ESR -

Tool to investigate Paramagnets

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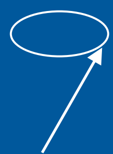
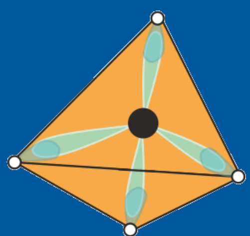


Electron Spin Resonance (ESR) – Basic Idea, Theory, Detection



ESR of ions in crystal - crystal fields

Ferromagnetic Resonance (FMR)

Magnetic Resonance - a Russian Discovery

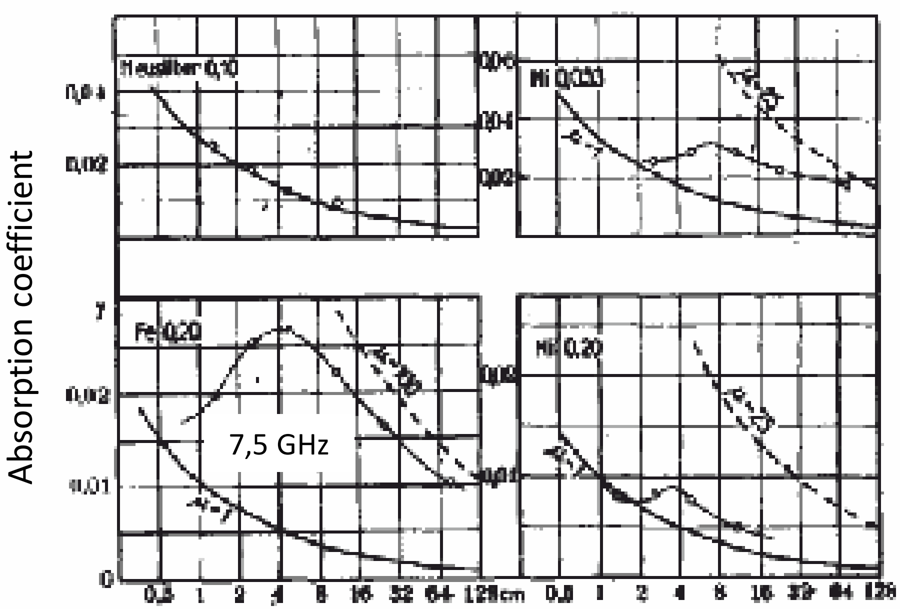
First report and proof of magnetic resonance performed in Moscow

1919. № 2.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 58.

1. *Über die Absorption elektromagnetischer Wellen an zwei parallelen Drähten;*
von W. Arkadiew.



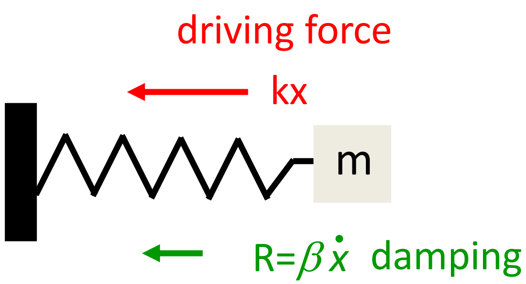
wavelength **Fig. 10.** wavelength

Key paper:

Quantitative theory:
Landau-Lifshitz-equation

L. Landau, E. Lifshitz,
Physik. Zeits. Sowjetunion 8, 153 (1935)

Recapitulation: Damped harmonic oscillator I



DGL

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

$$\gamma = \frac{\beta}{m};\ \omega_0^2 = \frac{k}{m}$$

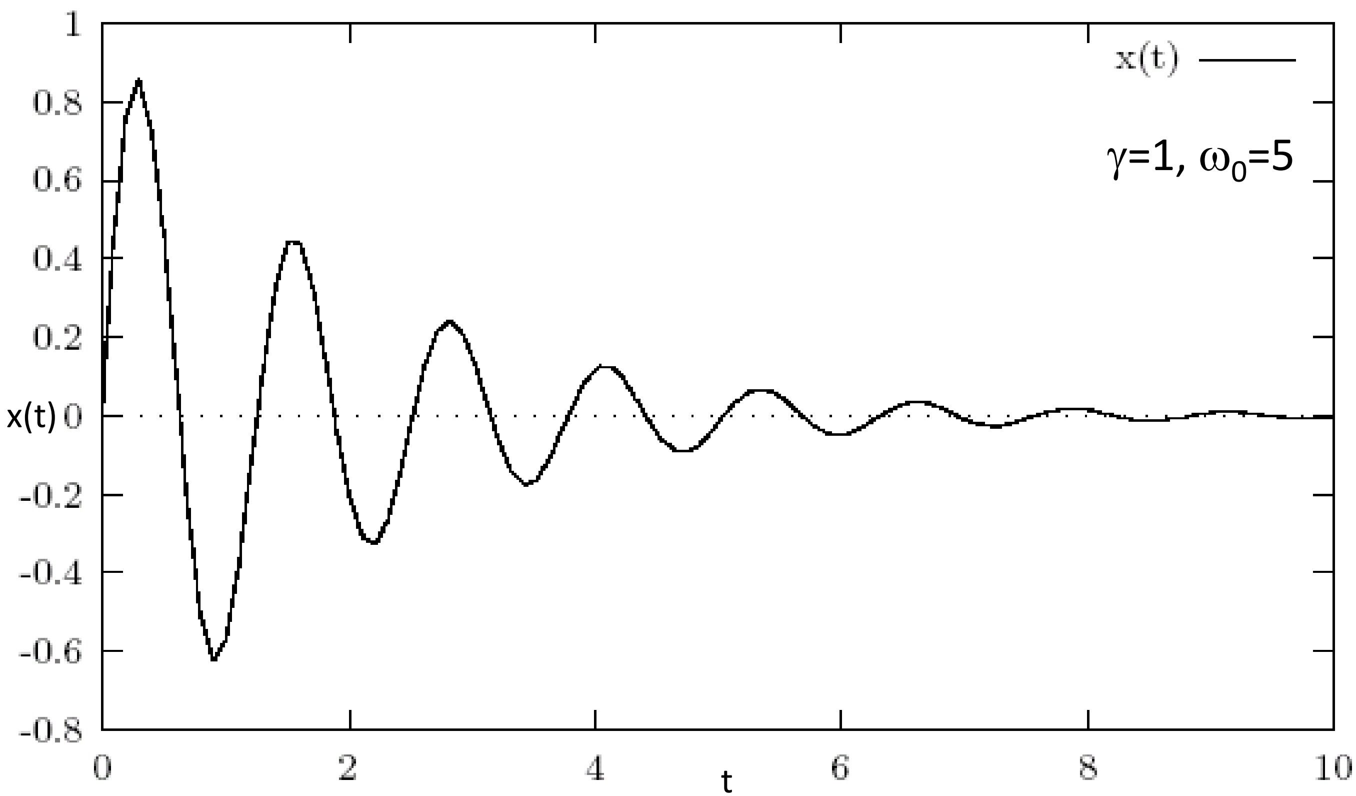
$$\begin{aligned} x &= Ae^{\lambda t} \\ \dot{x} &= \lambda A e^{\lambda t} \\ \ddot{x} &= \lambda^2 A e^{\lambda t} \end{aligned}$$

$$\begin{aligned} \lambda^2 A e^{\lambda t} + \gamma\lambda A e^{\lambda t} + \omega_0^2 A e^{\lambda t} &= 0 \\ \lambda^2 + \gamma\lambda + \omega_0^2 &= 0 \end{aligned}$$

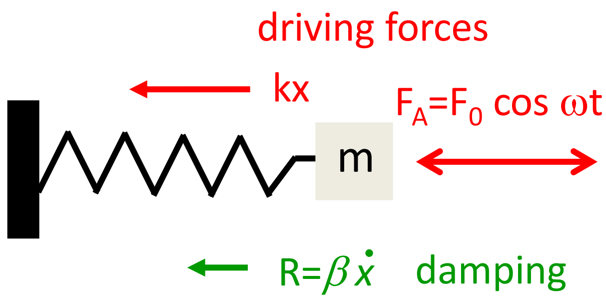
$$\lambda_{1,2} = -\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - \omega_0^2}$$

Recapitulation: Damped harmonic oscillator II

$$\lambda_{1,2} = -\frac{\gamma}{2} \pm i\sqrt{\omega_0^2 - \frac{\gamma^2}{4}} \qquad \frac{\gamma^2}{4} < \omega_0^2 \longrightarrow \text{‚underdamping‘}$$



Recapitulation: Externally excited damped harmonic oscillator I



DGL

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t + \delta)$$

$$\gamma = \frac{\beta}{m};\ \omega_0^2 = \frac{k}{m}$$

harmonic excitation

General solution:

$$x = x_{\mathrm{H}} + x_{\mathrm{S}}$$

Solution of homogeneous DGL
Plus particular solution of inhomogeneous DGL

$$F_{\mathrm{A}} = F_0(\cos(\omega t + \delta) + i\sin(\omega t + \delta)) = F_0 e^{i(\omega t + \delta)}$$

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i(\omega t + \delta)}$$

Real part of solution of
komplex DGL
is solution of original DGL

Recapitulation: Externally excited damped harmonic oscillator II

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i(\omega t+\delta)} \longrightarrow F' = \frac{F_0}{m} e^{i\delta} \longrightarrow \ddot{z} + \gamma\dot{z} + \omega_0^2 z = F' e^{i\omega t}$$

$$z_S = A' e^{i\omega t}$$

$$\dot{z}_S = i\omega A' e^{i\omega t}$$

$$\ddot{z}_S = -\omega^2 A' e^{i\omega t}$$

$$-\omega^2 A' e^{i\omega t} + i\omega\gamma A' e^{i\omega t} + \omega_0^2 A' e^{i\omega t} = F' e^{i\omega t}$$

$$A'\left(\omega_0^2 - \omega^2 + i\omega\gamma\right) = F'$$

amplitude of response

$$\chi = A' e^{-i\delta}$$

‚amplitude' that neglects excitation Phase'

$$\chi = \frac{\frac{F_0}{m}}{\omega_0^2 - \omega^2 + i\omega\gamma}$$

complex amplitude

Recapitulation: Externally excited damped harmonic oscillator III

complex amplitude
(susceptibility)

$$\chi = \frac{\frac{F_0}{m}}{\omega_0^2 - \omega^2 + i\omega\gamma} \rightarrow$$

$$\chi = \frac{\frac{F_0}{m}(\omega_0^2 - \omega^2 - i\omega\gamma)}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}$$

$$= \underbrace{\frac{\frac{F_0}{m}(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}_{\chi'} + i\underbrace{\frac{-\frac{F_0}{m}\omega\gamma}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}_{\chi''}$$

phase
of response

$$\chi = |\chi|\, e^{i\varphi}$$

$$|\chi| = \sqrt{\chi'^2 + \chi''^2}$$

$$\tan\varphi = \frac{\chi''}{\chi'}$$

$$|\chi| = \frac{\frac{F_0}{m}\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}$$

$$= \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}$$

$$\tan\varphi = \frac{-\frac{F_0}{m}\omega\gamma}{\frac{F_0}{m}(\omega_0^2 - \omega^2)}$$

$$= \frac{-\omega\gamma}{\omega_0^2 - \omega^2}$$

Recapitulation: Externally excited damped harmonic oscillator IV

$$x = x_\mathrm{H} + x_\mathrm{S}$$

stationary solution

$$z_\mathrm{S} = A' e^{i\omega t} = \chi \, e^{i\delta} e^{i\omega t} = |\chi| e^{i\varphi} e^{i\delta} e^{i\omega t} = |\chi| e^{i(\omega t + \varphi + \delta)}$$

$$z_\mathrm{S} = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}} e^{i\left(\omega t + \arctan\left(\frac{-\omega\gamma}{\omega_0^2 - \omega^2}\right) + \delta\right)}$$

$$\begin{aligned} x_\mathrm{S} &= \mathrm{Re}[z_\mathrm{S}] \\ &= \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}} \cos\left(\omega t + \arctan\left(\frac{-\omega\gamma}{\omega_0^2 - \omega^2}\right) + \delta\right) \end{aligned}$$

oscillation with driving frequency plus phase shift
that changes sign depending on whether $\omega < \omega_0$ or $> \omega_0$

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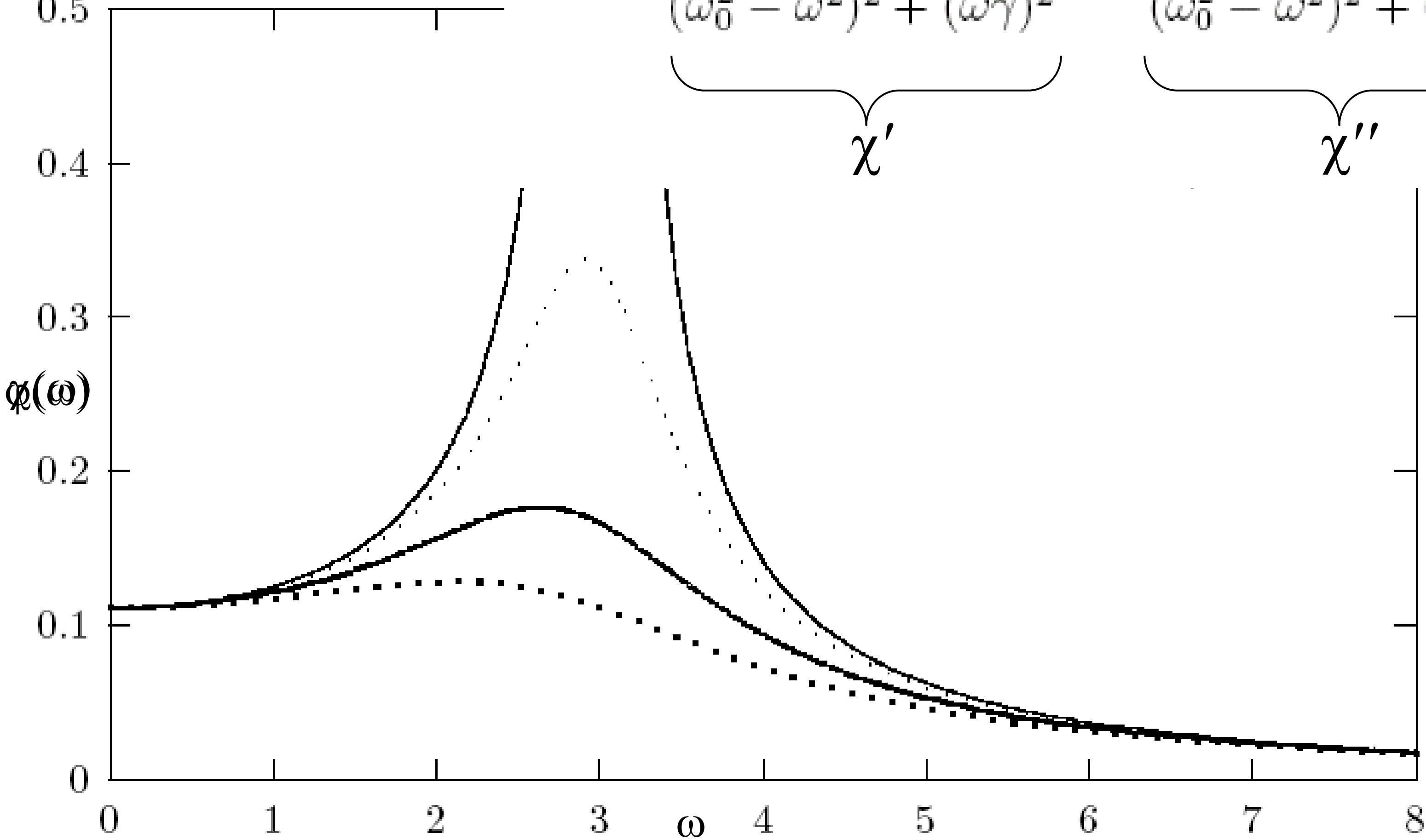
Recapitulation: Externally excited damped harmonic oscillator V

‚resonance curve'

$$\chi = \underbrace{\frac{\frac{F_0}{m}(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}_{\chi'} + i\underbrace{\frac{-\frac{F_0}{m}\omega\gamma}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}}_{\chi''}$$



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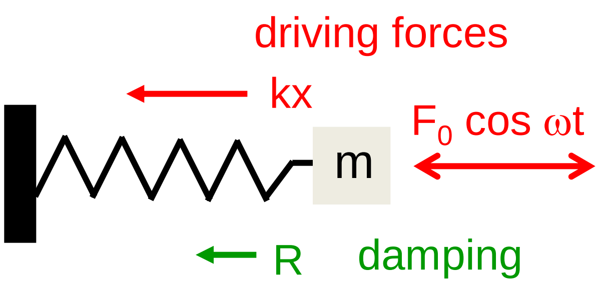
Comparison: Mechanical Oscillator vs. Electron Spin Resonance (ESR)

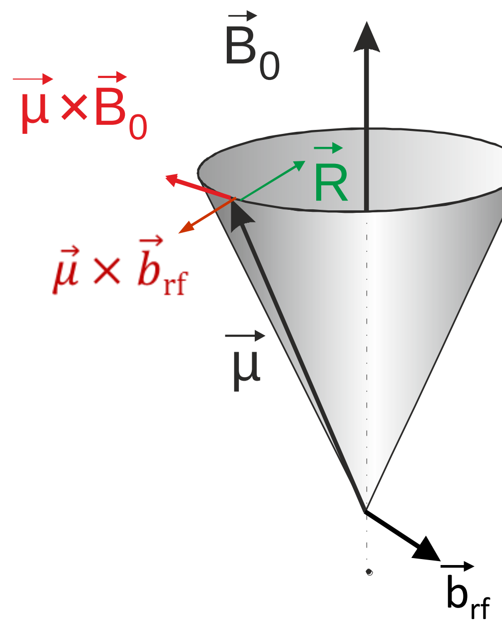
Mechanical system



$$\frac{d^2\vec{x}}{dt^2} = -\frac{k}{m}x + F_0 \cos\omega t + \vec{R}$$

driving torque damping

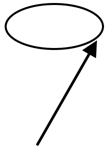
Magnetic moment



Landau-Lifshitz-Eq.

$$\frac{d\vec{\mu}}{dt} = -\gamma\,\vec{\mu} \times \left(\vec{B}_0 + \vec{b}_{rf}\right) + \vec{R}$$

driving torque damping



Spin in magnetic field – Quantum mechanical treatment I

Classical physics (spin $\hbar/2$ leads to magnetic moment μ_B)

$$\vec{\mu} = -\gamma\,\vec{s} \qquad\qquad \gamma = g_e \frac{\mu_B}{\hbar}$$

magnetic moment | electron spin | gyromagnetic ratio

Energy of magnetic moment in external magnetic field:

$$E = -\vec{\mu}\cdot\vec{B}_0$$

$\downarrow$ Quantum mechanics

Schrödinger Equation

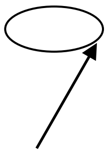
$$\gamma\vec{B}_0\cdot\hat{s}\,\varphi_i = \gamma\left(B_x\hat{s}_x + B_y\hat{s}_y + B_z\hat{s}_z\right)\varphi_i = E\varphi_i$$

Pauli spin matrices

$$\hat{s}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{s}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\hat{s}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Spin in magnetic field – Quantum mechanical treatment II

$$\gamma \vec{B}_0 \cdot \hat{\mathrm{s}}\,\varphi_{\mathrm{i}} = \gamma\left(B_{\mathrm{x}}\hat{\mathrm{s}}_{\mathrm{x}} + B_{\mathrm{y}}\hat{\mathrm{s}}_{\mathrm{y}} + B_{\mathrm{z}}\hat{\mathrm{s}}_{\mathrm{z}}\right)\varphi_{\mathrm{i}} = E\varphi_{\mathrm{i}}$$

field along z-direction $(0,0,B_0)$

$$\hat{\mathrm{s}}_{\mathrm{z}} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma B_0 \hat{\mathrm{s}}_{\mathrm{z}}\varphi_{\mathrm{i}} = E\varphi_{\mathrm{i}} = \pm g_{\mathrm{e}}\frac{\mu_{\mathrm{B}}}{\hbar}B_0\frac{\hbar}{2}\varphi_{\mathrm{i}} = \pm\mu_{\mathrm{B}}B_0\varphi_{\mathrm{i}}$$

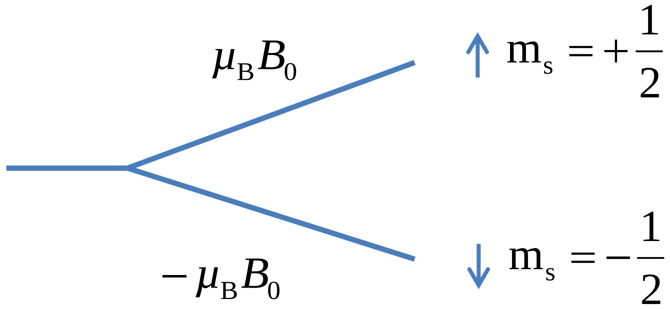
$$\varphi_{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \varphi_{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\longrightarrow \quad E = \pm\mu_{\mathrm{B}}B_0 \left(+\text{für } \varphi_{\uparrow}, -\text{ für } \varphi_{\downarrow}\right)$$

$$\hat{\mathrm{s}}_{\mathrm{z}}\varphi_{\mathrm{i}} = \hbar \mathrm{m}_s \varphi_{\mathrm{i}}$$

$$\mathrm{m}_s = \pm 1/2$$

expectation values
of Hamilton-Operator



Spin in magnetic field – Quantum mechanical treatment III - time dependent behavior

Time dependent Schrödinger Equation

field along z-direction $(0,0,B_0)$

$$\gamma B_0 \hat{s}_z \, \varphi = i\hbar \frac{d\varphi}{dt}$$

solution:

$$\varphi(t) = a e^{-i\omega_0 t/2} \varphi_\uparrow + b e^{i\omega_0 t/2} \varphi_\downarrow = \alpha \varphi_\uparrow + \beta \varphi_\downarrow$$

$$\text{with } \omega_0 = \gamma B_0$$

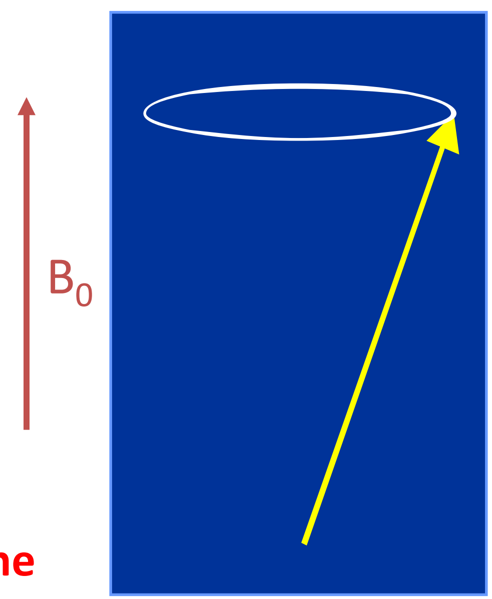
Interpretation (expectation value of $\hat{s}_i$, *i.e.* $< \varphi^* \hat{s}_i \varphi >$):

$$\hat{s}_z \varphi = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$$

$$< \hat{s}_z > = \varphi^* \hat{s}_z \varphi = \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} \alpha \\ -\beta \end{pmatrix} = \frac{\hbar}{2} \left(|\alpha|^2 - |\beta|^2 \right) = \frac{\hbar}{2} \left(a^2 - b^2 \right)$$

constant in time

Larmor precession



$$< \hat{s}_x > = ab\hbar \cos \omega_0 t \qquad < \hat{s}_y > = ab\hbar \sin \omega_0 t$$

rotating with ω_0 in x/y-plane

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Spin in magnetic field – Classical treatment

Landau-Lifshitz-Eq. $\frac{\mathrm{d}\vec{\mu}}{\mathrm{d}t} = -\gamma\, \vec{\mu} \times \vec{B}_0$

Time dep. of angular momentum — Torque in external magnetic field

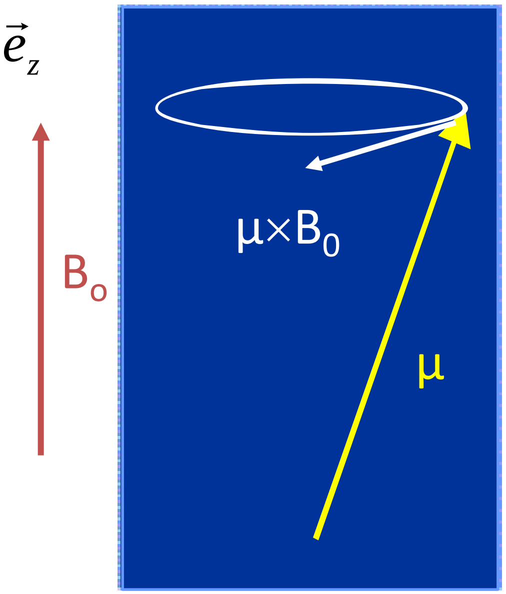
$$\text{Ansatz (harmonic oscillator):}$$

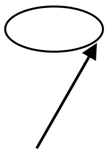
$$\vec{\mu} = \underbrace{\left(\mu_x \vec{e}_x + \mu_y \vec{e}_y + \mu_z \vec{e}_z\right) e^{i\omega t}}_{\text{time dep.}} + \underbrace{|\mu|\,\vec{e}_z}_{\text{constant}} \text{ und } \vec{B}_0 = B_0 \vec{e}_z$$

$$\begin{aligned} & i\omega\mu_x = -\gamma B_0 \mu_y \\ \Rightarrow\quad & i\omega\mu_y = \gamma B_0 \mu_x \\ & i\omega\mu_z = 0 \end{aligned}$$

$$\Rightarrow\quad \mu(t) = \left(\mu_x \vec{e}_x - i\mu_x \vec{e}_y\right) e^{i\omega_0 t} + |\mu|\,\vec{e}_z$$

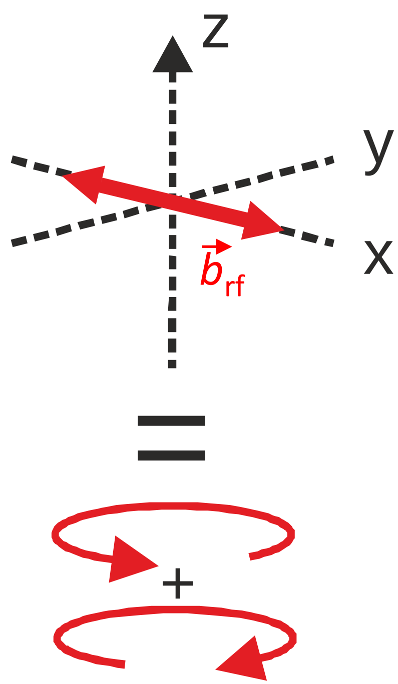
$$\text{with } \omega_0 = \gamma B_0 \text{ (Larmor frequency)}$$



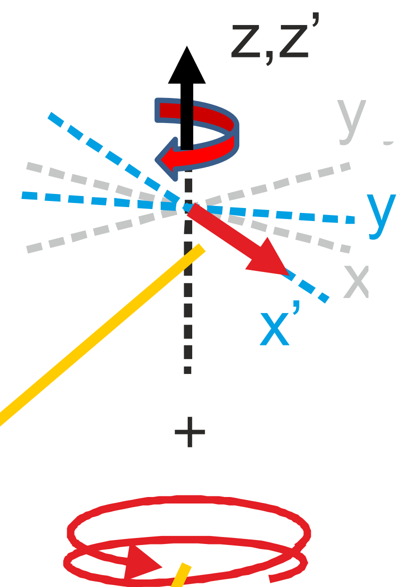


Influence of rf-field on magnetization precession I

Laboratory system



system that
rotates with rf-field



linearly polarized

$$\vec{b}_{\mathrm{rf}} = (2b_{\mathrm{rf}},0,0)\cdot\cos(\omega t) \text{ or}$$

$$\vec{b}_{\mathrm{rf}} = b_{\mathrm{rf}}[(\cos(\omega t),\sin(\omega t),0) + (\cos(\omega t),-\sin(\omega t),0)]$$

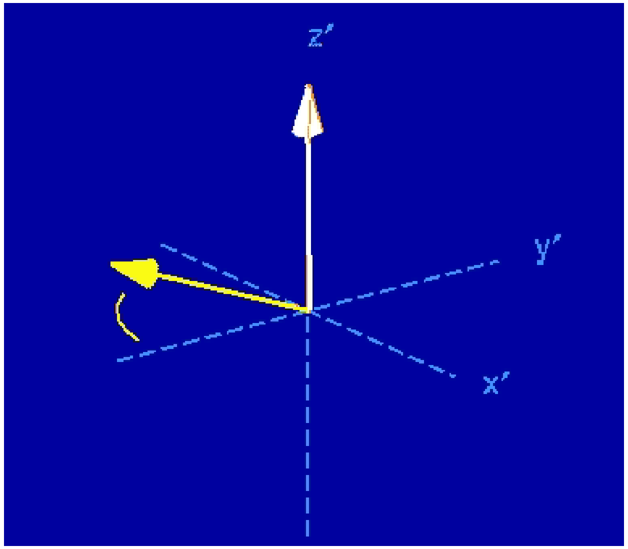
circularly polarized

pulls magnetization into plane (most effectively for $\omega=\omega_0$)

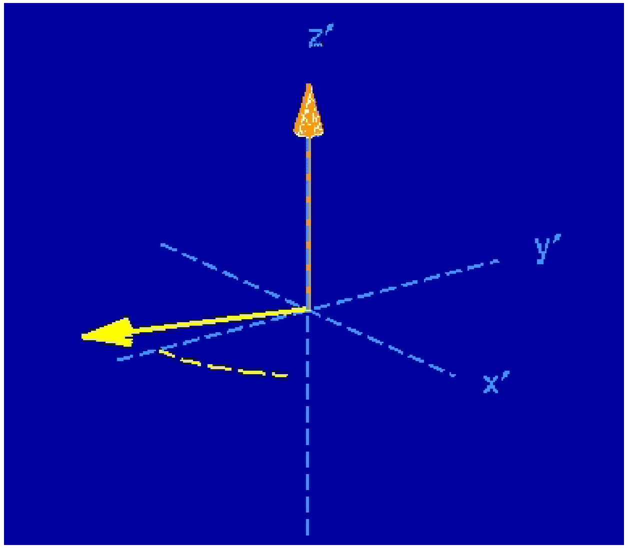
on average no effect

Influence of rf-field on magnetization precession II

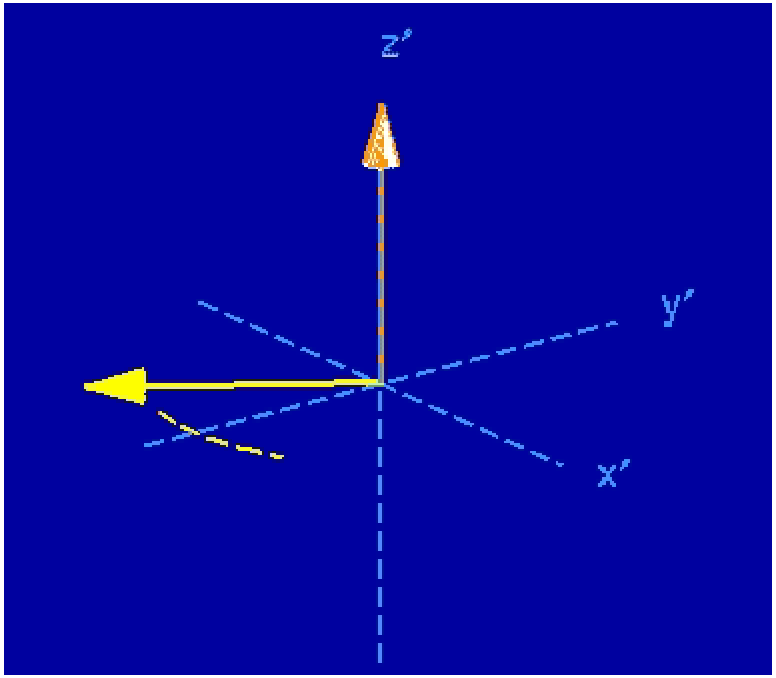
$\omega=2\omega_0$



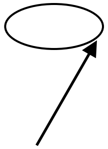
$\omega=0.5\omega_0$



$\omega=\omega_0$



rf-field opens precession cone for $\omega=\omega_0$



Conventional ESR detection III

$$\tilde{m} = \tilde{\chi} \cdot \tilde{h} = (\chi' - \mathrm{i}\chi'') \cdot \tilde{h}$$

$$\tilde{h}(t) = a e^{\mathrm{i}\omega t} = a(\cos\omega t + \mathrm{i}\sin\omega t) \longrightarrow m(t) = \mathrm{Re}(\tilde{m}(t)) = a\left[\chi' \cos\omega t + \chi'' \sin\omega t\right]$$

$$\mathrm{Re}(\tilde{h}(t)) = a\cos\omega t$$

$$P(t) = \mu_0 h \cdot \frac{\mathrm{d}m}{\mathrm{d}t} = \mu_0 a^2 \omega \cos\omega t \left(-\chi' \sin\omega t + \chi'' \cos\omega t\right)$$

$$\begin{aligned}\langle P\rangle_T = \frac{\langle w\rangle_T}{T} &= \frac{\mu_0\omega}{2\pi}\int h \cdot \frac{\mathrm{d}m}{\mathrm{d}t} \\ &= \frac{\mu_0\omega}{2\pi}\int_0^{2\pi/\omega} a^2\omega\chi'' \cos^2\omega t\,\mathrm{d}t \\ &= \frac{\mu_0\omega^2}{2\pi} a^2 \chi'' \left[\frac{1}{2}t + \frac{1}{4\omega}\sin 2\omega t\right]_0^{2\pi/\omega} = \frac{1}{2}\mu_0\omega\chi'' a^2\end{aligned}$$

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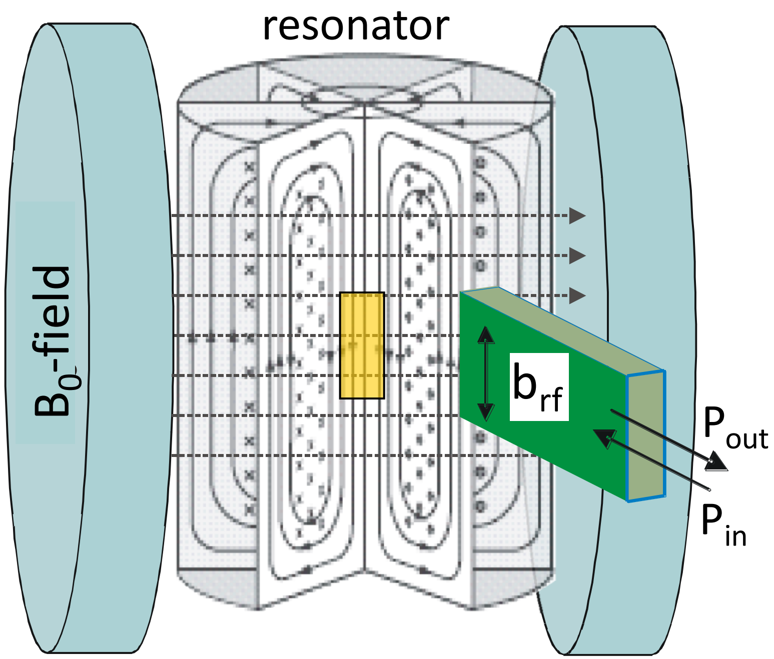
Experimental detection of ESR (conventional method)



$$Q_{Res}^{u} = \frac{P_{res}}{P_{loss}}$$

quality factor resonator: ratio of input power to power loss

$$P_{Res} = \frac{\omega_0}{2\mu_0} \int_{\text{volume resonator}} b_{rf}^2 \, dV$$

power coupled into resonator (per period)

$$P_{abs} = \frac{\omega_0}{2\mu_0} \chi'' \int_{\text{Sample}} b_{rf}^2 \, dV$$

power absorbed by sample from rf-field (per period)

$$Signal \sim \frac{P_{abs}}{P_{loss}} = \frac{\chi'' \int_{\text{Sample}} b_{rf}^2 \, dV}{\frac{1}{Q_{Res}^{u}} \int_{\text{volume resonator}} b_{rf}^2 \, dV} = \underbrace{\chi''\eta}_{\text{Prob}} \cdot \underbrace{Q_{Res}^{u}}_{\text{Res.}}$$

$$\eta := \frac{\int_{\text{Sample}} b_{rf}^2 \, dV}{\int_{\text{volume resonator}} b_{rf}^2 \, dV}$$ filling factor

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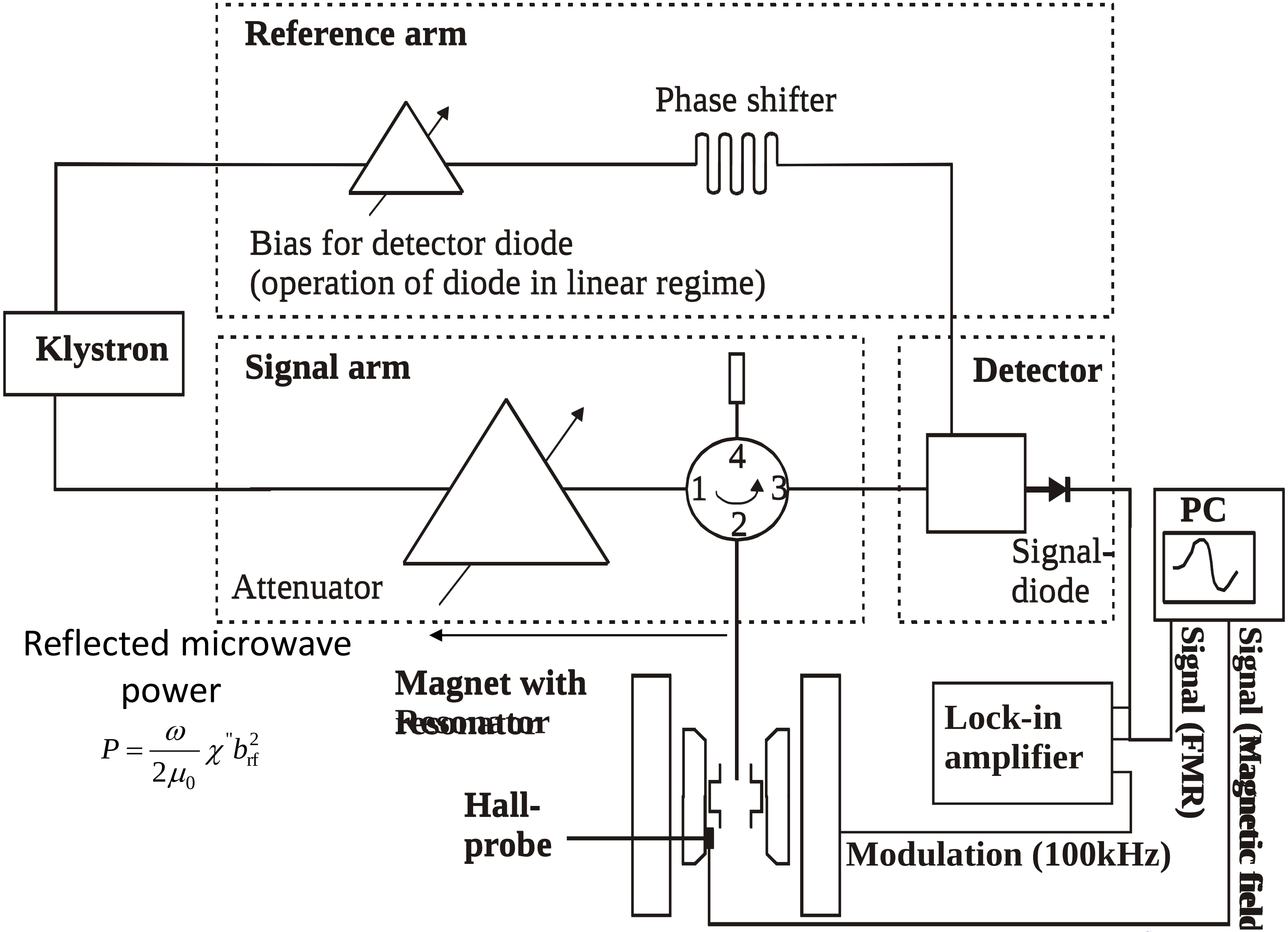
Conventional ESR detection I



Reflected microwave power

$$P = \frac{\omega}{2\mu_0}\chi'' b_{\mathrm{rf}}^2$$

Paramagnetic ion in a crystal

Hamilton-Operator for electrons of paramagnetic ion (no external field):

$$\hat{H} = \left\{ \left(\hat{p}_i^2 / 2m\right) - \left(Ze^2 / r_i\right) + \left(e^2 / r_{ij}\right) + \lambda_{ij} \widehat{l}_i \cdot \hat{s}_i + a_i \hat{j}_i \cdot \hat{I} - e_i \Phi_c\left(\vec{r}_i\right) \right\}$$

Kinetic energy of ion

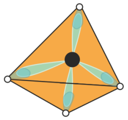
Coulomb repulsion between electron pairs

Magnetic interaction between electrons and nucleus with nuclear spin I

Crystal field Interaction

Coulomb attraction between electrons and nucleus

Spin-orbit interaction between electrons



Reminder: Cubic Harmonics I

Solution of Schrödinger Equation for H-Atom

$$\Psi_{\mathrm{n,l,m}}(r,\vartheta,\varphi) = R_{n,l}(r)\, Y_{l,m}(\vartheta,\varphi)$$ Hydrogen wavefunctions

Radial part

$n=1$	$l=0$	$R_{1,0}(r) = 2\left(\frac{Z}{a}\right)^{\frac{3}{2}} e^{-\frac{Z\,r}{a}}$
$n=2$	$l=0$	$R_{2,0}(r) = \frac{1}{2\sqrt{2}}\left(\frac{Z}{a}\right)^{\frac{3}{2}}\left(2-\frac{Z\,r}{a}\right) e^{-\frac{Z\,r}{2a}}$
	$l=1$	$R_{2,1}(r) = \frac{1}{2\sqrt{6}}\left(\frac{Z}{a}\right)^{\frac{3}{2}}\frac{Z\,r}{a} e^{-\frac{Z\,r}{2a}}$
$n=3$	$l=0$	$R_{3,0}(r) = \frac{2}{81\sqrt{3}}\left(\frac{Z}{a}\right)^{\frac{3}{2}}\left(27-18\frac{Z\,r}{a}+2\left(\frac{Z\,r}{a}\right)^2\right) e^{-\frac{Z\,r}{3a}}$
	$l=1$	$R_{3,1}(r) = \frac{2}{81\sqrt{6}}\left(\frac{Z}{a}\right)^{\frac{3}{2}}\left(6\frac{Z\,r}{a}-\left(\frac{Z\,r}{a}\right)^2\right) e^{-\frac{Z\,r}{3a}}$
	$l=2$	$R_{3,3}(r) = \frac{2}{81\sqrt{30}}\left(\frac{Z}{a}\right)^{\frac{3}{2}}\left(\frac{Z\,r}{a}\right)^2 e^{-\frac{Z\,r}{3a}}$

Angular dependent part

$l=0$	$m=0$	$Y_{0,0}(\vartheta,\varphi) = \frac{1}{\sqrt{4\pi}}$
$l=1$	$m=0$	$Y_{1,0}(\vartheta,\varphi) = \sqrt{\frac{3}{4\pi}}\cos\vartheta$
	$m=\pm1$	$Y_{1,\pm1}(\vartheta,\varphi) = \mp\sqrt{\frac{3}{8\pi}}\sin\vartheta\; e^{\pm i\varphi}$
$l=2$	$m=0$	$Y_{2,0}(\vartheta,\varphi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\vartheta - 1)$
	$m=\pm1$	$Y_{2,\pm1}(\vartheta,\varphi) = \mp\sqrt{\frac{15}{8\pi}}\sin\vartheta\cos\vartheta\; e^{\pm i\varphi}$
	$m=\pm2$	$Y_{2,\pm2}(\vartheta,\varphi) = \sqrt{\frac{15}{8\pi}}\cdot\frac{1}{2}\sin^2\vartheta\; e^{\pm 2i\varphi}$

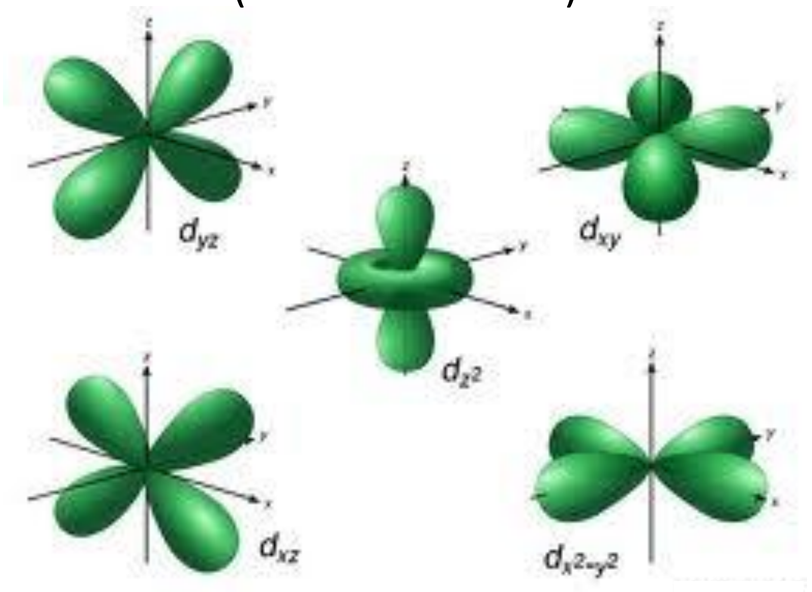
Reminder: Cubic Harmonics II

Schrödinger Equation is a linear equation
If $\Psi 1$ and $\Psi 2$ are solutions, then
$\Psi 3 = a\Psi 1 + b\Psi 2$ is solution, too.

One obtains REAL solutions for:

$-\frac{i}{\sqrt{2}}\left(Y_{2,2}-Y_{2,-2}\right)$	$-\frac{1}{\sqrt{2}}\left(Y_{2,1}-Y_{2,-1}\right)$	$Y_{2,0}$
d_{xy}	d_{yz}	d_{z^2}
$\frac{1}{\sqrt{2}}\left(Y_{2,2}+Y_{2,-2}\right)$	$\frac{i}{\sqrt{2}}\left(Y_{2,1}+Y_{2,-1}\right)$	
$d_{x^2-y^2}$	d_{xz}	

d-orbitals
(cubic harmonics)



Example:

$$Y_{2,\pm 2}=\sqrt{\frac{15}{8\pi}}\frac{1}{2}\sin^2\vartheta\cdot e^{\pm 2i\varphi} \longrightarrow \frac{1}{\sqrt{2}}\left(Y_{2,2}+Y_{2,-2}\right)\sim\sin^2\vartheta\cos 2\varphi=(\sin\vartheta\cos\varphi)^2-(\sin\vartheta\sin\varphi)^2$$

With (unit sphere, i.e. r=1) $\sin\vartheta\sin\varphi = x, \sin\vartheta\sin\varphi = y, \cos\vartheta = z$

$$\longrightarrow \quad \frac{1}{\sqrt{2}}\left(Y_{2,2}+Y_{2,-2}\right)\sim x^2-y^2$$

Crystal field interaction I

$$\hat{H} = \left\{ \left(\hat{p}_i^2 / 2m\right) - \left(Ze^2 / r_i\right) + \left(e^2 / r_{ij}\right) + \lambda_{ij} \hat{l}_i \cdot \hat{s}_i + a_i \hat{j}_i \cdot \hat{I} \boxed{- e_i \Phi_c\left(\vec{r}_i\right)} \right\}$$

Assumption: Crystal field creates electrostatic potential at the site of the ion
Symmetry of this field is determined by symmetry of the crystal

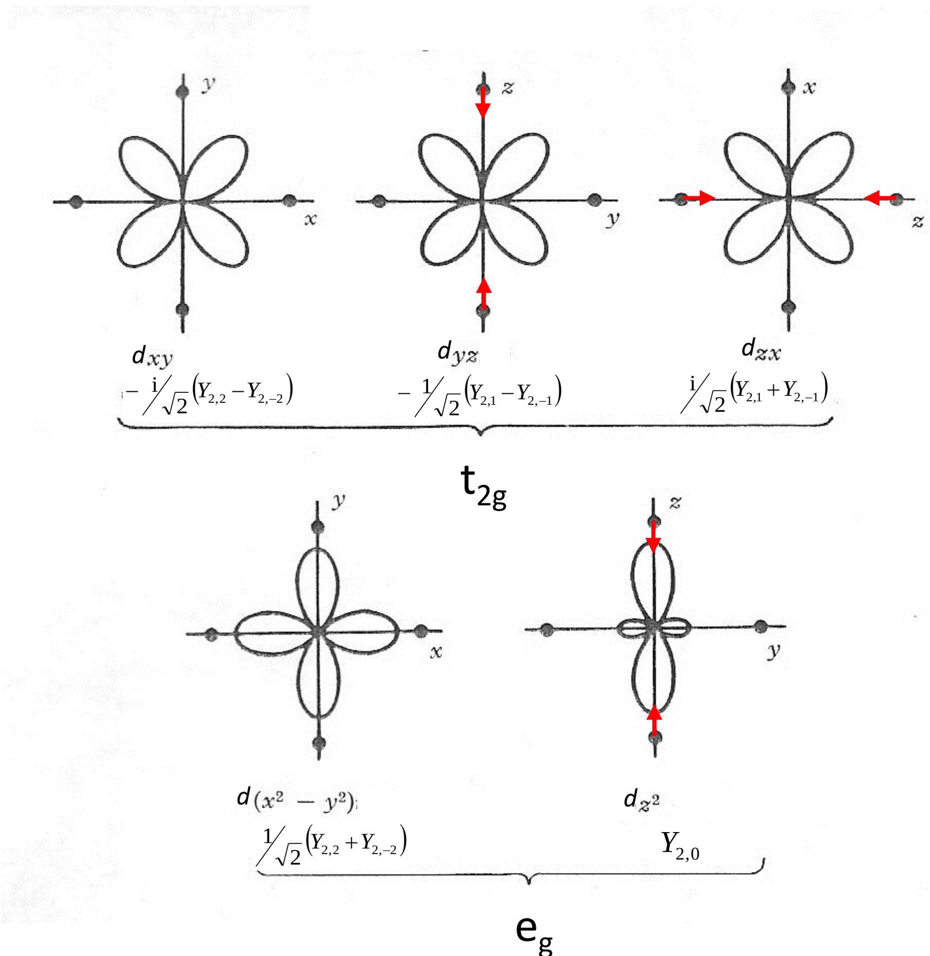
Weak crystal field: Crystal field weaker than spin-orbit interaction
Rare earth ions with low lying f-shell

Intermediate crystal field: Crystal field stronger than spin-orbit interaction
3d group ions with outer shell 3d electrons

Strong crystal field: Covalent bonding to neighbored ions or atoms
Crystal field theory not applicable, since sources of crystal field are not outside of ion under consideration

Crystal field interaction II –

Instructive example: d-orbitals in cubic/tetragonal crystal field

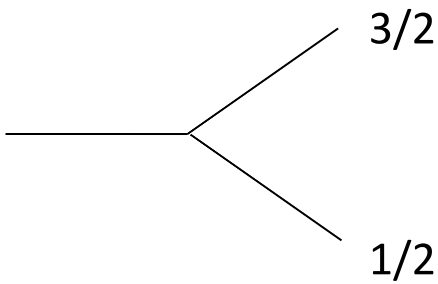


Zero-field splitting (influence of spin-orbit coupling)

Removal spin microstate degeneracy for systems with $S > 1/2$ in absence of an applied field.

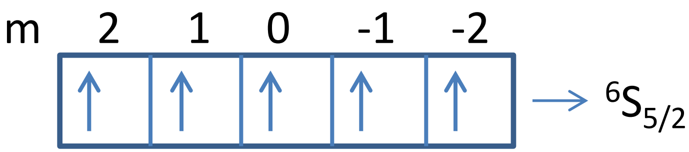
Reason are spin-spin interactions like dipolar interaction
or spin-orbit interaction between different electrons

Example: $^4S_{3/2}$



Example I: Fe^{3+}- and Mn^{2+}-impurities in ZnO

Electronic configuration of free Fe^{3+}/Mn^{2+}ion



S=5/2; L=0; J=S=5/2

No effect of crystal field, due to spherical charge distribution

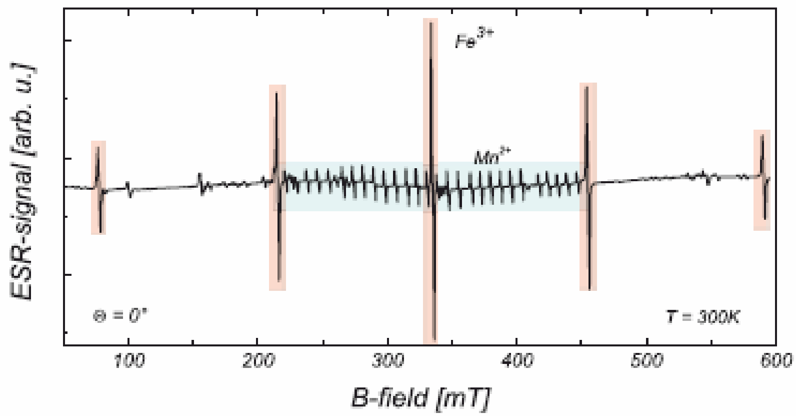
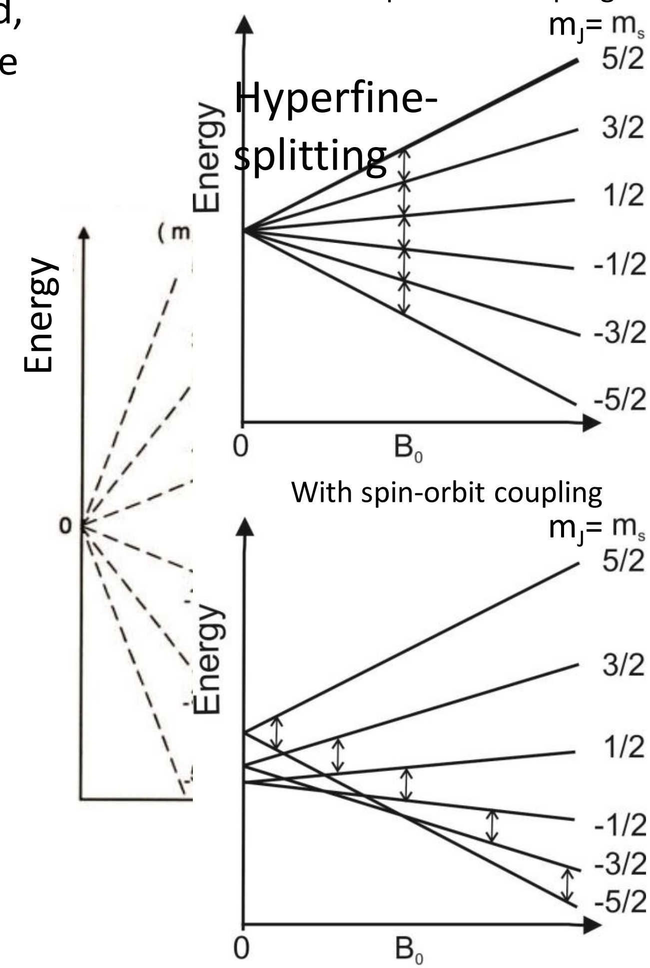


Figure 3.3: Fe^{3+}(red bars) and Mn^{2+}(blue bar) impurities in ZnO measured by ESR at $\nu = 9.3$ GHz. The magnetic field is oriented parallel to the crystal c-axis (0001). Both ions have a fivefold fine structure due to $S = \frac{5}{2}$. Contrary to the Fe lines the Mn resonances are sixfold hyperfine split by the nuclear moment $I_{Mn} = \frac{5}{2}$ leading to a spectrum of in total 30 lines.

T. Kammermeier, PhD thesis Uni Duisburg-Essen (2010)

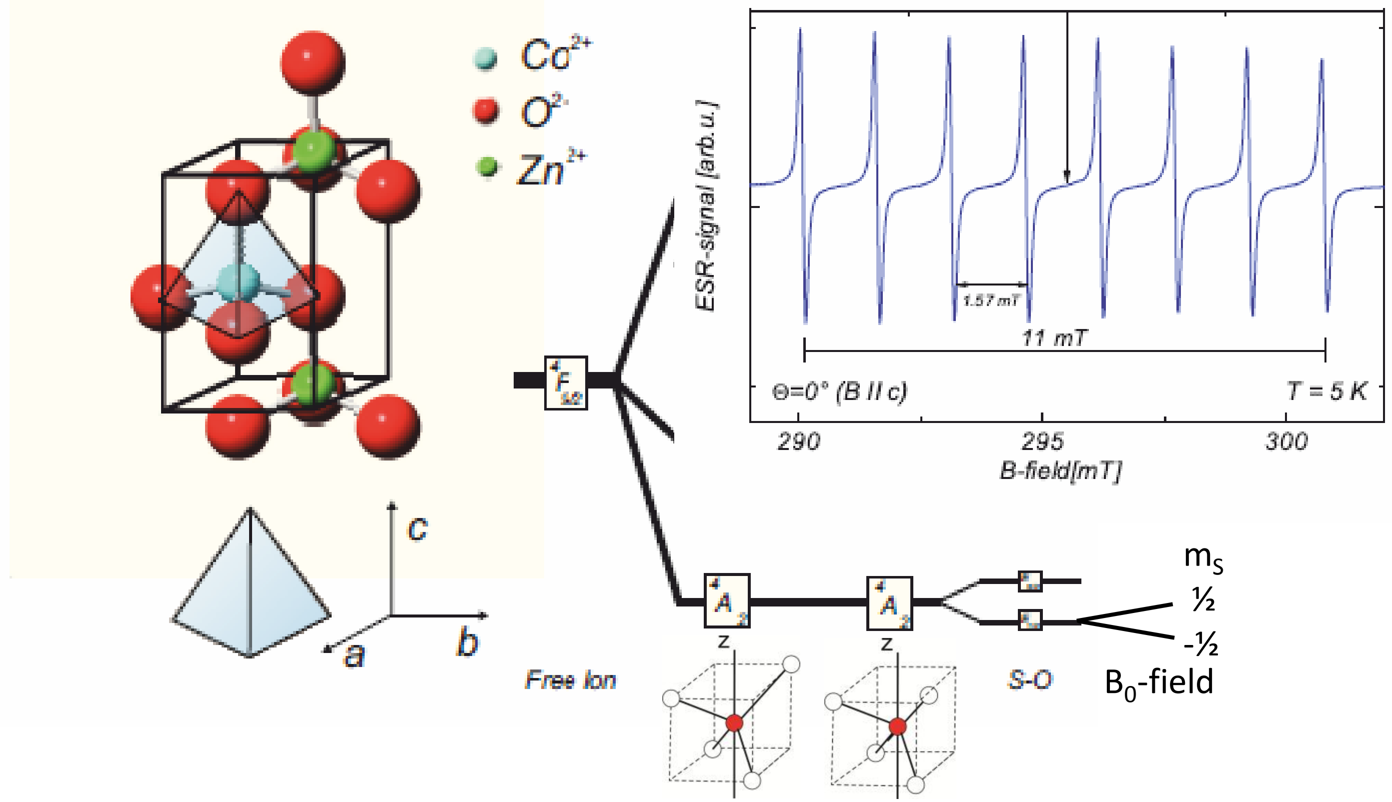


Example II: Co^{2+} in ZnO

m	2	1	0	-1	-2	
	↑↓	↑↓	↑	↑	↑	→ $^4F_{9/2}$

S=3/2; L=3; J=L+S=9/2

T. Kammermeier, PhD thesis Uni Duisburg-Essen (2010)



Tetrahedral crystal field

Trigonal distortion

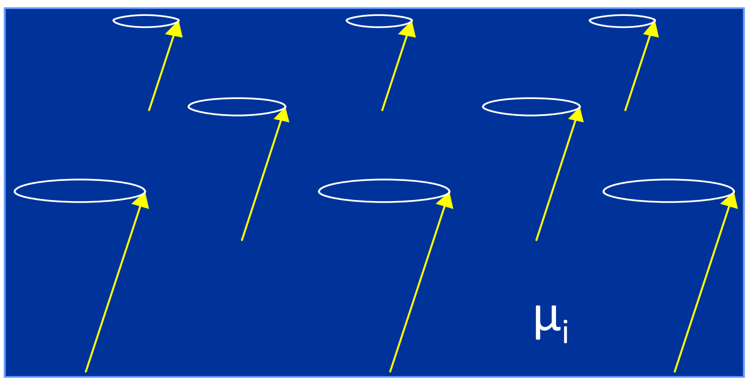
→

Hyperfine-splitting

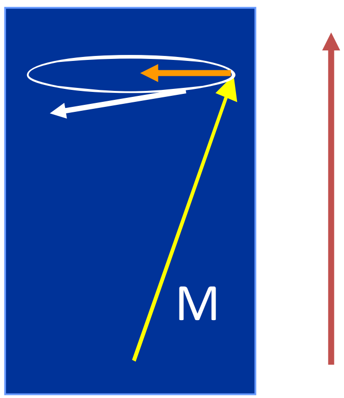
Co has isotope with I=7/2

Magnetic Resonance in Ferromagnets (coupled spin system)

Ensemble of coupled spins

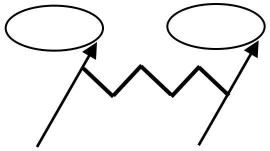


Macrospin model

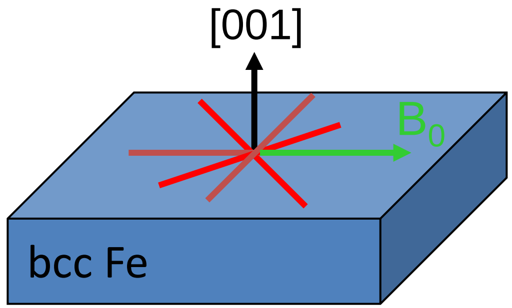


B_{eff}: Anisotropy field,
dipolar fields
exchange fields
etc.

Magnetisation in
effective field
+
damping

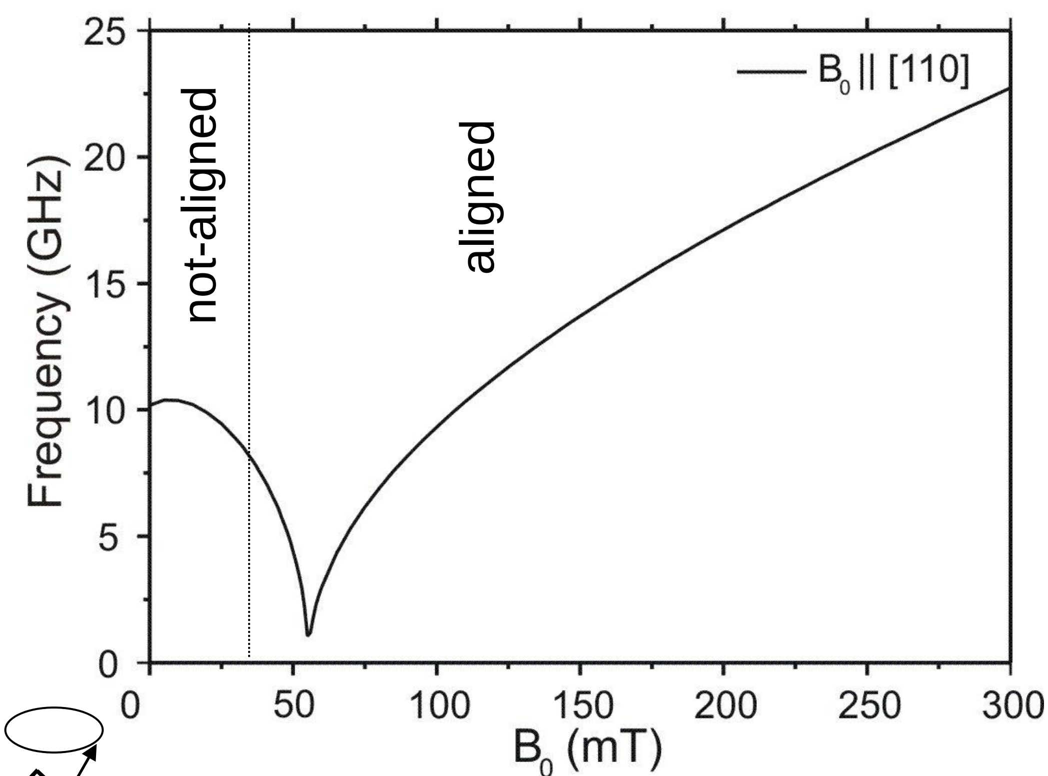


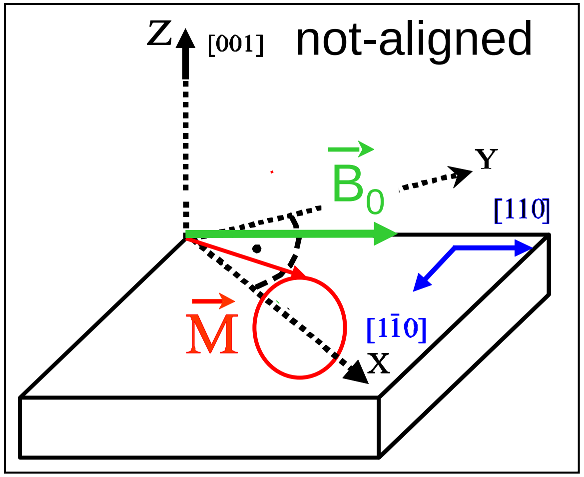
FMR: Uniform mode - Influence of magnetic anisotropy



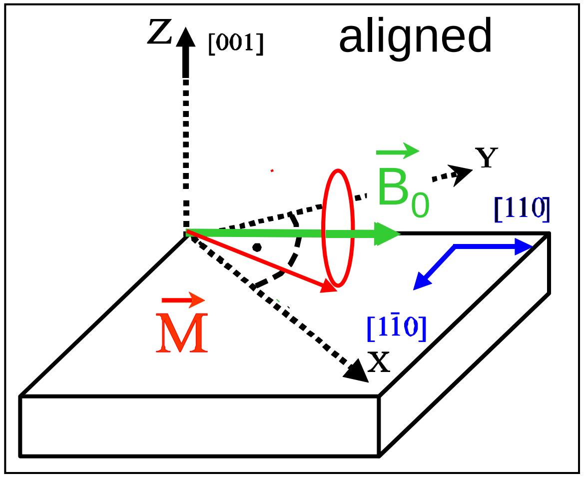
<110>-directions (hard axes)

<100>-directions (easy axes)

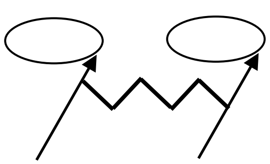




Picture: Kh. Zakeri

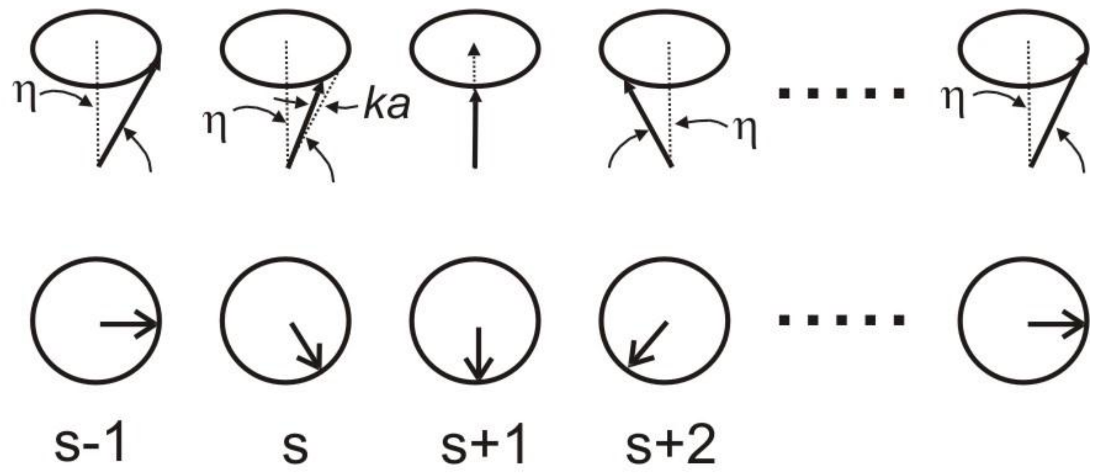


Picture: Kh. Zakeri



FMR: Spinwave excitations due to coupling of spins (non-uniform modes) I

Elementary excitation of coupled spins



dipolar interaction

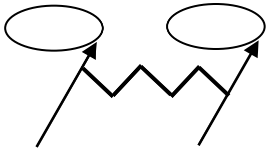
Spins coupled mainly by

exchange

Magnetostatic modes
control magneto-dynamics

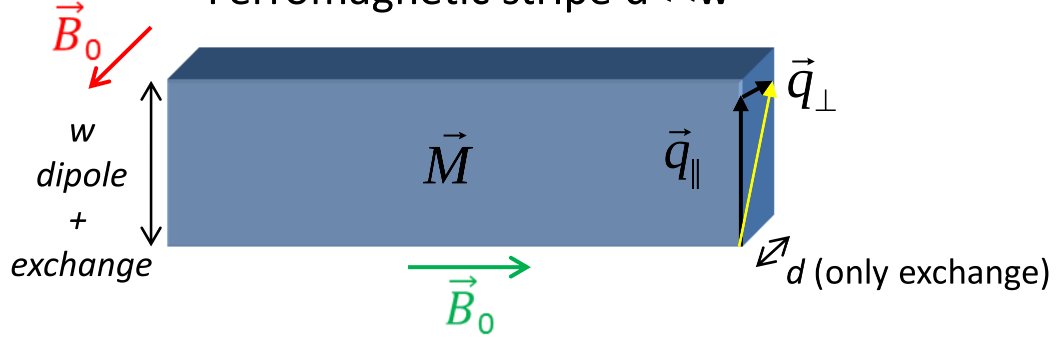
ground state excitations
at small energies <1meV
(e.g. by microwaves)

Thermal modes
control thermo-dynamics
Excitations at high energies >1meV
(e.g. by temperature)



FMR: Spinwave excitations due to coupling of spins (non-uniform modes) II

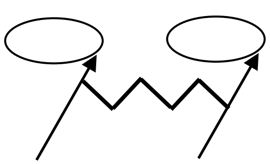
Ferromagnetic stripe d<<w



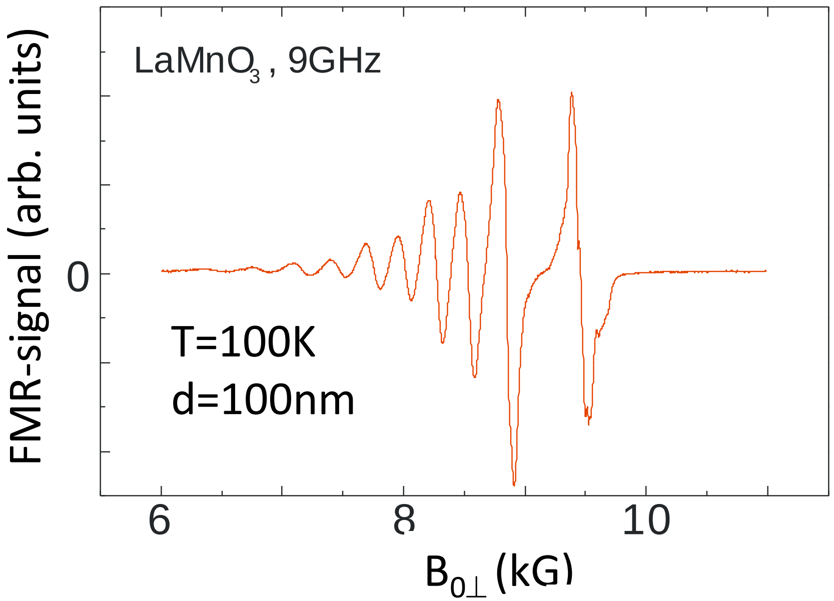
exchange exchange dipole

$$\omega = \gamma\left[\left(B_{0\parallel} + \frac{2A}{M}q_\perp^2\right)\left(B_{0\parallel} + \frac{2A}{M}q_\perp^2 + \mu_0 M \cdot F_{pp}\left(q_\parallel d\right)\right)\right]^{1/2} \text{with } q^2 = q_\parallel^2 + q_\perp^2 = q_\parallel^2 + \left(\frac{p\pi}{d}\right)^2$$

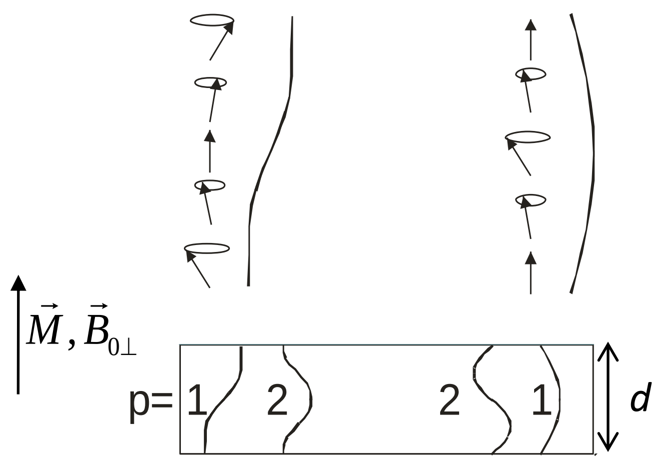
$$d = {}^{p\lambda}\!/_{2} \qquad q_\perp = {}^{2\pi}\!/_{\lambda} = {}^{p\pi}\!/_{d}$$

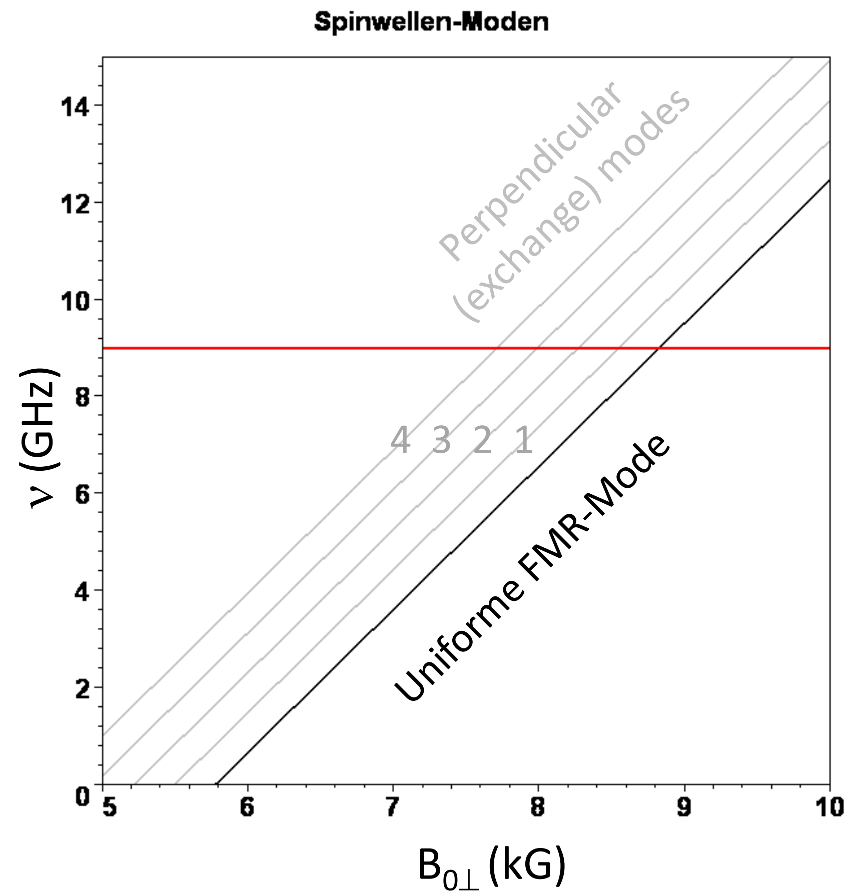


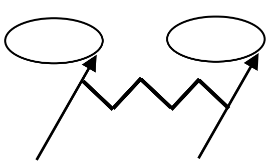
Exchange dominated spinwaves in thin films



$$\omega = \gamma\left(B_{0\perp} - \mu_0 M + \frac{2A}{M} q_\perp^2\right)$$

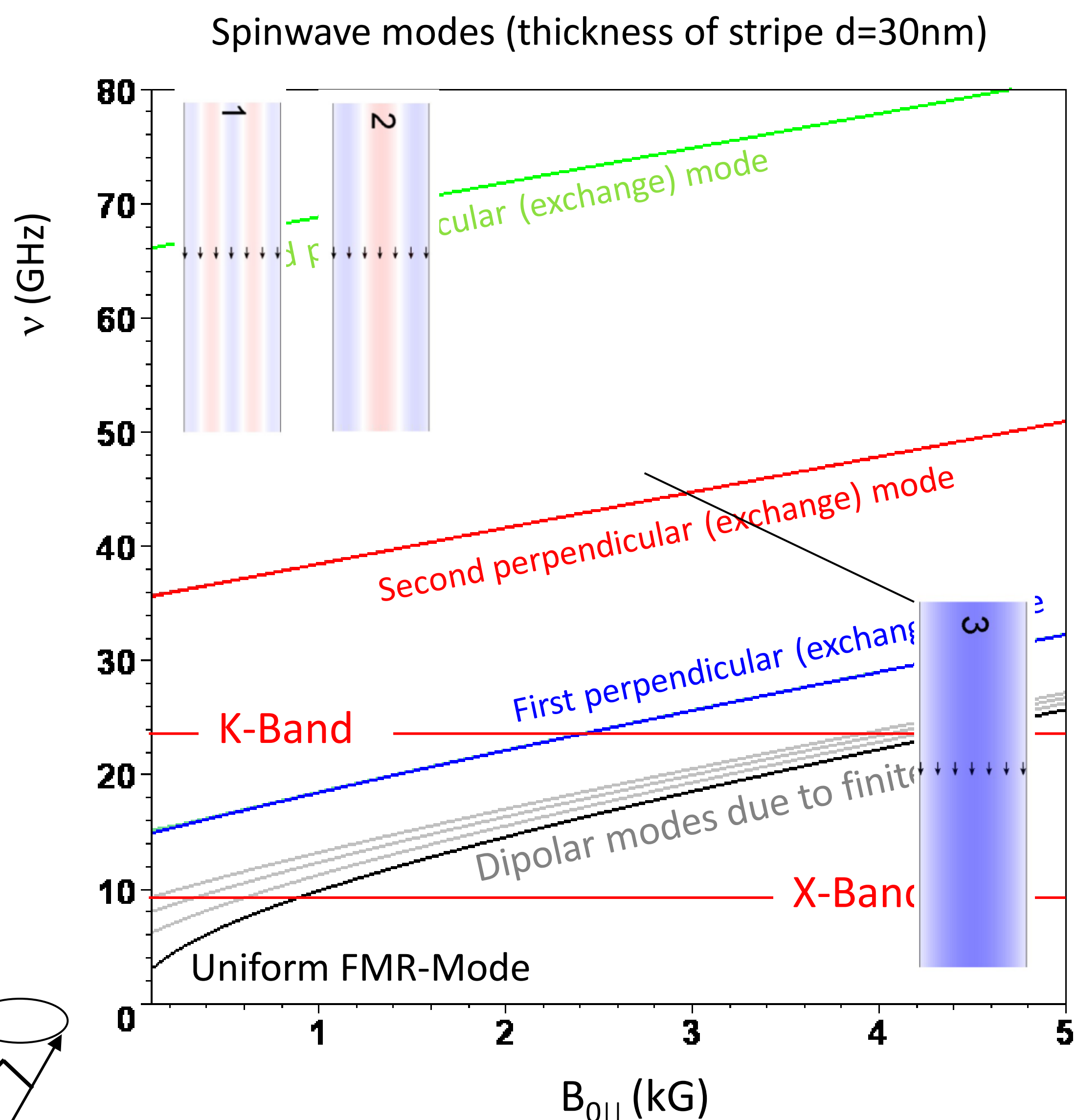






P. Aleshkevych et al., Phys. stat. sol. (a) ***196****, 93 (2003)*

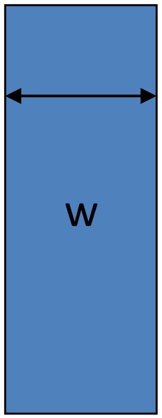
Dipolar spinwave modes in ferromagnetic stripe

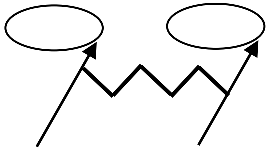


stripe (length → ∞):

w=1.8 µm
M=0.8 kG
$B_{0||}$,M || long axis

w=500nm-5000nm (parameter)
d=30nm

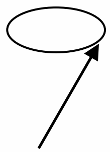


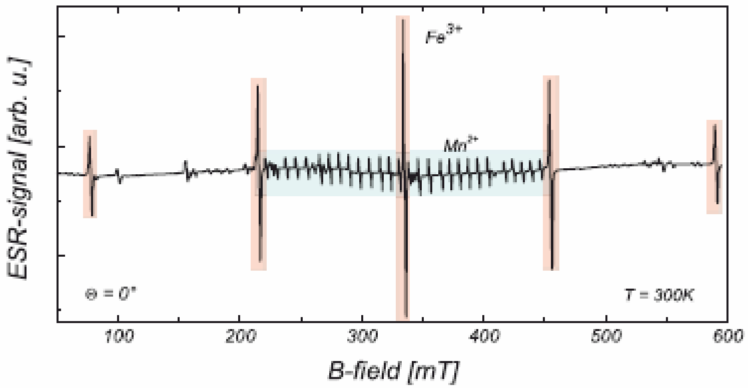


Summary

Magnetic Resonance in

Paramagnets

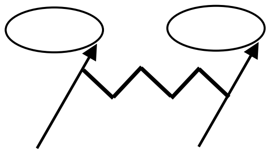


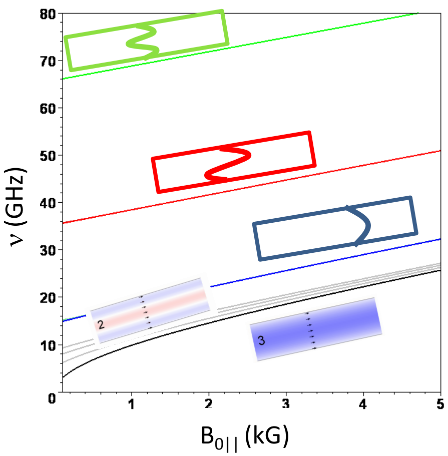


Identification of species and environment (crystal symmetry)

Indirect way of investigating lattice sites that are occupied

Ferromagnets





Measurement of magnetic anisotropy, internal fields (dipolar, exchange)

Investigating spindynanics in nanostructures

Extra slides

$$\frac{\mathrm{d}\vec{\mu}}{\mathrm{d}t} = -\gamma\, \vec{\mu}\times\vec{B}_0 + \frac{\eta}{\mu}\left(\vec{\mu}\times\frac{\mathrm{d}\vec{\mu}}{\mathrm{d}t}\right)$$

$$\vec{h}_{rf} = \left(h_{rf,x}\,\vec{e}_x + h_{rf,y}\,\vec{e}_y + h_{rf,z}\,\vec{e}_z\right)\cdot e^{i\omega t}$$

$$\begin{aligned}
i\omega\mu_x &= -(\gamma B_0 + i\omega\eta)\mu_y - \gamma\mu b_{\mathrm{rf,\,y}} \\
i\omega\mu_y &= \gamma\mu b_{\mathrm{rf,\,x}} + (\gamma B_0 + i\omega\eta)\mu_x \\
i\omega\mu_z &= 0
\end{aligned}$$