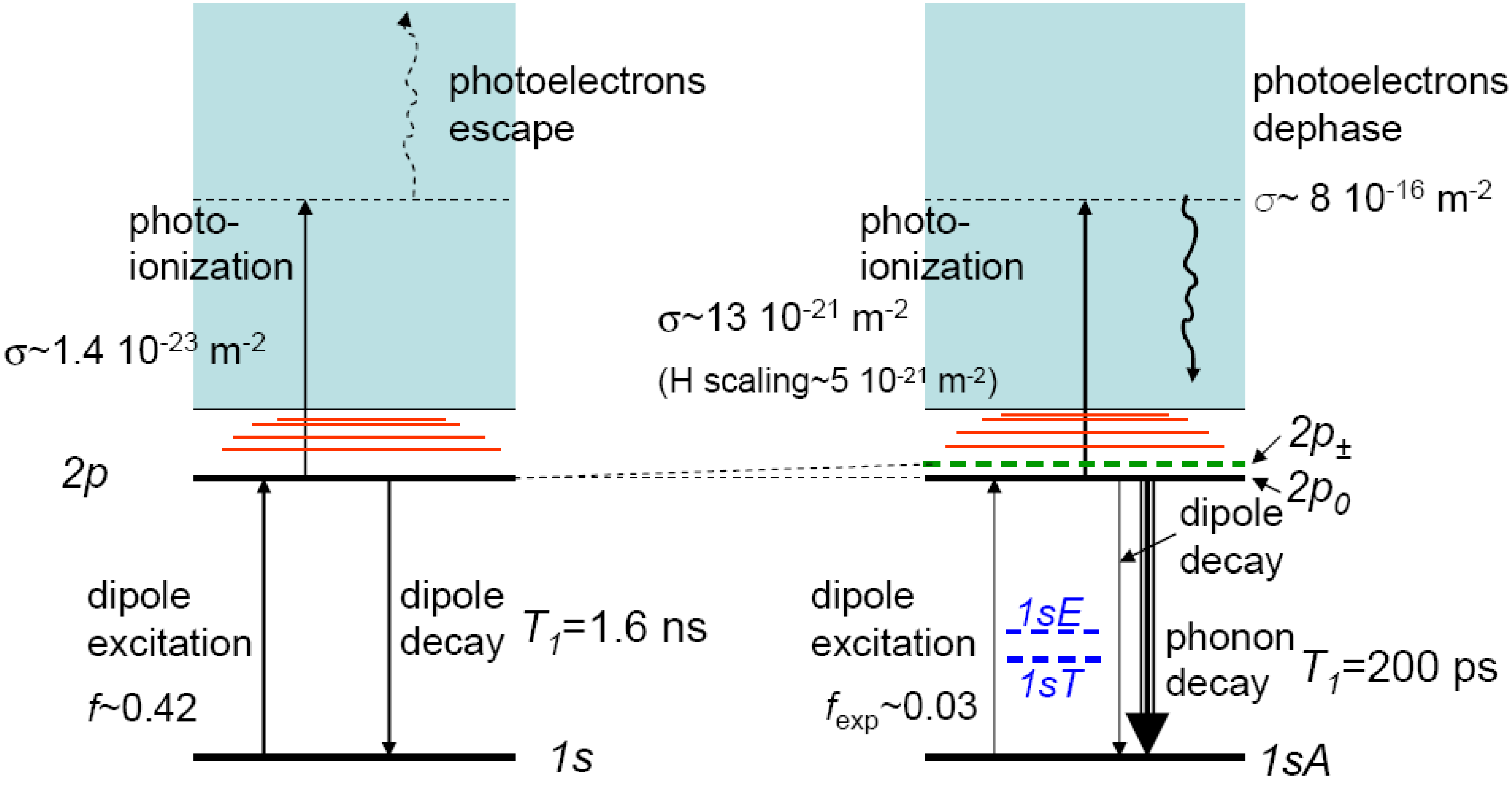
2-level density matrix model

- Theory includes photoionisation dependent dephasing and spatial beam profile
- Requires 3 parameters:
  - dipole matrix element,
  - photoionisation cross-section,
  - collision probability for free electron and excited neutral donor
- Comparison between expt and theory of
  - first Rabi oscillation peak height
  - Intensity of peak position



Summary

Comparison of H and Si:P



photoelectrons escape

photo-ionization

σ~1.4 10^{-23} m^{-2}

2p

dipole excitation

f~0.42

dipole decay

T_1=1.6 ns

1s

photoelectrons dephase

σ~ 8 10^{-16} m^{-2}

photo-ionization

σ~13 10^{-21} m^{-2}

(H scaling~5 10^{-21} m^{-2})

$2p_\pm$

$2p_0$

dipole decay

dipole excitation

f_{exp}~0.03

1sE

1sT

phonon decay

T_1=200 ps

1sA

Summary

- We measured incoherent dynamics of shallow donor impurities in silicon – for the first time
- The P lifetimes are long and close to the inverse linewidth of ^{28}Si:P
  - in the sense that ion traps remove sources of decoherence, the silicon host is like a very cheap ion trap!
- We demonstrated quantum coherence effects in Si:P – the first steps towards quantum computation
- Photoionisation is annoying
  - so reduce the density
- Laser beam profile is annoying
  - So reduce the doped area

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Konstantin Litvinenko, Steve Clowes, Ellis Bowyer, Matt Pang, Nicole Li,
Advanced Technology Institute, University of Surrey, Guildford, England

Gabriel Aeppli, Thornton Greenland Byron Villis, Morteza Erfani
London Centre for Nanotechnology, University College London

Vinh Nguyen, Lex van der Meer, Britta Redlich
FOM Institute for Plasma Physics, Nieuwegein, The Netherlands

Carl Pidgeon
Department of Physics, Heriot-Watt University, Edinburgh

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