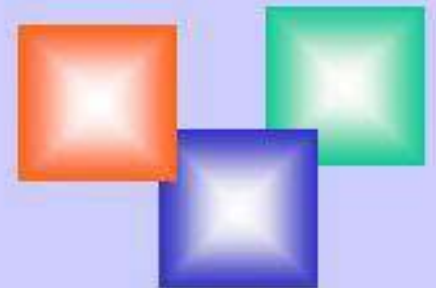




# Spin dependent electronic transport (Magnetotransport properties of metals and semiconductors: introduction)

**Alexander Granovsky**

*Lomonosov Moscow State University, Moscow 119991, Russia*



1755

40 Faculties  
50.000 students  
10.000 staff  
2 ships, 6 subdiv.  
Supercomputer  
Sputnik



Magnetism Department was founded more than 70 years ago.

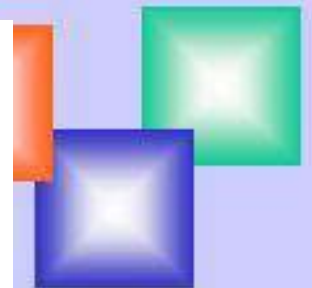
The hysteresis loop was measured in MSU by Prof. Stoletov in 1872.

Stoletov, Arkad'ev, Landau, Kapitza, Kondorski

The main topics:

- Advanced Magnetic Materials (nanostructures, soft and hard, amorphous and nanocrystalline, thin films, ribbons, microwires, carbon nanotubes, multilayers “ferromagnet/superconductor” etc)
- Spintronics
- Magnetophotonics (magneto-optics, magnetophotonic crystals)
- Room temperature dilute magnetic semiconductors and oxides
- Magnetic liquids and polymers
- Magnetic sensors
- Biomagnetism

36 departments,  
2500 students  
400 PhD students  
800 staff

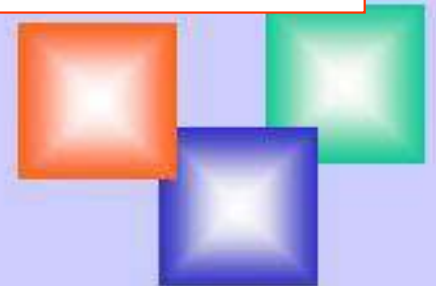




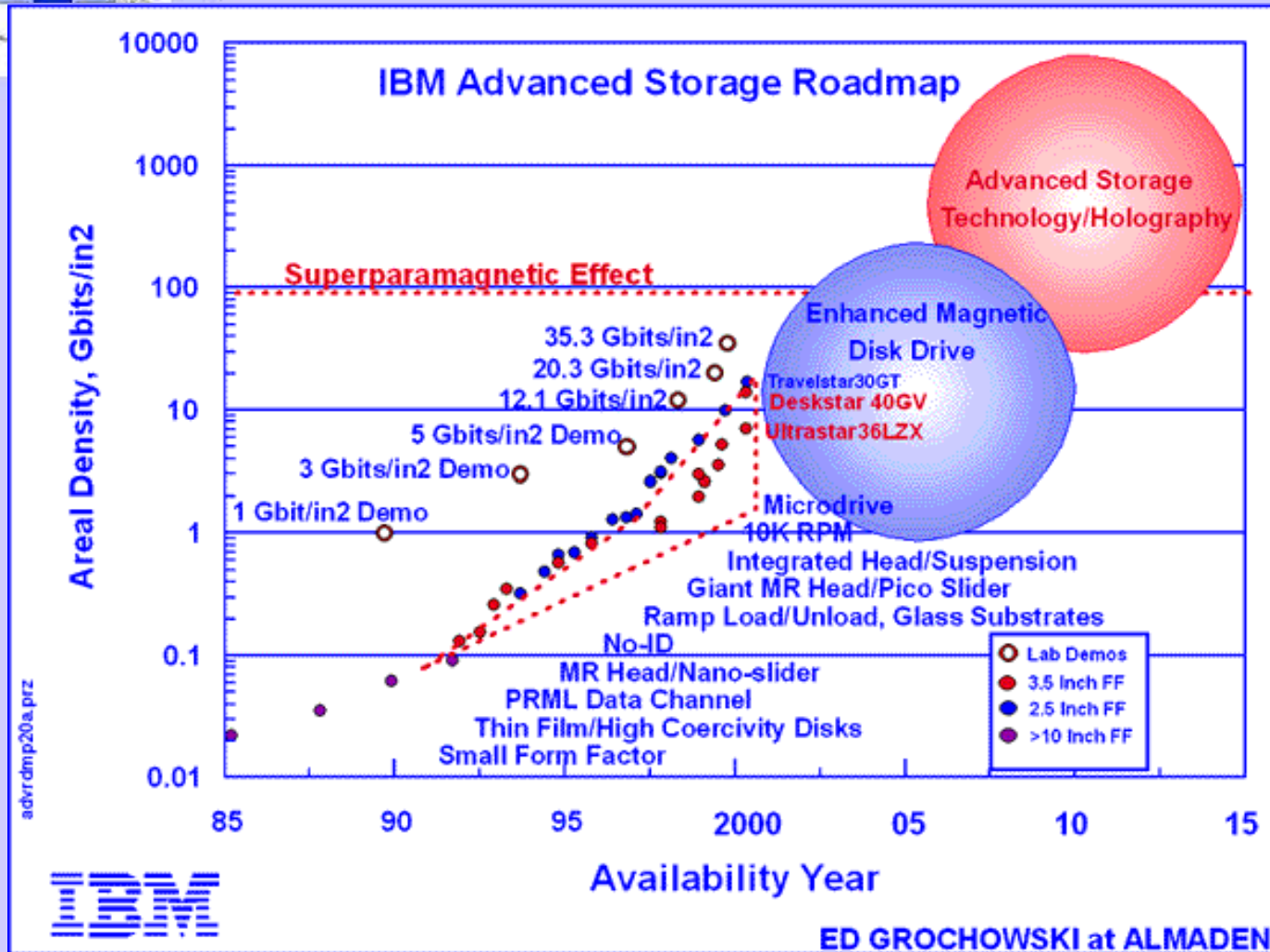
## Outline

- Introduction
- Resistivity (Mooij correlation)
- Magnetoresistance (AMR, injection MR, organic MR)
- Anomalous Hall effect : Basic mechanisms
- Anomalous Hall effect in heterogeneous alloys
- Anomalous Hall effect in diluted magnetic semiconductors.
- Can the presence of anomalous Hall effect serve as an evidence of spin polarization of current carriers ?
- Recent experimental data on  $\text{TiO}_2\text{:Co}$
- Conclusions

*“I swear to tell the truth, all the truth and nothing but the truth”*



# Present and Future of Hard-Disks

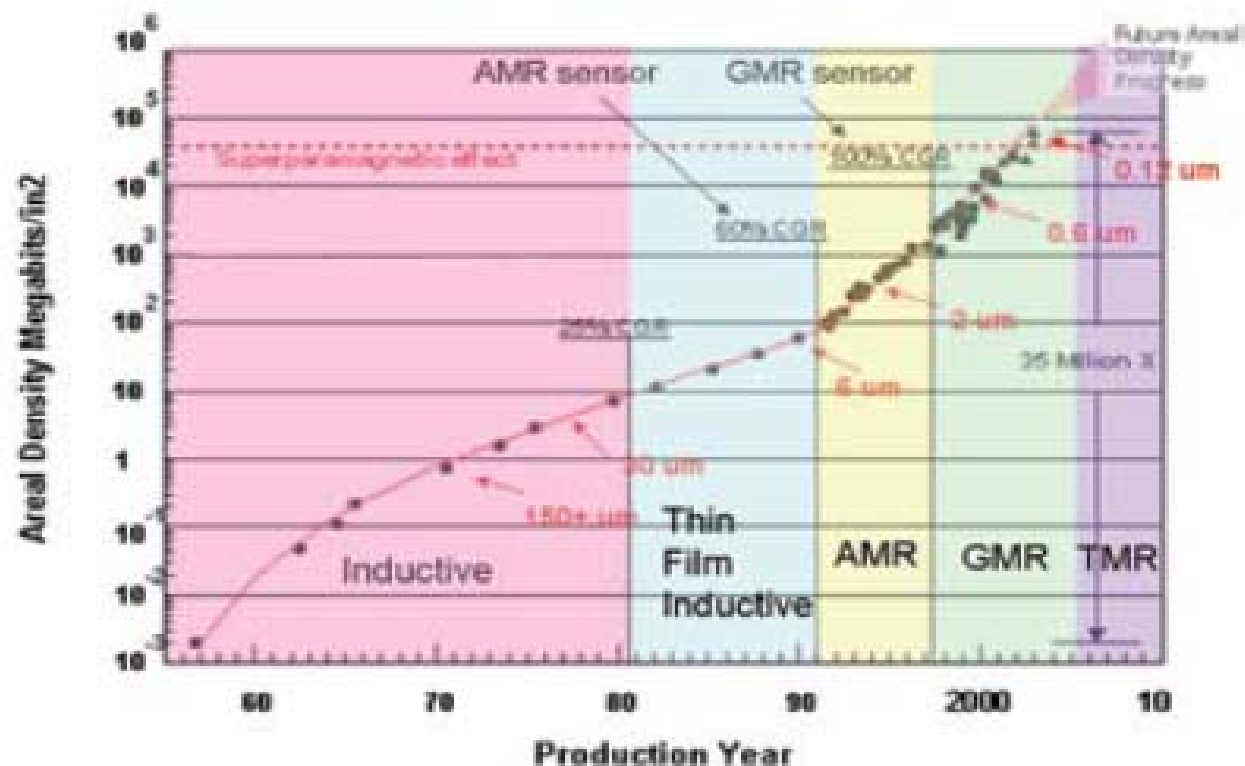


IBM has demonstrated a GMR head with an areal density capability greater than 35.3 billion bits per square inch, and laboratory demonstrations beyond 50 billion bits per square inch have been reported, indicating that future disk drives could exhibit capacities at least two times higher than today.



Disk drives will continue to be enhanced through the use of MEMS micro-actuators, fluid bearing spindle motors and even split or multiple actuators. Also, new data storage techniques, as holographic storage are on horizon.



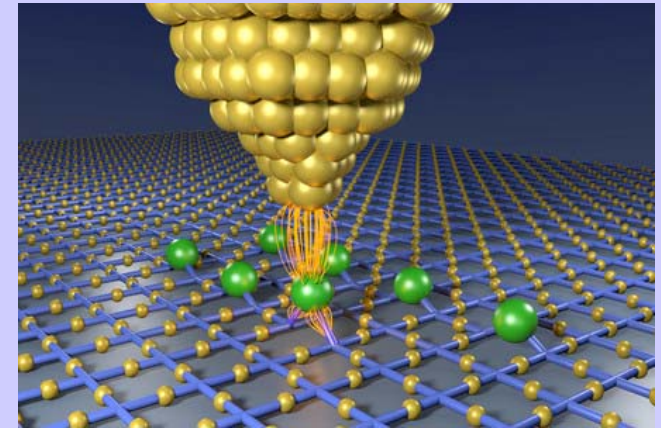


*Fig. 7: Magnetic Recording Areal Density vs. year of product introduction, showing the evolution of sensor technologies. The introduction of the GMR spin valve in 1997 is the first commercially successful use of spintronics.*



12 atoms of Fe – artificial antiferromagnet

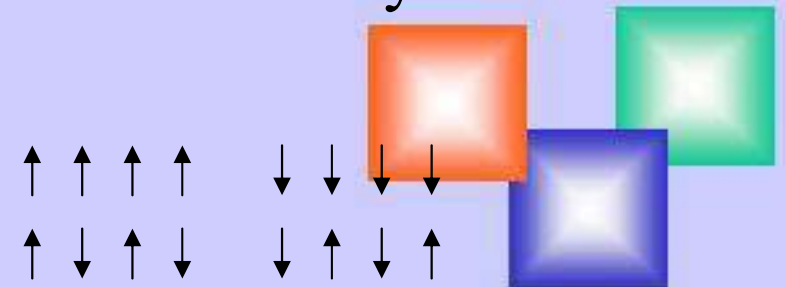
The smallest magnetic memory cell



Tunnel microscope manipulation

1 byte = 8 bit = 96 atoms

Nowaday -  $10^8$  atoms



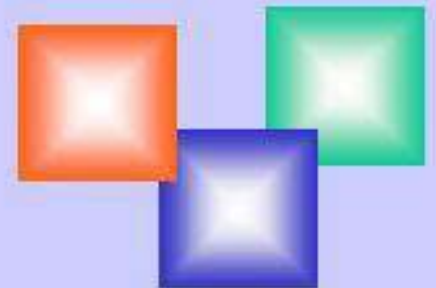
Electronics, Micro- and Nanoelectronics → Charge of Electron

**SPINTRONICS** = SPIN + TRANSPORT + ELECTRONICS  
(1992) →  
Spintronics → Charge + Spin of Electron

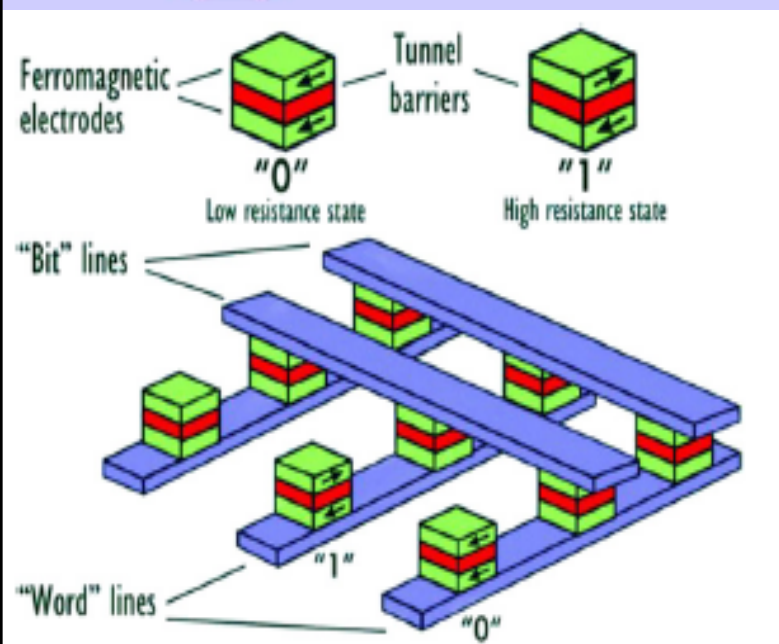
Spin control and manipulation

Spin current without dissipation!!!!?

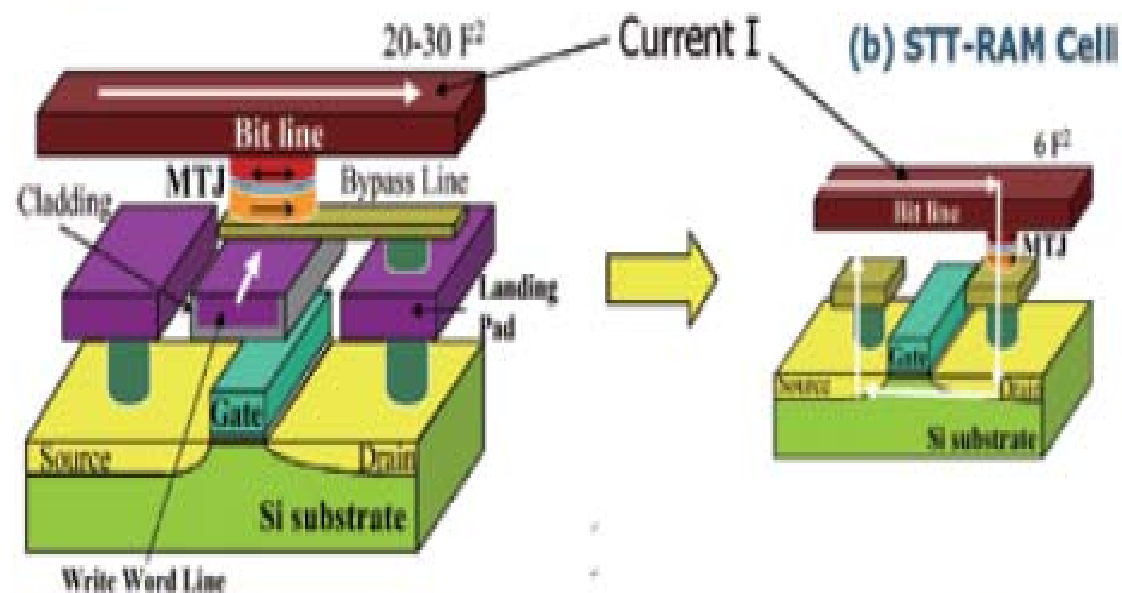
Quantum Computers







(a) Conventional MRAM Cell



Write Current:  $I_{sw} \sim 1 / \text{Volume}$

$I_{sw} \sim \text{Volume}$

Fig. 1: Comparison of memory cell architecture between conventional field switching MRAM (a) and spin-transfer torque MRAM (STT-MRAM) (b).

## Future: From charge current to pure spin current

$$J_e = J_{\uparrow} + J_{\downarrow} \quad (1)$$

$$J_s = J_{\uparrow} - J_{\downarrow} \quad (2)$$

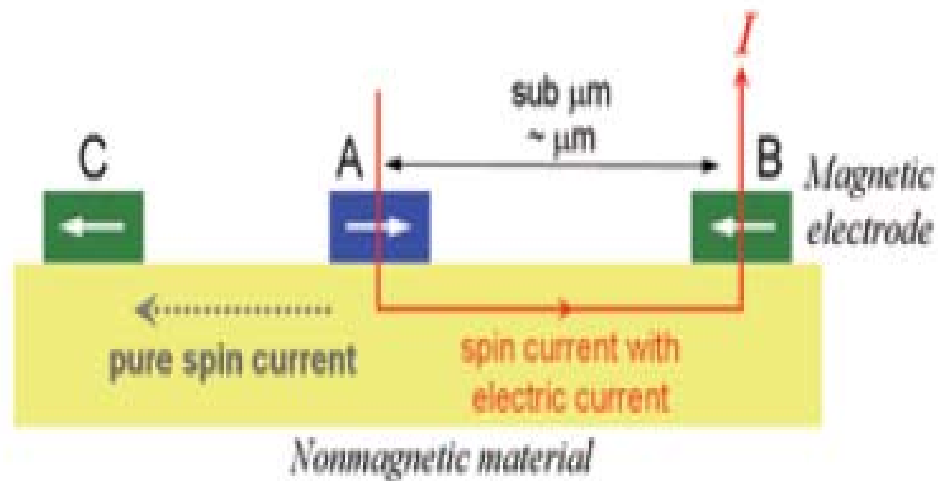


Fig. 1: A basic device structure for the study of spin current.

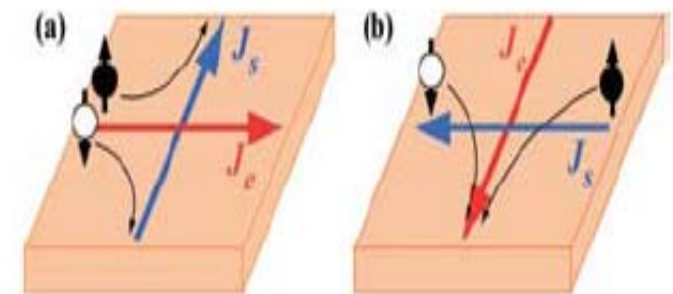
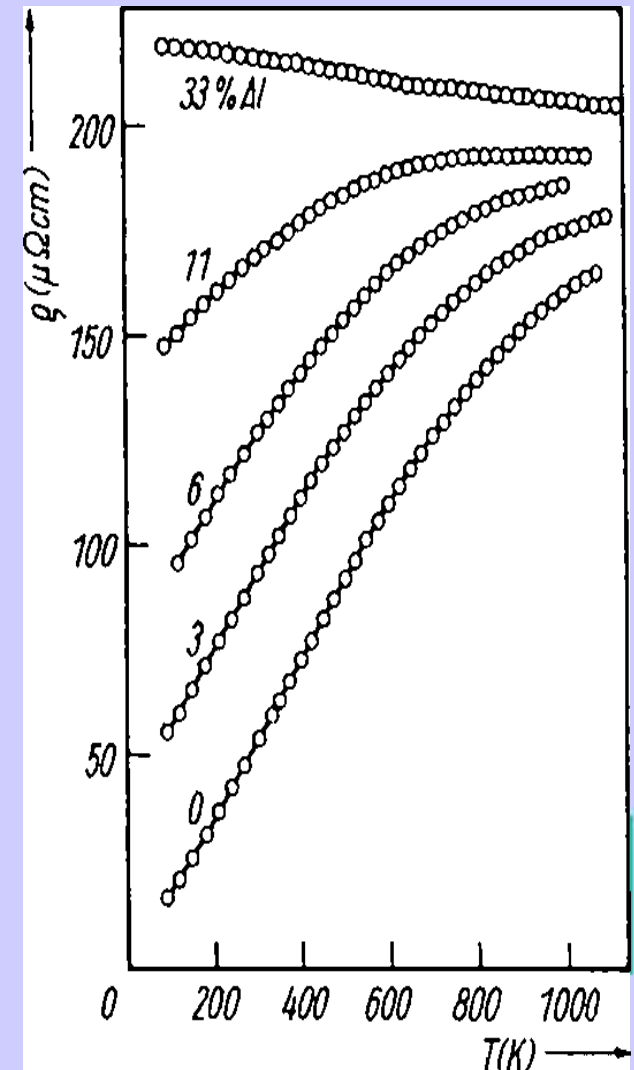
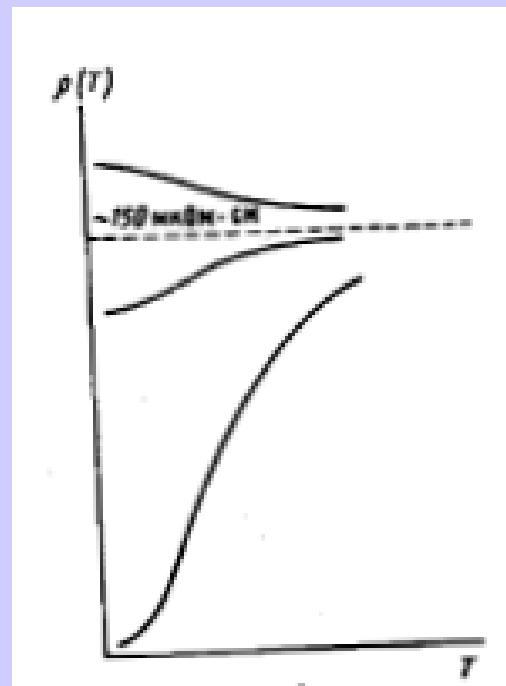
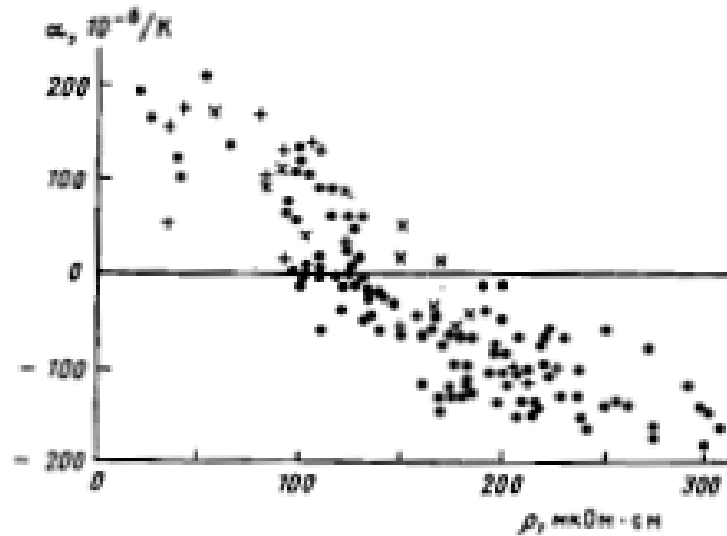


Fig. 6: Schematic illustration of (a) direct and (b) inverse spin-Hall effects in a nonmagnetic material.  $J_e$  and  $J_s$  are the charge and spin currents, respectively.

## Spin Hall Effect

# Mooij correlation in metals

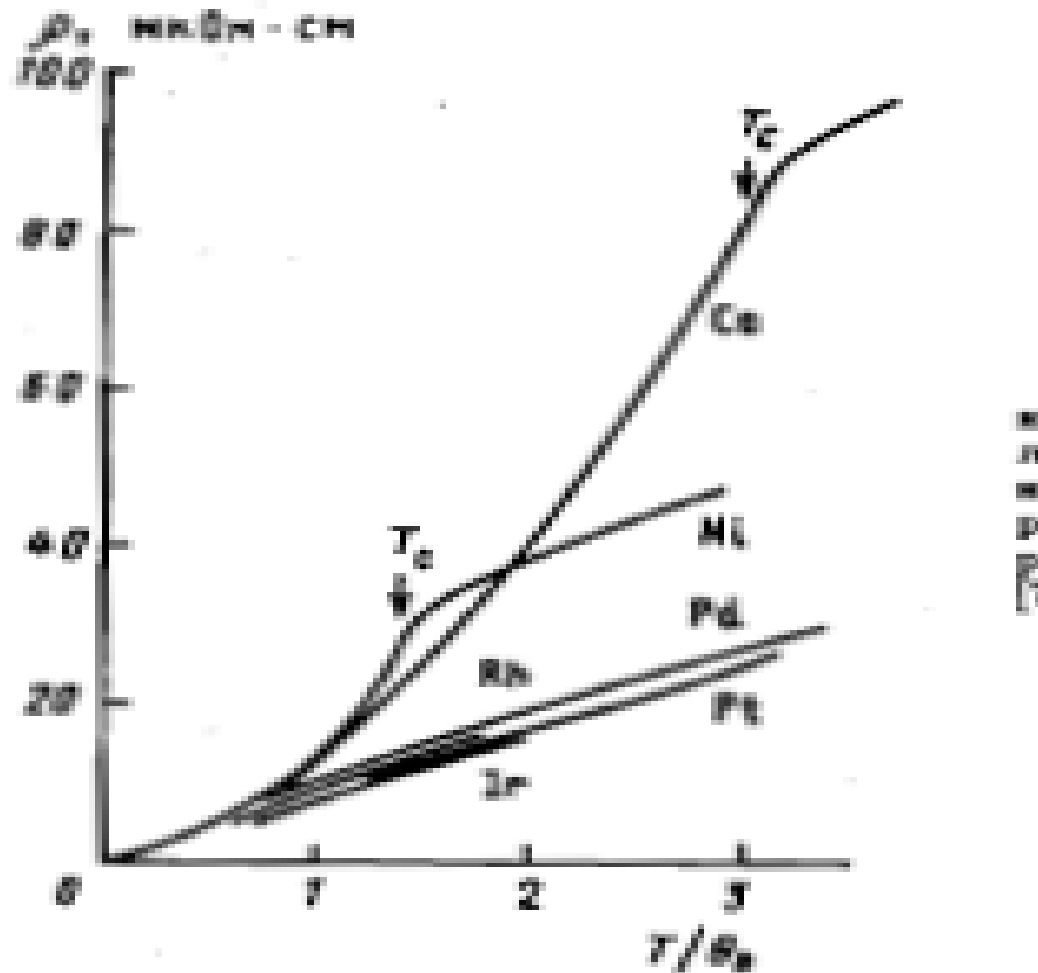


$$\rho = \rho_0 + \rho_1(T), \quad \rho_1(T) \ll \rho_0,$$

$$\frac{\partial \rho_1}{\partial T} > 0 \quad \text{при} \quad \rho_0 < \rho^*,$$

$$\frac{\partial \rho_1}{\partial T} < 0 \quad \text{при} \quad \rho_0 > \rho^*,$$

## Resistivity in ferromagnetic metals



# Resistivity: GaAs:Mn

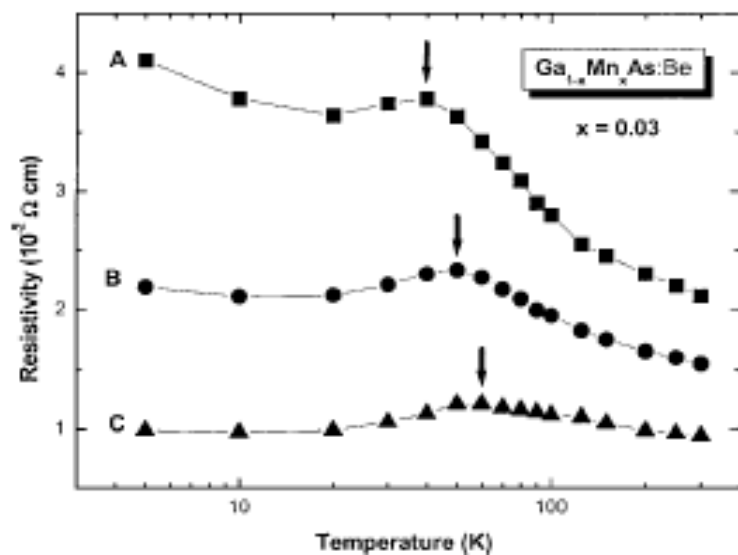


FIG. 1. Temperature dependence of the resistivity for undoped (A) and Be-doped (B and C) samples of  $\text{Ga}_{1-x}\text{Mn}_x\text{As}$  ( $x=0.03$ ) at zero magnetic field.

$$\rho = \frac{m^*}{e^2 p \langle \tau_k \rangle} = \frac{e^2 m^{*2} N_I}{8 \pi k_F^3 \hbar^3 p \varepsilon_0^2 (1 - \Gamma)^2} \left[ \ln(1 + \alpha) - \frac{\alpha}{1 + \alpha} \right],$$

$$\alpha = 4 k_{\text{Fe}}^2 v_s^2,$$

$$\Gamma \approx \frac{J_{pd}^4 S^2 (S+1)^2 m^* N^2 k_F p}{4 \pi^2 E_F^2 \hbar^2 k_B T} \left[ 1 - \frac{3 k_F a}{4 \pi} \right],$$

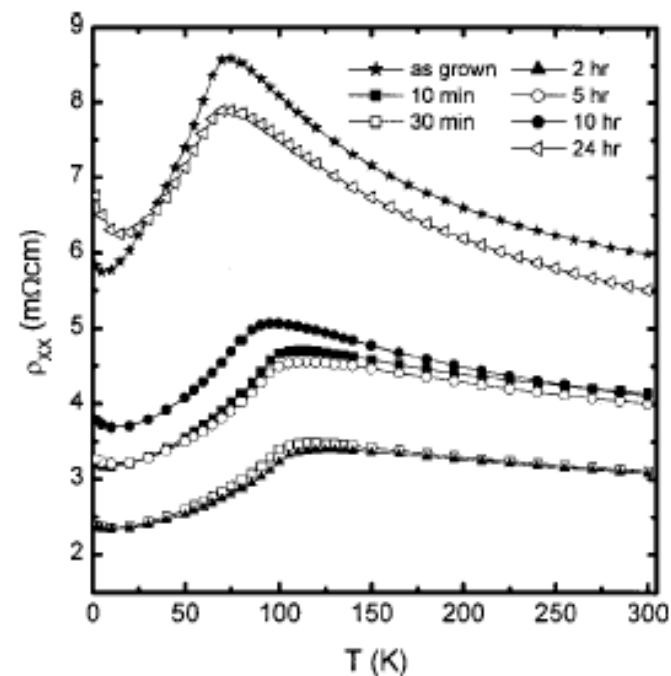


FIG. 38 Experimental resistivity of  $\text{Ga}_{1-x}\text{Mn}_x\text{As}$  for  $x = 8\%$  vs. temperature for various annealing times. From (Potashnik *et al.*, 2001).

See, reviews of T. Jungwirth



## Resistivity in $\text{TiO}_2\text{:Co}$

$$\sigma = \sigma_0 \exp(-E_a/kT)$$

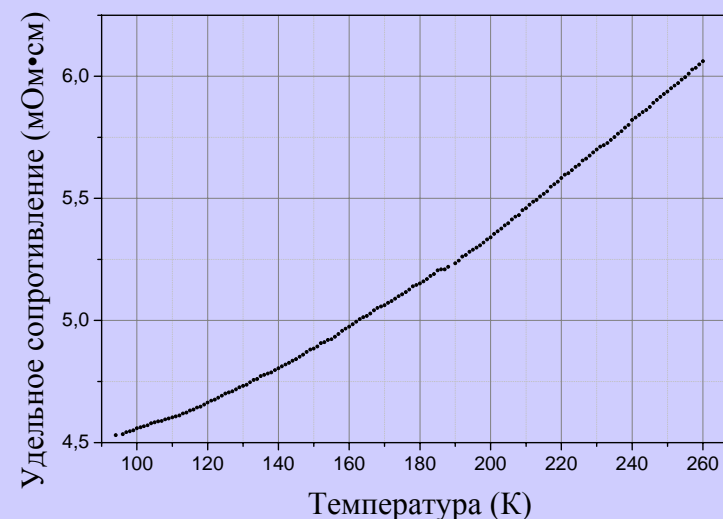
$$\sigma = \sigma_0 \exp(-T_0/T)^s$$

$s=1/4$  Mott (VRH)

$s=1/2$  Efros-Shklovski (VRH + Gap)

$s=1$  Activation

SPH = small polaron hopping ( $T > \Theta_D/4$ )



Yu. Mikhailovski

$$\sigma = \sigma_0 T^{-\alpha} \exp(-W/kT)$$

A.Yildis et al. JAP 2010

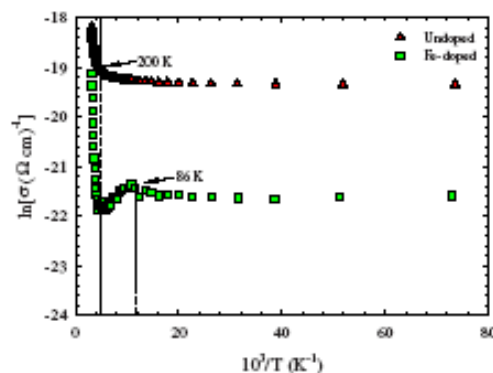
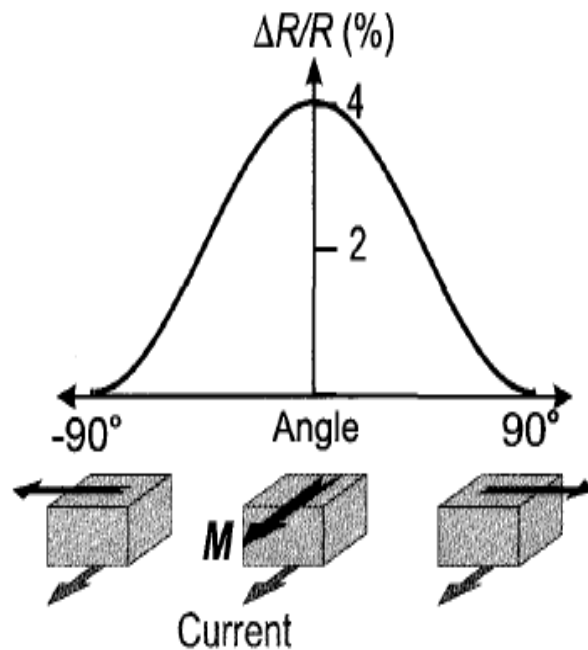


FIG. 2. (Color online) Temperature dependence of the conductivity plotted as  $\ln(\sigma)$  vs  $10^3/T$  in the temperature range 13–320 K.

## Anisotropic magnetoresistance



Permalloy: AMR=4-5% at 300 K,  $H=5$  Oe

$$\frac{MR, \%}{H} = 1\% / \text{Oe}$$

$$\rho_{ij} = \begin{vmatrix} \rho_{\perp}(B) & -\rho_H(B) & 0 \\ \rho_H(B) & \rho_{\perp}(B) & 0 \\ 0 & 0 & \rho_{\parallel}(B) \end{vmatrix}$$

$$\frac{\Delta \rho}{\rho} = \frac{\rho_{\parallel} - \rho_{\perp}}{\frac{1}{3}\rho_{\parallel} + \frac{2}{3}\rho_{\perp}}$$

$$\vec{E} = \rho_{\perp}(B)\vec{j} + [\rho_{\parallel}(B) - \rho_{\perp}(B)](\vec{\alpha} \cdot \vec{j}) \cdot \vec{\alpha} + \rho_H(B)[\vec{\alpha} \times \vec{j}],$$

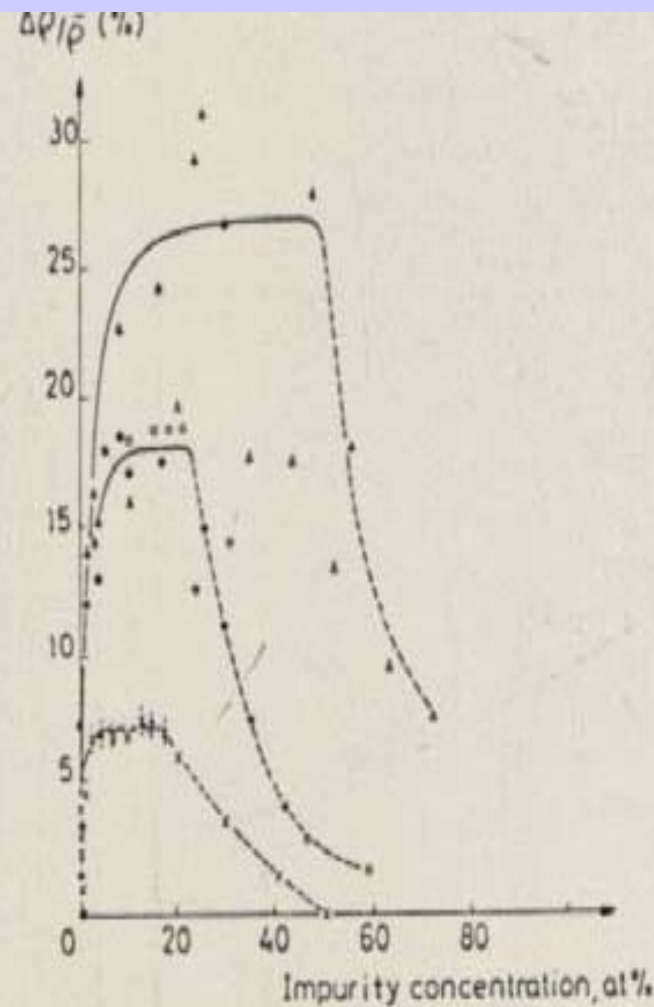
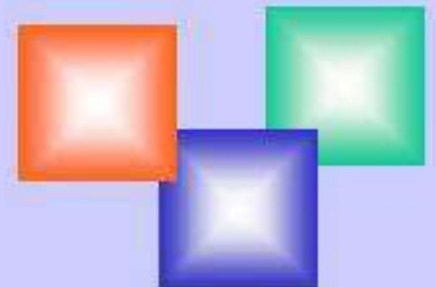


Fig. 13. Concentration dependence of the resistivity anisotropy at 4.2 K for several nickel based alloys.  
 $\triangle$ :  $\underline{\text{NiCo}}$ ,  $\bullet$ :  $\underline{\text{NiFe}}$ ,  $\times$ :  $\underline{\text{NiCu}}$  (after Jaoul et al. 1977).

$$\Delta\rho/\rho=0.01(\alpha-1)$$

$$\alpha=\rho_{\downarrow}/\rho_{\uparrow}$$



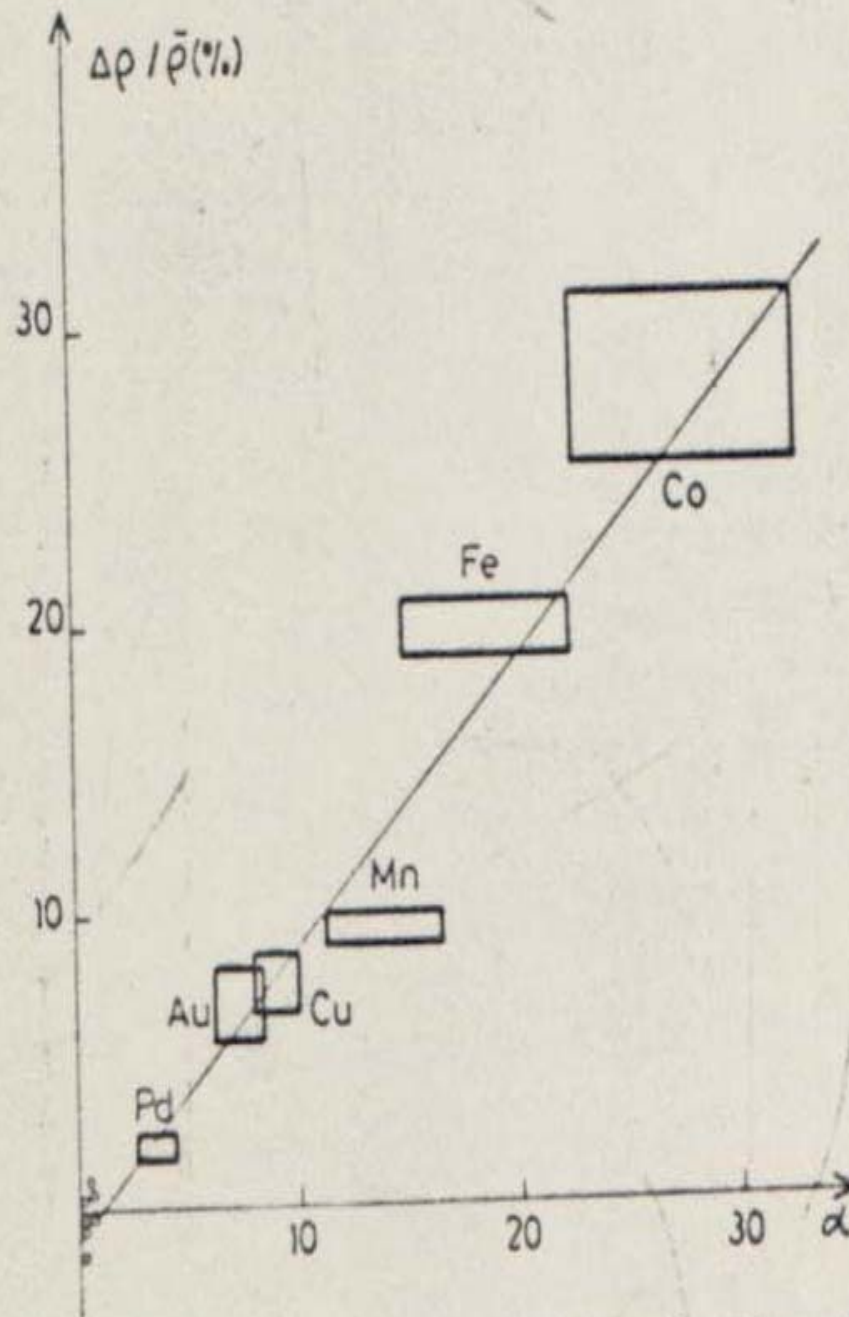


Fig. 14. Resistivity anisotropy of Ni based alloys at 4.2 K as a function of  $\alpha = \rho_{01}/\rho_{02}$ . The straight line is  $\Delta\rho/\bar{\rho} = 0.01 (\alpha - 1)$  (after Jaoul et al. 1977).

## AMR in GaAs:Mn

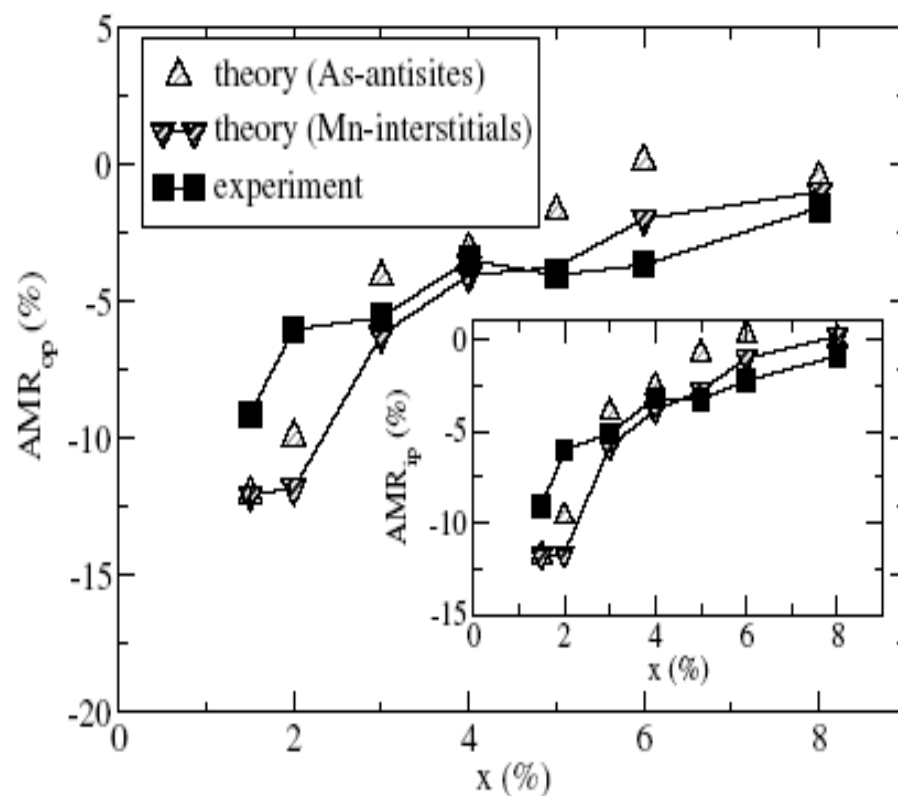


Fig. 3. Experimental (filled symbols) AMR coefficients and theoretical data obtained assuming As-antisite compensation (open symbols) and Mn-interstitial compensation (semi-filled symbols) of  $AMR_{op}$  and  $AMR_{ip}$ . (After Ref. 52.)

No data in TiO<sub>2</sub>:Co

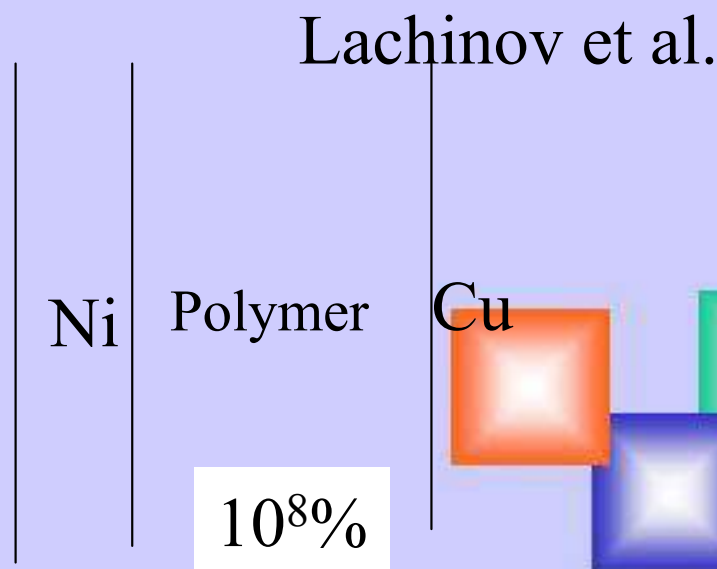
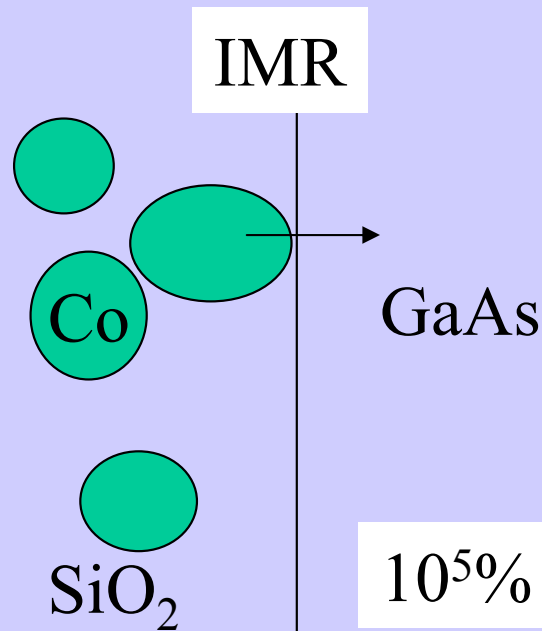


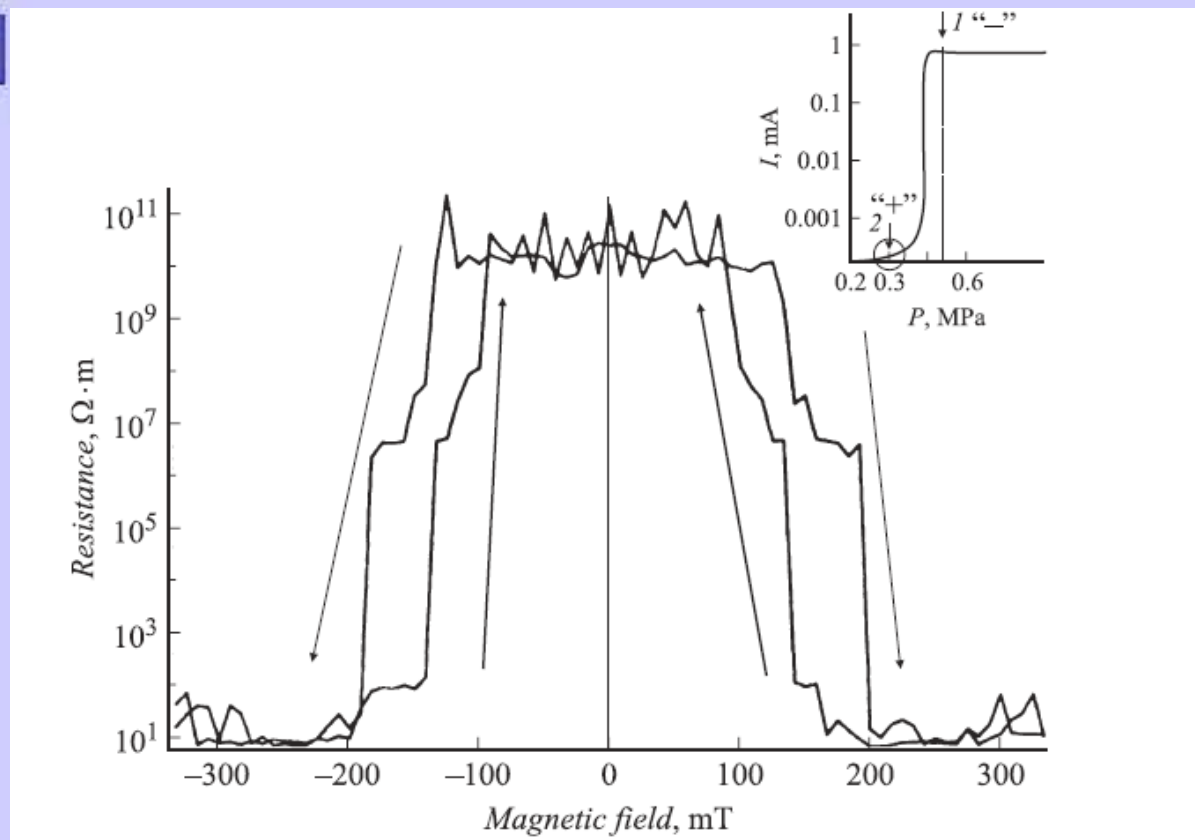
## Novel types of magnetoresistance

### Giant magnetoresistance in semiconductor / granular film heterostructures with cobalt nanoparticles

L.V. Lutsev<sup>1</sup>, A.I. Stognij<sup>2</sup>, and N.N. Novitskii<sup>2</sup>

In this paper, we study the magnetoresistance in  $\text{SiO}_2(\text{Co})/\text{GaAs}$  and  $\text{SiO}_2(\text{Co})/\text{Si}$  heterostructures, where the  $\text{SiO}_2(\text{Co})$  is the granular  $\text{SiO}_2$  film with Co nanoparticles. Sample preparation and experimental results are presented in section 2. The effect is more expressed, when electrons are injected from the granular film into the SC, therefore, the magnetoresistance has been called the injection magnetoresistance (IMR) [38, 39]. For  $\text{SiO}_2(\text{Co})/\text{GaAs}$  heterostructures the IMR value is of the magnetoresistance effect in perature, which is two-three orders higher than maximum value by magnetic-field-controlled permalloy multilayers and the TMR in MTJ structures. On the contrary, for  $\text{SiO}_2(\text{Co})/\text{Si}$  heterostructures the magnetoresistance





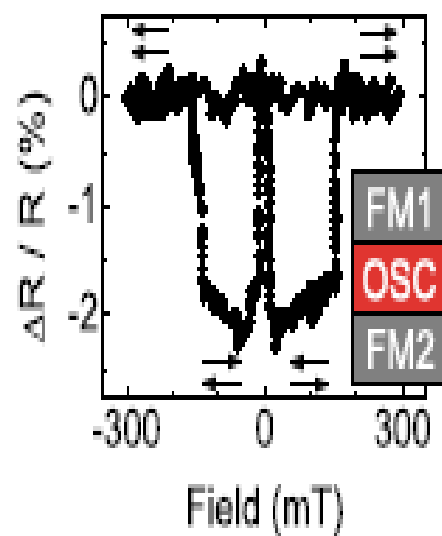
Ni-polymer-Cu

800 nm

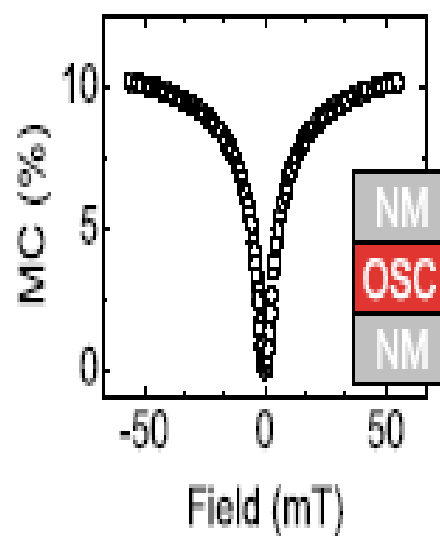
Lachinov 2009

# ORGANIC MAGNETORESISTANCE

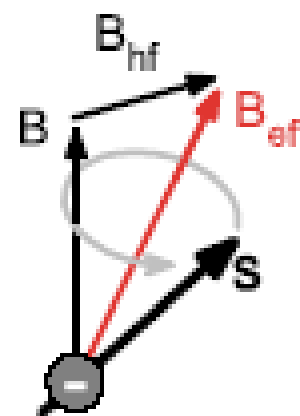
Francis 2004



(a)



(b)



W. Wagemans et al. SPIN, vol.1. no.1 p.94 ( 2011)

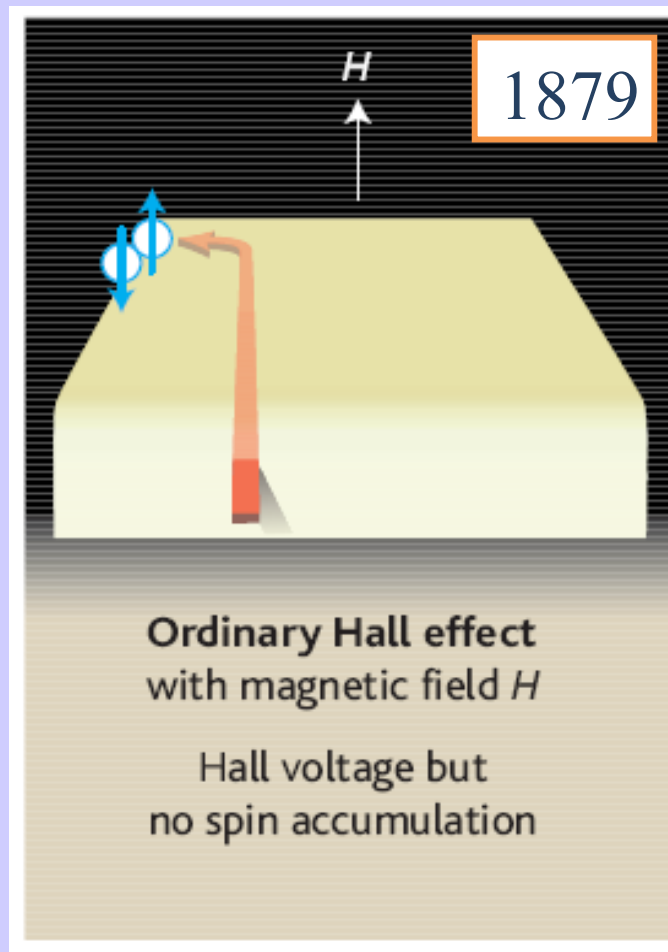
## Ordinary Hall effect

$$\rho_{xy} = R_0 B$$

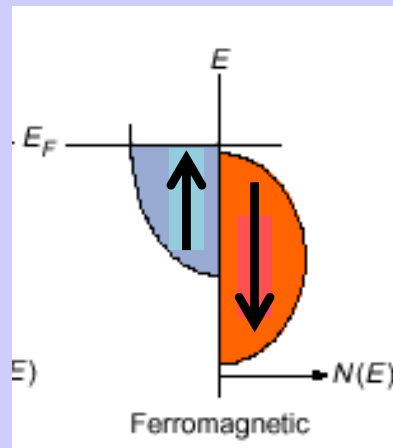
$$\rho_{ah} = 4\pi M_z$$

## Anomalous Hall effect

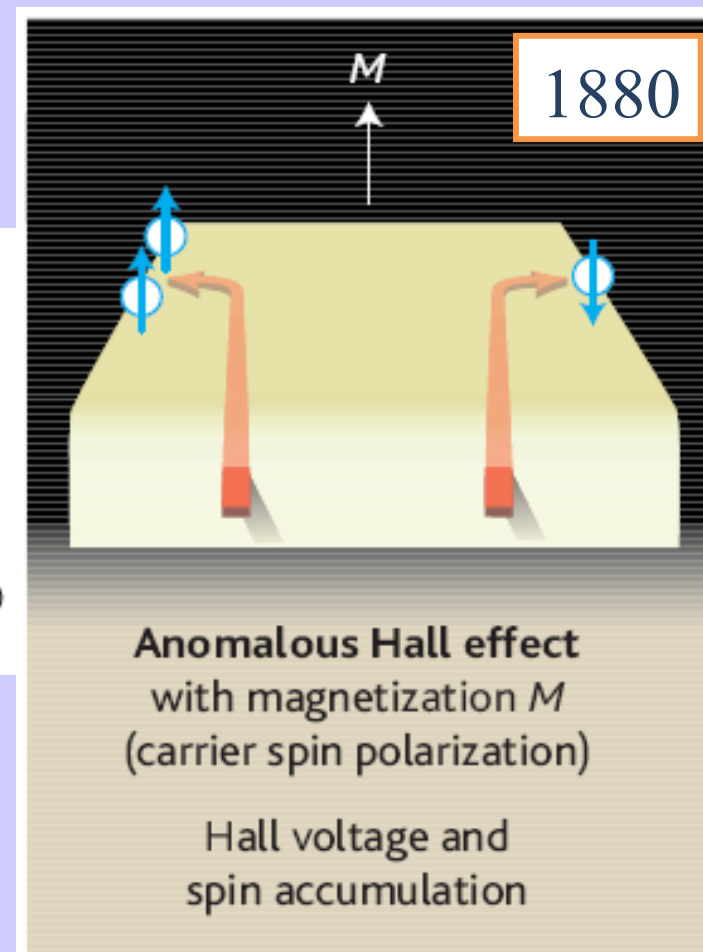
$$\rho_{xy} = R_0 B + \rho_{ah}$$



Lorentz Force

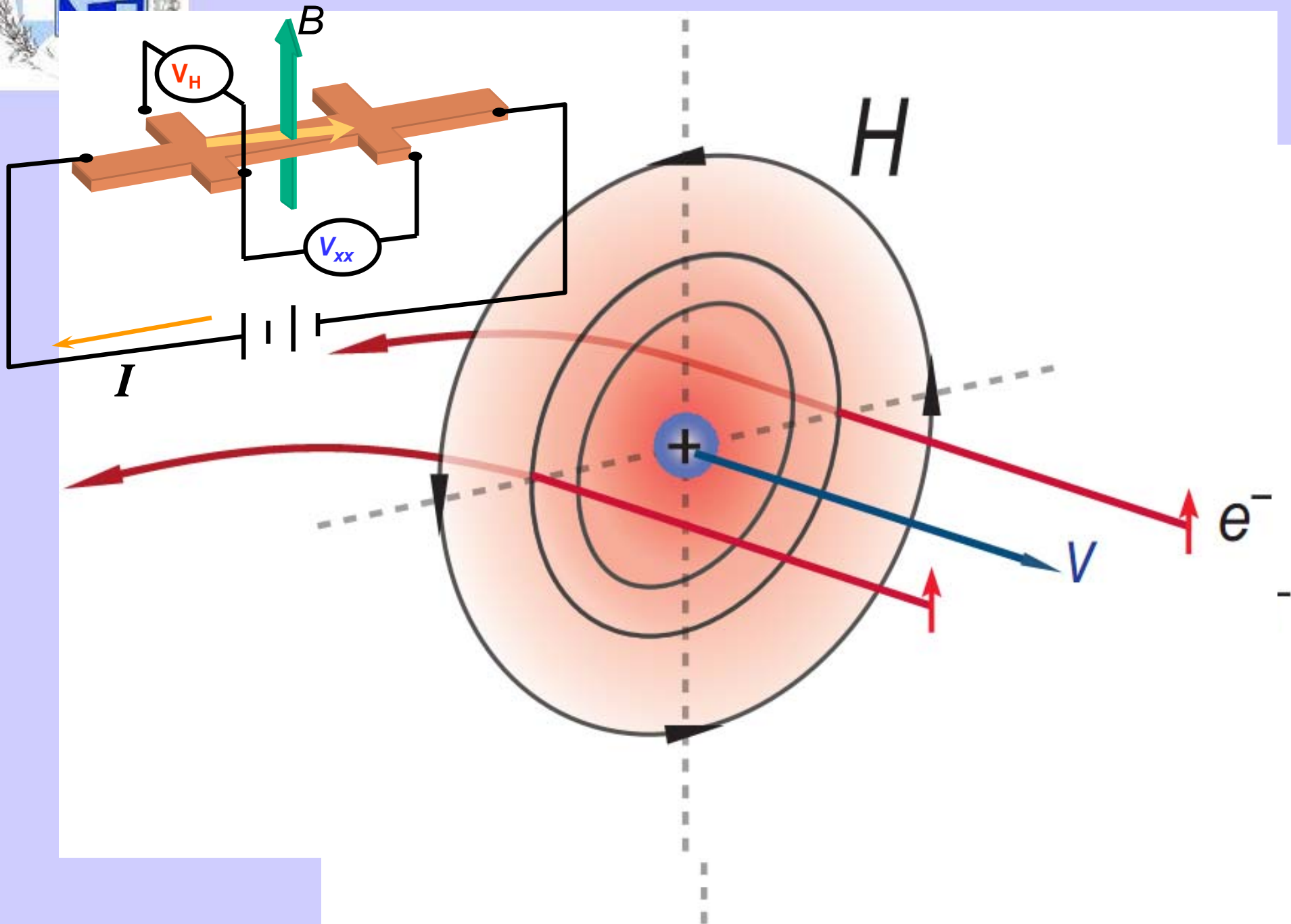


$$B = H + 4\pi M(1 - N)$$



Spin-Orbit Coupling

# Spin-orbit coupling





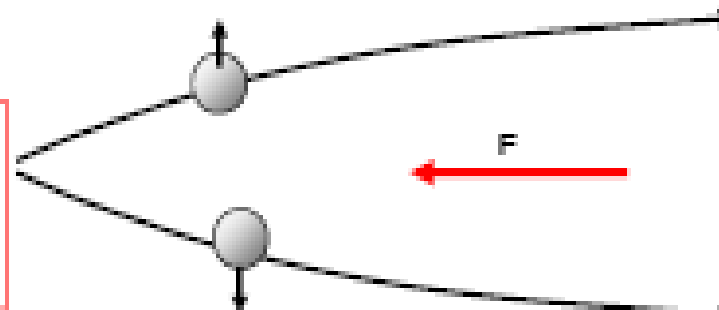
$$\rho_H = R_0 B_z + 4\pi R_s M_z$$

### a) Intrinsic deflection

Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

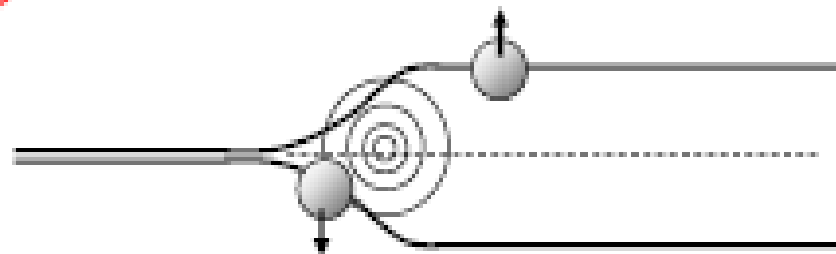
$$\frac{d\langle \vec{r} \rangle}{dt} = \frac{\partial E}{\hbar \partial \vec{k}} + \frac{e}{\hbar} \vec{E} \times \vec{b}_n$$

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature



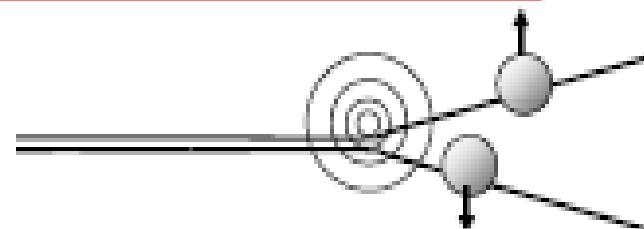
### b) Side jump

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.



### c) Skew scattering

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.



# (1) Karplus-Luttinger Intrinsic (1954)



$$\frac{d\vec{r}(t)}{dt} = \frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}}$$

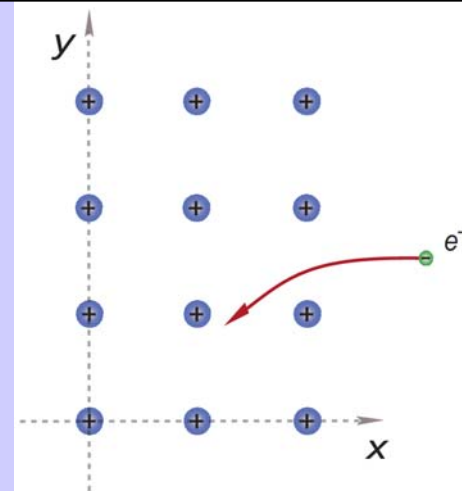
Anomalous velocity

$$-\vec{\Omega}_n(\vec{k}) \times \frac{d\vec{k}(t)}{dt}$$

k- space curvature

$$\frac{d\vec{k}(t)}{dt} = \frac{\partial V(\vec{r})}{\partial \vec{r}} - \vec{B}(\vec{r}) \times \frac{d\vec{r}(t)}{dt}$$

r- space curvature



G. Sundaram and Q. Niu,  
Phys. Rev. B 59 (1999) 14915.

Jungwirth, Niu, MacDonald (2002), Onoda & Nagaosa (2002)

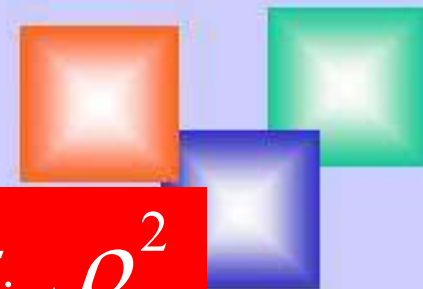
$$\sigma_{xy} = -\frac{e^2}{\hbar} \int d^3\mathbf{k} \sum_n f(\varepsilon_n(\mathbf{k})) \Omega_n^z(\mathbf{k})$$

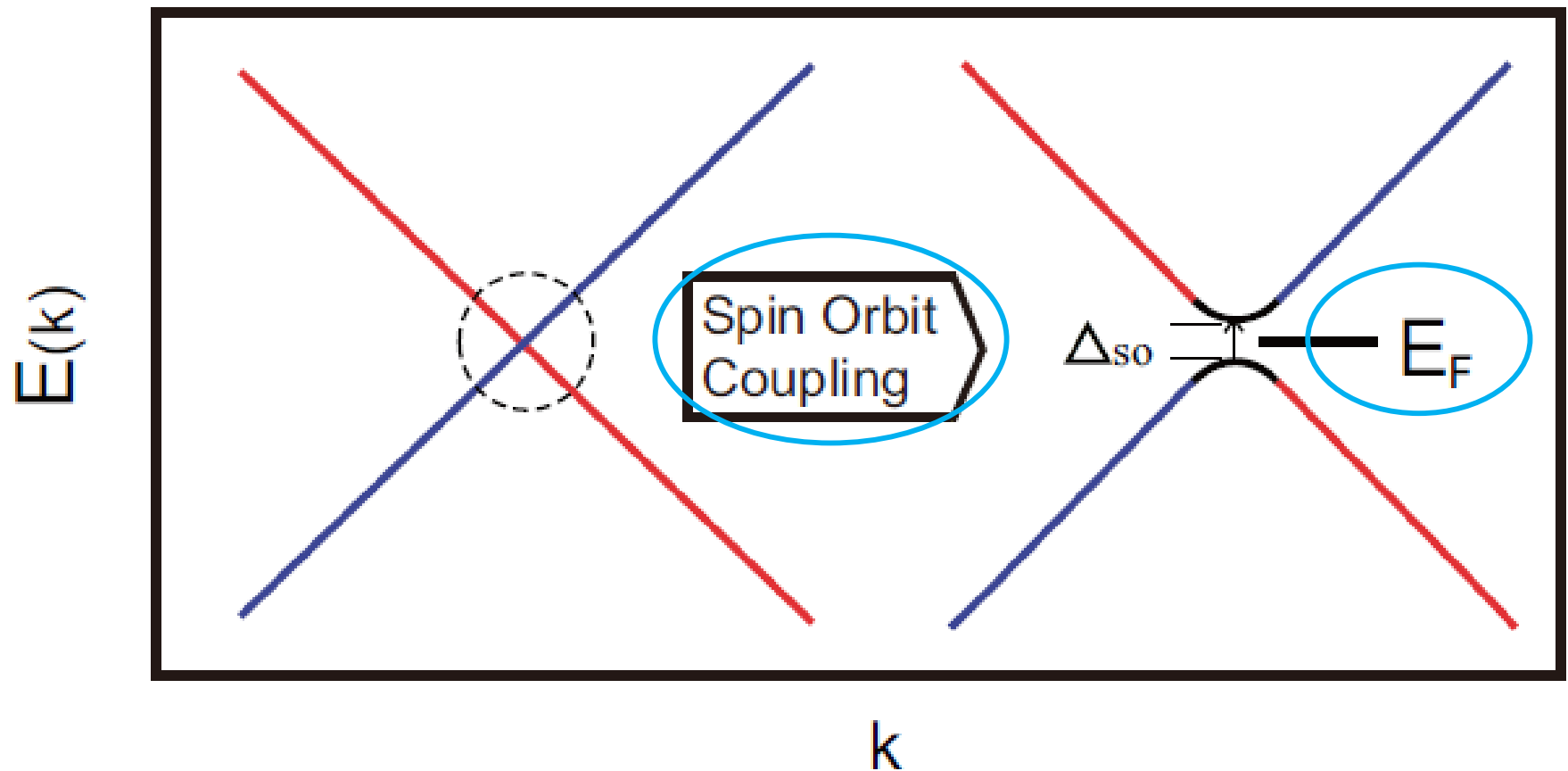
Berry curvature

$$\Omega_n^z(\mathbf{k}) = -\sum_{n' \neq n} \frac{2 \operatorname{Im} \langle \mathbf{k}n | v_x | \mathbf{k}n' \rangle \langle \mathbf{k}n' | v_y | \mathbf{k}n \rangle}{(\omega_{\mathbf{k}n'} - \omega_{\mathbf{k}n})^2}$$

$$\sigma_{\text{int}} = \text{constant}$$

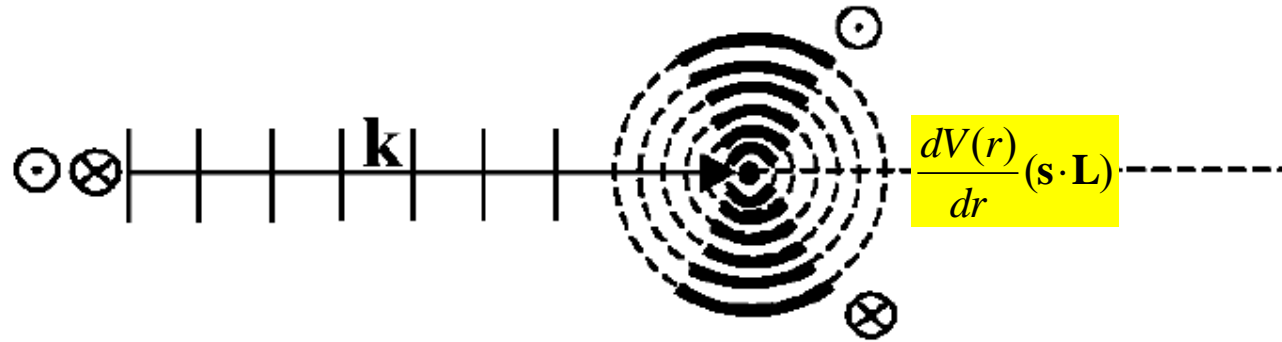
$$\rho_{\text{int}} = \sigma_{\text{int}} \rho_{xx}^2$$





$$\Omega_n^z(\mathbf{k}) = - \sum_{n' \neq n} \frac{2 \operatorname{Im} \langle \mathbf{k}n | v_x | \mathbf{k}n' \rangle \langle \mathbf{k}n' | v_y | \mathbf{k}n \rangle}{(\omega_{\mathbf{k}n'} - \omega_{\mathbf{k}n})^2}$$

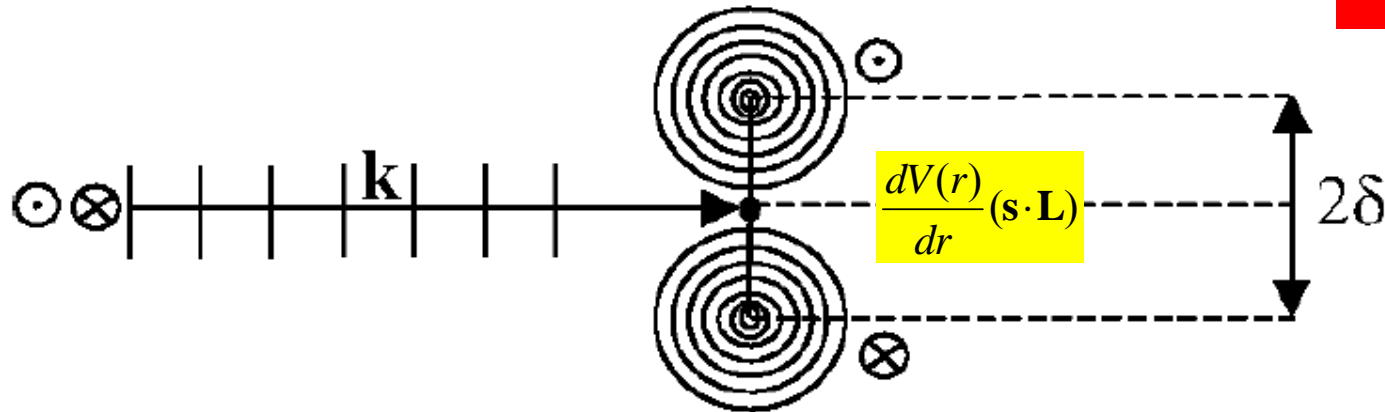
## (2) Skew-scattering (Smit, 1955)



$$\rho_{ah} = \alpha \rho_{xx}$$

$$c \ll 1 \\ (T=0)$$

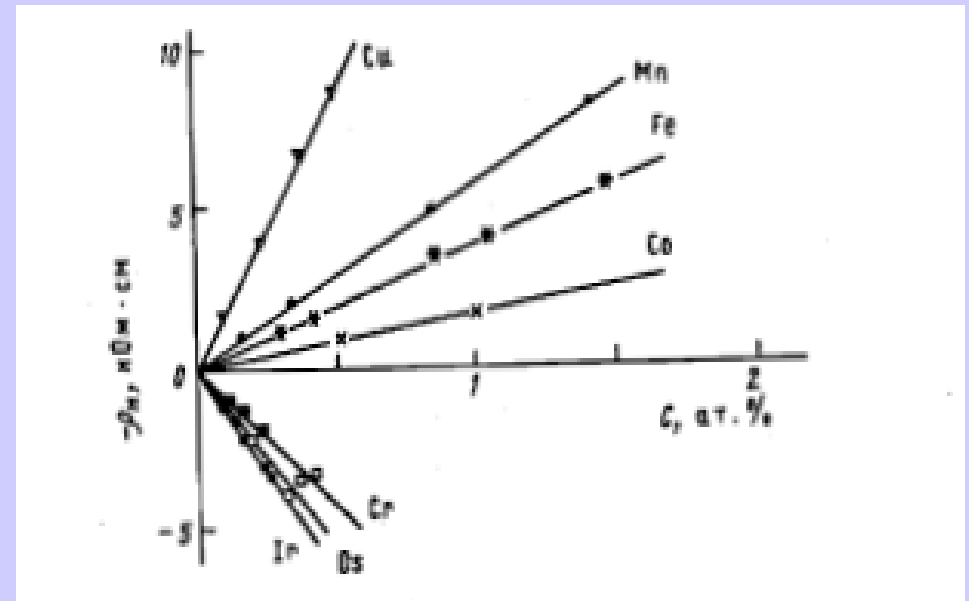
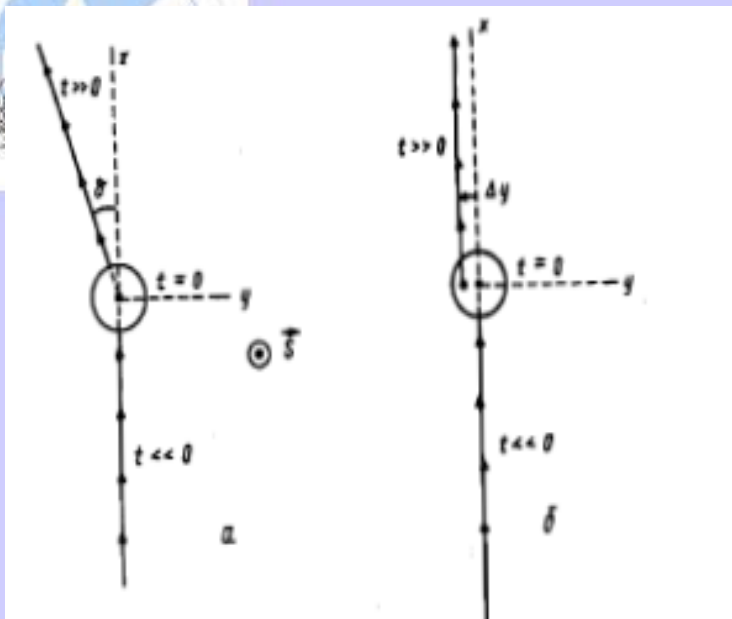
## (3) Side-jump (Berger, 1970)



$$\rho_{ah} = \beta \rho_{xx}^2$$

$$\rho_{ah} = a \rho_{xx} + b \rho_{xx}^2$$

Intrinsic or Extrinsic ?!



Size-effects

Weak localization

Electron-electron interaction

Short-range order

No doubt that at  $T=0$  and low impurity concentration AHE is due to skew scattering

$$(R_s)^{sc} = a\rho_0 + b\rho_0^2$$

Only  $T=0$  and low impurity concentration



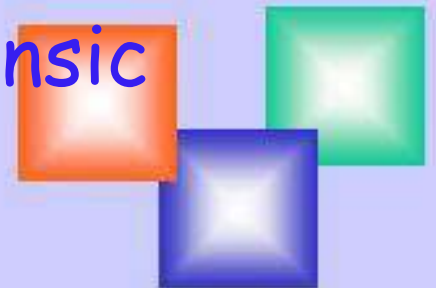


## Intrinsic and Extrinsic ?

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

$$\rho_{ah} = \underbrace{\alpha\rho_{xx0}}_{\text{Extrinsic}} + \underbrace{\beta\rho_{xx0}^2 + b\rho_{xx}^2}_{\text{Intrinsic}}$$

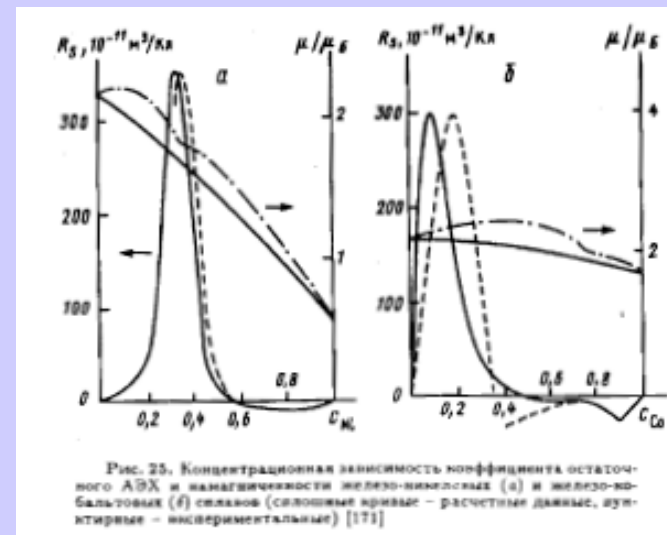
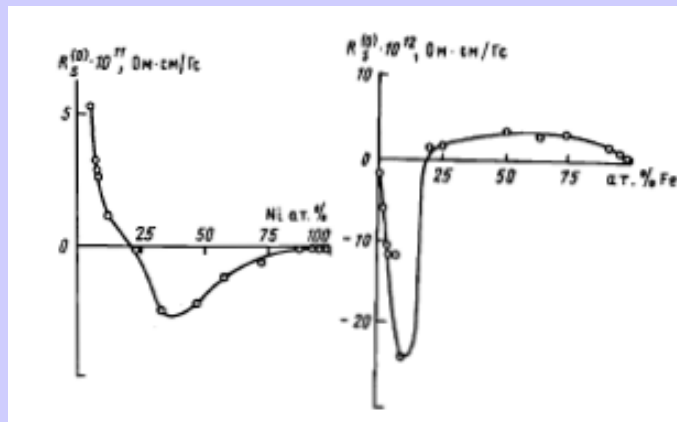
Jin et al. 2012 –ICM2012





$$R_s = c(1-c)(1-2c+f)$$

Skew scattering for disordered alloys at  $T=0$



$$(R_s)^{sc} = a\rho_0 + b\rho_0^2$$

$$(R_s)^{KL} = A\rho^2$$

$$(R_s)^{sj} = B\rho^2$$

$$(R_s) = D\rho^{0.4-0.2}$$

$$(\sigma_{xy}) \propto \sigma_{xx}^{1.6-1.8}$$

$$(\sigma_{xy}) \propto \sigma_{xx}^{1.5}$$

AEH in hopping

A.Vedyaev and A.Granovsky, Phys. Solid State **28**, 2310 (1986).

size [6], we derive the parametric relationship between the Hall  $\rho_h$  and the longitudinal  $\rho_{xx}$  resistivities (the parameter is temperature):  $\rho_h \propto \rho_{xx}^m$ ;  $m \approx 0.6$ . This relationship fits the experimental results well.

$< 1 \mu\Omega\text{cm}$

$< 100 \mu\Omega\text{cm}$

$> 100 \mu\Omega\text{cm}$

$\rho$

JETP Lett. 2000

# Extraordinary Hall effect (EHE) of ferromagnetic composites in the effective medium approximation

A.B. Granovsky <sup>a,\*</sup>, A.V. Vedyayev <sup>a</sup>, F. Brouers <sup>b</sup>

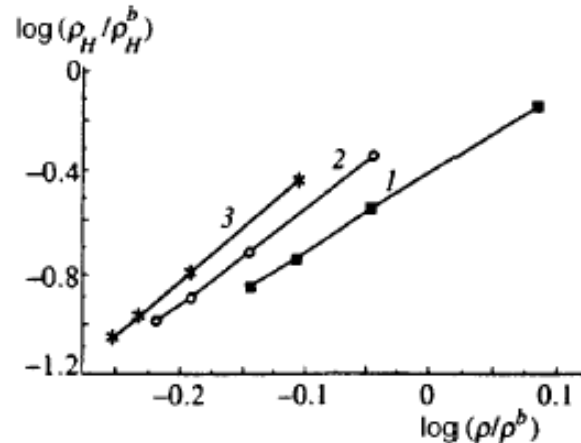


FIG. 2. Correlation between the Hall resistivity  $\rho_H/\rho_H^b$  and total resistivity  $\rho/\rho^b$  of a granular alloy, having the form  $(\rho_H/\rho_H^b) \sim (\rho/\rho^b)^n$ ;  $c=0.2$ ,  $p_b=0.2$ ,  $p_s=0.52$ ,  $l_{m(s)}=120$  Å,  $l_{m(d)}=20$  Å,  $l_{nm}=200$  Å,  $\rho_H^s/\rho_H^b=1$ ,  $r_0=20$ – $80$  Å. The power  $n$  depends on the nature of scattering by the surfaces of the grains:  $l_s/a_0=2$ ,  $n=3.1$  (curve 1);  $l_s/a_0=4$ ,  $n=3.8$  (curve 2);  $l_s/a_0=6$ ,  $n=4.3$  (curve 3).

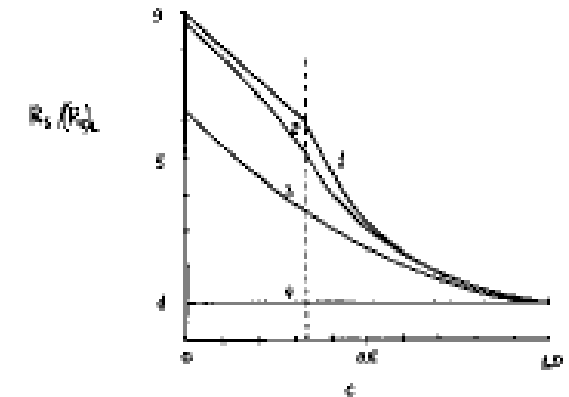
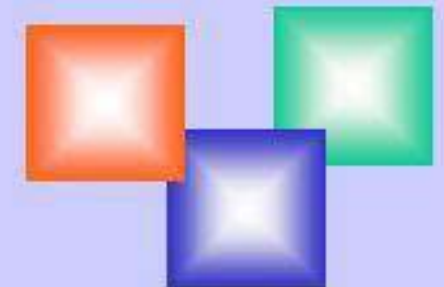


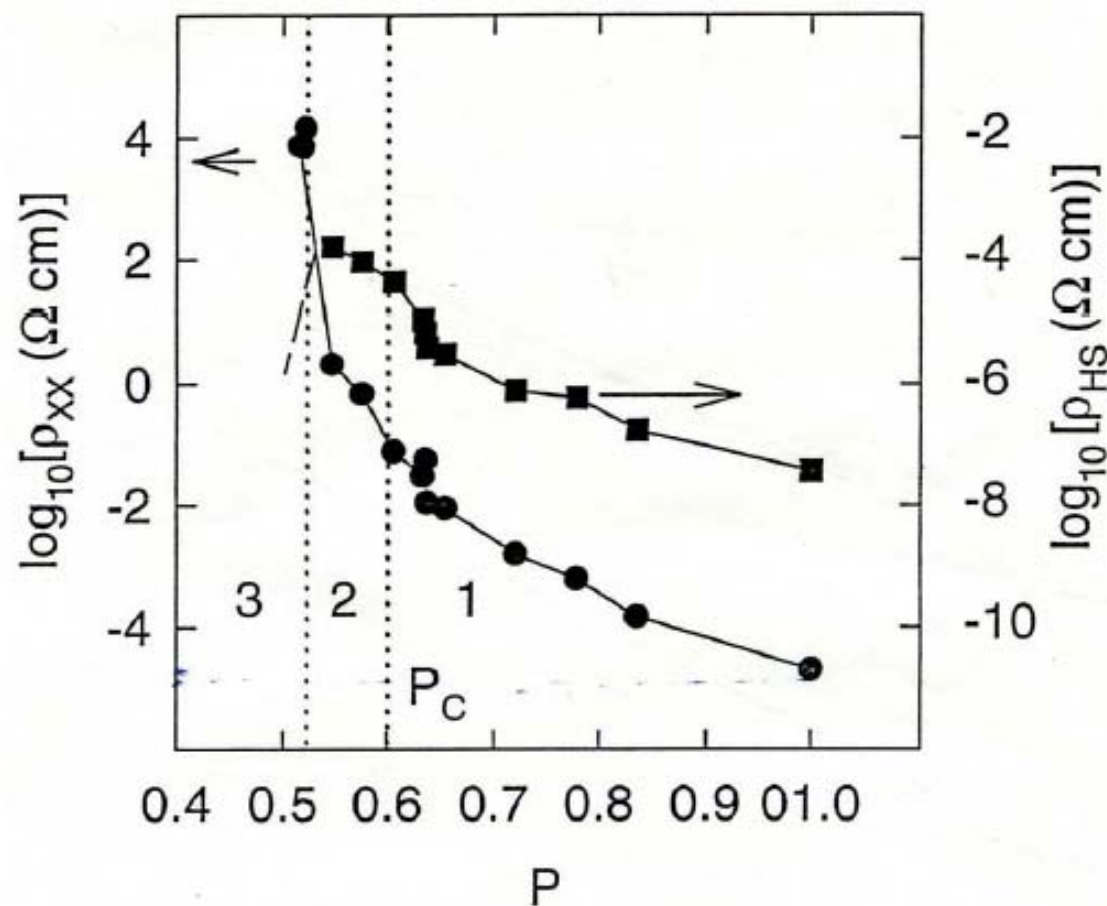
Fig. 1. Reduced EHE coefficient  $R_s/(R_s)_1$  for composites versus the concentration  $x$  of the ferromagnetic component ( $(R_s)_2=0$ ,  $M=cM_1$ ; 1,  $x=\sigma_2/\sigma_1=0$ ; 2,  $x=10^{-2}$ ; 3,  $x=10^{-1}$ ; 4,  $x=1$ ).

$$\rho_H = \frac{x}{x + y \left( \frac{\rho_2}{\rho_1} \right)^2 \left( \frac{\rho + 2\rho_1}{\rho + 2\rho_2} \right)^2} \left[ R_{01} B_1 + 4\pi R_{s1} M_1 \right] \left( \frac{\rho}{\rho_1} \right)^2 +$$

$$+ \frac{y}{y + x \left( \frac{\rho_1}{\rho_2} \right)^2 \left( \frac{\rho + 2\rho_2}{\rho + 2\rho_1} \right)^2} \left[ R_{02} B_2 + 4\pi R_{s2} M_2 \right] \left( \frac{\rho}{\rho_2} \right)^2$$



## Giant Hall effect in NiFe-SiO<sub>2</sub>



A. Pakhomov 1994

## Conclusion 1-bad news

If magnetization in the sample is due to ferromagnetic or superparamagnetic clusters the sample exhibits AHE and AHE resistivity is linear to magnetization.

Observation of AHE can not be considered as a strict evidence of intrinsic ferromagnetism in diluted magnetic semiconductors

### Co-occurrence of Superparamagnetism and Anomalous Hall Effect in Highly Reduced Cobalt Doped Rutile $\text{TiO}_{2-x}$ Films

S. R. Shinde<sup>1,\*</sup>, S. B. Ogale<sup>1,2</sup>, J. S. Higgins<sup>1</sup>, H. Zheng<sup>2</sup>, A. J. Millis<sup>3</sup>, V.N. Kulkarni<sup>1,\*</sup>, R. Ramesh<sup>1,2</sup>, R. L. Greene<sup>1</sup>, and T. Venkatesan<sup>1</sup>

<sup>1</sup>Center for Superconductivity Research, Department of Physics, University of Maryland, College Park, MD 20742-4111

<sup>2</sup>Department of Materials and Nuclear Engineering, University of Maryland, College Park, MD 20742-42111

<sup>3</sup>Department of Physics, Columbia University, 538 West 120th Street, New York, New York 10027

We report a detailed magnetic and structural analysis of highly reduced Co doped rutile  $\text{TiO}_{2-x}$  films displaying an anomalous Hall effect (AHE). The temperature and field dependence of magnetization, and transmission electron microscopy clearly establish the presence of nano-sized superparamagnetic cobalt clusters of ~8–10 nm size in the films at the interface. The co-occurrence of superparamagnetism and AHE raises questions regarding the use of the AHE as a test of the intrinsic nature of ferromagnetism in diluted magnetic semiconductors.

There is no correlation between AHE and resistivity in heterogeneous alloys

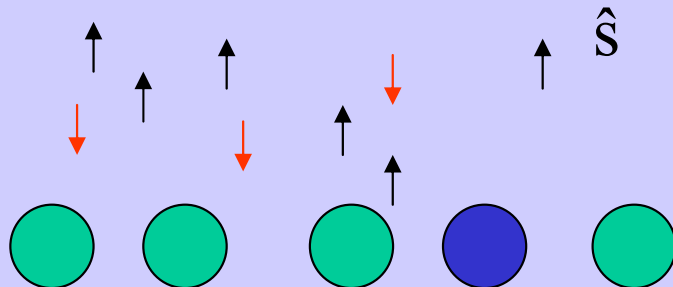




## Two basic models and 4 types of SOI

### Itinerant Magnetism

$$H_{so}^U = \frac{\hbar}{2m^2c^2} [\vec{\nabla} U(\vec{r}) \times \vec{p}] \cdot \vec{s}$$

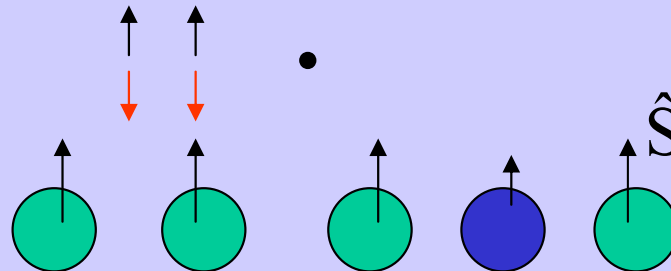


$$H_{so}^V = \frac{\hbar}{2m^2c^2} [\vec{\nabla} V(\vec{r}) \times \vec{p}] \cdot \vec{s}$$

Spin of current carrier  
(electron or hole) with its  
orbital moment

### Localized Magnetism

$$(H_{so}^U)_{oc}^i = \sum_{j \neq i} \frac{\hbar}{2m^2c^2} [\vec{\nabla} U_{ij} \times \vec{p}_j] \cdot \vec{s}_j$$



$$(H_{so}^V)_{oc}^i = \sum_{j \neq i} \frac{\hbar}{2m^2c^2} [\vec{\nabla} V_{ij} \times \vec{p}_j] \cdot \vec{s}_j$$

Orbital moment of current carrier  
with localized spin



## Conclusion 2 – bad news

AHE may exist even if there is no spin polarization of charge current carriers

Observation of AHE in diluted magnetic semiconductors and oxides is not strict evidence of spin polarization of charge carriers



$$R_s = \frac{\sigma_{xy}(M_z)}{4\pi M_z (\sigma_{xx} + \sigma_{xy})^2} \approx \frac{\sigma_{xy}(M_z)}{4\pi M_z} \rho^2,$$

$$R_o = \frac{\sigma_{xy}(B_z)}{B_z (\sigma_{xx} + \sigma_{xy})^2} \approx \frac{\sigma_{xy}(B_z)}{B_z} \rho^2,$$

$$\frac{\Delta\rho}{\rho}(H_z) = \frac{\rho(H_z) - \rho(0)}{\rho(0)} = \beta \frac{M_z^2}{M_s^2}$$

$$\frac{\sigma_{xy}(M_z)}{4\pi M_z} = A \left(1 + \alpha \frac{M_z^2}{M_s^2}\right),$$

$$\frac{\sigma_{xy}(B_z)}{B_z} = C \left(1 + \gamma \frac{M_z^2}{M_s^2}\right),$$

$$R_s = R_s(0) \left[1 + \alpha \frac{M_z^2}{M_s^2}\right] \left[1 + \beta \frac{M_z^2}{M_s^2}\right]^2$$

$$R_o = R_o(0) \left[1 + \gamma \frac{M_z^2}{M_s^2}\right] \left[1 + \beta \frac{M_z^2}{M_s^2}\right]^2$$

Conclusion 3 – be carefull

# Anomalous Hall Effect in Ferromagnetic Semiconductors

T. Jungwirth,<sup>1,2</sup> Qian Niu,<sup>1</sup> and A. H. MacDonald<sup>1</sup>

<sup>1</sup>Department of Physics, The University of Texas, Austin, Texas 78712

<sup>2</sup>Institute of Physics ASCR, Cukrovarnická 10, 162 53 Praha 6, Czech Republic

(Received 3 October 2001; published 6 May 2002)

We present a theory of the anomalous Hall effect in ferromagnetic (III,Mn)V semiconductors. Our theory relates the anomalous Hall conductance of a homogeneous ferromagnet to the Berry phase acquired by a quasiparticle wave function upon traversing closed paths on the spin-split Fermi surface. The quantitative agreement between our theory and experimental data in both (In, Mn)As and (Ga, Mn)As systems suggests that this disorder independent contribution to the anomalous Hall conductivity dominates in diluted magnetic semiconductors. The success of this model for (III,Mn)V materials is unprecedented in the longstanding effort to understand origins of the anomalous Hall effect in itinerant ferromagnets.

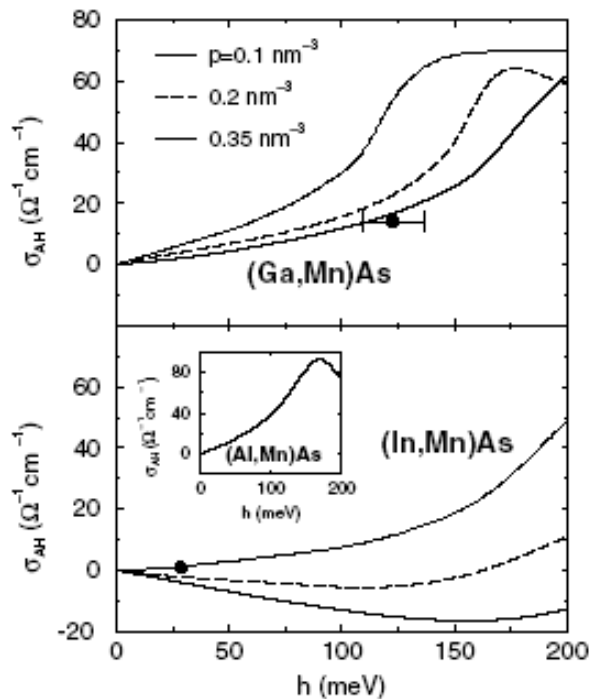


FIG. 2. Full numerical simulations of  $\sigma_{AH}$  for GaAs host (top panel), InAs host (bottom panel), and AlAs host (inset) with hole densities  $p = 0.1 \text{ nm}^{-3}$  (dotted lines),  $p = 0.2 \text{ nm}^{-3}$  (dashed lines), and  $p = 0.35 \text{ nm}^{-3}$  (solid lines). Luttinger parameters of the valence bands were obtained from Ref. [32]. Filled circles in the top and bottom panels represent measured AHE [1,3,9]. The saturation mean-field  $h$  values for the two points were estimated from nominal sample parameters [1,3,9]. Horizontal error bars correspond to the experimental uncertainty of the  $J_{pd}$  coupling constant. Experimental hole density in the (Ga, Mn)As sample is  $p = 0.35 \text{ nm}^{-3}$ ; for (In, Mn)As,  $p = 0.1 \text{ nm}^{-3}$  was determined indirectly from sample's transition temperature.

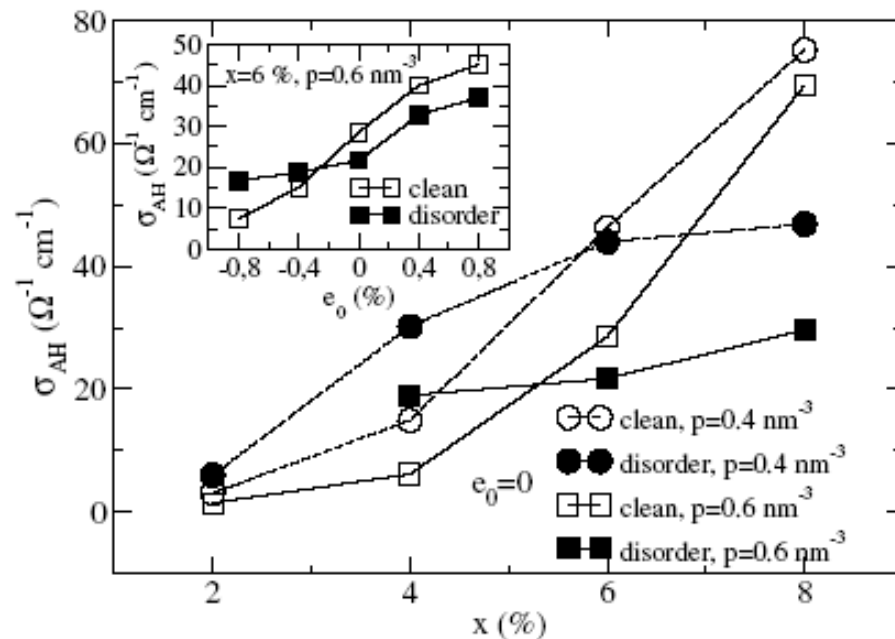


Fig. 4. Theoretical anomalous Hall conductivity of  $\text{Mn}_x\text{Ga}_{1-x}\text{As}$  DMS calculated in the clean limit (open symbols) and accounting for the random distribution of Mn and As-antisite impurities (filled symbols). (After Ref. 52.)

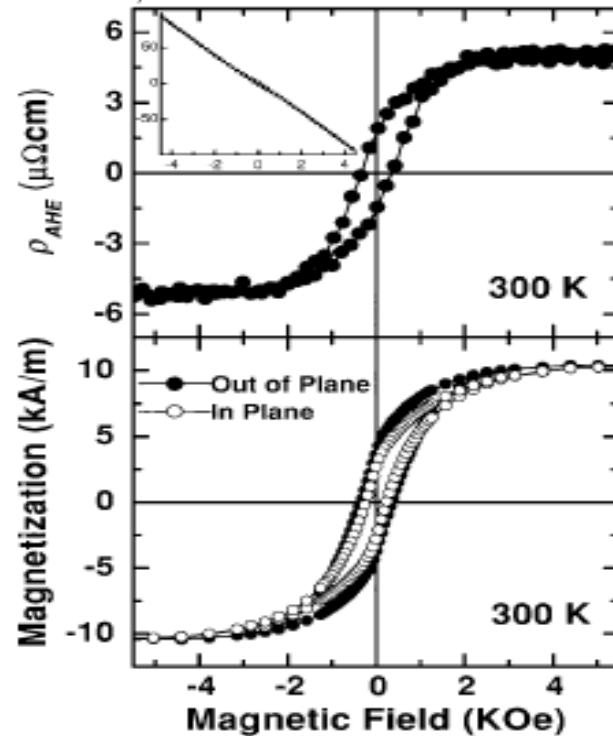
Anomalous Hall effect in anatase Co:TiO<sub>2</sub> ferromagnetic semiconductorR. Ramaneti, J. C. Lodder, and R. Jansen<sup>a)</sup>

FIG. 1. Top panel: Anomalous Hall resistivity vs the out-of-plane applied magnetic field for a 550 nm Co:TiO<sub>2</sub> thin film. The data are obtained from the total Hall resistivity shown in the inset by subtracting the linear term due to the OHE. The inset has the same units as the main panel. Bottom panel: Magnetization with the field applied in plane (open circles) and out of plane (solid circles) of the same sample. All measurements were done at room temperature.

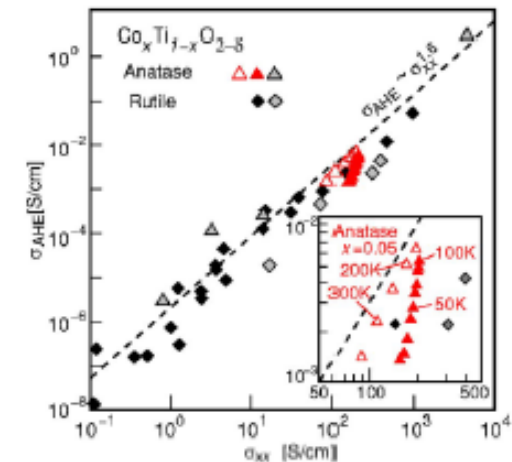
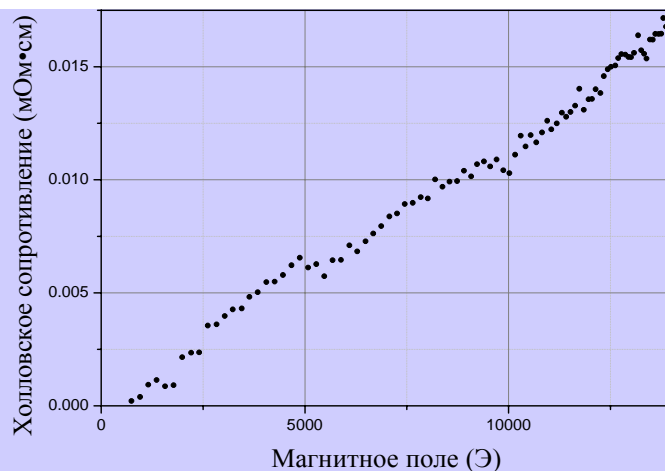


FIG. 20 Plot of AHE conductivity  $\sigma_{AHE}$  vs. conductivity  $\sigma$  for anatase Ti<sub>1-x</sub>Co<sub>x</sub>O<sub>2-δ</sub> (triangles) and rutile Ti<sub>1-x</sub>Co<sub>x</sub>O<sub>2-δ</sub> (diamonds). Grey symbols are data taken by other groups. The inset shows the expanded view of data for anatase with  $x=0.05$  (the open and closed triangles are for  $T > 150$  K and  $T < 100$  K, respectively). [From Ref. Ueno *et al.*, 2007.]

In thin-film samples of the ferromagnetic semiconductor anatase Ti<sub>1-x</sub>Co<sub>x</sub>O<sub>2-δ</sub>, Ueno *et al.* (Ueno *et al.*, 2008) have reported scaling between the AHE resistance and the magnetization  $M$ . The AHE conductivity  $\sigma_{xy}^{AH}$  scales with the conductivity  $\sigma_{xx}$  as  $\sigma_{xy}^{AH} \propto \sigma_{xx}^{1.6}$  (Fig. 20). A similar scaling relation was observed in another polymorph rutile. See also Ref. Ramaneti *et al.*, 2007 for related work on Co-doped TiO<sub>2</sub>.

Yu. Mikhailovsky

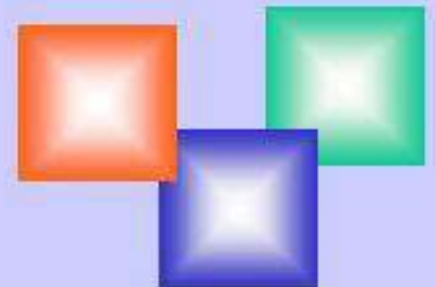


Magnetism in Semiconducting Quilts, 2007: 69-86 | ISBN: 81-7875-254-5  
Editor: Nguyen Hoa Hong

# 4

## Anomalous Hall effect in magnetic semiconductors

Jinke Tang  
Department of Physics, University of New Orleans, New Orleans, LA 70148  
USA.







THANK YOU !

