Partial Dynamical Symmetry and Odd-Even Staggering in Deformed Nuclei

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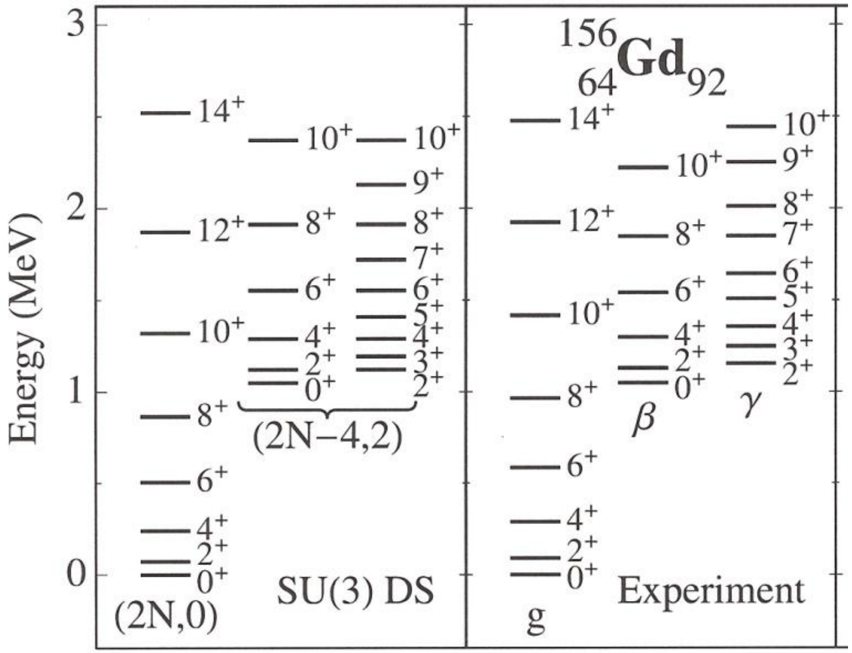

Transition	Exp	DS
$2_1^+ \to 0_1^+$	0.933 *25*	0.933
$4_1^+ \to 2_1^+$	1.312 *25*	1.313
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$2_\beta^+ \to 0_\beta^+$	0.26 *11*	0.679
$4_\beta^+ \to 2_\beta^+$	1.40 *75*	0.951
$0_\beta^+ \to 2_1^+$	0.04 *2*	0.034
$2_\beta^+ \to 0_1^+$	0.0031 *3*	0.0055
$2_\beta^+ \to 2_1^+$	0.0165 *15*	0.0084
$2_\beta^+ \to 4_1^+$	0.0204 *20*	0.020
$4_\beta^+ \to 2_1^+$	0.0065 *35*	0.0067
$4_\beta^+ \to 6_1^+$	0.0105 *55*	0.021

SU(3) Dynamical Symmetry

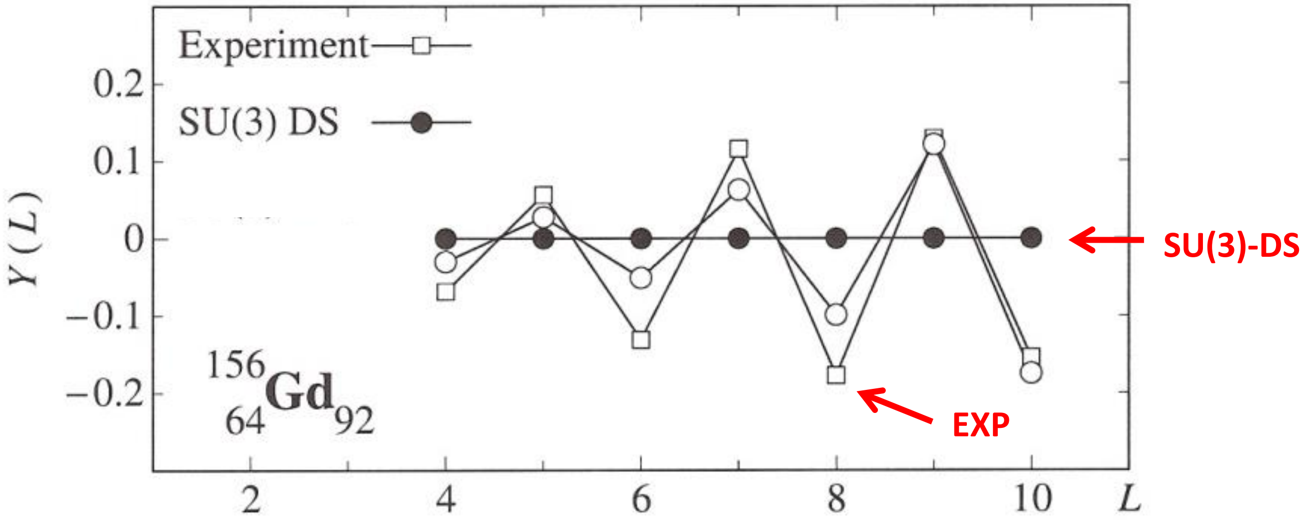
$$U(6) \supset SU(3) \supset SO(3)$$
$$[N] \quad (\lambda,\mu) \quad K \quad L$$

$$\hat{H}_{\mathrm{DS}} = A\,\hat{C}_{\mathrm{SU(3)}} + B\,\hat{C}_{\mathrm{SO(3)}}$$

$$E_{DS} = A\, f(\lambda,\mu) + B\, L(L+1)$$

- Vib. bands: $(\lambda,\mu) = (2N,0)$ $g(K=0_1)$
  $(\lambda,\mu) = (2N-4,2)$ $\beta(K=0_2)$
  $\gamma(K=2_1)$
- Rot. splitting: **L(L+1)** rigid rotor
- ^{156}Gd a good example of SU(3)-DS
- SU(3)-DS provides a good description of **ground (2N,0)** and **β (2N-4,2)** bands

Odd-even staggering in the γ-band



Staggering index
$$Y(L) = \frac{(2L-1)}{L}\left[\frac{E(L)-E(L-1)}{E(L)-E(L-2)}\right] - 1$$

rigid rotor: Y(L)=0 independent of L

• SU(3)-DS: **poor** description of odd-even staggering in the **γ-band**

- SU(3)-DS: **good** description of states in the ground **(2N,0)** and beta **(2N-4,2)** bands
  poor description of odd-even staggering in the gamma band

- IBM: **signature splitting in the γ-band** can be incorporated by the inclusion of at least cubic terms

  17 possible three-body interactions

- Need to select suitable higher-order terms that can **break** the SU(3)-DS in the gamma band but **preserve** it in the ground and beta bands

$\Rightarrow$ **Partial Dynamical Symmetry (PDS)**

Dynamical Symmetry

$$
\begin{array}{ccccc}
G_{\rm dyn} \supset & G & \supset \cdots \supset & G_{\rm sym} \\
\downarrow & \downarrow & & \downarrow \\
[N] & \langle\Sigma\rangle & & \Lambda
\end{array}
$$

$$\hat{H} = \sum_G a_G \hat{C}_G$$

- *Solvability of the **complete** spectrum*
- *Quantum numbers for **all** eigenstates*

$$E = E_{[N]\langle\Sigma\rangle\ldots\Lambda}$$

eigenstates $\quad |[N]\langle\Sigma\rangle\Lambda\rangle$

operators $\quad \hat{T}_{[n]\langle\sigma\rangle\lambda}$

Partial Dynamical Symmetry

- Only **part** of these properties are obeyed

Leviatan, Prog. Part. Nucl. Phys. **66**, 93 (2011)

Partial Dynamical Symmetry

$$G_{\rm dyn} \supset G \supset \cdots \supset G_{\rm sym}$$

$$[N] \quad \langle\Sigma\rangle \quad \Lambda$$

n-particle annihilation operator

$$\boxed{\hat{T}_{[n]\langle\sigma\rangle\lambda} | [N] \langle\Sigma_0\rangle \Lambda\rangle = 0}$$

for **all** possible Λ contained in the irrep $\langle\Sigma_0\rangle$ of G

Equivalently:

$$\boxed{\hat{T}_{[n]\langle\sigma\rangle\lambda} | [N] \langle\Sigma_0\rangle \rangle = 0}$$

| Lowest weight state ⟩

- Condition is satisfied if $\langle\sigma\rangle \otimes \langle\Sigma_0\rangle \notin [N\text{-}n]$

n-body $\hat{H}' = \sum_{\alpha,\beta} A_{\alpha\beta} \hat{T}^{\dagger}_{\alpha} \hat{T}_{\beta}$

$$\hat{H}_{PDS} = \hat{H}_{DS} + \hat{H}'$$

DS is **broken** but **solvability** of states with $\langle\Sigma\rangle = \langle\Sigma_0\rangle$ is **preserved**

Garcia-Ramos, Leviatan, Van Isacker, PRL **102**, 112502 (2009)

Construction of Hamiltonians with SU(3) PDS

U(6) ⊃ SU(3) ⊃ SO(3)
[N] (λ,μ) K L

$$\hat{B}^{\dagger}_{[n](\lambda,\mu)K;\ell m} \qquad \hat{H}_{PDS} = \sum_{\alpha\beta} u_{\alpha\beta}\hat{B}^{\dagger}_{\alpha}\hat{B}_{\beta}$$

$\hat{B}_{\alpha}\,|$**[N] (2N,0)K=0,L**$\rangle$ = 0

for **all** **L** contained in the SU(3) irrep **(λ,μ)** = **(2N,0)**

n = 2 **(λ,μ)** = (0,2)

$$\hat{\theta}_2 \equiv P^{\dagger}_0 P_0 + P^{\dagger}_2 \cdot \tilde{P}_2 = \left[-\hat{C}_2 + 2\hat{N}(2\hat{N}+3)\right]$$

$$P^{\dagger}_0 = d^{\dagger}\cdot d^{\dagger} - 2(s^{\dagger})^2\ ,\ P^{\dagger}_{2,\mu} = 2s^{\dagger}d^{\dagger}_{\mu} + \sqrt{7}(d^{\dagger}d^{\dagger})^{(2)}_{\mu}$$

$$\hat{C}_2 = 2\hat{Q}\cdot\hat{Q} + \frac{3}{4}\hat{L}\cdot\hat{L}$$

n = 3 **(λ,μ)** = (2,2)

$$W^{\dagger}_0 = 5P^{\dagger}_0 s^{\dagger} - P^{\dagger}_2\cdot d^{\dagger} \quad , \quad W^{\dagger}_{2,\mu} = P^{\dagger}_0 d^{\dagger}_{2,\mu} + 2P^{\dagger}_{2,\mu}s^{\dagger}$$

$$V^{\dagger}_{2,\mu} = 6P^{\dagger}_0 d^{\dagger}_{2,\mu} - P^{\dagger}_{2,\mu}s^{\dagger} \ , \ W^{\dagger}_{\ell,\mu} = (P^{\dagger}_2 d^{\dagger})^{(\ell)}_{\mu} \quad \ell = 3,4$$

n = 3 **(λ,μ)** = (0,0)

$$2\Lambda^{\dagger}\Lambda = \hat{C}_3 - 3(2\hat{N}+3)\hat{C}_2 + 4\hat{N}(2\hat{N}+3)(\hat{N}+3)$$

$$\Lambda^{\dagger} = P^{\dagger}_0 s^{\dagger} + P^{\dagger}_2\cdot d^{\dagger}$$

$$\hat{C}_3 = -4\sqrt{7}\,\hat{Q}\cdot(\hat{Q}\times\hat{Q})^{(2)} - \frac{9}{2}\sqrt{3}\,\hat{Q}\cdot(\hat{L}\times\hat{L})^{(2)}$$

$$\hat{H}_{DS} = \xi_1 \hat{\theta}_2 + \xi_2 \Lambda^\dagger \Lambda + \rho \hat{L} \cdot \hat{L}$$

full DS $|[N](\lambda,\mu)K,L\rangle \quad (\lambda,\mu) = (2N-4k-6m, 2k)$ **solvable**

$$\hat{H}_{PDS} = \sum_{\alpha\beta} u_{\alpha\beta} \hat{B}^\dagger_\alpha \hat{B}_\beta$$

PDS $\hat{B}_\alpha |[N](2N,0), K=0, L\rangle = 0$ **g(K=0) solvable, other bands mixed**

$$\hat{H}_{PDS-1} = \hat{H}_{DS} + \xi_3 P^\dagger_0 P_0 + \xi_4 P^\dagger_0 s^\dagger s P_0 + \xi_5 \left(\Lambda^\dagger s P_0 + P^\dagger_0 s^\dagger \Lambda\right)$$

PDS-1

$$\left.\begin{array}{l} P_0 |[N](2N-4k, 2k), K=2k, L\rangle = 0 \\ \Lambda |[N](2N-4k, 2k), K, L\rangle = 0 \end{array}\right\}$$

g(K=0), γ^k(K=2k) solvable

$$\hat{H}_{PDS-2} = \hat{H}_{DS} + \xi_6 W^\dagger_2 \cdot \tilde{W}_2 + \xi_7 W^\dagger_3 \cdot \tilde{W}_3$$

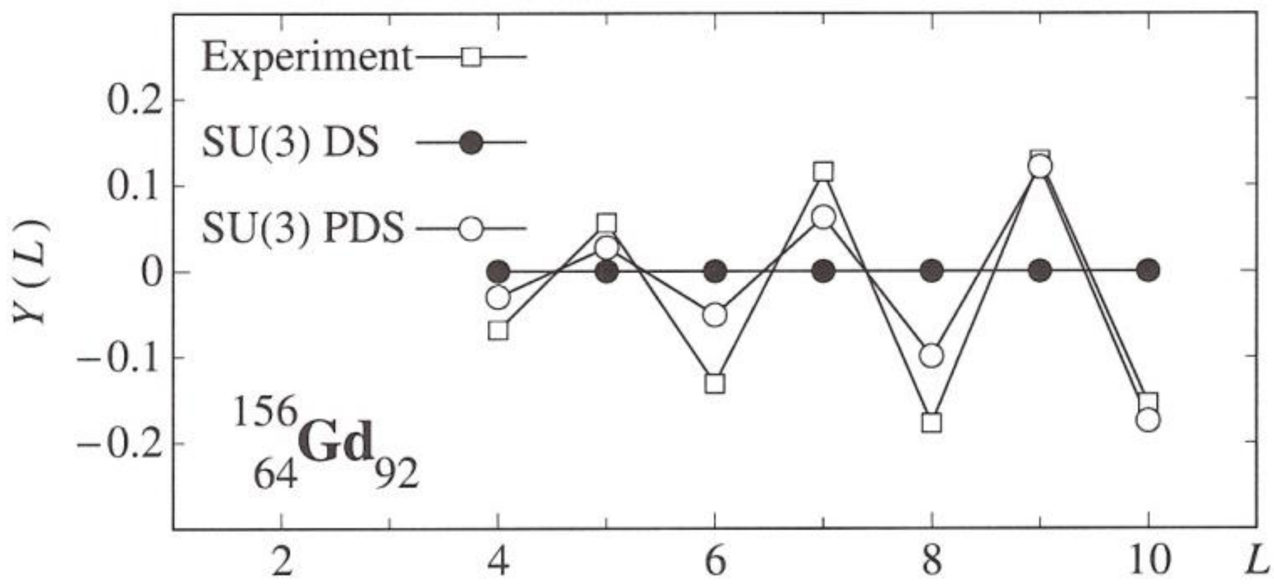
PDS-2 $W_{\ell,\mu} |[N](2N-4,2), K=0, L\rangle = 0 \quad \ell = 2,3$ **g(K=0), β(K=0) solvable**

$\hat{\Omega} = -4\sqrt{3}\,\hat{Q} \cdot (\hat{L} \times \hat{L})^{(2)}$ **solvable,** diagonal in (λ,μ), lifts **K**-degeneracy!

can be expressed in terms of $\hat{H}_{PDS-1}$ and $\hat{H}_{PDS-2}$

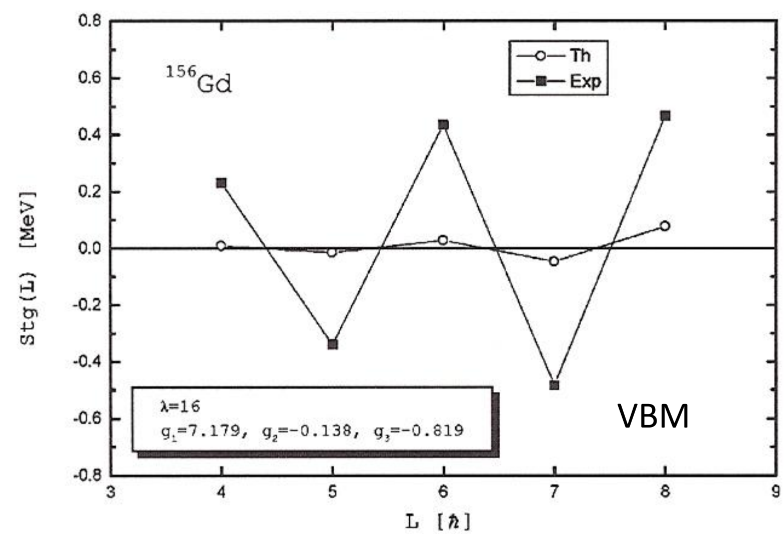
$$\hat{H}_{PDS-2} = \hat{H}_{DS} + \xi_6 W_2^\dagger \cdot \tilde{W}_2 + \xi_7 W_3^\dagger \cdot \tilde{W}_3$$

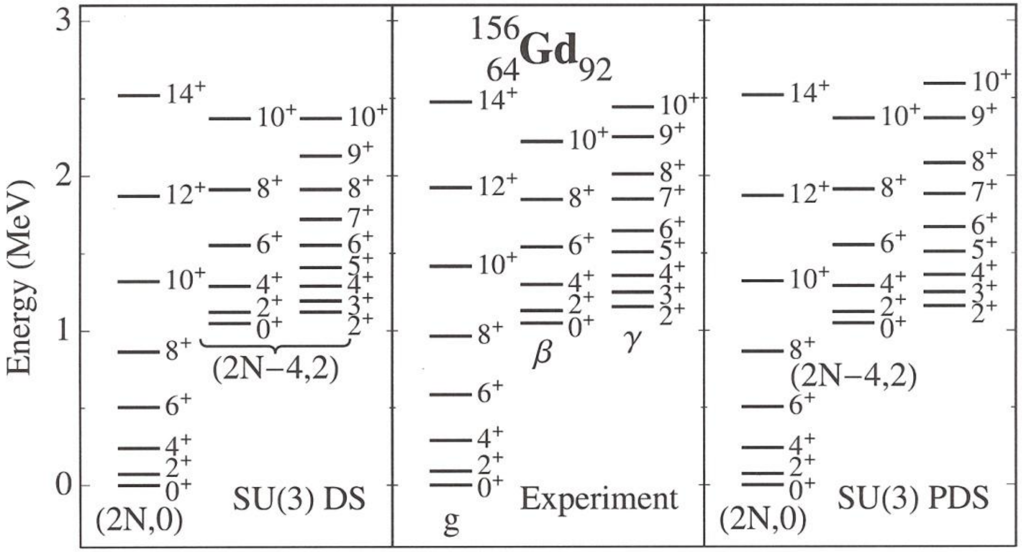
g(2N,0), β(2N-4,2) solvable
γ-band mixed



odd even staggering (OES) in terms of band mixing

- SU(3) conserving: $\hat{H} = \hat{H}_{DS} + \hat{\Omega}$ $\in$ (2N-4,2) (Bonatsos, 1988)
  cannot describe OES in nuclei for which $E_\beta < E_\gamma$ (e.g. ^{156}Gd)
- VBM: **g ,γ** ∈ (λ,2)
  cannot reproduce OES in ^{156}Gd (Minkov et al., 2000)
- SU(3)-PDS: **γ band** mixed with **higher bands**
  (Leviatan, Garcia-Ramos, Van Isacker, 2013)

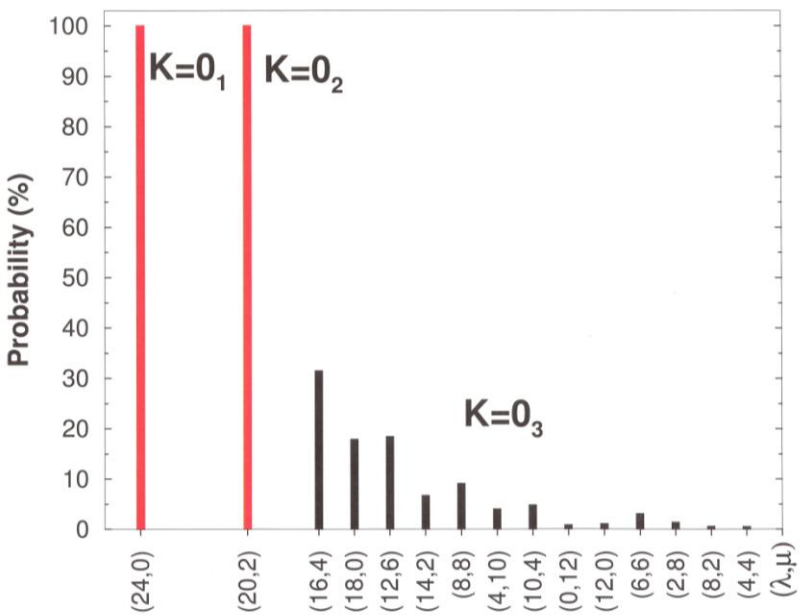




$$\hat{H}_{PDS-2} = \hat{H}_{DS} + \xi_6\, W_2^{\dagger} \cdot \tilde{W}_2 + \xi_7\, W_3^{\dagger} \cdot \tilde{W}_3$$

- **g(2N,0), β(2N-4,2)** **solvable**
  same energy **$E=E_{DS}$** and
  same w.f. $\Rightarrow$ **same B(E2)**

- **Other bands** **strongly mixed**
  γ(K=2_1), K=0_3, etc. non-perturbative



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$4_\beta^+ \to 6_1^+$	0.0105 *55*	0.021	0.021
$2_\gamma^+ \to 0_1^+$	0.0233 *8*	0.035	0.030
$2_\gamma^+ \to 2_1^+$	0.0361 *12*	0.056	0.048
$2_\gamma^+ \to 4_1^+$	0.0038 *2*	0.0037	0.0031
$3_\gamma^+ \to 2_1^+$	0.0364 *70*	0.062	0.053
$3_\gamma^+ \to 4_1^+$	0.0254 *50*	0.032	0.028
$4_\gamma^+ \to 2_1^+$	0.0090 *25*	0.017	0.015
$4_\gamma^+ \to 4_1^+$	0.050 *15*	0.067	0.057
$4_\gamma^+ \to 6_1^+$	—	0.0089	0.0076
$4_\gamma^+ \to 2_\beta^+$	0.0214 *80*	0.0033	0.0096

SU(3) PDS and n-body terms

a) Solvable g(K=0) (2N,0) and β(K=0) (2N-4,2) [n=3]

b) Solvable g(K=0) (2N,0) and γ^k(K=2k) (2N-4k,2k) [n=2, 3]

c) Solvable, diagonal in (λ,μ), lifts K-degeneracy [n=3] $\hat{\Omega} = -4\sqrt{3}\,\hat{Q}\cdot(\hat{L}\times\hat{L})^{(2)}$

- PDS as a selection criterion for higher-order terms

More than half of all possible IBM interactions have SU(3) PDS

Order	Number of interactions		
	General	SU(3) DS	SU(3) PDS
1	$2 \mapsto 1$	$1 \mapsto 0$	$1 \mapsto 0$
2	$7 \mapsto 5$	$3 \mapsto 2$	$4 \mapsto 3$
3	$17 \mapsto 10$	$4 \mapsto 1$	$10 \mapsto 6$
$1+2+3$	$26 \mapsto 16$	$8 \mapsto 3$	$15 \mapsto 9$

Leviatan, Garcia-Ramos, Van Isacker, PRC **87**, 021303(R) (2013)

Summary and Outlook

- Systematic procedure for **identifying** and **selecting** interactions of a given **order** with PDS

- PDS **breaks** the DS but **retains** selected subsets of solvable states with **good symmetry**, while other states are **strongly mixed**

- Higher-order terms with a PDS can be introduced **without destroying** results previously obtained with a DS for a segment of the spectrum

- Several **classes of SU(3)-PDS**
  a) Solvable g(K=0), β(K=0) bands (A.L. et al. PRC 2013; **this talk**)
  b) Solvable g(K=0) and γ^k(K=2k) bands (A.L. PRL 1996; Casten et al. PRL 2014)
  c) Integrity basis term Ω (Draayer et al. NPA 1985, Bonatsos PLB 1988)

- PDS as a **selection criterion for higher-order terms**
  SU(3)-PDS: odd-even staggering ^{156}Gd
  SO(6)-PDS: band anharmonicities ^{196}Pt (Ramos et al., PRL 2009)
  Relevance to extension of beyond MF methods to heavy nuclei

Thank you