



Isospin transfer modes in exotic nuclei

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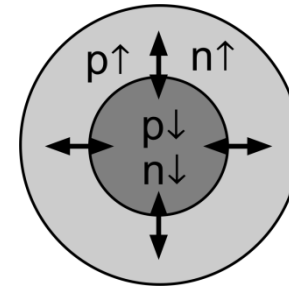
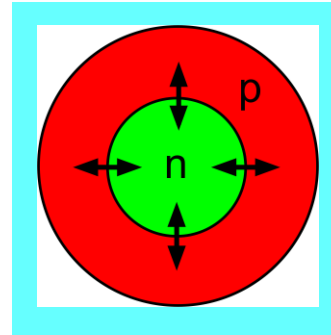
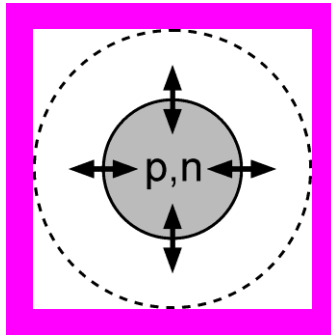


MICHIGAN STATE
UNIVERSITY

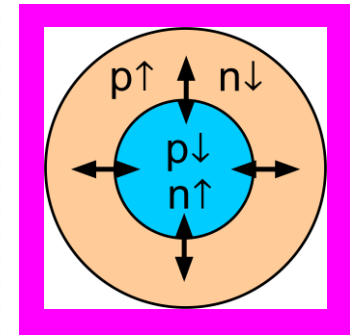
- **Introduction:** modes of nuclear excitations; isospin transfer modes
- **Theory:** building blocks
- **Covariant nuclear field theory (NFT):**
response theory with non-local (in space & time) effective interaction
- **Applications** to nuclear spin-isospin response:
 - Gamow-Teller resonance
 - spin-dipole resonance
- **Summary and outlook**

Nuclear vibrational motion

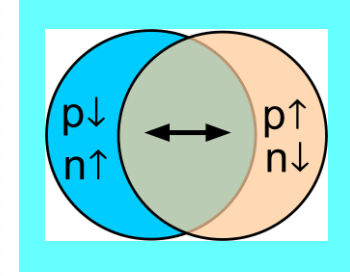
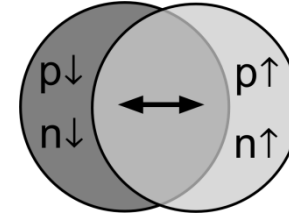
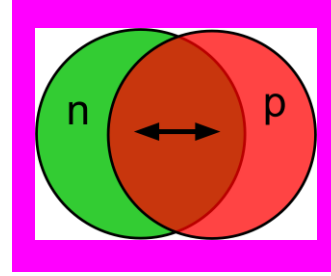
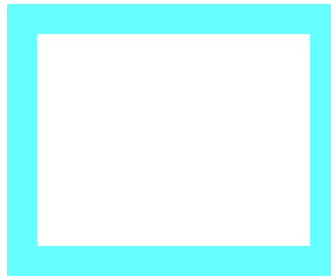
Monopole
 $\Delta L = 0$



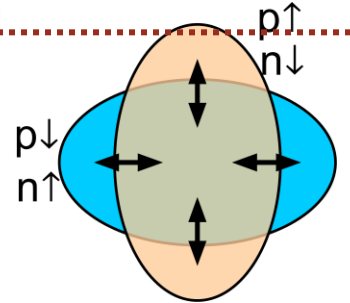
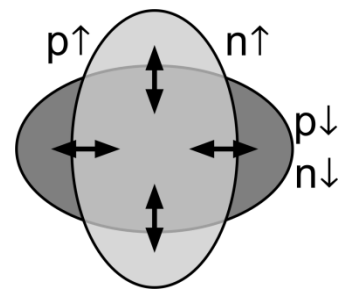
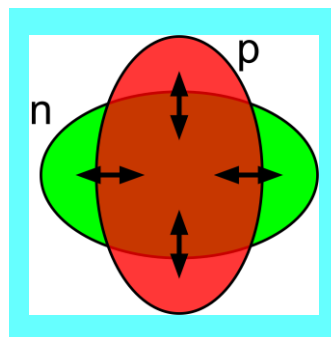
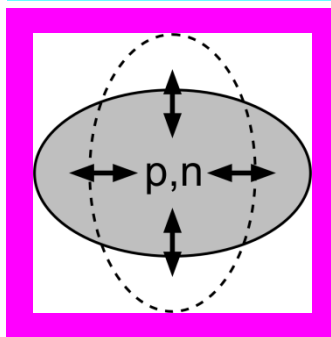
Gamow-Teller



Dipole
 $\Delta L = 1$



Quadrupole
 $\Delta L = 2$



$\Delta T = 0$
 $\Delta S = 0$

$\Delta T = 1$
 $\Delta S = 0$

$\Delta T = 0$
 $\Delta S = 1$

$\Delta T = 1$
 $\Delta S = 1$

Building blocks of nuclear structure models

❖ Degrees of freedom

at ~1-50 MeV excitation energies:

single-particle & collective (vibrational, rotational)

NO complete separation of the scales!

-Coupling between single-particle and collective:

-Coupling to continuum

as nuclei are open quantum systems

❖ Symmetries -> Eqs. of motion

Galilean inv. -> Schrödinger Eq.

Lorentz inv. -> Dirac & Klein-Gordon Eqs.

❖ Interaction V_{NN} : 3 basic concepts

Ab initio: from vacuum V_{NN} -> in-medium V_{NN}

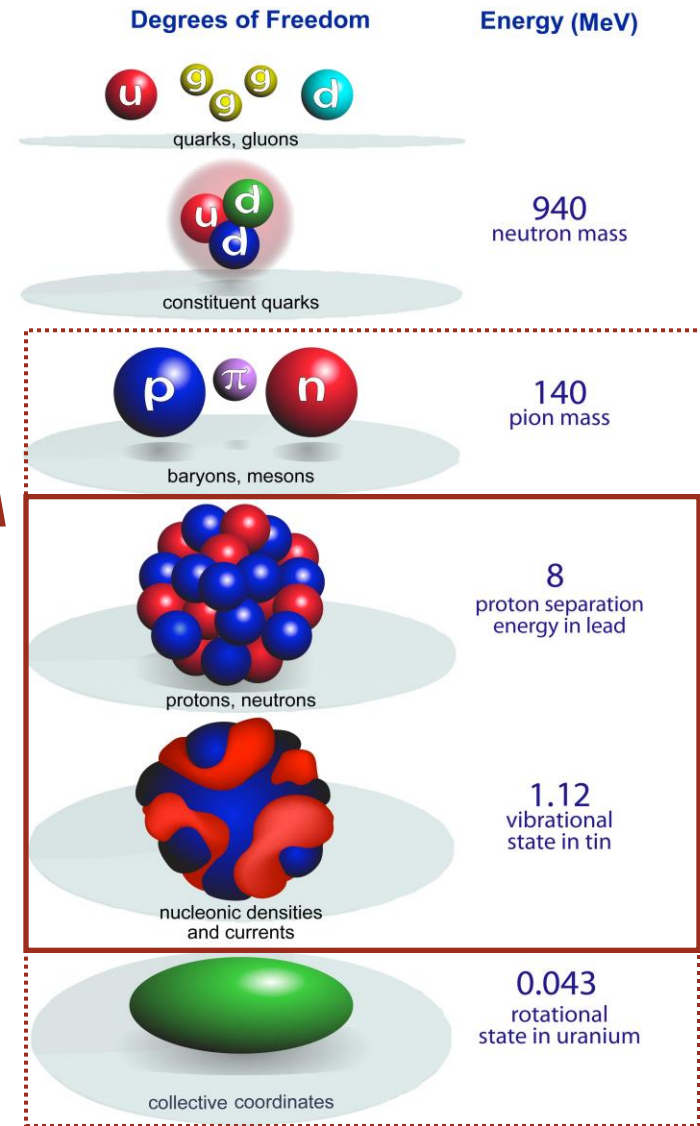
Density functional: an ansatz for in-medium V_{NN}

Configuration interaction: matrix elements for in-medium V_{NN}

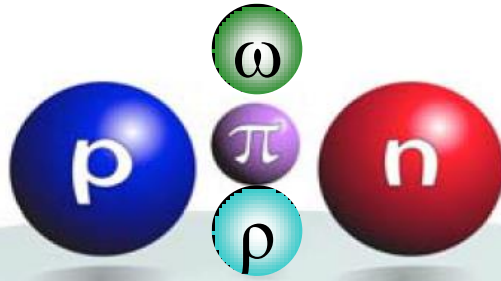
Separation of the scales

Physics of Hadrons

Physics of Nuclei

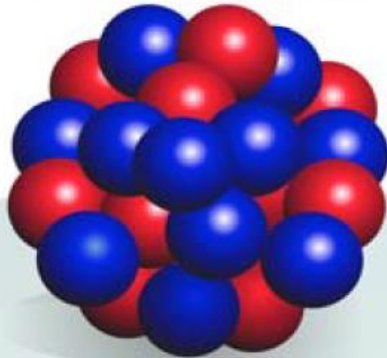


Nucleons, mesons, phonons

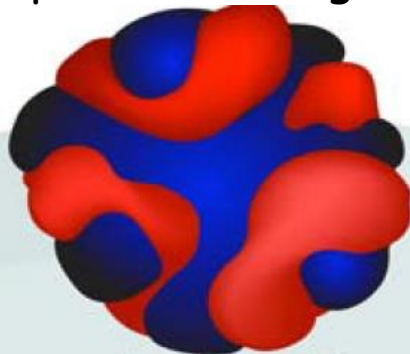


$m_\pi \sim 140 \text{ MeV}$, $m_\rho \sim 770 \text{ MeV}$, $m_\omega \sim 783 \text{ MeV}$

+ superfluidity!

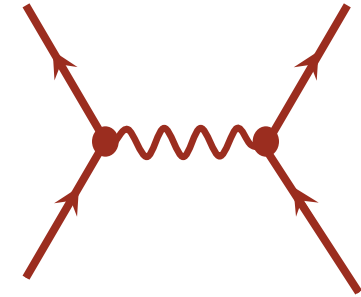


Nucleon separation energies: $\sim 1\text{-}10 \text{ MeV}$

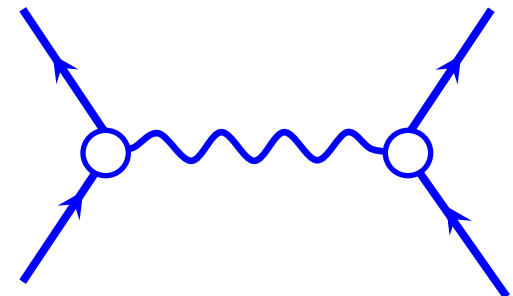


Emerging collective phonons: $\sim 1\text{-}10 \text{ MeV}$

Short range:
Mean-field approximation



Strong coupling:
non-perturbative techniques

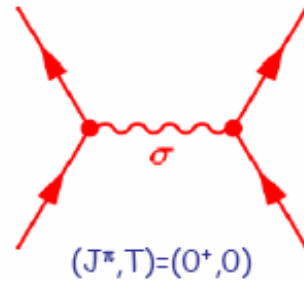


Long range:
Time blocking

Covariant density functional theory

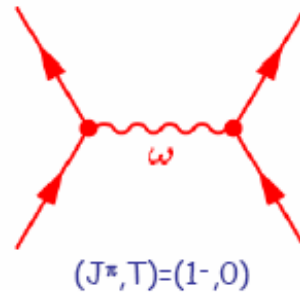
The nuclear fields are obtained by coupling the nucleons through the exchange of effective mesons through an **effective Lagrangian**.

Walecka model
+ later modifications
(P. Ring et al.)



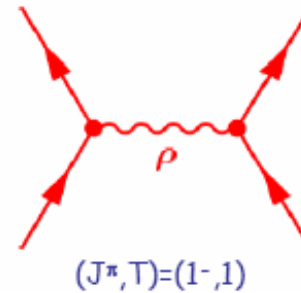
$$S(\mathbf{r}) = g_\sigma \sigma(\mathbf{r})$$

Sigma-meson:
attractive scalar field



$$V(\mathbf{r}) = g_\omega \omega(\mathbf{r}) + g_\rho \vec{\tau} \vec{\rho}(\mathbf{r}) + e A(\mathbf{r})$$

Omega-meson:
short-range repulsive



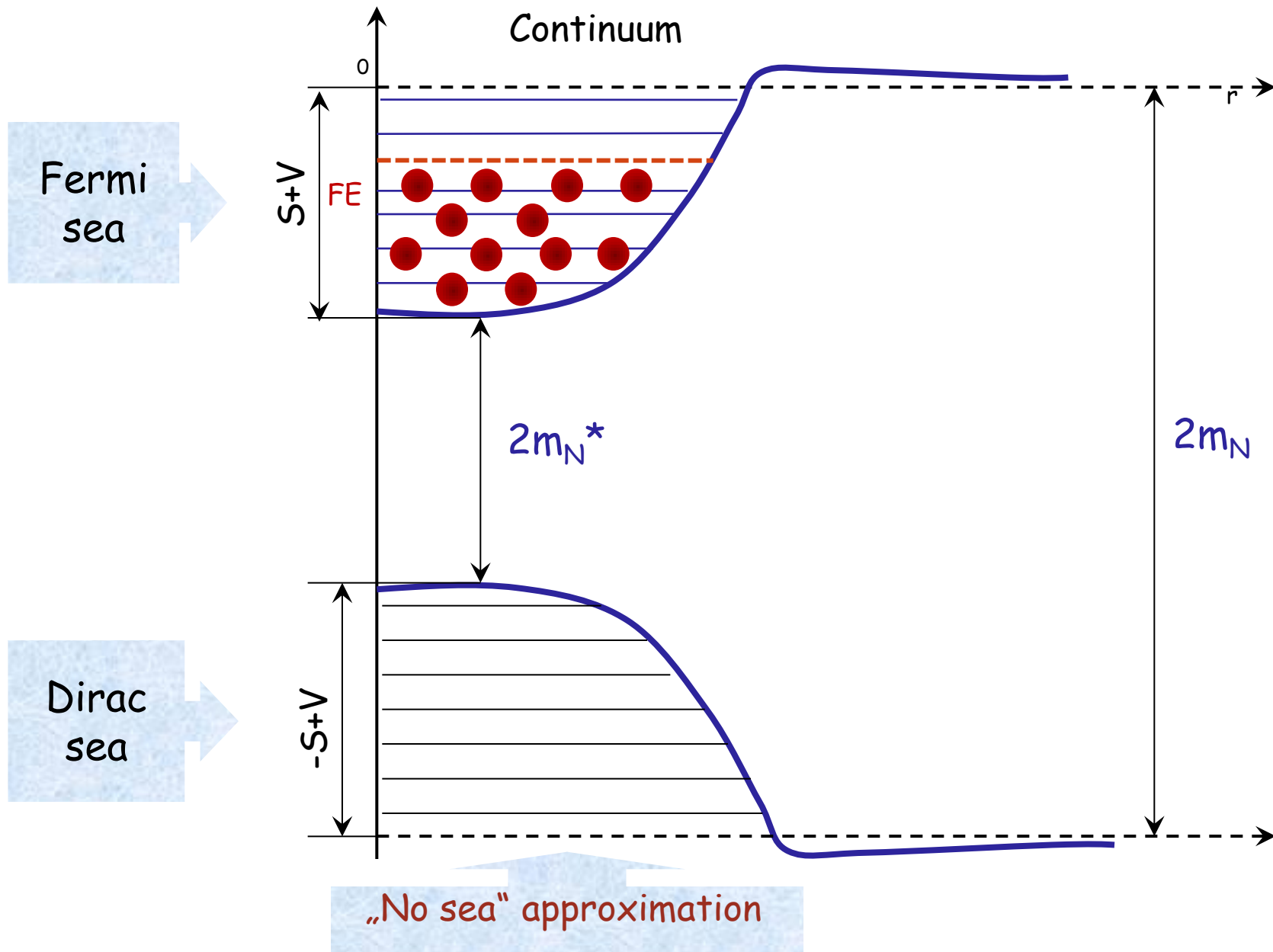
Rho-meson:
isovector field

Lorentz
symmetry

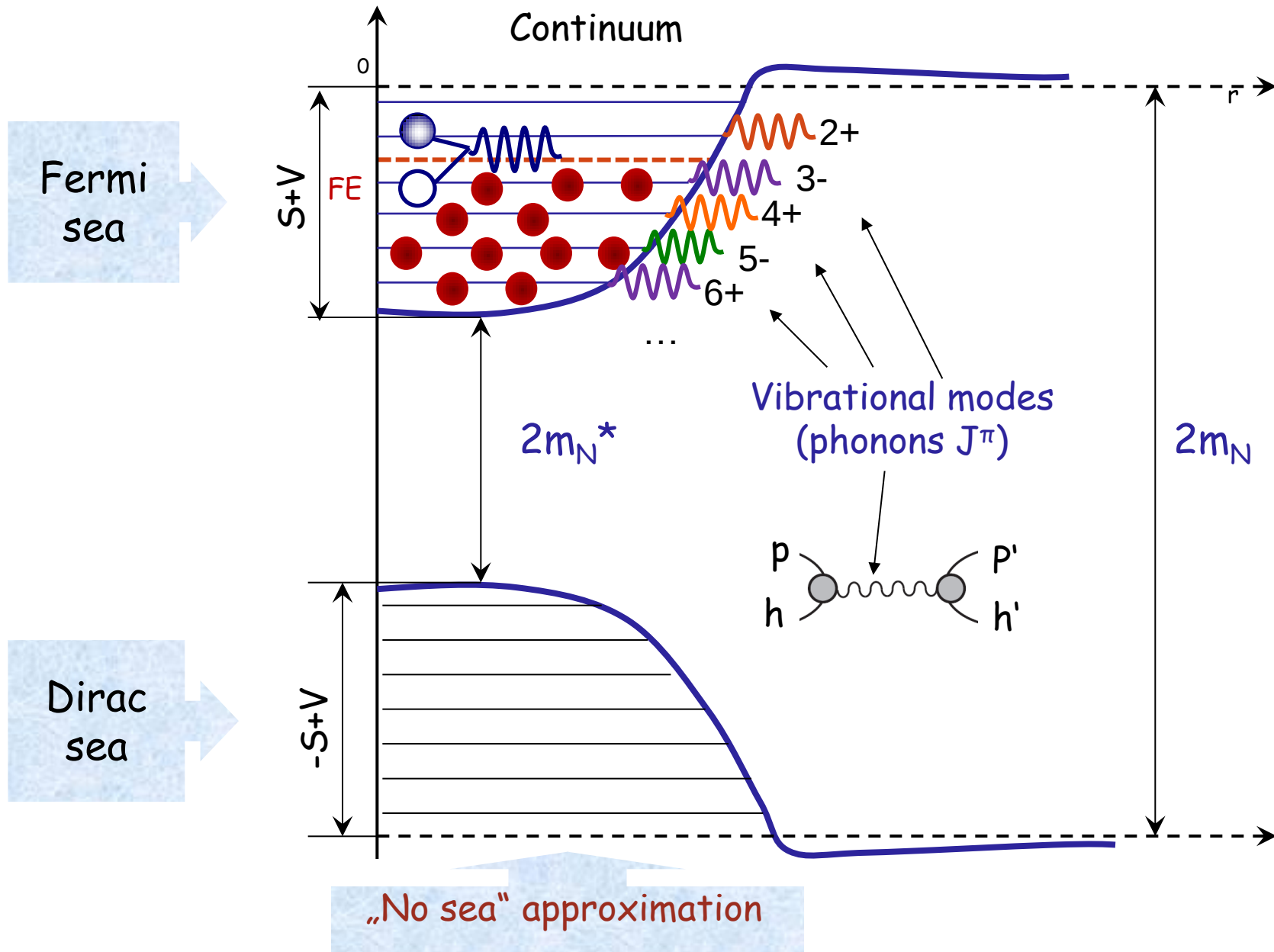
$$E_{RHB}[\hat{\rho}, \hat{\mathcal{U}}, \hat{\mathcal{U}}^*, \phi] = E_{RMF}[\hat{\rho}, \phi] + E_{pair}[\hat{\mathcal{U}}, \hat{\mathcal{U}}^*]$$

$$E_{RMF}[\hat{\rho}, \phi] = \underbrace{Tr[(\alpha \mathbf{p} + \beta m) \hat{\rho}]}_{\text{nucleons}} + \underbrace{\sum_m \left\{ Tr[(\beta \Gamma_m \phi_m) \hat{\rho}] \mp \int \left[\frac{1}{2} (\nabla \phi_m)^2 + U(\phi_m) \right] d^3 r \right\}}_{\text{interaction}} \underbrace{\quad}_{\substack{\text{mesons} \\ \text{(as classical fields)}}}$$

Uncorrelated ground state as the zero approximation: Relativistic Mean Field (RMF)

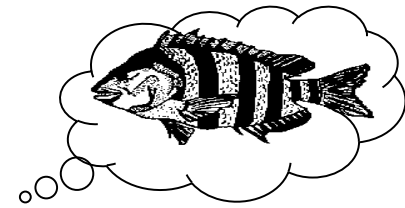
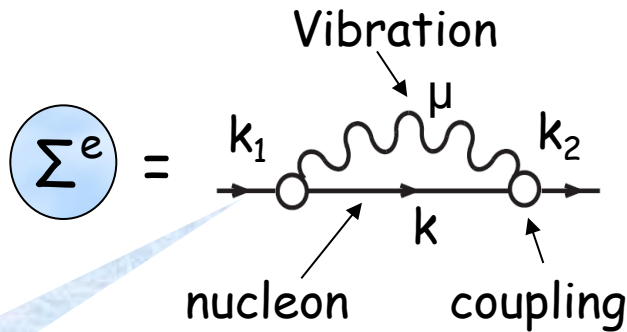
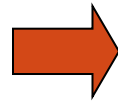


Small perturbation => Coherent oscillations of the mean nuclear potential



First step beyond relativistic mean field: quasiparticles coupled to vibrations

Additional "potential"
= "self-energy" =
= "mass operator"
with energy dependence



"Fish" diagram

Nambu-Gor'kov
2 x 2 matrices:

$$\rightarrow = \begin{pmatrix} \rightarrow & \rightarrow \\ \leftarrow & \leftarrow \end{pmatrix}$$

One-body propagator G :
Dyson's equation

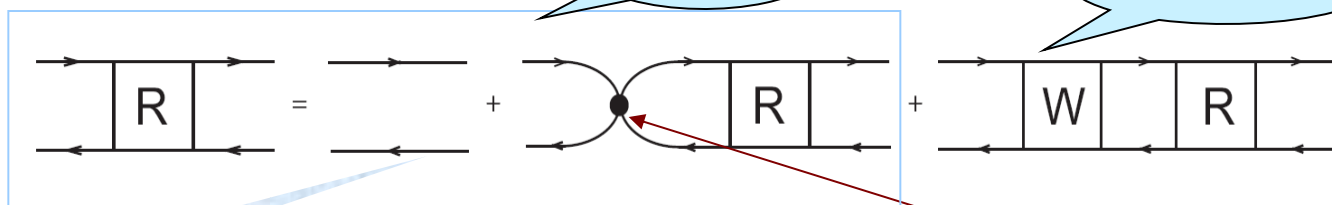
$$(\varepsilon - \hat{\mathcal{H}}_{RHB} - \hat{\Sigma}^{(e)}(\varepsilon)) \hat{G}(\varepsilon) = 1$$

Energy dependence

$$\begin{array}{c} \text{--- } k \quad k' \text{ ---} \\ G \end{array} = \begin{array}{c} \text{--- } k \quad k' \text{ ---} \\ G_0 \end{array} + \begin{array}{c} \text{--- } k \quad k_1 \text{ ---} \\ G_0 \end{array} \begin{array}{c} \text{--- } \Sigma^e \text{ ---} \\ \Sigma^e \end{array} \begin{array}{c} \text{--- } k_2 \quad k' \text{ ---} \\ G \end{array}$$

Excited states: linear response function

Bethe-Salpeter Equation (BSE):



QRPA

Extension

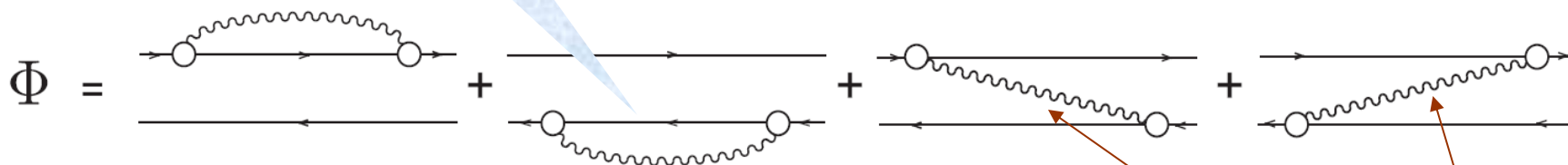
$$\rightarrow = \begin{bmatrix} \rightarrow & \rightarrow \\ \leftarrow & \leftarrow \end{bmatrix}$$

$$R(\omega) = A(\omega) + A(\omega) [V + W(\omega)] R(\omega)$$

$$V = \frac{\delta \Sigma^{\text{RMF}}}{\delta \rho}$$

$$W(\omega) = \Phi(\omega) - \Phi(0)$$

Self-consistency



$$i \frac{\delta}{\delta G} \rightarrow \text{diagram with } G \text{ and a red X} = i \frac{\delta \Sigma^e}{\delta G} = \text{diagram with a wavy line and a vertex}$$

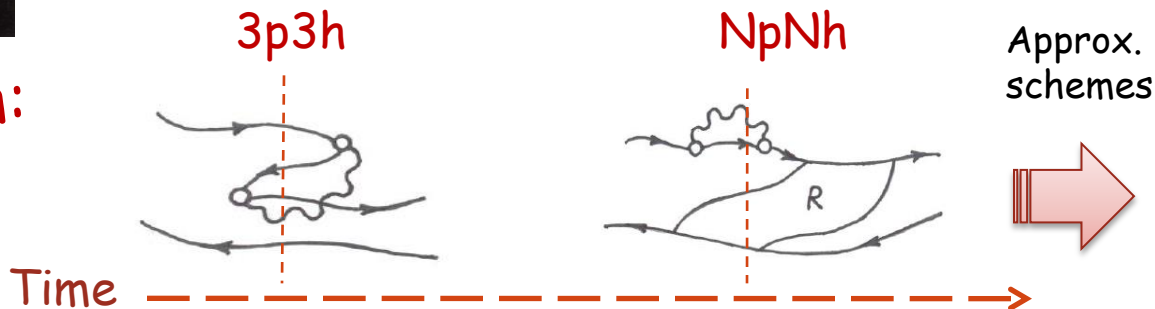
$$U^e = i \frac{\delta \Sigma^e}{\delta G}$$

Consistency on 2p2h-level

Time blocking

Problem:

'Melting' diagrams



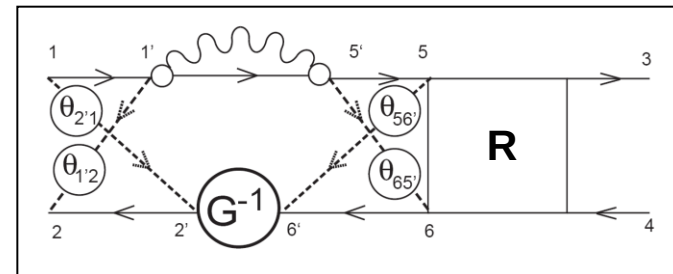
Unphysical result:
negative
cross sections

Solution:

Time-
projection
operator:

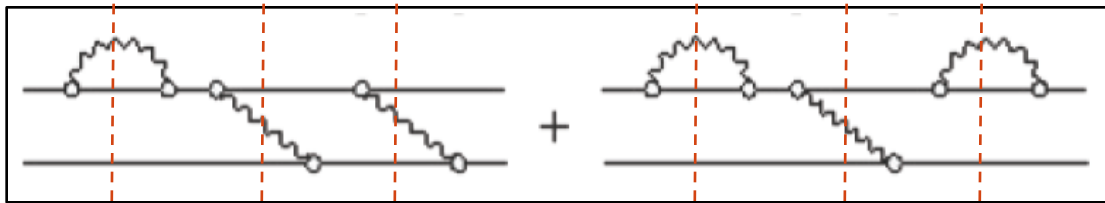
$$\delta_{\sigma_1 - \sigma_2} \theta(\sigma_1 t_{2'1}) = 1 \rightarrow \text{diagram with } \theta_{2'1}$$

$$\delta_{\sigma_2 - \sigma_{1'}} \theta(\sigma_1 t_{1'2}) = 2 \leftarrow \text{diagram with } \theta_{1'2}$$

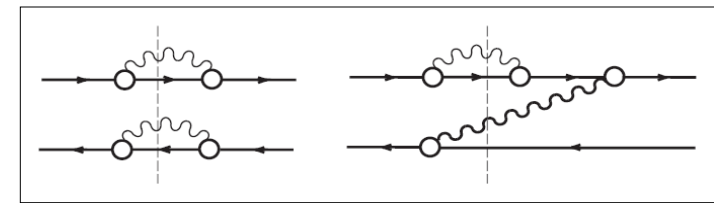


Partially
fixed

Allowed terms: 1p1h, 2p2h



Blocked terms: 3p3h, 4p4h,...



Time blocking approximation =
= „one-fish“ approximation!

- Separation of the integrations in the BSE kernel
- R has a simple-pole structure (spectral representation)
 - »» Strength function is positive definite!

Spin-isospin response function

response

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega) \bar{W}(\omega) R(\omega)$$

interaction

$$\bar{W}(\omega) = \underbrace{V_\rho + V_\pi + V_{\delta\pi}}_{\text{Static: RRPA}} + \underbrace{\Phi(\omega)}_{\text{Dynamic: particle-vibration coupling}} - \underbrace{\Phi(0)}_{\text{Subtraction to avoid double counting}}$$

Subtraction
to avoid double
counting

Static:
RRPA

$$V_\rho(1, 2) = g_\rho^2 \vec{\tau}_1 \vec{\tau}_2 (\beta \gamma^\mu)_1 (\beta \gamma_\mu)_2 D_\rho(\mathbf{r}_1, \mathbf{r}_2)$$

$$V_\pi(1, 2) = -\left(\frac{f_\pi}{m_\pi}\right)^2 \vec{\tau}_1 \vec{\tau}_2 (\boldsymbol{\Sigma}_1 \nabla_1) (\boldsymbol{\Sigma}_2 \nabla_2) D_\pi(\mathbf{r}_1, \mathbf{r}_2),$$

$$V_{\delta\pi}(1, 2) = g' \left(\frac{f_\pi}{m_\pi}\right)^2 \vec{\tau}_1 \vec{\tau}_2 \boldsymbol{\Sigma}_1 \boldsymbol{\Sigma}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

Dynamic:

particle-
vibration
coupling

in time blocking
approximation

$$\begin{aligned} \Phi_{k_1 k_4, k_2 k_3}^\eta(\omega) = & \\ = \sum_{\mu\xi} \delta_{\eta\xi} \bigg[& \delta_{k_1 k_3} \sum_{k_6} \frac{\gamma_{\mu; k_6 k_2}^{\eta; -\xi} \gamma_{\mu; k_6 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_1} - E_{k_6} - \Omega_\mu} + \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma_{\mu; k_1 k_5}^{\eta; \xi} \gamma_{\mu; k_3 k_5}^{\eta; \xi*}}{\eta\omega - E_{k_5} - E_{k_2} - \Omega_\mu} \\ & - \left(\frac{\gamma_{\mu; k_1 k_3}^{\eta; \xi} \gamma_{\mu; k_2 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_3} - E_{k_2} - \Omega_\mu} + \frac{\gamma_{\mu; k_3 k_1}^{\eta; \xi*} \gamma_{\mu; k_4 k_2}^{\eta; -\xi}}{\eta\omega - E_{k_1} - E_{k_4} - \Omega_\mu} \right) \bigg] \end{aligned}$$

Response to an external field: strength function

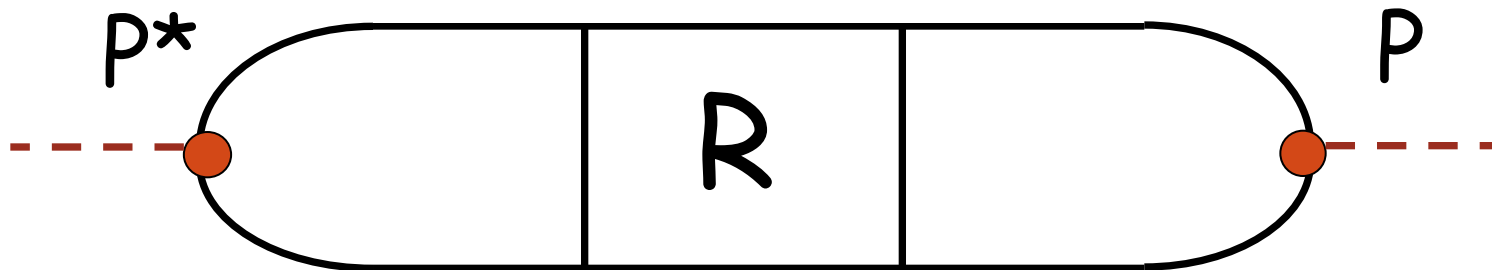
Strength function:

$$S(E) = -\frac{1}{\pi} \lim_{\Delta \rightarrow +0} \text{Im} \Pi_{PP}(E + i\Delta)$$

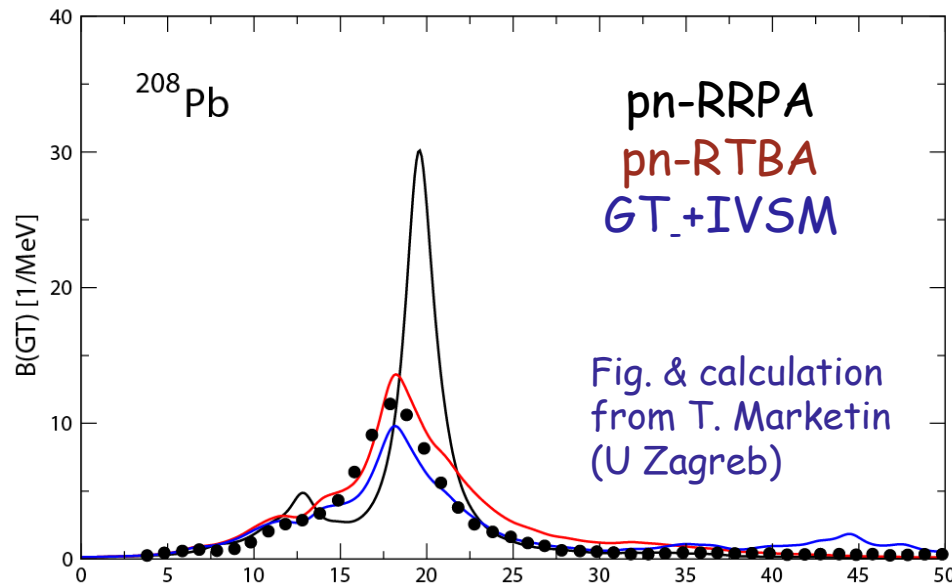
Polarizability:

$$\Pi_{PP}(\omega) = P^\dagger R(\omega) P := \sum_{k_1 k_2 k_3 k_4} P_{k_1 k_2}^* R_{k_1 k_4, k_2 k_3}(\omega) P_{k_3 k_4}$$

External
field

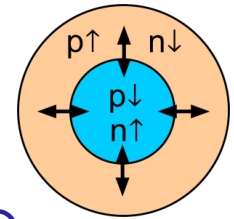


Gamow-Teller Resonance with finite momentum transfer



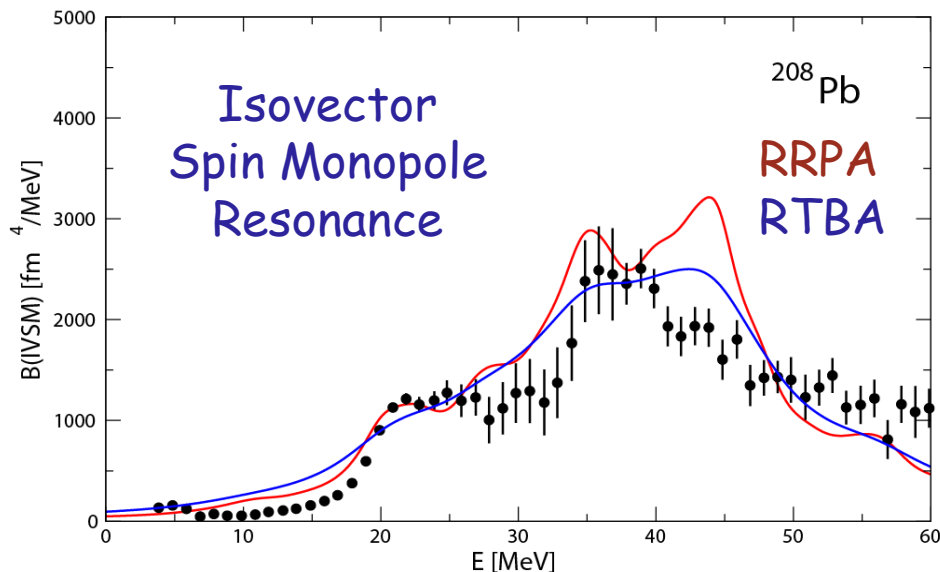
$$P = \sum_i \sigma^{(i)} \tau_{\pm}^{(i)}$$

Finite q :
a correction for
Isovector spin monopole
resonance
(IVSMR) - overtone of GTR



$$\begin{aligned} \Delta L &= 0 \\ \Delta T &= 1 \\ \Delta S &= 1 \end{aligned}$$

$$i_0(qr) \approx 1 - \frac{q^2 r^2}{6} + \dots$$

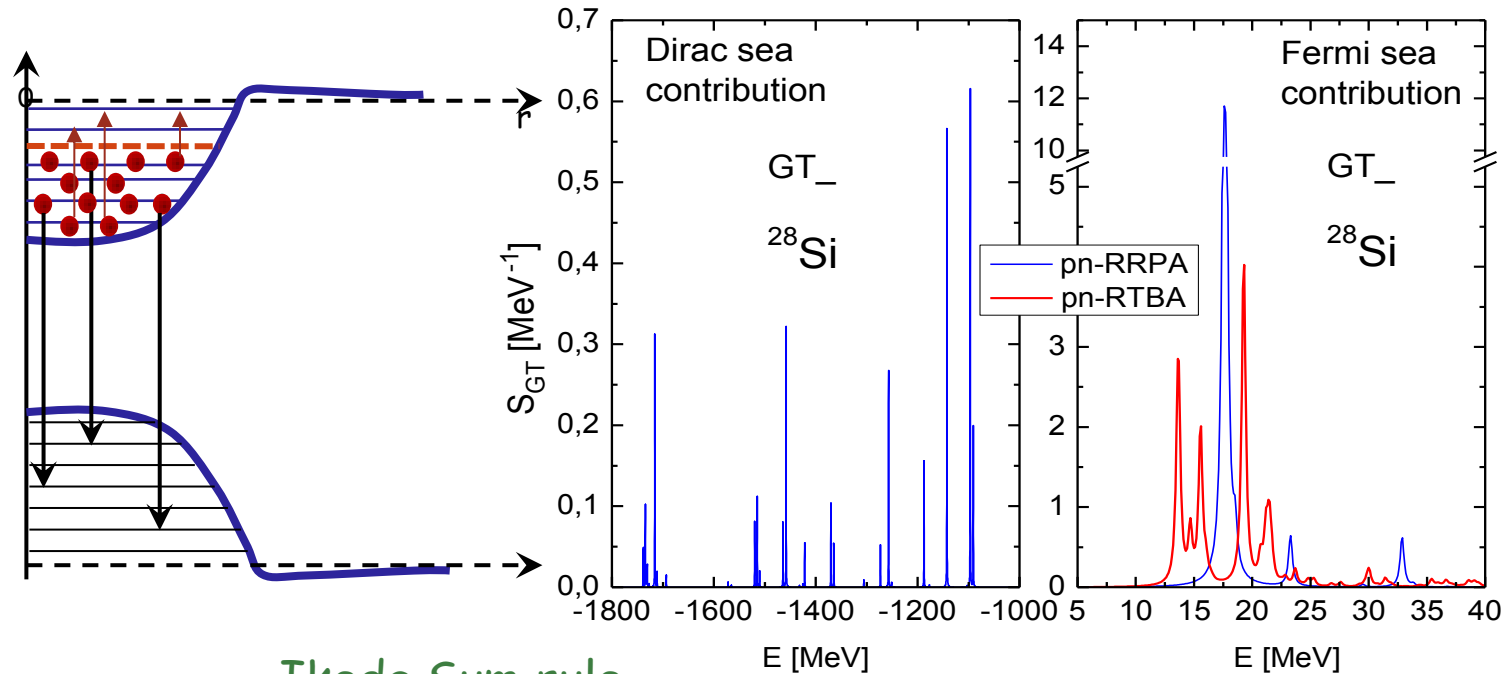


„Microscopic“ quenching of $B(GT)$:

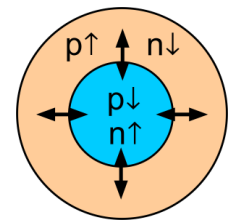
- (i) relativistic effects,
- (ii) ph+phonon configurations,
- (iii) finite momentum transfer

Spin-isospin response: Gamow-Teller Resonance in ^{28}Si

„Proton-neutron“ relativistic time blocking approximation (pn-RTBA): ρ , π , phonons



$$P = \sum_i \sigma^{(i)} \tau_{\pm}^{(i)}$$

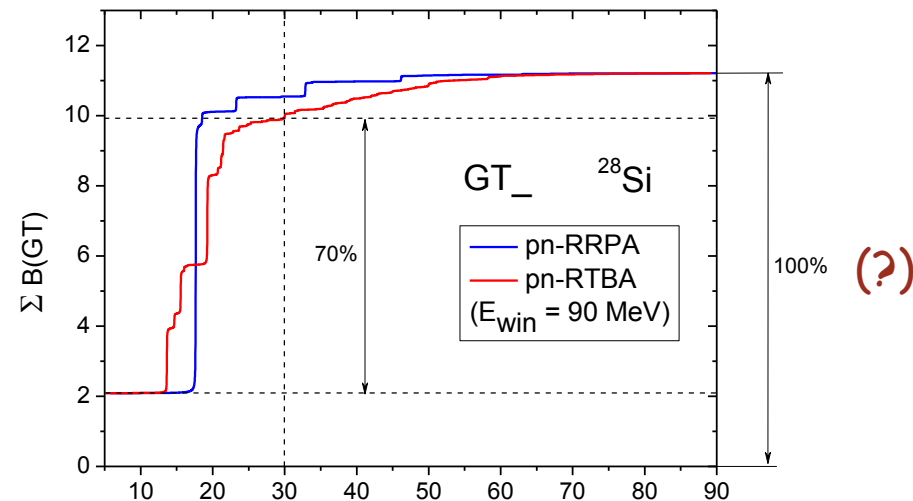


$$\begin{aligned} \Delta L &= 0 \\ \Delta T &= 1 \\ \Delta S &= 1 \end{aligned}$$

Ikeda Sum rule
(model independent):

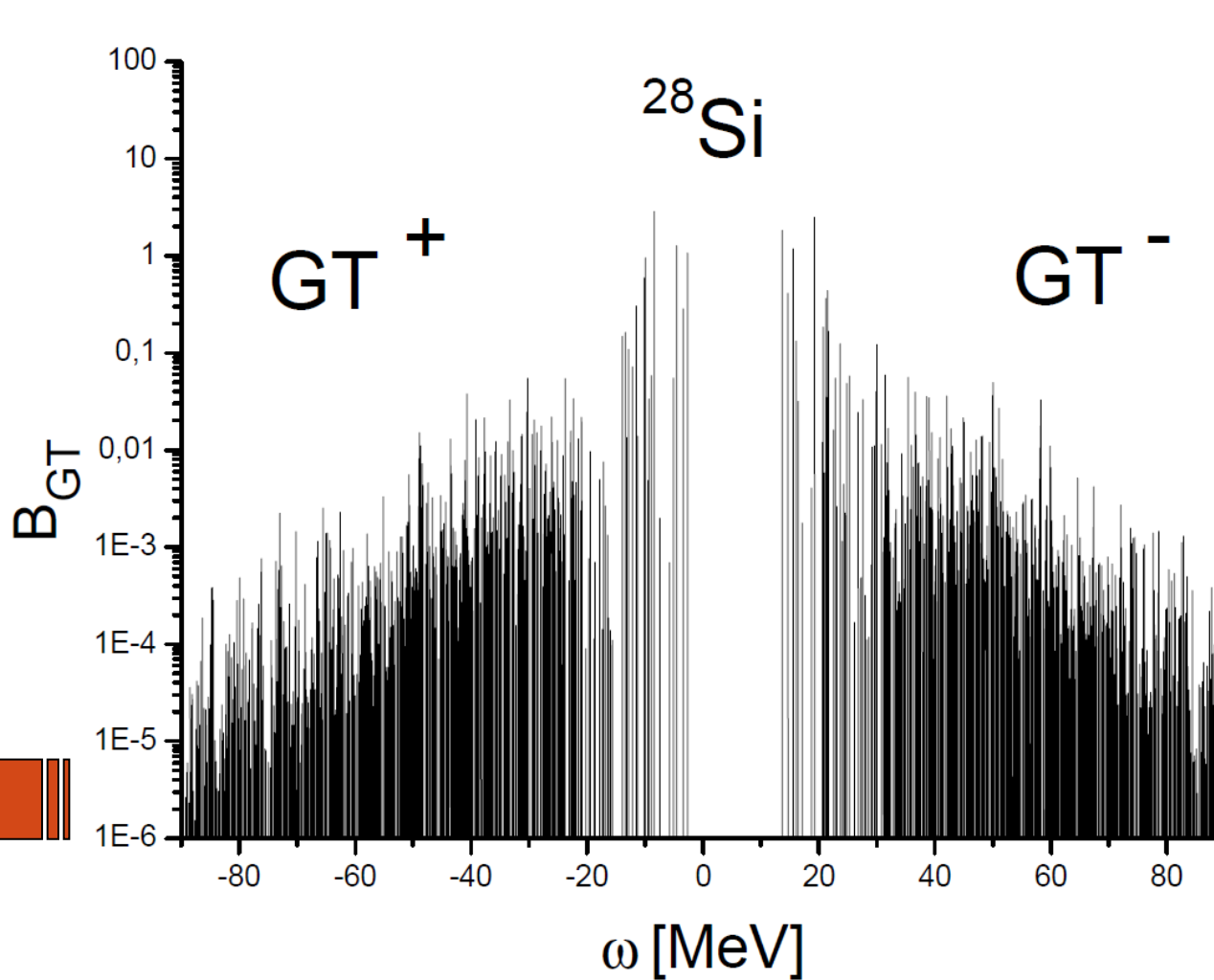
$$\begin{aligned} S_- - S_+ &= 3(N - Z), \\ S_{\pm} &= \sum B(\text{GT} \pm) \end{aligned}$$

„Microscopic“ quenching of $B(\text{GT})$:
(i) relativistic effects,
(ii) ph+phonon configurations,

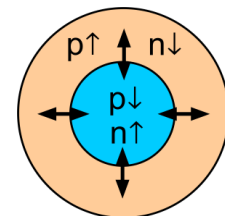




$^{28}\text{Si}: N=Z$



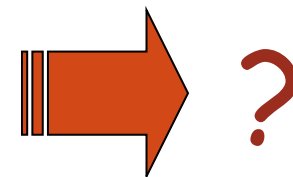
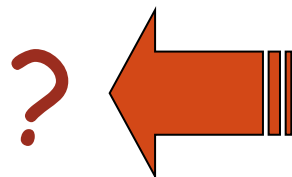
$$P = \sum_i \sigma^{(i)} \tau_{\pm}^{(i)}$$



$$\Delta L = 0$$

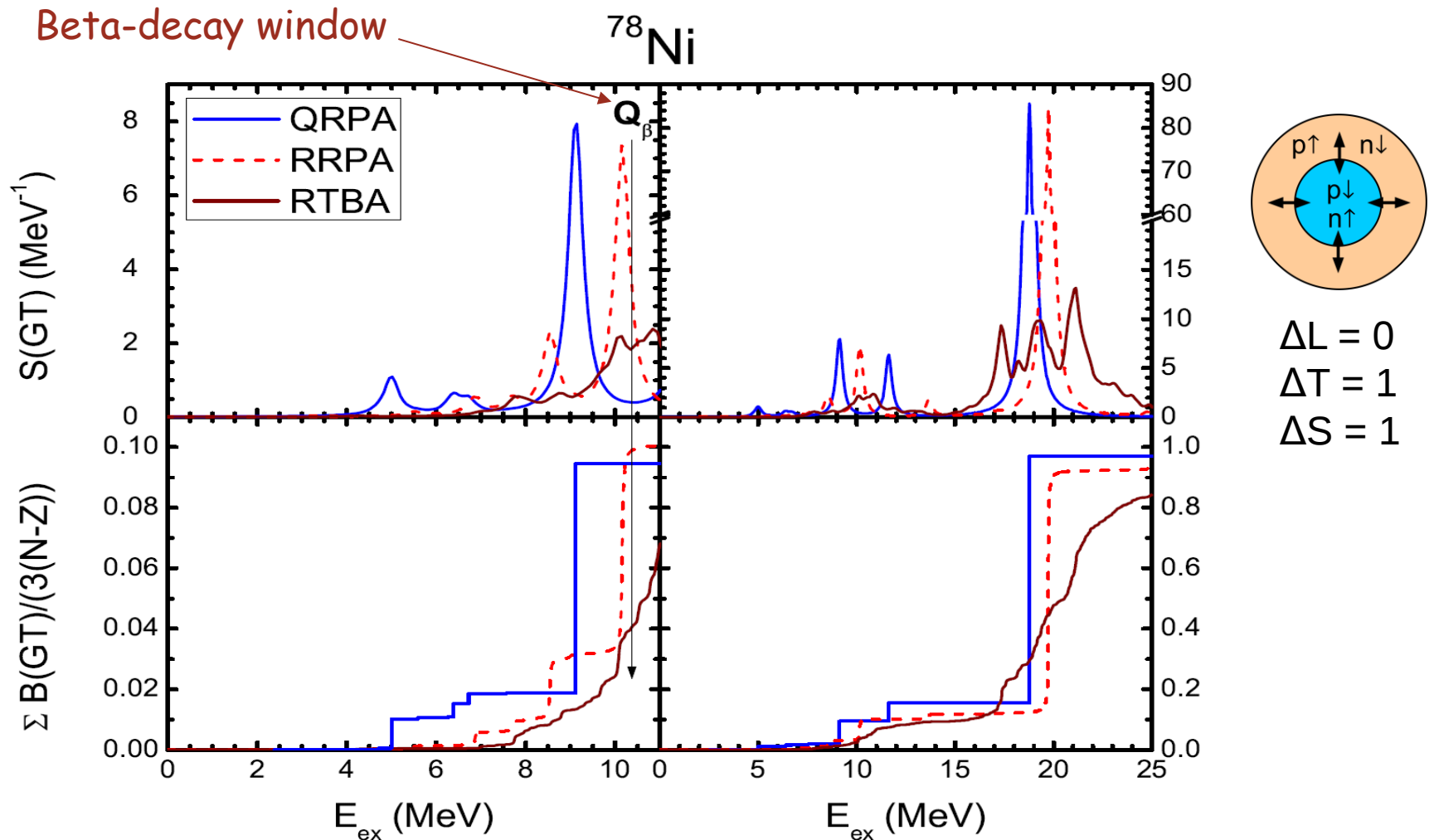
$$\Delta T = 1$$

$$\Delta S = 1$$



Problem: finite basis

GTR in ^{78}Ni : G-matrix+QRPA, RRPA and RTBA



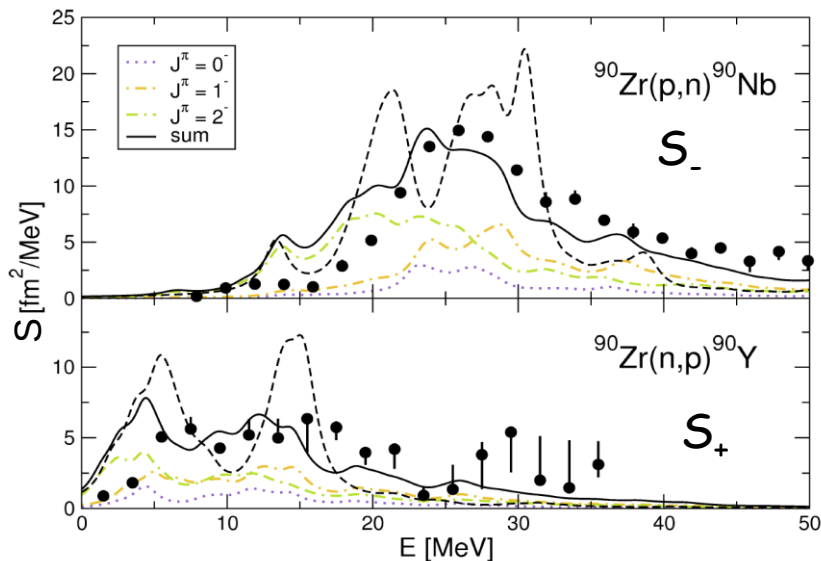
G-matrix+QRPA based on Skyrme DFT with $m^* = 1$ (D.-L. Fang & A. Fässler & B.A. Brown)

RTBA: Relativistic RPA + phonon coupling (T. Marketin & E.L.)

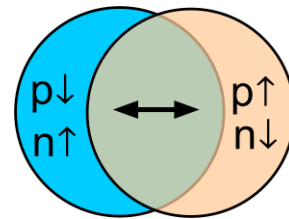
E.L., B.A. Brown, D.-L. Fang, T. Marketin, R.G.T. Zegers, PLB 730, 307 (2014)

Spin-dipole resonance: beta-decay, electron capture

T. Marketin, E.L., D. Vretenar, P. Ring,
PLB 706, 477 (2012).



$$P_{\pm}^{\lambda} = \sum_i r_i \left[\sigma^{(i)} \otimes Y_1(\hat{r}_i) \right]_{\lambda} \tau_{\pm}^{(i)}$$



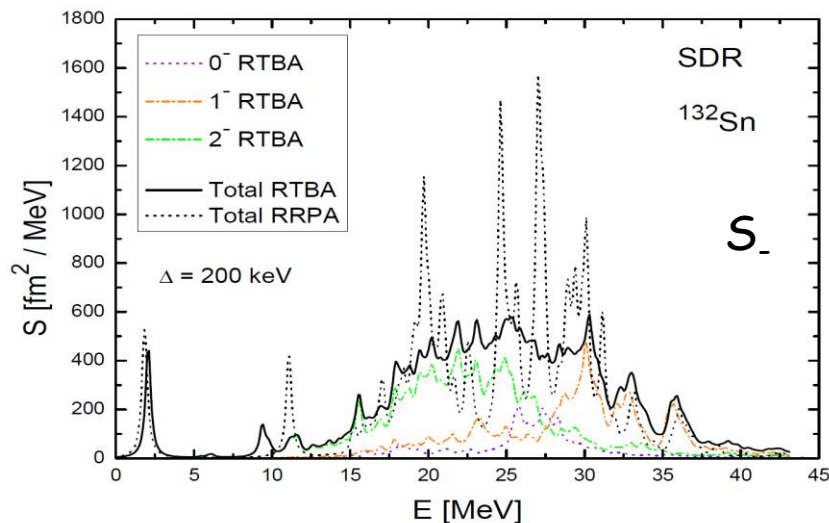
----- RQRPA
—— RQTBA

$$\begin{aligned} \Delta L &= 1 \\ \Delta T &= 1 \\ \Delta S &= 1 \end{aligned} \quad \lambda = 0, 1, 2$$

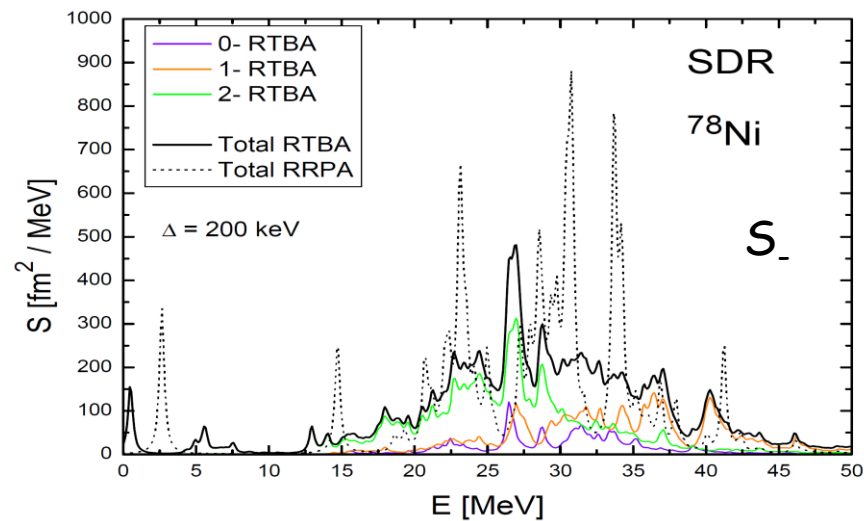
Skin thickness: $\delta_{np} = \sqrt{\langle r^2 \rangle_n} - \sqrt{\langle r^2 \rangle_p}$

Sum rule: $S_{-}^{\lambda} - S_{+}^{\lambda} = \frac{2\lambda + 1}{4\pi} \left(N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p \right)$

Measured recently at RIKEN:



To be measured in the future (?)





Summary and outlook

Conclusions:

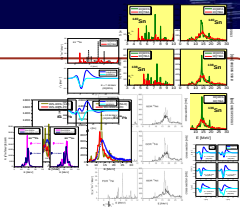
- Fully self-consistent and covariant response theory (pn-RTBA) is formulated and applied to spin-isospin excitations in ordinary and exotic nuclei
- The approach is tested and benchmarked by comparison to data and to other existing approaches (non-rel. QRPA & shell-model)
- The pn-RTBA has a great potential to describe simultaneously the overall strength distribution up to high excitation energy, fine structure and quenching of spin-isospin strength
- **Applications** to beta-decay, double beta-decay, r-process etc.

Perspectives:

- Extension to superfluid systems (underway)
- Implementation of higher-order correlations, two-body currents
- Understanding the role of 2- states in the phonon basis, connecting QVC and tensor forces
- Solution for the finite-basis problem, ...

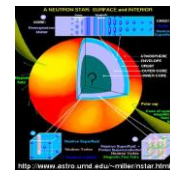
Outlook

New Consistent input
for r-process
nucleosynthesis



Applications

Nuclear matter,
Neutron stars, ...

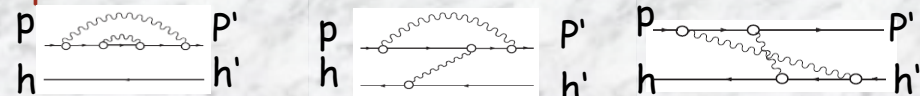


Data => Constraints
from RIB facilities

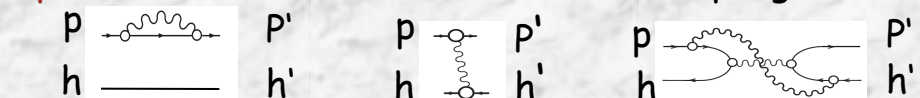
np-nh

$$\Phi^{(n+1)} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]}$$

3p3h excitations: iterative PVC



2p2h excitations: Particle-Vibration Coupling



1p1h excitations: RQRPA



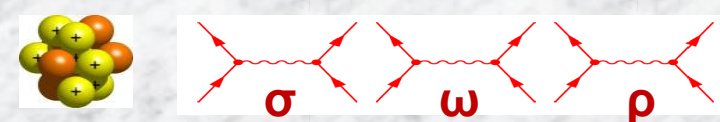
Self-consistency

$$V = \frac{\delta^2 E[R]}{\delta R^2}$$

Data => Constraints
from RIB facilities

Time-dependent
CEDFT ???

Ground state: Covariant EDFT



$E[R]$

Generalized
CEDFT ???

DD-ME δ CEDFT:
Ab initio Brückner +
4 adjustable parameters
PRC 84, 054309 (2011)

Toward „ab initio“

UNEDF:
DFT from EFT
...

Many thanks for collaboration:

Theory & computing:

Peter Ring (Technische Universität München)

Victor Tselyaev (St. Petersburg State University)

Tomislav Marketin (U Zagreb)

Applications:

B.A. Brown (NSCL),

D.-L. Fang (NSCL)

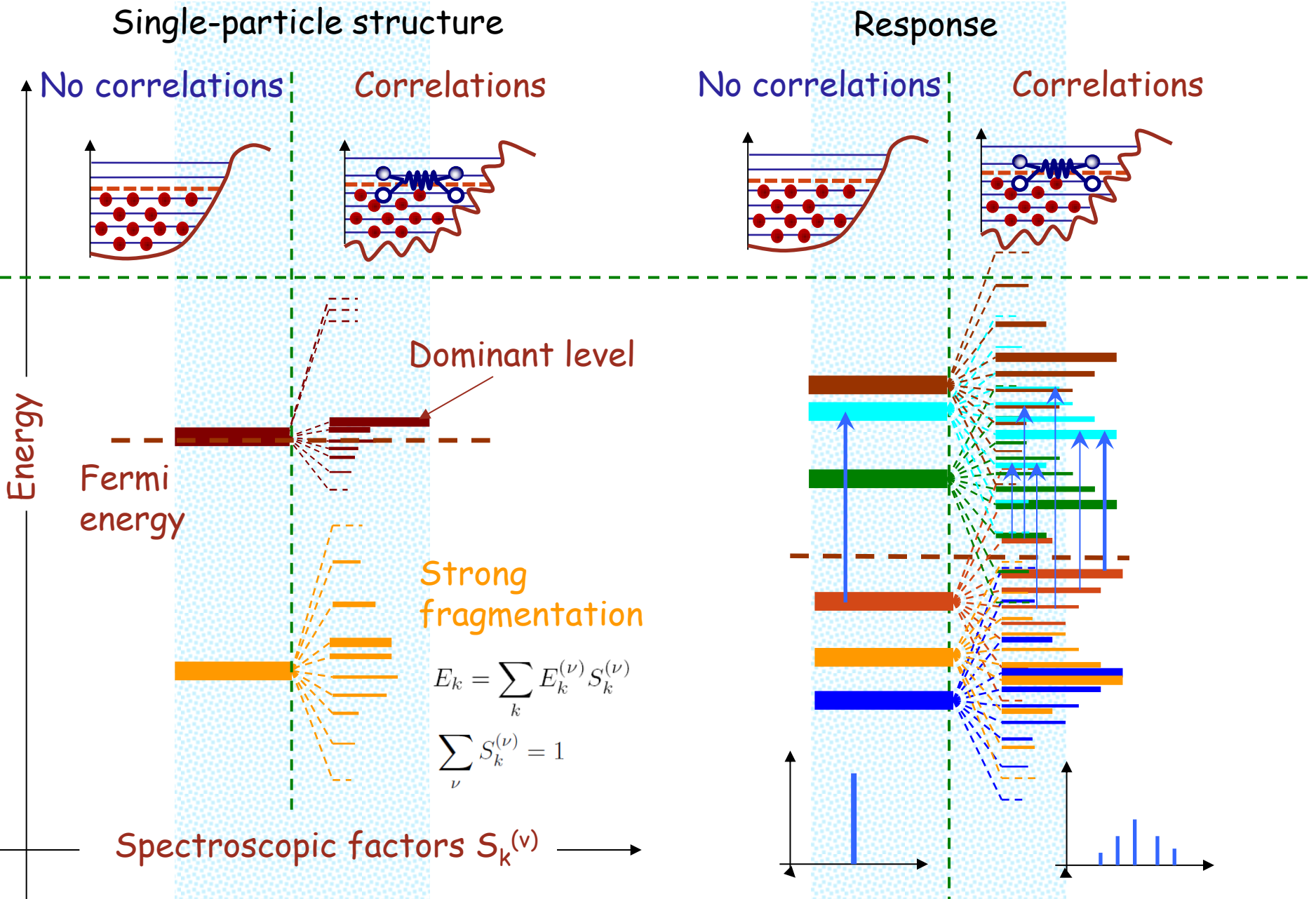
R.G.T. Zegers (NSCL)

V. G. Zelevinsky (NSCL)



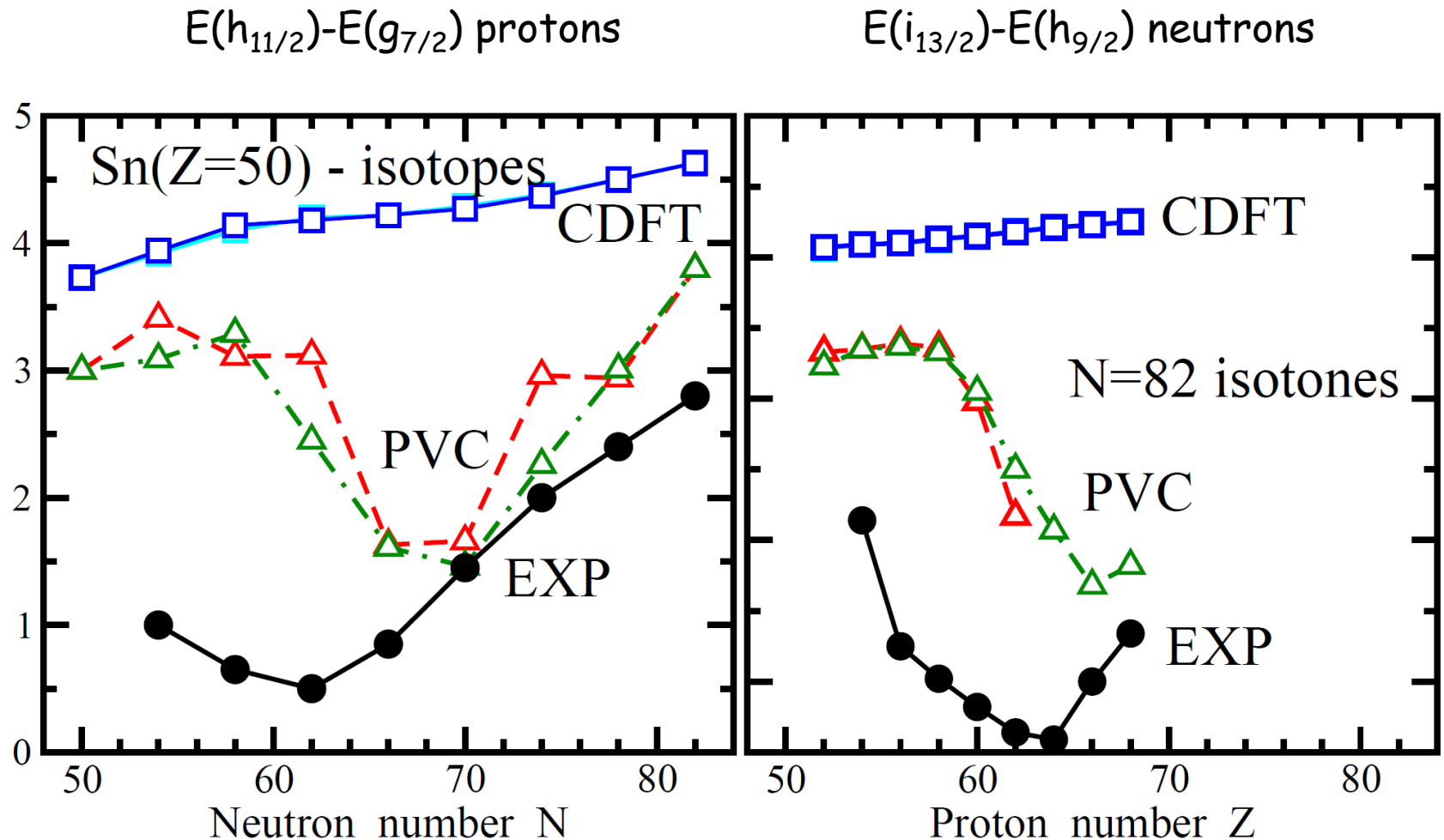
***This work supported by NSCL @ Michigan State University and
by US-NSF Grants PHY-1204486, PHY-1404343***

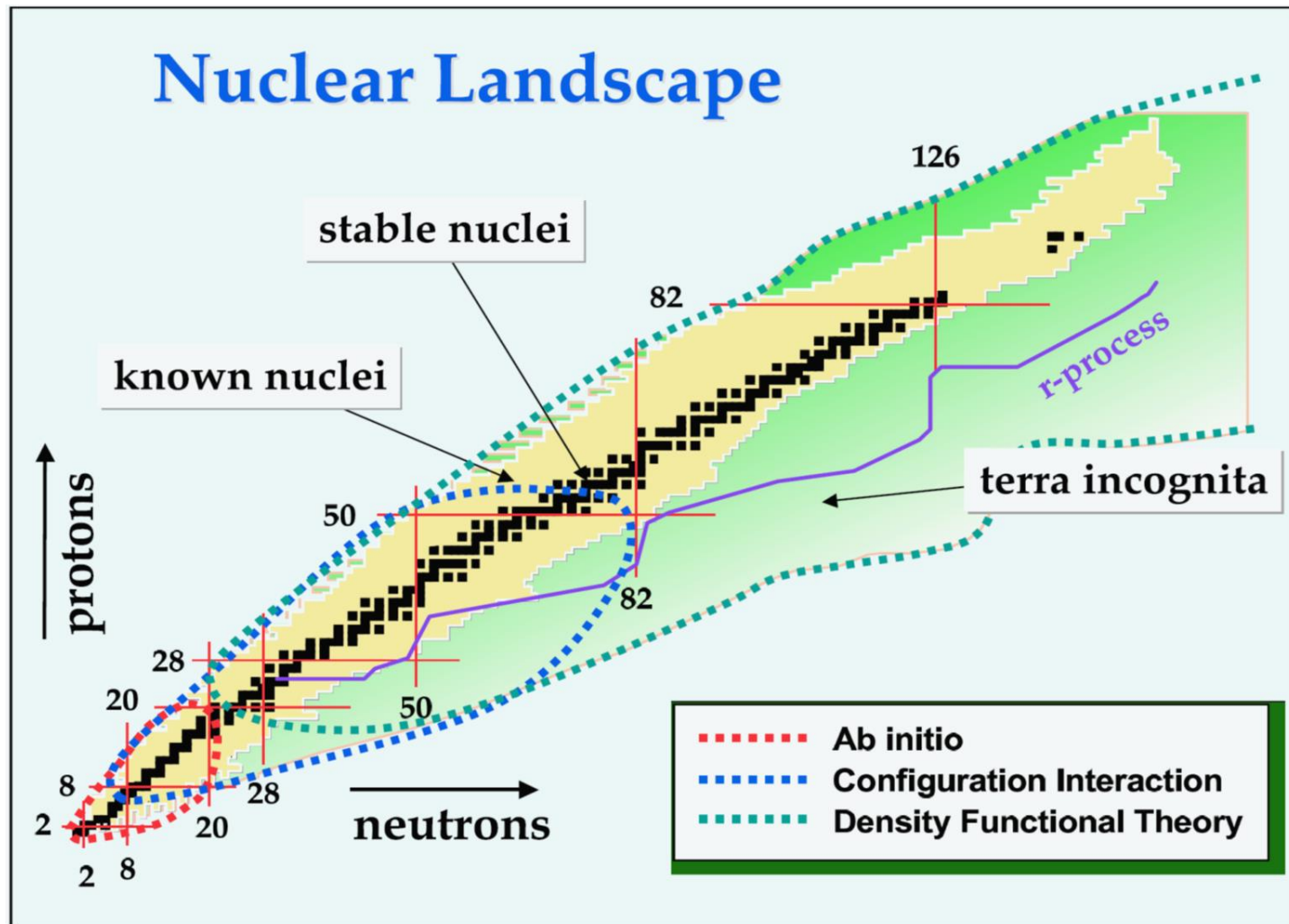
Fragmentation of states in odd and even systems (schematic)



Properties of single-particle states in Z=50 and N=82 nuclei

Particle-vibration coupling improves the description of the single-particle states considerably and if the low-lying phonons are described well the single-particle states are reproduced as well.





DFT: major part of the nuclear chart => input for astrophysical modeling

Figure from: G.F. Bertsch, J. Phys.: Conf. Ser. 78, 012005 (2007)

Single-quasiparticle Green's function

Doubled quasiparticle space: $\eta = \pm 1$

$$G_k^\eta(\varepsilon) = \sum_{\nu, \eta'} \frac{\tilde{S}_k^{\eta'(\nu)}}{\varepsilon - \eta\eta' E_k^{(\nu)}}$$

Spectroscopic factors
Energies

One-body Green's function in N-body system (Lehmann):

$$G(\xi, \xi'; \varepsilon) = \sum_n \frac{(\Psi(\xi))_{0n} (\Psi^\dagger(\xi'))_{n0}}{\varepsilon - (E_n^{(N+1)} - E_0^{(N)}) + i\delta} + \sum_m \frac{(\Psi^\dagger(\xi'))_{0m} (\Psi(\xi))_{m0}}{\varepsilon + (E_m^{(N-1)} - E_0^{(N)}) - i\delta},$$

Model
dependence
of S!

Excited state (N+1)

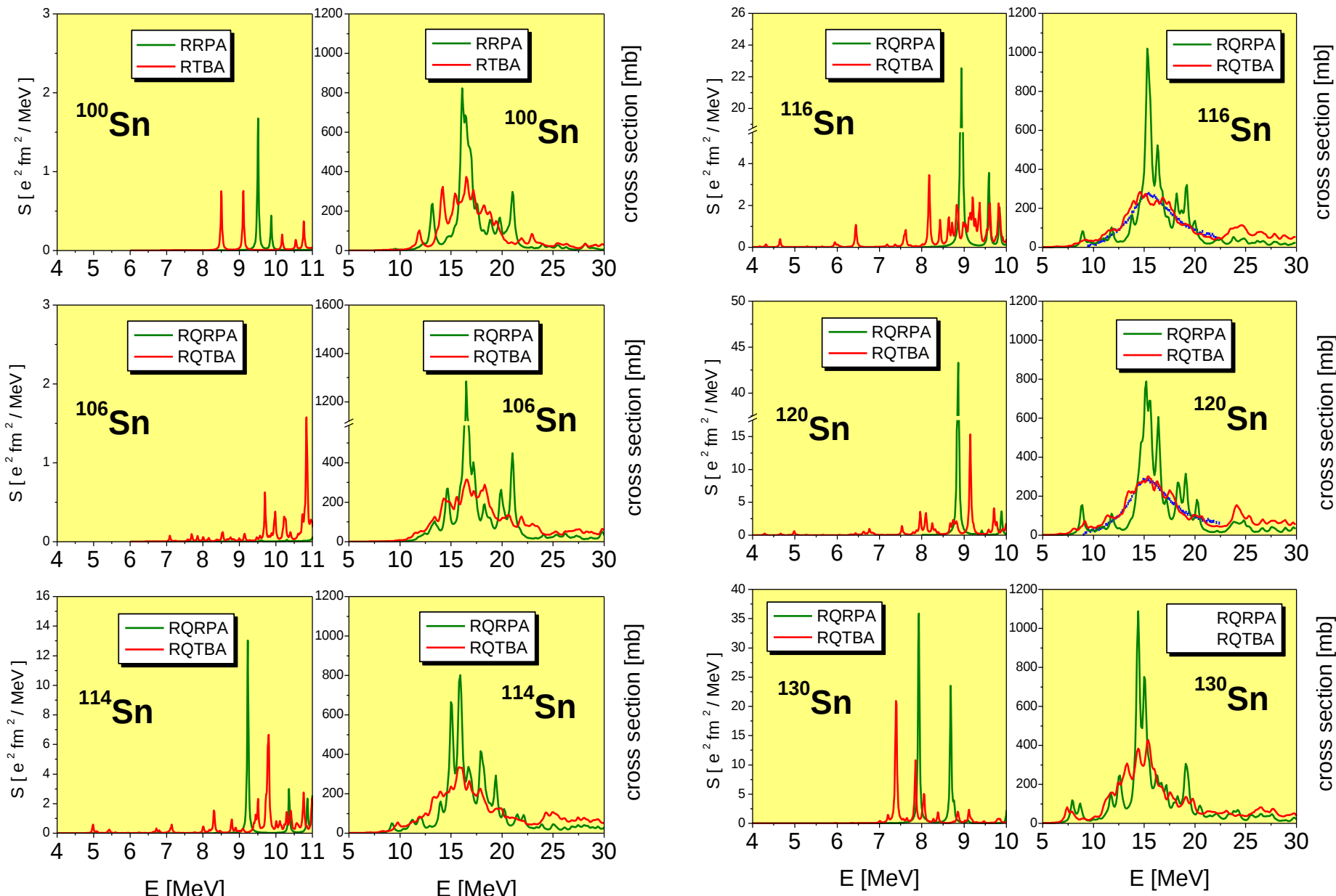
$$(\Psi^\dagger(\xi))_{n0} = \langle \Phi_n^{(N+1)} | \Psi^\dagger(\xi) | \Phi_0^{(N)} \rangle,$$

$$(\Psi(\xi))_{n0} = \langle \Phi_n^{(N-1)} | \Psi(\xi) | \Phi_0^{(N)} \rangle,$$

Ground state
(N)

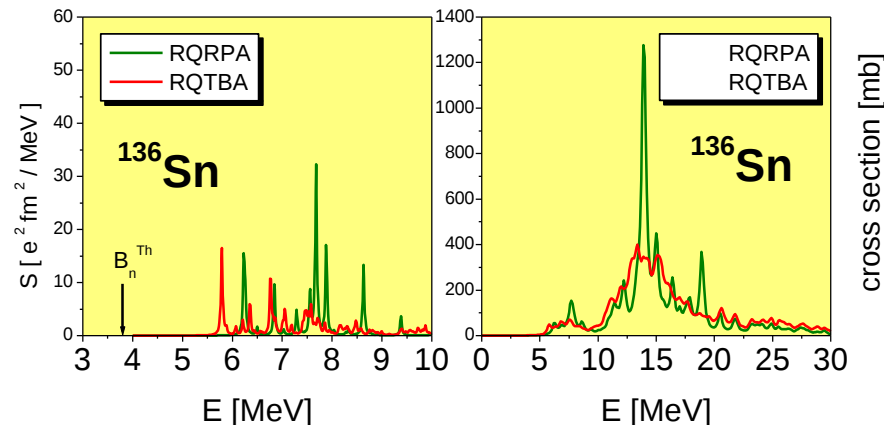
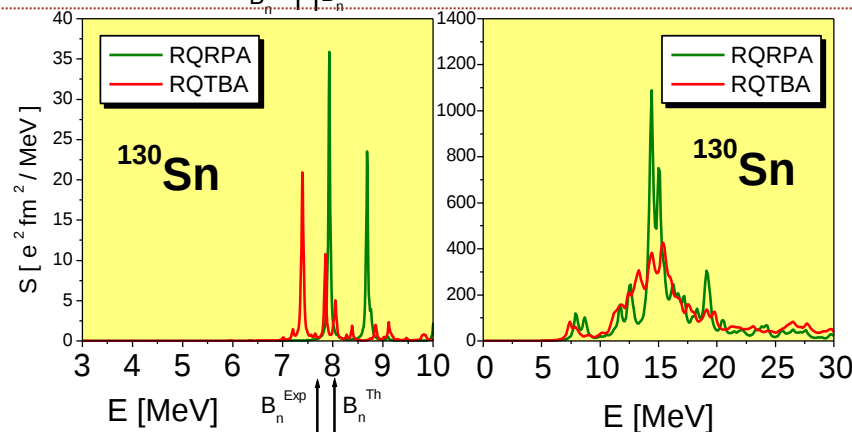
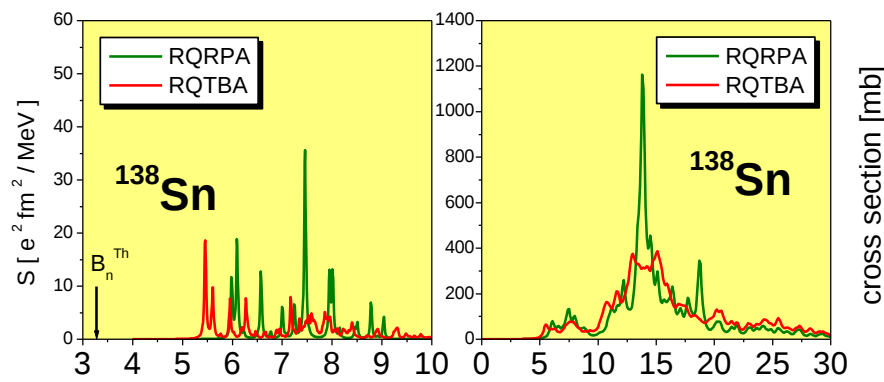
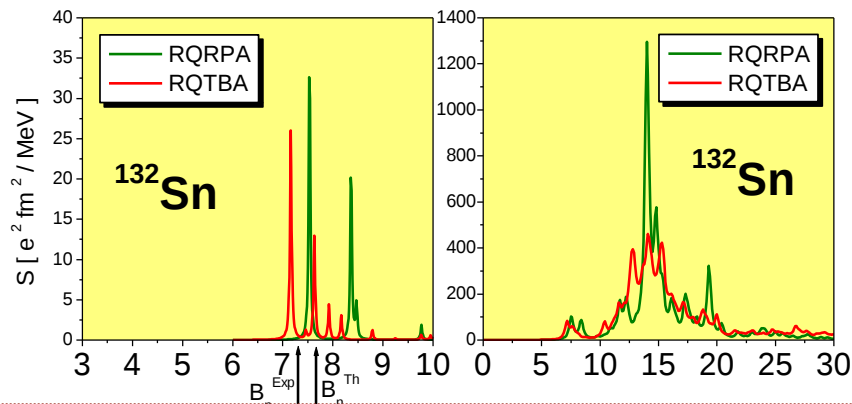
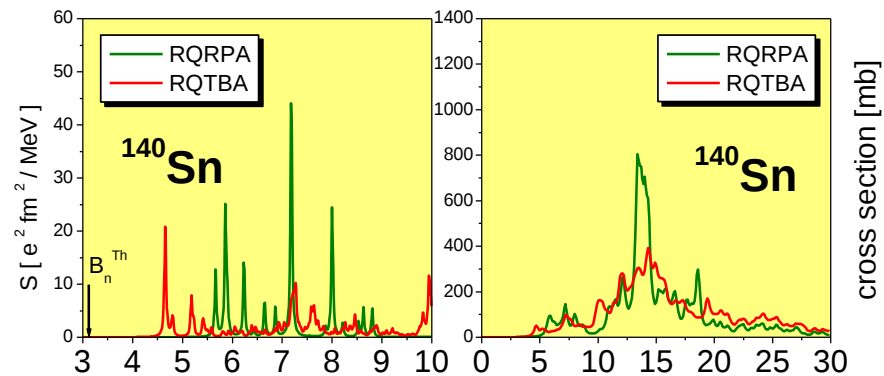
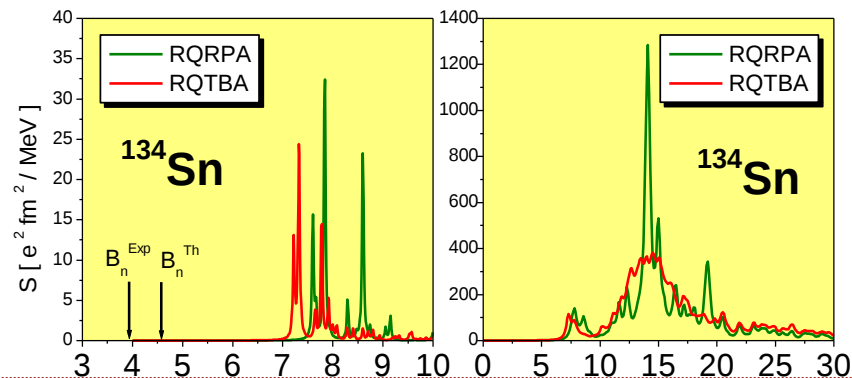
Dipole strength in Sn isotopes

E.L. et al, PRC 79, 054312 (2009)

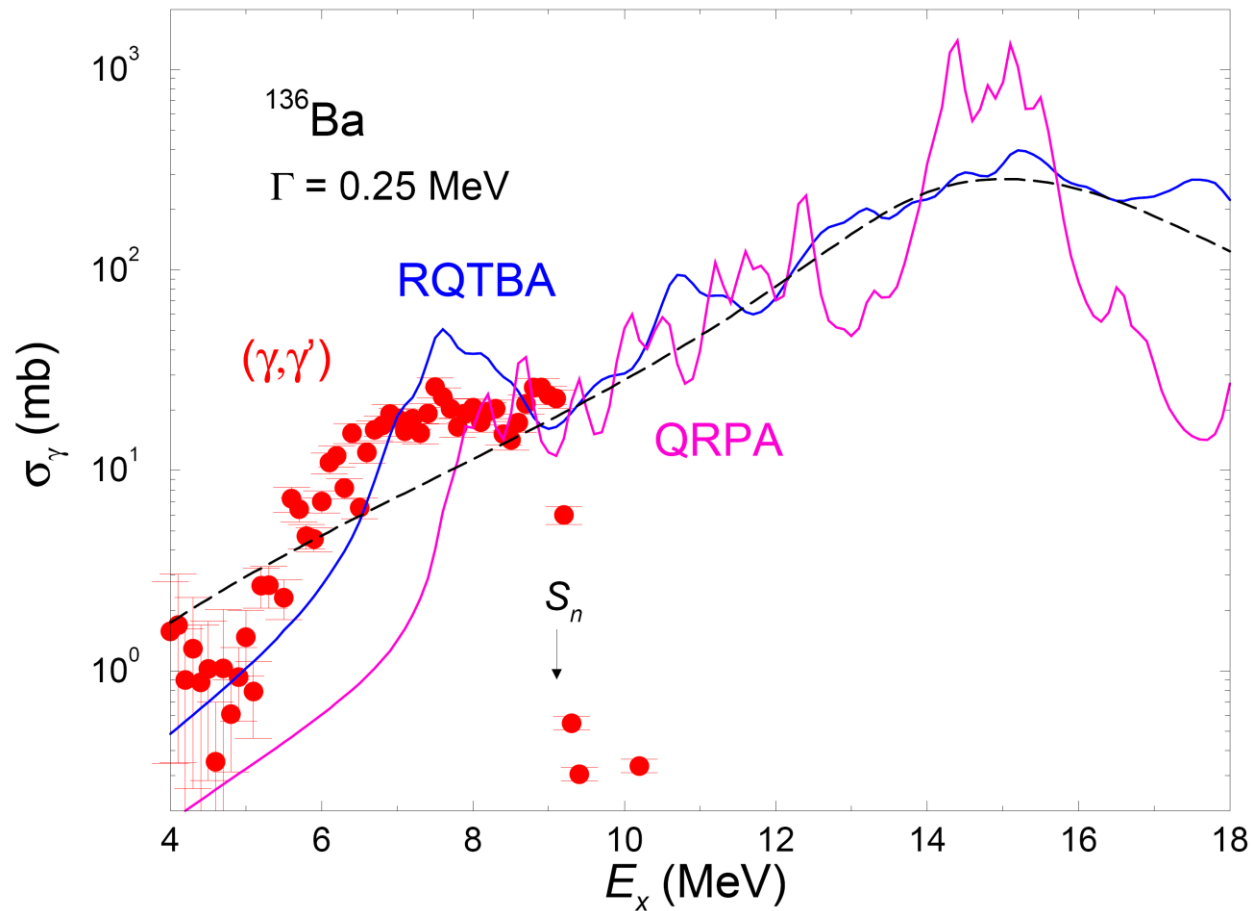


Dipole strength in Sn isotopes

E.L. et al, PRC 79, 054312 (2009)



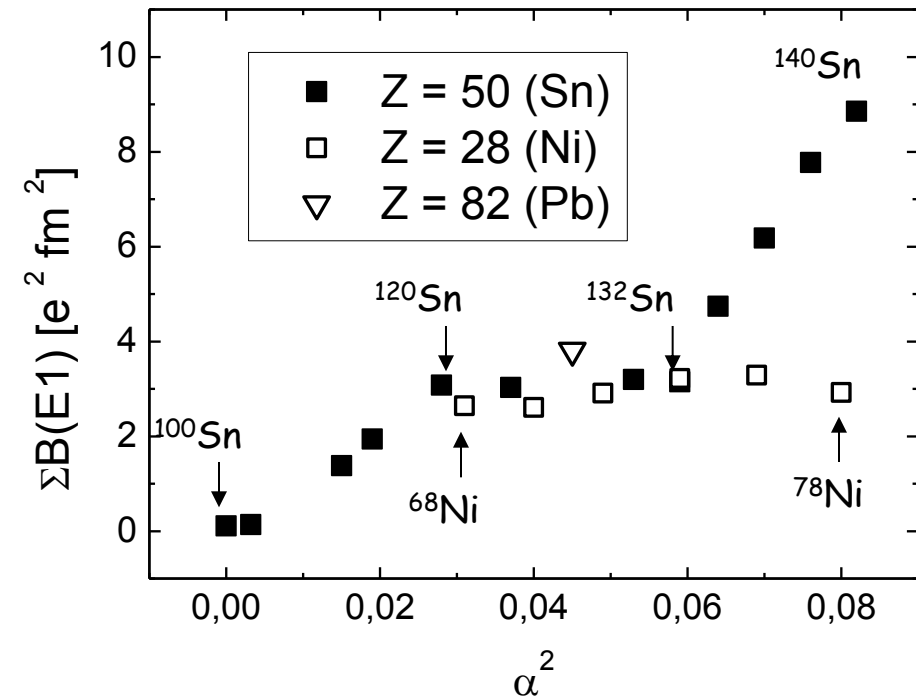
Low-lying dipole strength in ^{136}Ba



R. Massarczyk, R. Schwengner, F. Doenau, E. Litvinova, G. Rusev et al.,
Phys. Rev. C 86, 014319 (2012)

RQTBA systematics for PDR:

A proper definition of Pygmy Dipole Resonance is important!
PDR = all states with the "isoscalar" underlying structure!

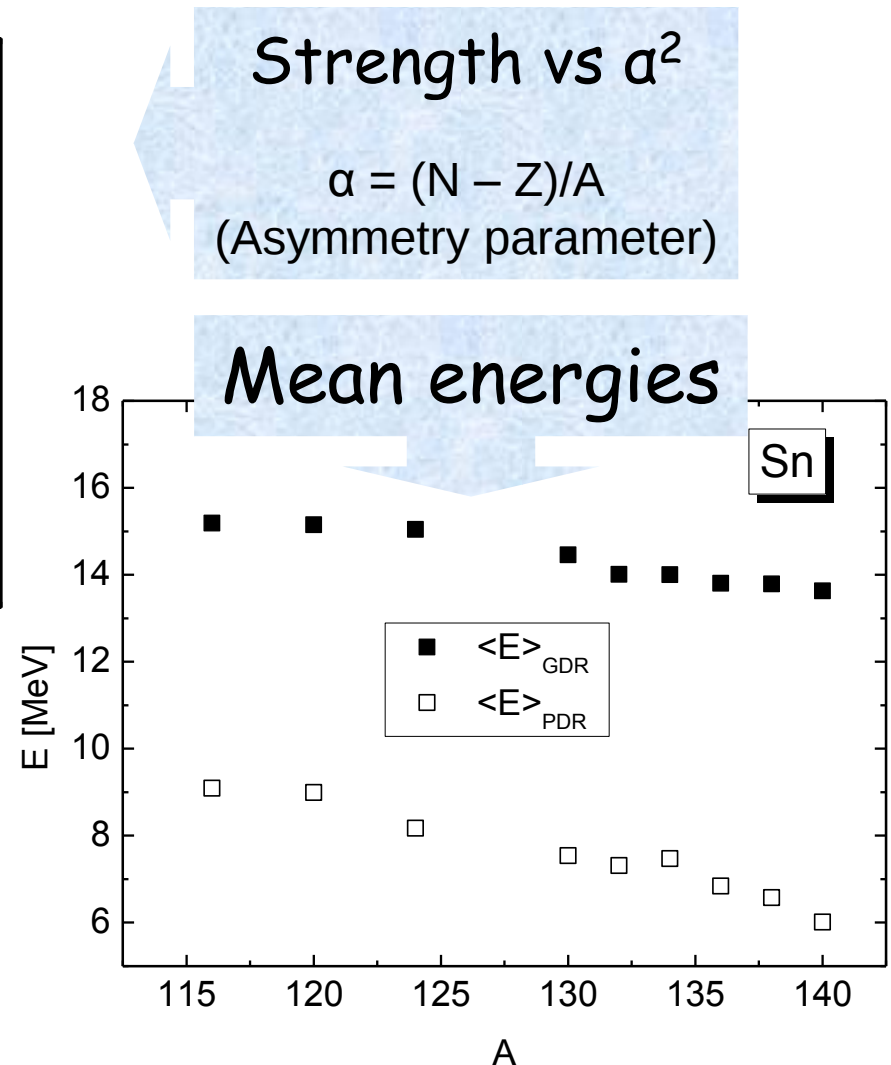


$^{120}\text{Sn} \rightarrow ^{132}\text{Sn}$: $1h_{11/2}$ (n)

$^{68}\text{Ni} \rightarrow ^{78}\text{Ni}$: $1g_{9/2}$ (n)

Intruder orbits !

E.L. et al. PRC 79, 054312 (2009)



Elimination of the spurious state

$$\delta\rho(x; \omega, T) = \frac{\zeta(x; T)}{\omega - \omega_0} + \delta\rho^{reg}(x; \omega, T), \quad \omega_0 \rightarrow 0 \quad \text{Condition 1}$$

$$F(x, x') = F^{LM}(x, x') + F^{rest}(x, x')$$

$$F^{LM}(x, x') = C_0 \left[f(x) + f'(x) \boldsymbol{\tau} \boldsymbol{\tau}' + (g(x) + g'(x) \boldsymbol{\tau} \boldsymbol{\tau}') \boldsymbol{\sigma} \boldsymbol{\sigma}' \right] \delta(\mathbf{r} - \mathbf{r}'),$$

$$f(x) = f_{ex} + (f_{in} - f_{ex}) \frac{\rho_0(x)}{\rho_0(0)}$$

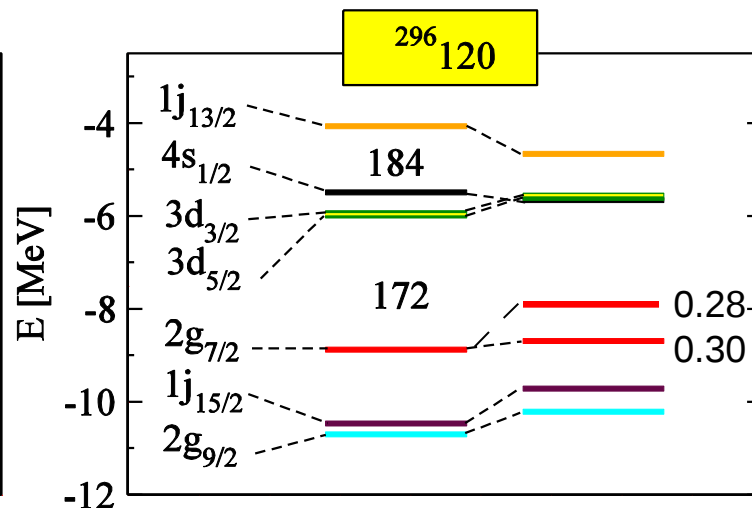
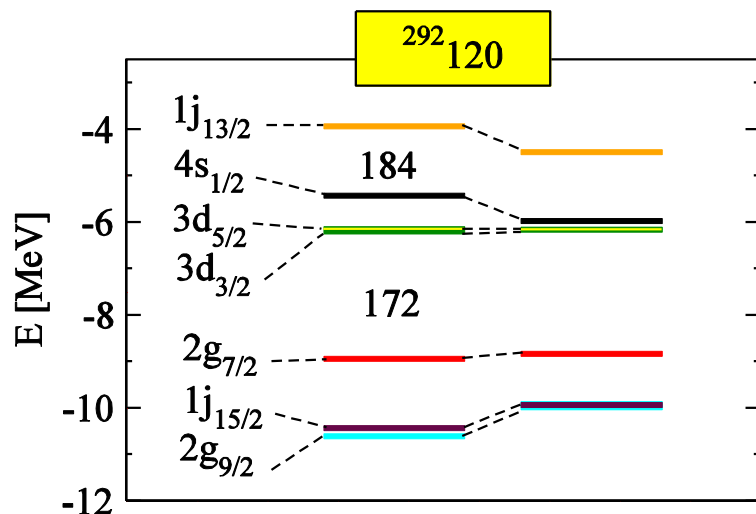
$$F^{rest}(r, r') = \sum_{k=1,2} \kappa_k f_k(x) f_k(x')$$

$$B_{E1}^{(0)} = \int dx \zeta(x, T) P_{E1}(x) = 0 \quad \text{Condition 2}$$

V. Tselyaev: S. Kamerdzhiev et al., PRC 58, 172 (1998)

M.I. Baznat et al., Sov. J. Nucl. Phys. 52, 627 (1990)

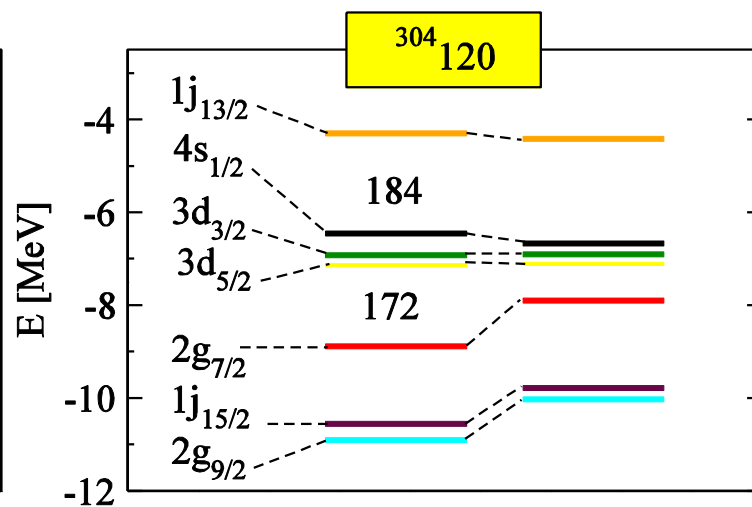
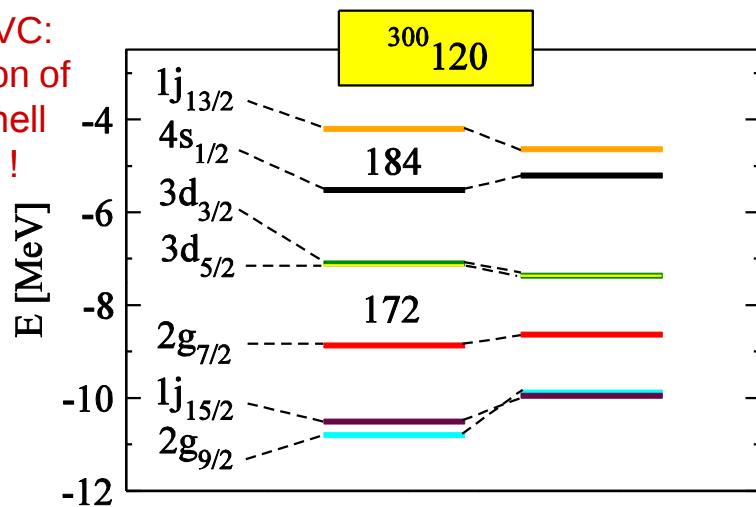
Dominant neutron states in $Z = 120$



Comparable
Spectroscopic
strengths



PC+QVC:
Formation of
the „shell
gap“ !



RMF+BCS

...+QVC

RMF+BCS

...+QVC

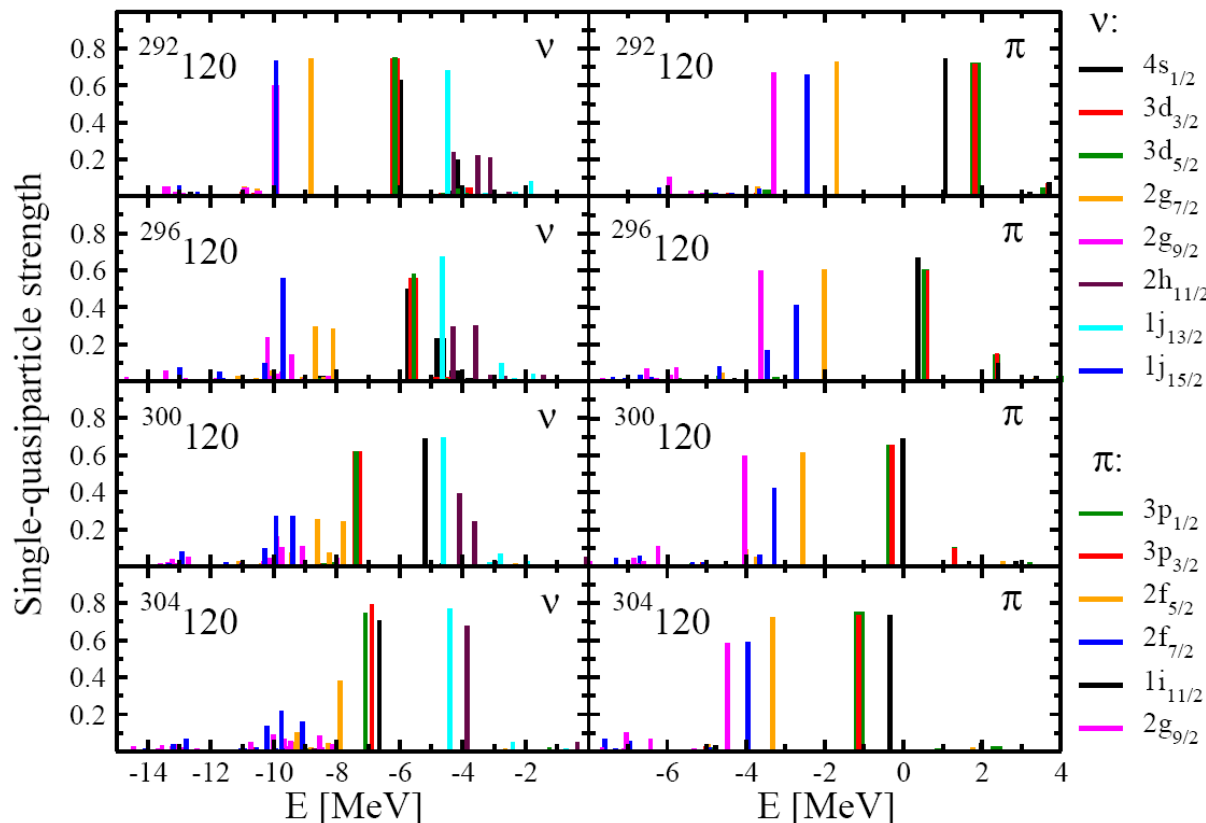
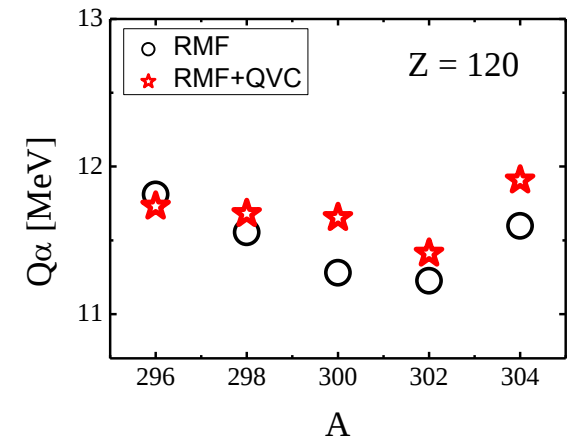
Shell evolution in superheavy $Z = 120$ isotopes: Quasiparticle-vibration coupling (QVC) in a relativistic framework

1. Relativistic Mean Field: **spherical minima**
2. π : collapse of pairing, **clear shell gap**
3. ν : survival of **pairing coexisting with the shell gap**
4. Very **soft** nuclei: large amount of low-lying collective vibrational modes (~ 100 phonons below 15 MeV)

Vibration corrections
to binding energy (RQRPA)

$$E_{VC} = - \sum_{\mu} \Omega_{\mu} \sum_{k_1 k_2} |Y_{k_1 k_2}^{\mu}|^2$$

Vibration corrections
to α -decay Q-values

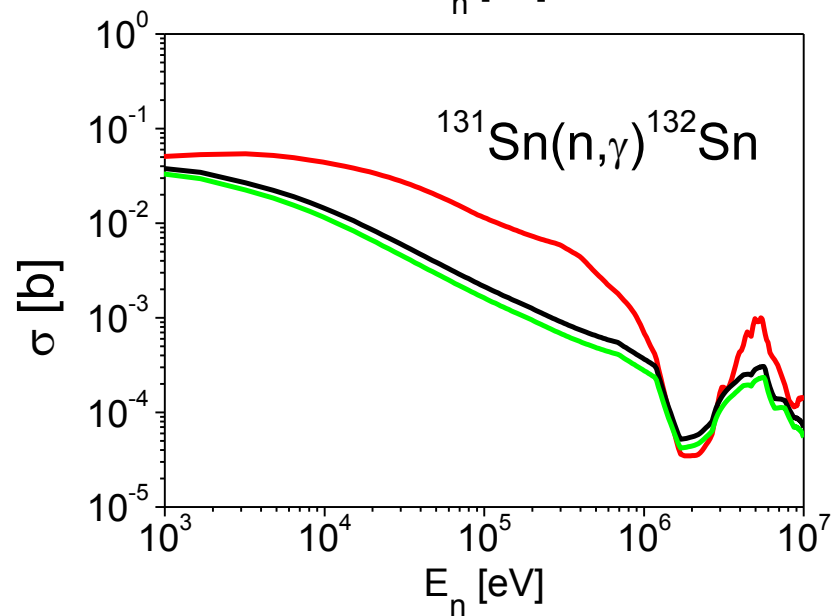
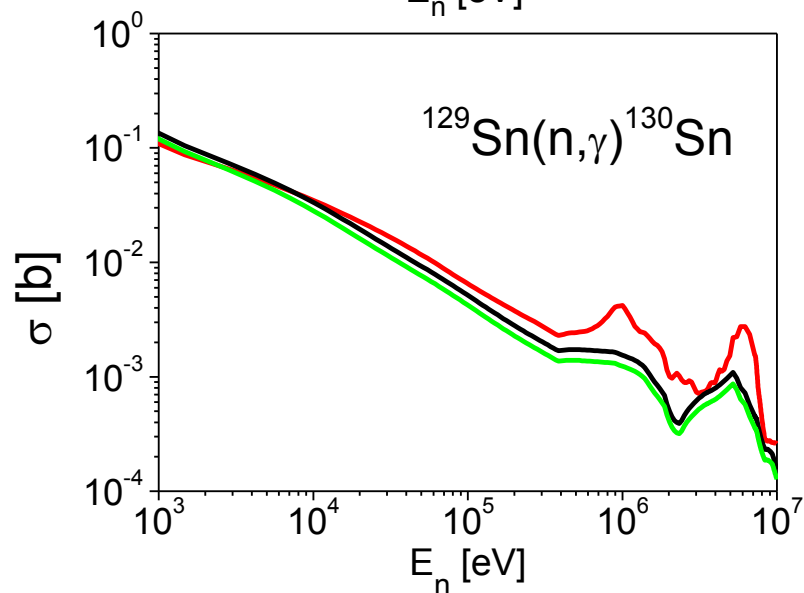
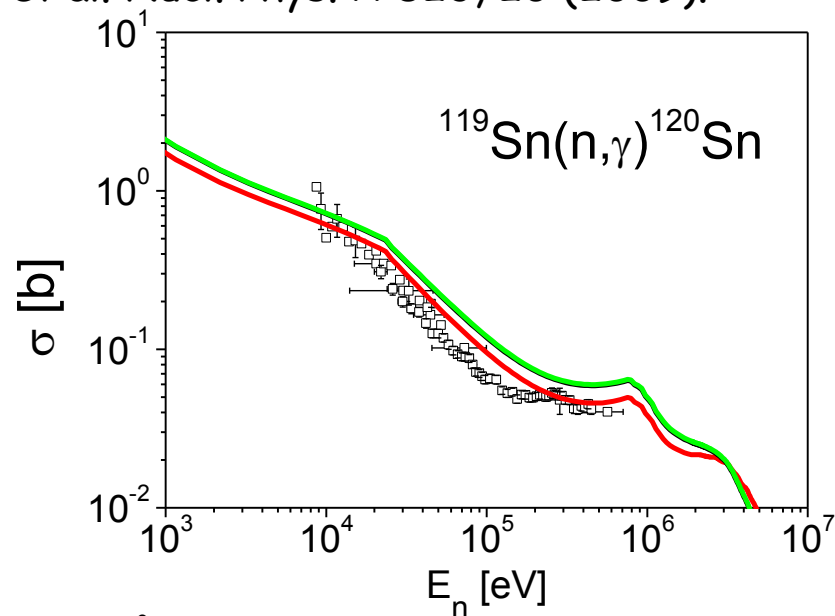
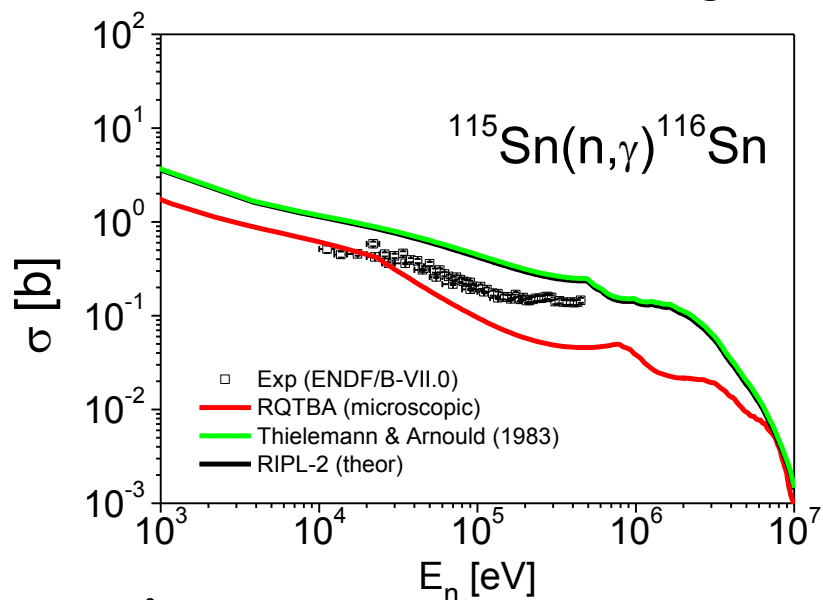


Vibrational corrections:

1. Impact on the shell gaps
2. Smearing out the shell effects

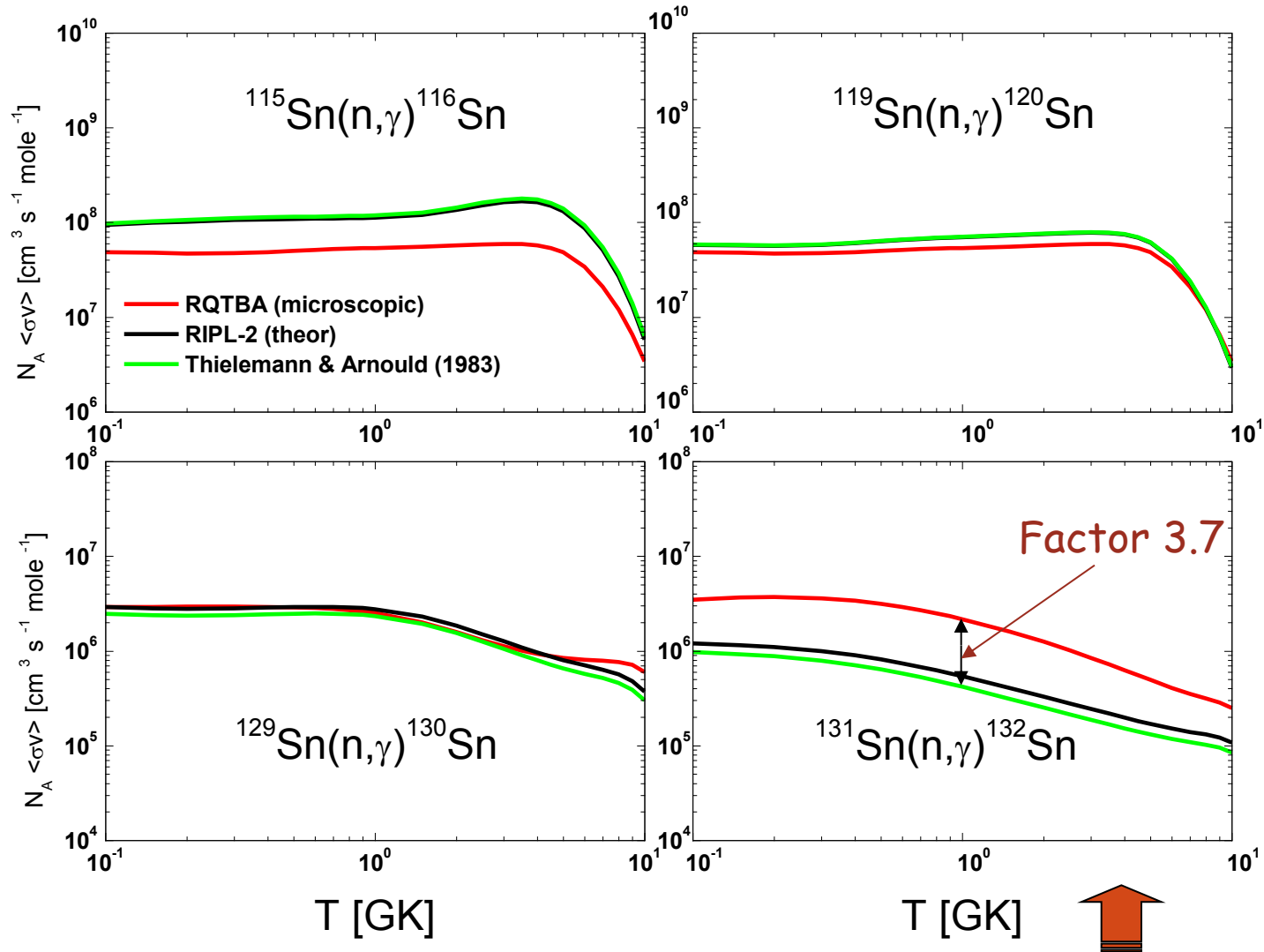
Radiative neutron capture in the Hauser-Feshbach model: standard Lorentzians and microscopic structure

E. L., H.P. Loens, K. Langanke, et al. Nucl. Phys. A 823, 26 (2009).



(n,γ) stellar reaction rates

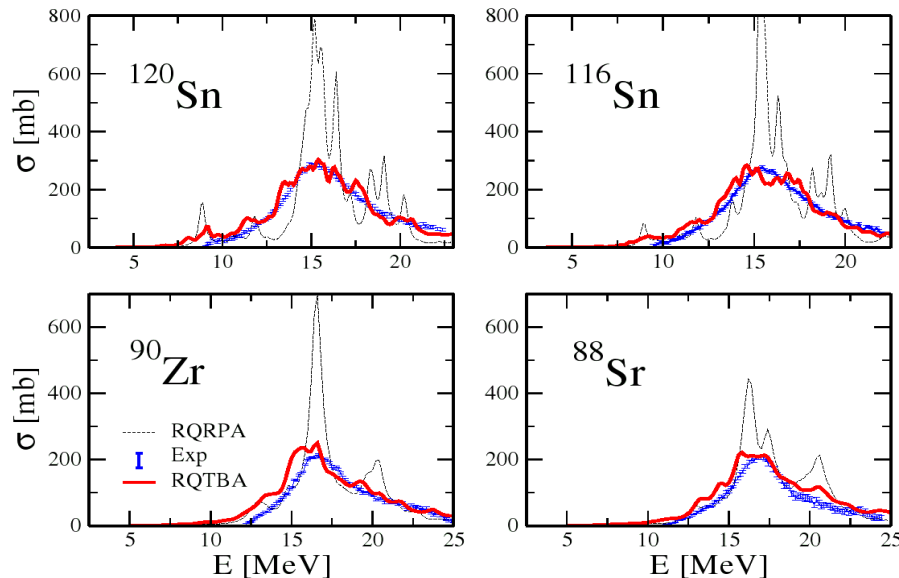
E. L., H.P. Loens, K. Langanke, et al. Nucl. Phys. A 823, 26 (2009).



Microscopic structure is important, e.g. PDR at the neutron threshold

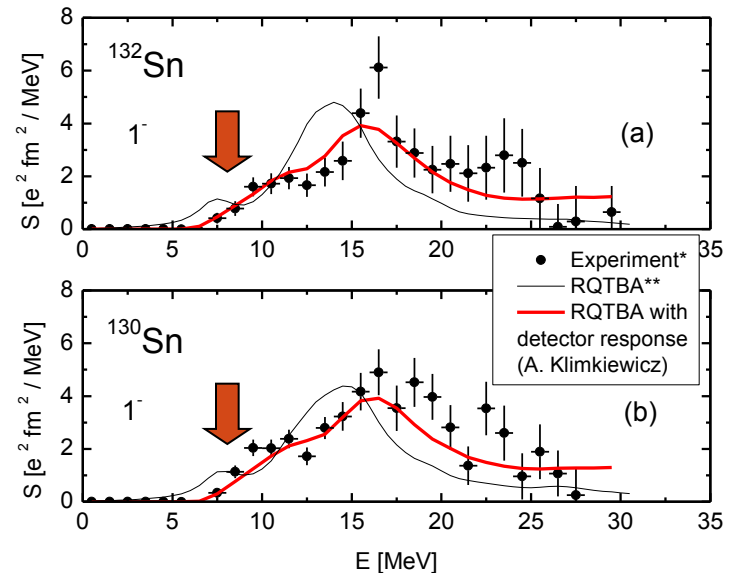
Dipole strength in neutron-rich nuclei within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)

Test case: E1 (IVGDR) stable nuclei



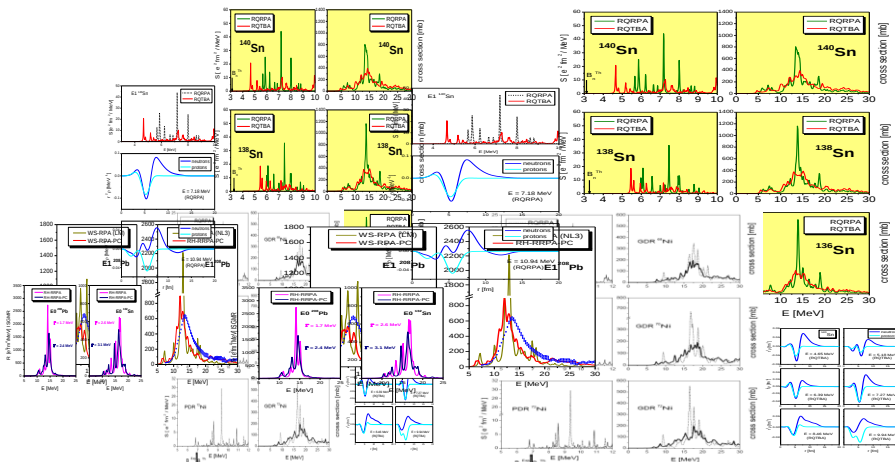
*E. L., P. Ring, and V. Tselyaev,
Phys. Rev. C 78, 014312 (2008)

Neutron-rich Sn



* P. Adrich, A. Klimkiewicz, M. Fallot et al.,
PRL 95, 132501 (2005)

**E. L., P. Ring, V. Tselyaev, K. Langanke
PRC 79, 054312 (2009)



Input for
r-process nucleosynthesis:
(n, γ) cross sections
and reaction rates

E. L., H.P. Loens, K. Langanke, et al.
Nucl. Phys. A 823, 26 (2009).

Nuclear response at finite temperature

Density matrix variation at $T > 0$:

$$\delta\mathcal{R}(x; \omega, T) = \delta\mathcal{R}^{(0)}(x; \omega, T) + \int dx' dx'' \mathcal{A}(x, x'; \omega, T) F(x', x'') \delta\mathcal{R}(x''; \omega, T)$$

Thermal „mean-field + pairing“ propagator in the continuum :

E.L. et al., Phys. Atomic Nuclei 66, 558 (2003)

$$\mathcal{A}(x, x'; \omega, T) = \sum_{1234} \varphi_1^*(x) \varphi_2(x) \varphi_3(x') \varphi_4^*(x') \int \frac{d\varepsilon}{2\pi i} \underbrace{G_{12}(\varepsilon, T) G_{34}(\varepsilon + \omega, T)}_{\text{Matsubara GF}}$$

Radiative strength function (RSF):

$$f_{E1}(E_\gamma, T) = -\frac{8e^2}{27(\hbar c)^3} \text{Im} \int dx P_{E1}^\dagger(x) \delta\mathcal{R}(x; \omega, T)$$

$$\omega = E_\gamma + i\Delta, \Delta \rightarrow 0 \quad \text{Exp. energy resolution}$$

Structure of the propagator in ph-channel

$$\begin{aligned}
 & \mathcal{A}_{LS,LS'}^{\text{cont}L}(r, r'; \omega; T) \\
 = & - \sum_1 \boxed{v_1^2(T)(1 - n_1(T))} R_1(r) R_1(r') \sum_{l_2 j_2} T_{12}^{LSS'} \\
 & \times \left[\mathcal{G}_{l_2 j_2}(r, r'; \mu(T) - E_1(T) + \omega) \right. \\
 & \left. + (-1)^{S+S'} \mathcal{G}_{l_2 j_2}(r, r'; \mu(T) - E_1(T) - \omega) \right] \\
 & - \sum_1 \boxed{u_1^2(T)n_1(T)} R_1(r) R_1(r') \sum_{l_2 j_2} T_{12}^{LSS'} \\
 & \times \left[\mathcal{G}_{l_2 j_2}(r, r'; \mu(T) + E_1(T) + \omega) \right. \\
 & \left. + (-1)^{S+S'} \mathcal{G}_{l_2 j_2}(r, r'; \mu(T) + E_1(T) - \omega) \right],
 \end{aligned}$$

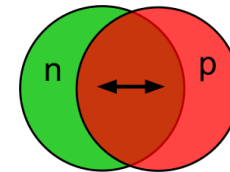
$$\begin{aligned}
 & \mathcal{A}_{LS,LS'}^{\text{disc}L}(r, r'; \omega; T) \\
 = & \sum_{12}^{\text{disc}} R_1(r) R_2(r') R_1(r') R_2(r) T_{21}^{LSS'} \\
 & \times \left[\mathcal{L}_{12}(\omega, T) + \frac{v_2^2(T)(1 - n_2(T))}{\omega + \mu(T) - \epsilon_1 - E_2(T)} \right. \\
 & - \frac{v_1^2(T)(1 - n_1(T))}{\omega - \mu(T) + \epsilon_2 + E_1(T)} \\
 & + \frac{u_2^2(T)n_2(T)}{\omega + \mu(T) - \epsilon_1 + E_2(T)} \\
 & \left. - \frac{u_1^2(T)n_1(T)}{\omega - \mu(T) + \epsilon_2 - E_1(T)} + (-1)^S \mathcal{M}_{12}(\omega, T) \right]
 \end{aligned}$$

$$\begin{aligned}
 n_1(T) &= \frac{1}{1 + \exp(E_1(T)/T)}, \\
 v_1^2(T) &= \frac{1}{2} \left(1 - \frac{\epsilon_1 - \mu(T)}{E_1(T)} \right), \\
 E_1(T) &= \sqrt{(\epsilon_1 - \mu(T))^2 + \Delta_1^2(T)} \\
 u_1^2(T) &= 1 - v_1^2(T)
 \end{aligned}$$

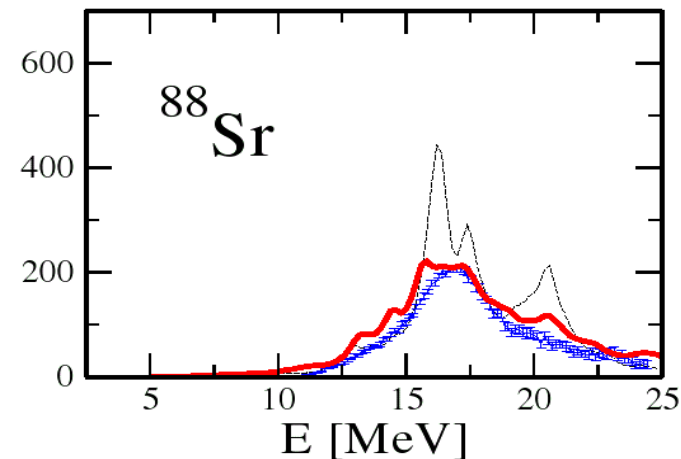
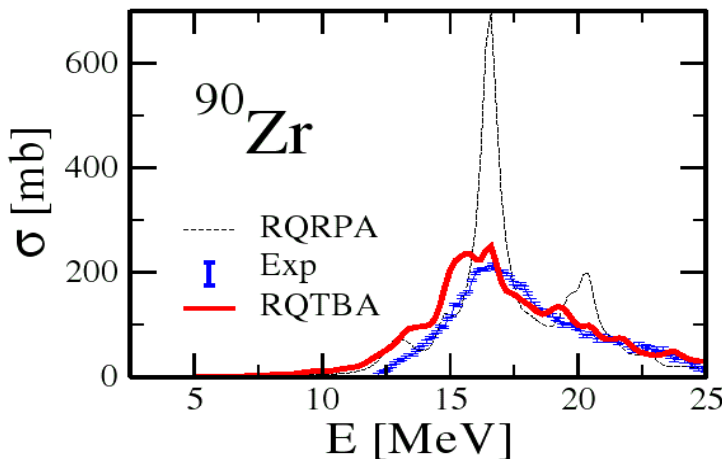
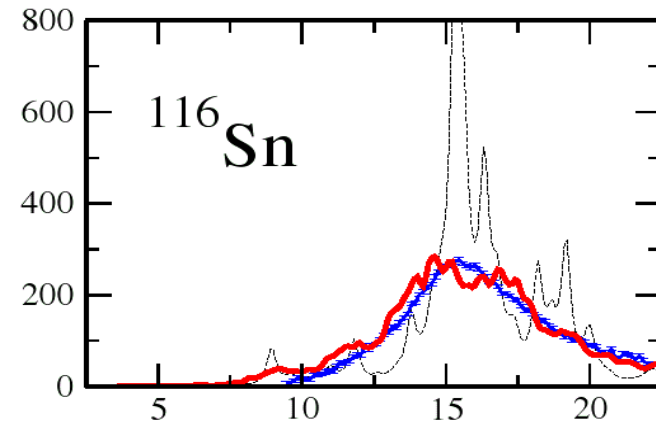
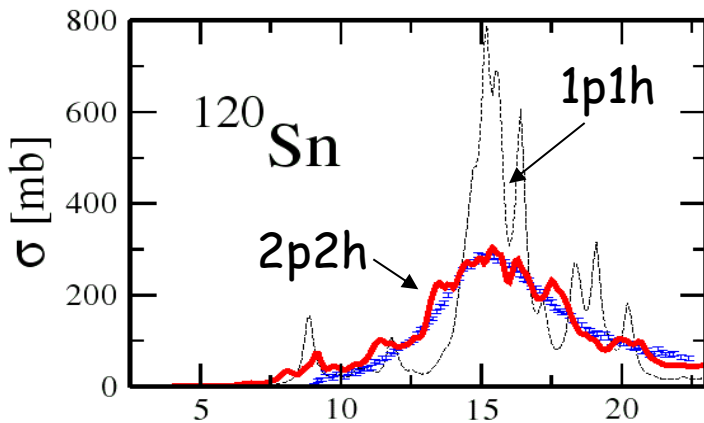
$$\begin{aligned}
 & \mathcal{L}_{12}(\omega, T) = -A_{ph}(\omega, T) \\
 & \times \left[(u_1^2 u_2^2 + v_1^2 v_2^2)(E_1 - E_2) - (u_1^2 u_2^2 - v_1^2 v_2^2)\omega \right] \\
 & - A_{pp}(\omega, T) \left[- (u_1^2 v_2^2 + v_1^2 u_2^2)(E_1 + E_2) \right. \\
 & \quad \left. + (u_1^2 v_2^2 - v_1^2 u_2^2)\omega \right], \\
 & \mathcal{M}_{12}(\omega, T) = \frac{\Delta_1 \Delta_2}{2E_1 E_2} \left[A_{ph}(\omega, T)(E_1 - E_2) \right. \\
 & \quad \left. + A_{pp}(\omega, T)(E_1 + E_2) \right], \\
 & A_{ph}(\omega, T) = \frac{n_1(T) - n_2(T)}{(E_1 - E_2)^2 - \omega^2}, \\
 & A_{pp}(\omega, T) = \frac{1 - n_1(T) - n_2(T)}{(E_1 + E_2)^2 - \omega^2};
 \end{aligned}$$

Giant Dipole Resonance within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)*

$$P = \sum_{i=1}^A \left(\tau_z^{(i)} - \frac{N-Z}{2A} \right) r_i Y_{1M}(\hat{r}_i)$$



$$\begin{aligned} \Delta L &= 1 \\ \Delta T &= 1 \\ \Delta S &= 0 \end{aligned}$$

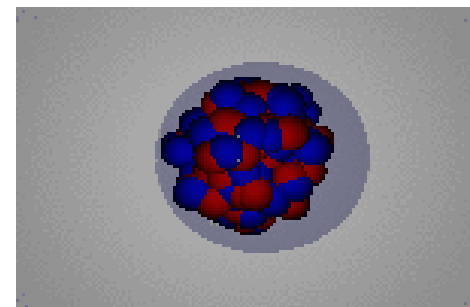


*E. L., P. Ring, and V. Tselyaev,
Phys. Rev. C 78, 014312 (2008)

Fragmentation of pygmy dipole resonance

E. L., P. Ring, and V. Tselyaev, Phys. Rev. C 78, 014312 (2008)

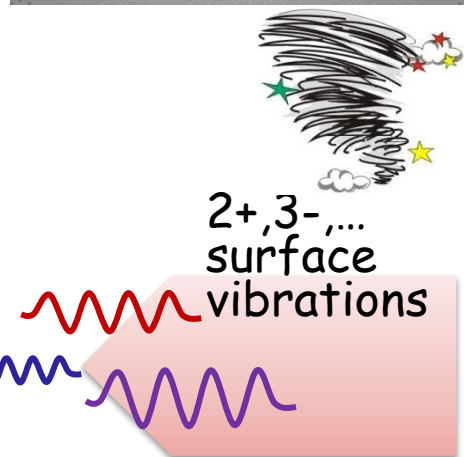
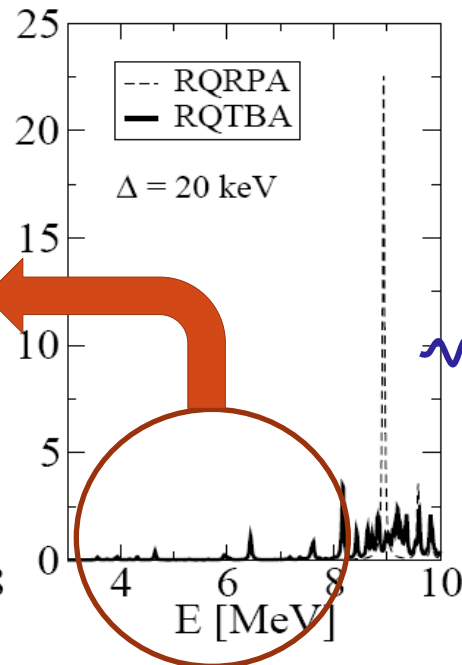
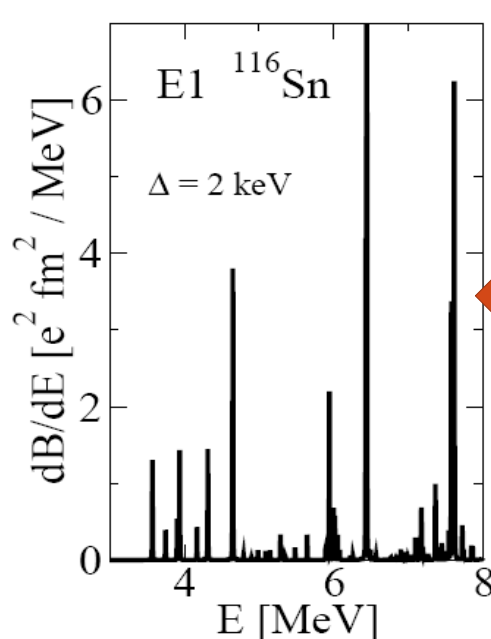
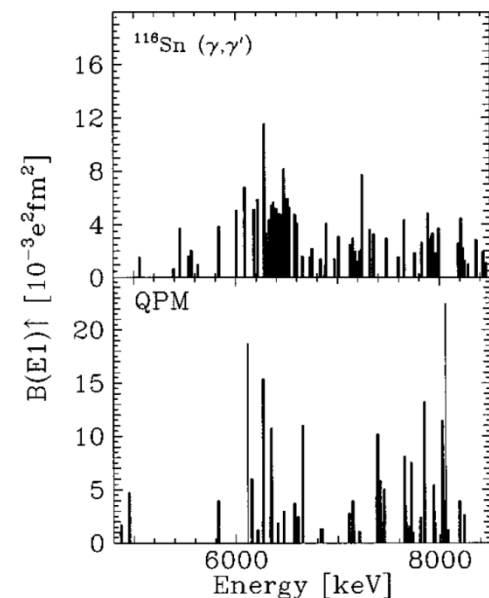
Low-lying dipole strength in ^{116}Sn



Experiment*

Fine structure

Gross structure



QPM up to 3p3h
(V.Yu. Ponomarev)

RQTBA 2p2h

RQRPA 1p1h vs
RQTBA 2p2h

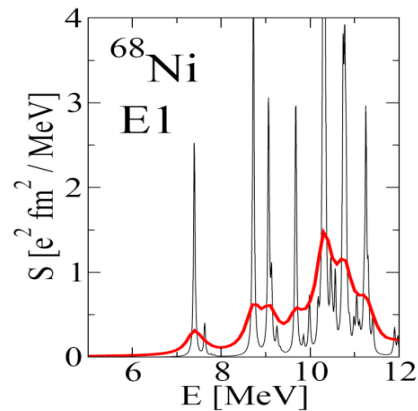
* K. Govaert et al.
PRC 57, 2229 (1998)

Integral
5-8 MeV:
 $\Sigma B(E1)_{\uparrow} [e^2 \text{ fm}^2]$

Exp.	0.204(25)
QPM	0.216
RQTBA	0.27

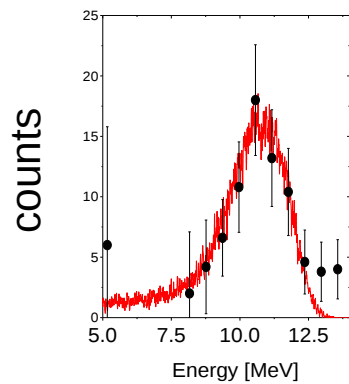
RQTBA dipole transition densities in ^{68}Ni at 10.3 MeV

Theory:



E.L., P.Ring, V.Tselyaev,
PRL 105, 02252 (2010)

Experiment:



O.Wieland et al.,
PRL 102, 092502 (2009)

Neutrons



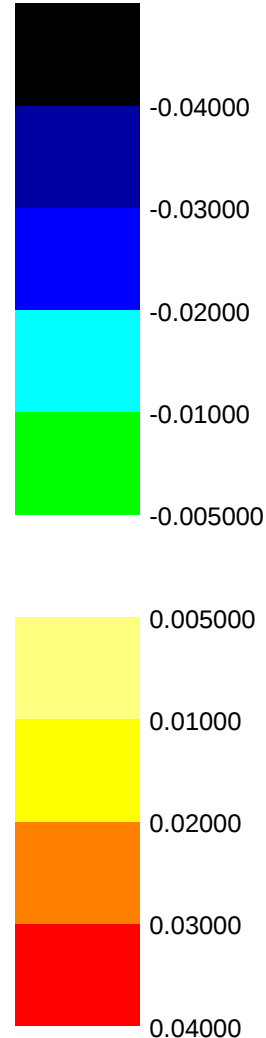
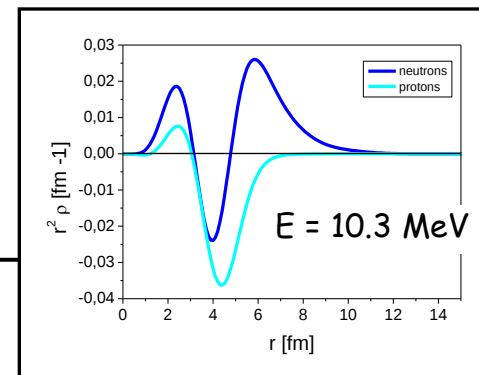
Protons



$$\rho(\mathbf{r}) = \rho(r)Y_{10}(\hat{\mathbf{r}}),$$

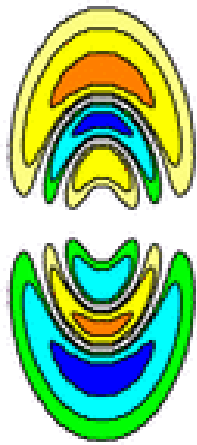
Experiment:

Coulomb excitation
of ^{68}Ni at 600 AMeV

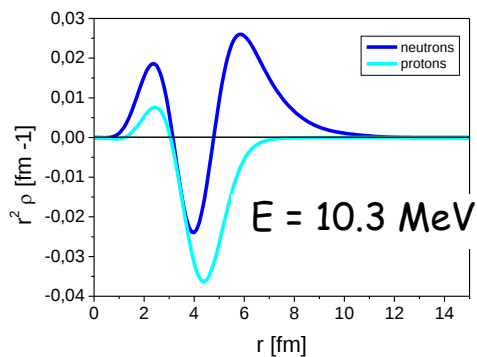


RQTBA dipole transition densities in ^{68}Ni at 10.3 MeV

Neutrons

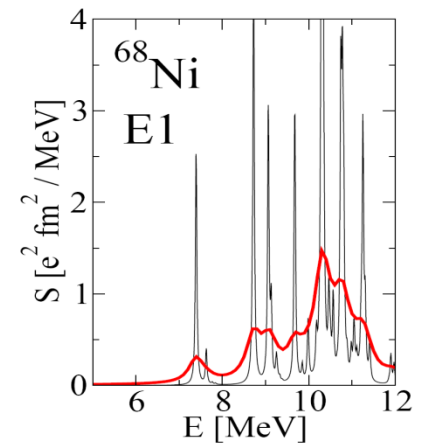


Protons



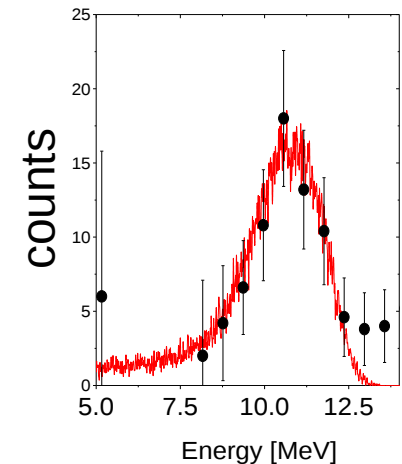
$$\rho(\mathbf{r}, t) = \rho(r) Y_{10}(\hat{\mathbf{r}}) e^{i\omega t}$$

Theory: RQTBA-2



E.L., P.Ring, V.Tselyaev,
PRL 105, 02252 (2010)

Experiment:



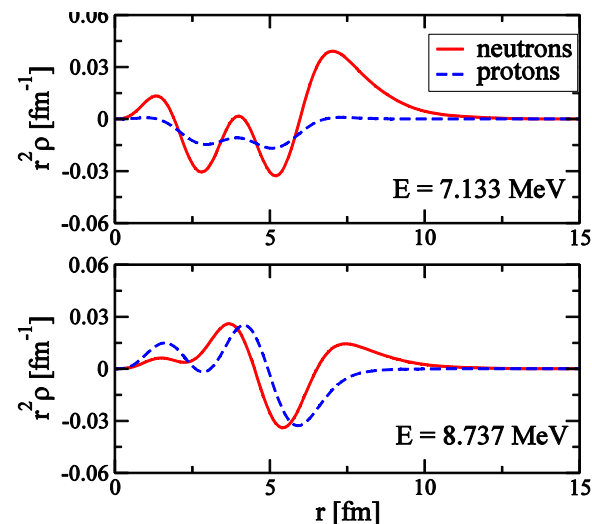
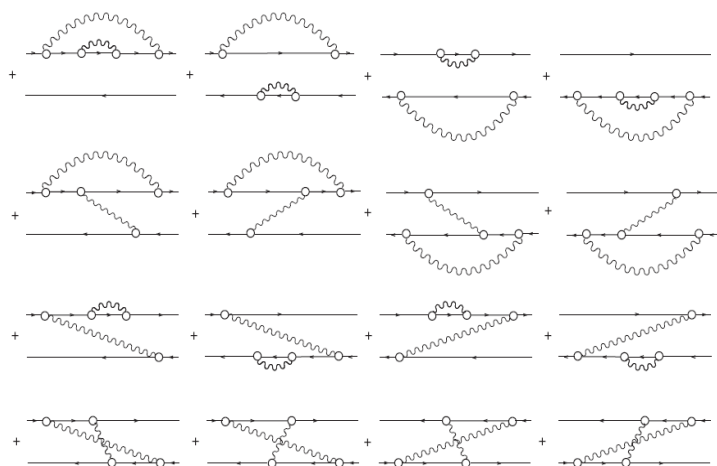
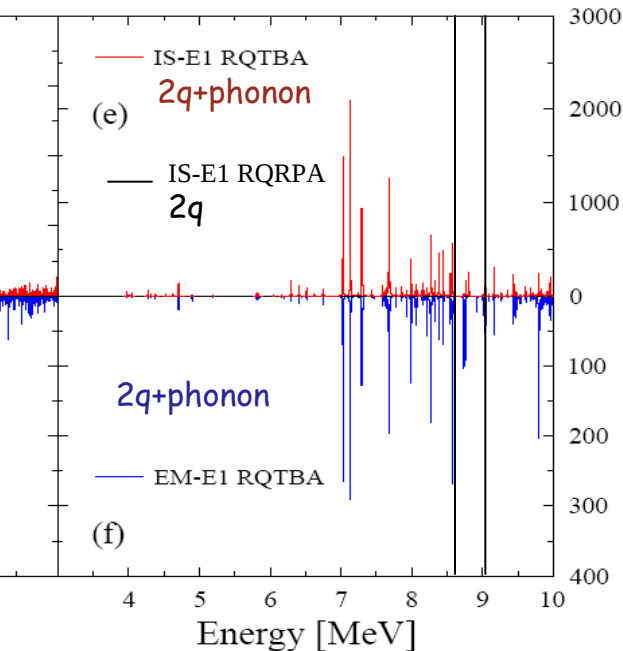
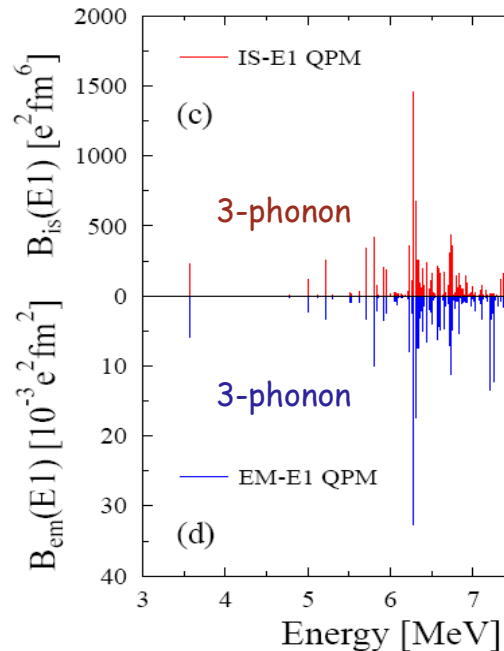
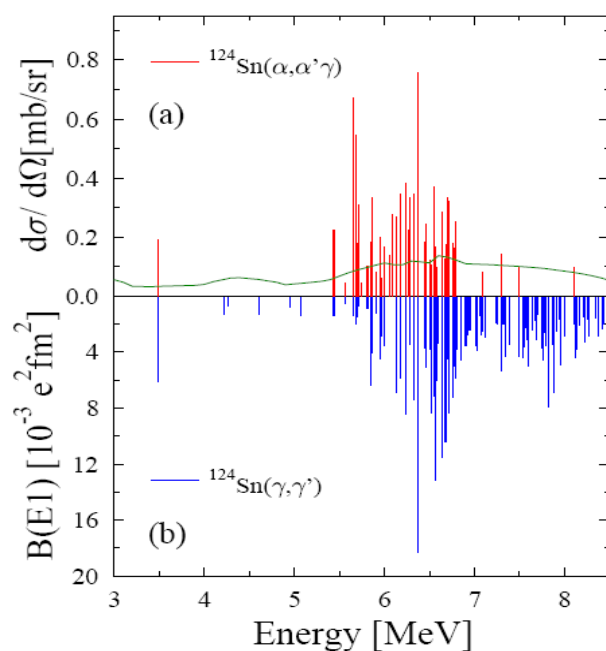
O.Wieland et al.,
PRL 102, 092502 (2009)

Isospin splitting of the pygmy dipole resonance in ^{124}Sn

J. Endres, E.L., D. Savran et al.,
PRL 105. 212503 (2010)

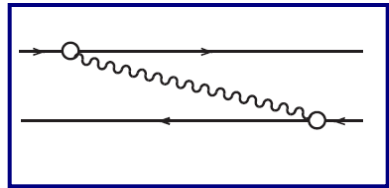
&

E. Lanza, A. Vitturi, E.L., D. Savran,
subm. to PRC (2013)

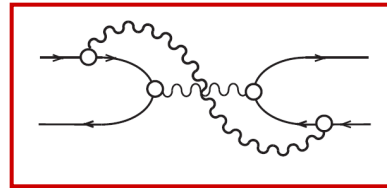


Fine features of dipole spectra: two-phonon effects

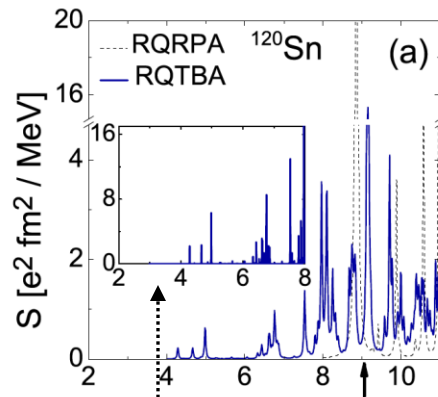
2q+phonon



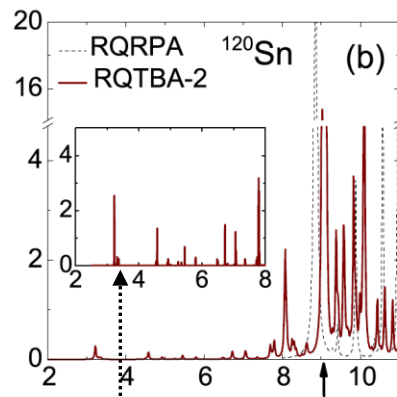
2 phonon



^{120}Sn



Does not exist



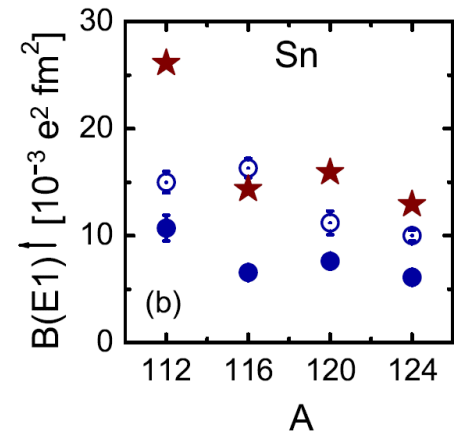
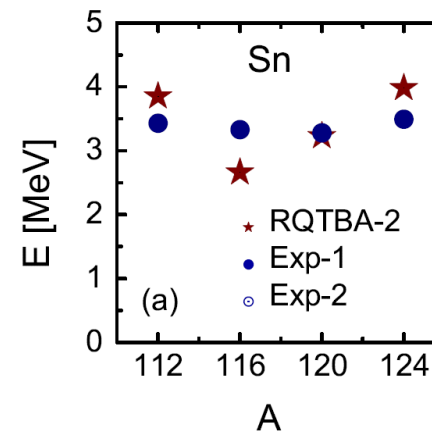
$3^- \otimes 2^+$

E.L., P.Ring, V.Tselyaev, PRL 105, 02252 (2010)
PRC 88, 044320 (2013)

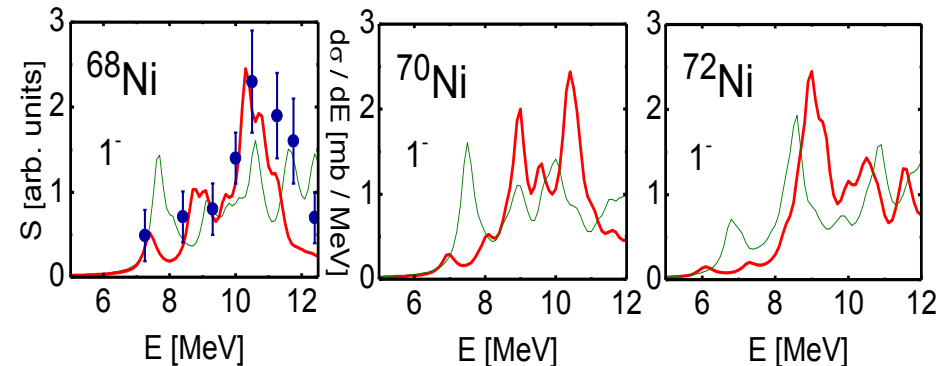
First two-phonon state 1^-_1 : $3^- \otimes 2^+$

$E(1^-_1)$

$B(E1) \uparrow$



Pygmy dipole resonance in neutron-rich Ni:
2q+phonon vs 2 phonon



Data: O. Wieland et al., PRL 102, 092502 (2009)

(n,γ) via compound nucleus: γ -transitions between excited states in the quasicontinuum

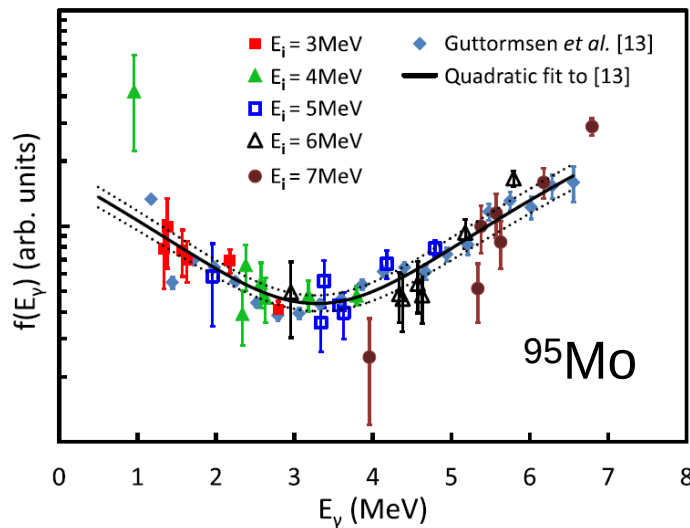
Low-energy enhancement of γ -strength

A. Voinov et al., PRL 93, 142504 (2004)

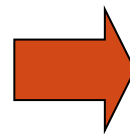
M. Guttormsen et al., PRC 71, 044307 (2005)

Oslo method

M. Wiedeking et al., PRL 108, 162503 (2012):

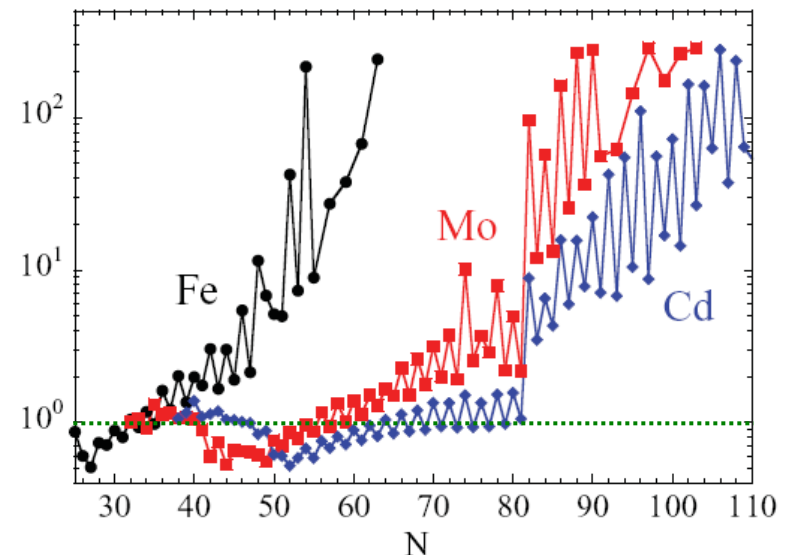
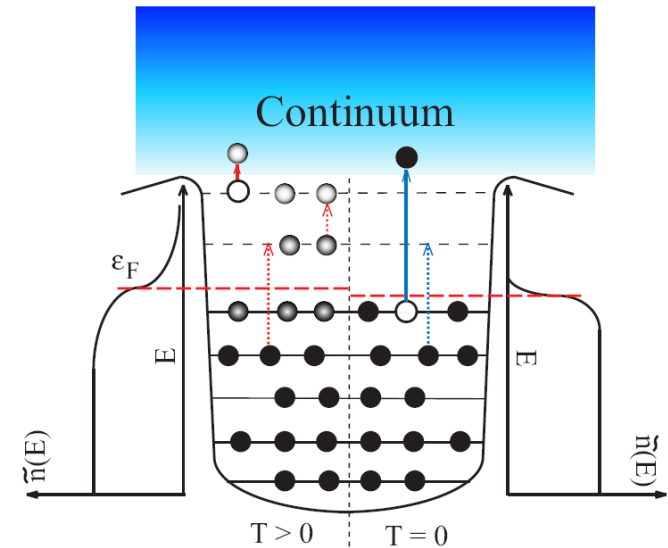


$$\frac{(n,\gamma) \text{ reaction rate (enh.)}}{(n,\gamma) \text{ reaction rate (no enh.)}} = \eta$$



A.C. Larsen, S. Goriely, PRC 014318 (2010)
Oslo data

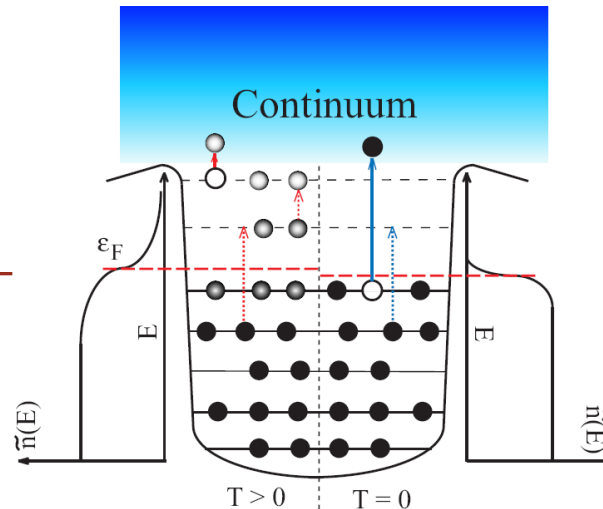
Correlations due to coupling to single-particle continuum



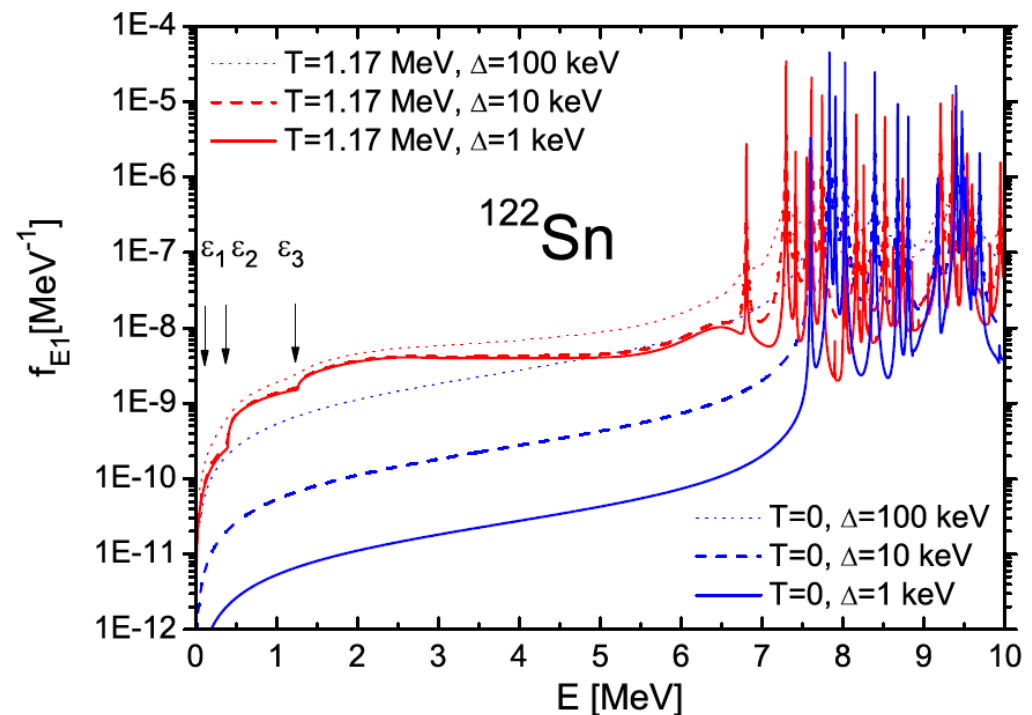
Mechanism of the RSF formation at low E_γ

$$\tilde{n}_i(E_i, T) = (1 - v_i^2(T)) n_i(E_i, T)$$

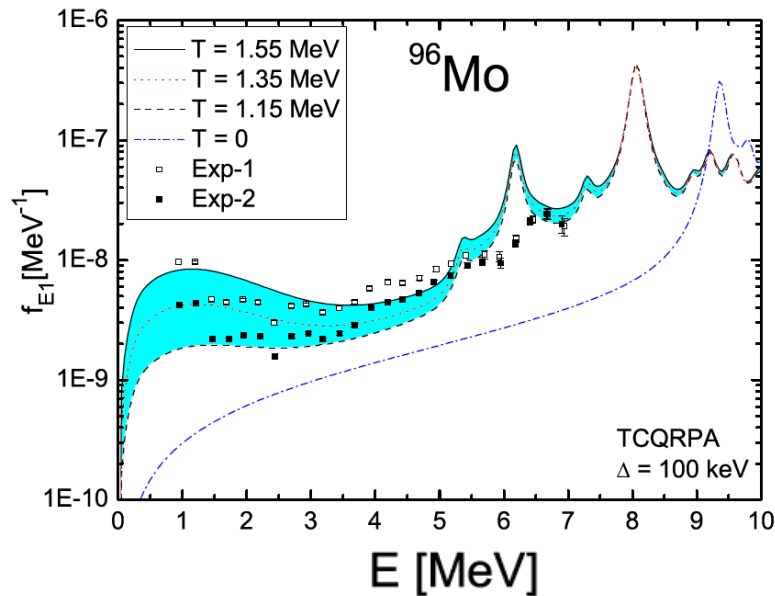
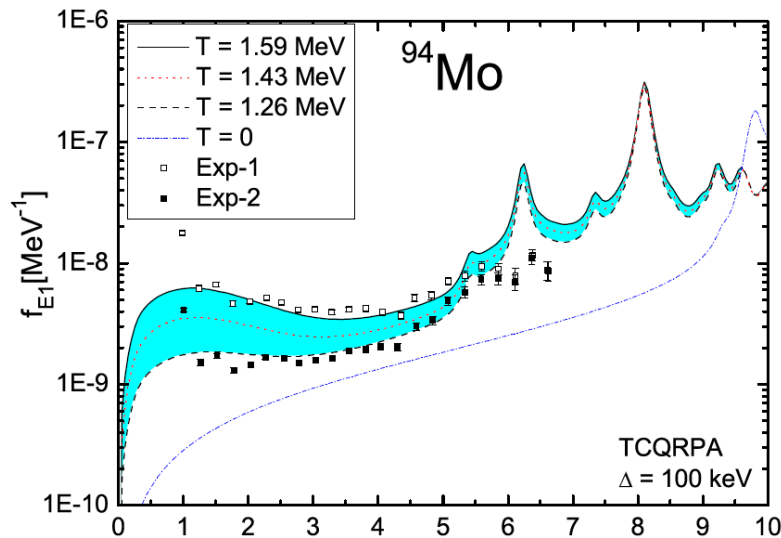
$$\tilde{n}_i(E_i, T) = v_i^2(T) (1 - n_i(E_i, T))$$



1. Saturation of RSF with Δ at $\Delta = 10$ keV for $T > 0$
2. The low-energy RSF is not a tail of the GDR and not a part of PDR!
3. The nature of RSF at $E_\gamma \rightarrow 0$ is continuum transitions from the thermally unblocked states
4. Spurious translation mode should be eliminated exactly



Low-energy limit of the RSF in even-even Mo isotopes



Theory: E. Litvinova, N. Belov,
PRC 88, 031302(R)(2013)

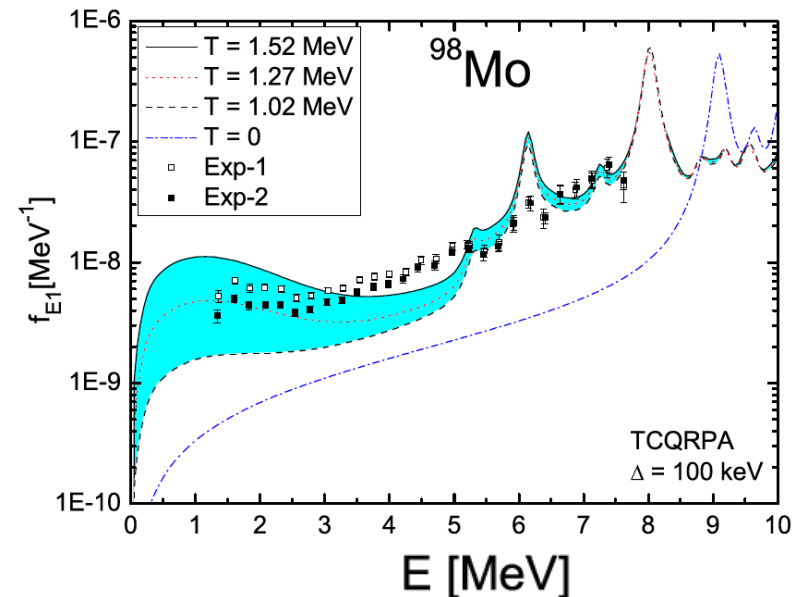
$$T = \sqrt{(E^* - \delta)/a}$$

$$a = a_{EGSF} \Rightarrow T_{min} \text{ (RIPL-3)}$$

$$a = \pi^2(g_\nu + g_\pi)/6 \Rightarrow T_{max} \text{ (microscopic)}$$

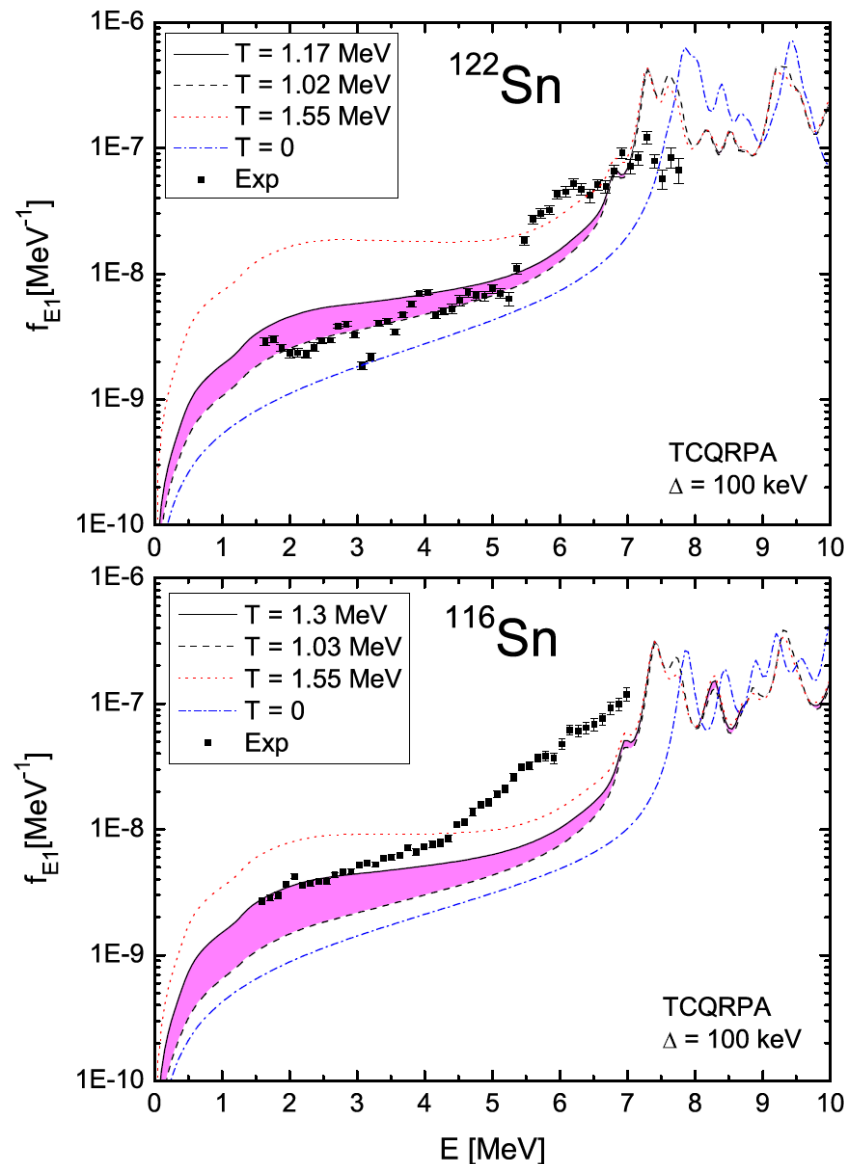
Exp-1: NLD norm-1,
M. Guttormsen et al., PRC 71, 044307 (2005)

Exp-2: NLD norm-2,
S. Goriely et al., PRC 78, 064307 (2008)



Data: A.C. Larsen, S. Goriely, PRC 014318 (2010)

Low-energy limit of the RSF in $^{116,122}\text{Sn}$



Theory: E. Litvinova, N. Belov,
PRC 88, 031302(R)(2013)

No upbend?

Larger microscopic level density
parameter a

$\Rightarrow$ Lower upper limit for the
temperature at S_n

Other effects

(ideally, all to be combined in one approach)

At 3-4 MeV:

- 2-phonon state (as above)

Above 4-5 MeV:

- Coupling to vibrations (as above),

- Thermal fluctuations:

M. Gallardo et al., NPA443, 415 (1985)

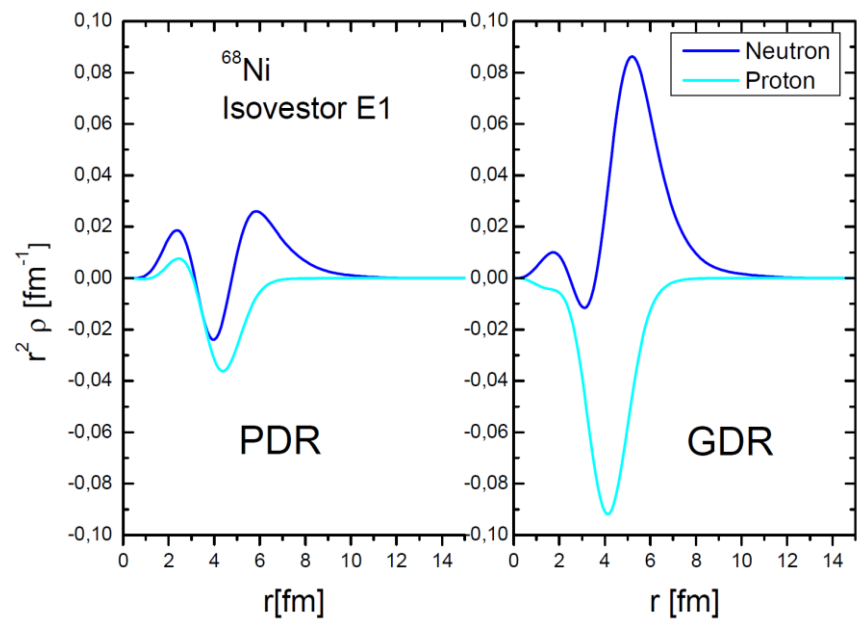
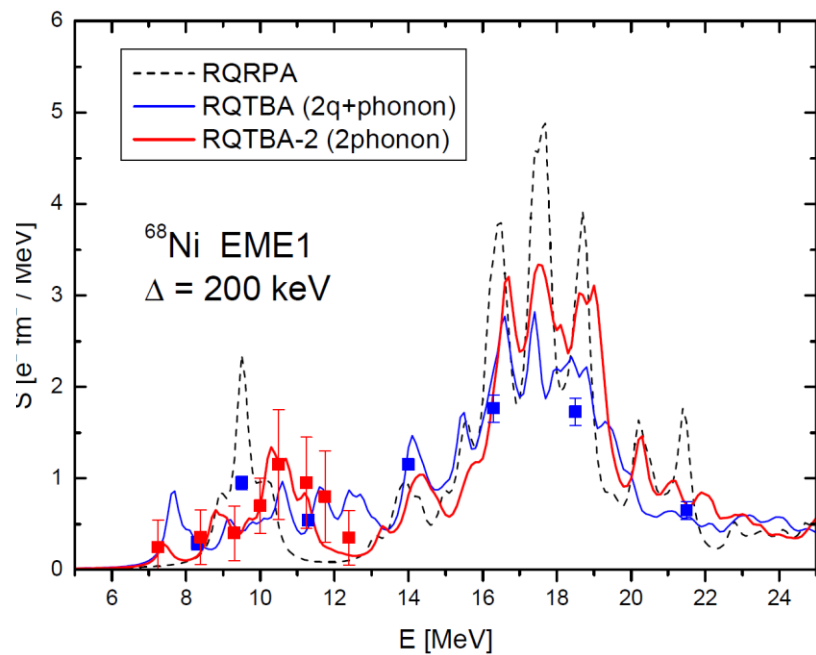
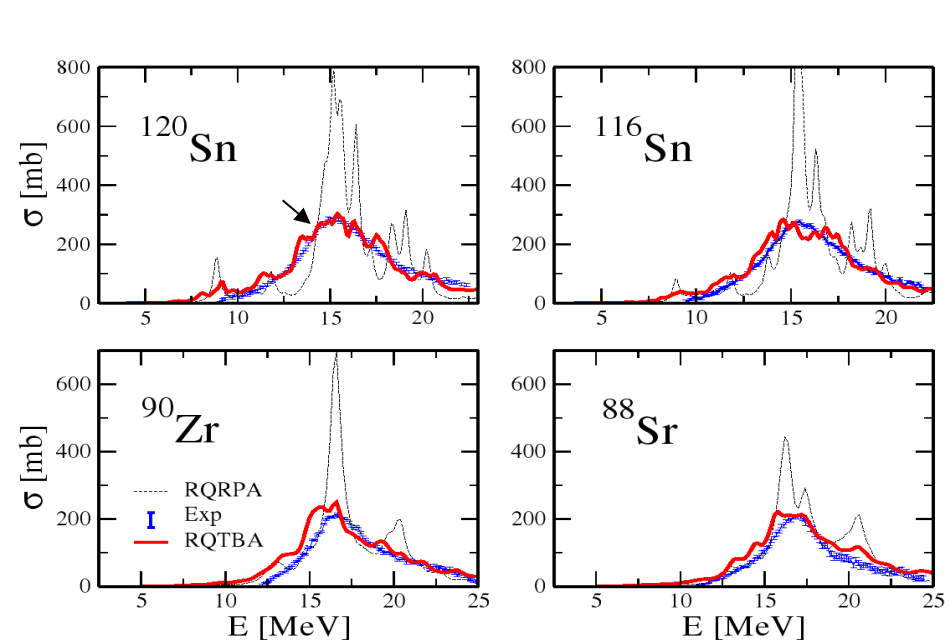
- More correct for γ -emission:

„final temperature“,

V. Plujko, NPA649, 209c (1999)

$$T_f = \sqrt{(E^* - \delta - E_\gamma)/a}$$

Data: H.K. Toft et al., PRC 81, 064311 (2010),
PRC 83, 044320 (2011)



$\Sigma_{p'p''}^e =$

$\Sigma_{h'h''}^e =$

Time arrow

Doubled quasiparticle space:

$$\eta = \pm 1$$

$$\Sigma_{k_1 k_2}^{(e)\eta_1 \eta_2}(\varepsilon) = \sum_{\eta=\pm 1} \sum_{k, \mu} \frac{\gamma_{\mu; k_1 k}^{\eta; \eta_1 \eta} \gamma_{\mu; k_2 k}^{\eta; \eta_2 \eta^*}}{\varepsilon - \eta(E_k + \Omega_\mu - i\delta)}$$

Relativistic mean field

$$E_{RMF}[\hat{\rho}, \phi] = Tr[(\boldsymbol{\alpha}\mathbf{p} + \beta m)\hat{\rho}] + \sum_m \left\{ Tr[(\beta \Gamma_m \phi_m)\hat{\rho}] \mp \int \left[\frac{1}{2}(\nabla \phi_m)^2 + U(\phi_m) \right] d^3r \right\}$$

$$\begin{cases} \mathcal{H}_{RHB} |\psi_k^\eta\rangle = \eta E_k |\psi_k^\eta\rangle, & \eta = \pm 1 \\ -\Delta \phi_m(\mathbf{r}) + U'(\phi_m(\mathbf{r})) = \mp \sum_k V_k^\dagger(\mathbf{r}) \beta \Gamma_m V_k^*(\mathbf{r}) \end{cases}$$

 Nucleons
 Mesons

no sea

RHB
Hamiltonian

$$\hat{\mathcal{H}}_{RHB} = \frac{\delta E_{RHB}}{\delta \mathcal{R}} = \begin{pmatrix} h^{\mathcal{D}} - m - \lambda & \Delta \\ -\Delta^* & -h^{\mathcal{D}*} + m + \lambda \end{pmatrix}$$

Dirac
Hamiltonian

$$h^{\mathcal{D}} = \boldsymbol{\alpha}\mathbf{p} + \beta(m + \underbrace{\sum_m \Gamma_m \phi_m(\mathbf{r})}_{\text{RMF self-energy}})$$

$\tilde{\Sigma}(\mathbf{r})$

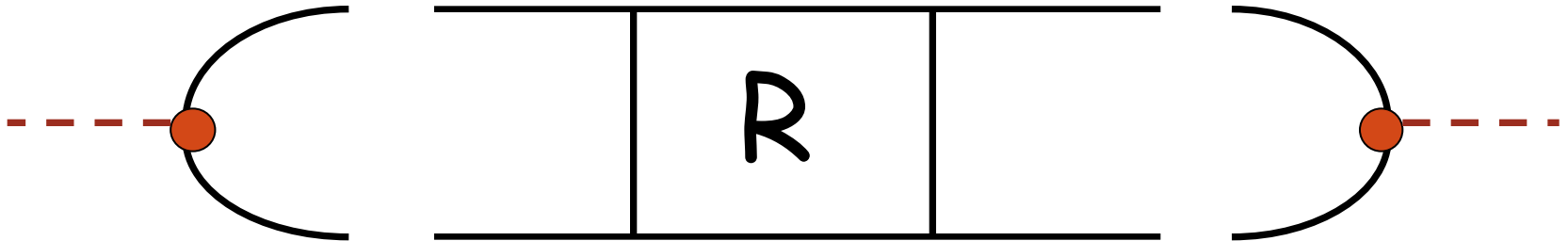
RMF
self-
energy

$$|\psi_k^+(\mathbf{r})\rangle = \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}$$

Eigenstates

$$|\psi_k^-(\mathbf{r})\rangle = \begin{pmatrix} V_k^*(\mathbf{r}) \\ U_k^*(\mathbf{r}) \end{pmatrix}$$

Nuclear Polarizability



Fine structure of spectra: next-order correlations from "2q+phonon" to "2 phonons"

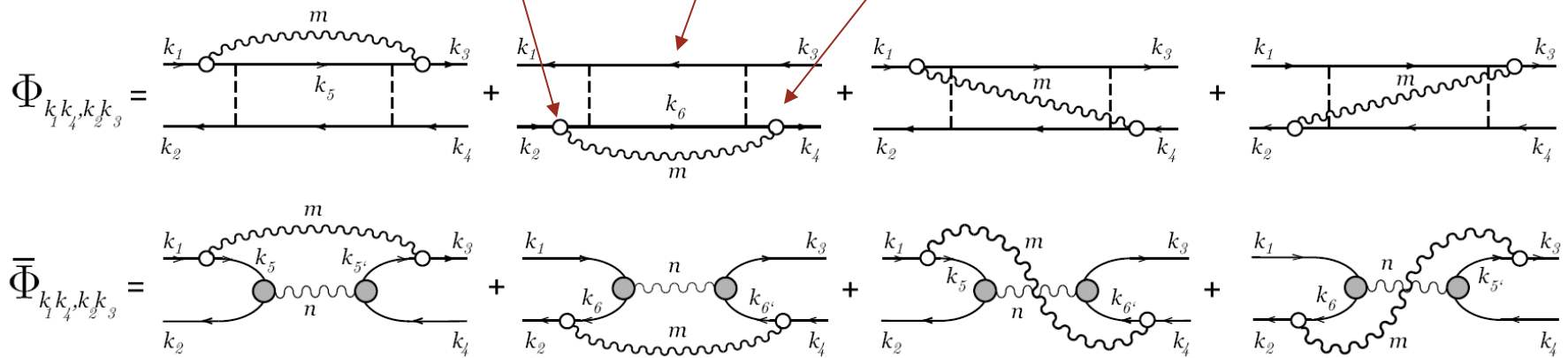
P. Schuck, Z. Phys. A 279, 31 (1976)

V.I. Tselyaev, PRC 75, 024306 (2007)

& Mode Coupling Theory
Time Blocking

$$\Phi_{12,34}(\omega) = - \sum_{5678, \eta, m} \gamma_{12}^{m56(\eta)} A_{56,78}^{(\eta)}(\omega - \eta \omega_m) \gamma_{34}^{m78(\eta)*}$$

Replacement of the uncorrelated propagator inside the Φ amplitude by QRPA response



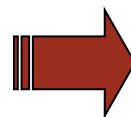
Nuclear response:

$$R = A + A (V + \bar{\Phi} - \bar{\Phi}_0) R$$

Poles may appear at lower energies:

'2q+phonon' response:

$$\Phi_{ij'j'}(\omega) \sim \sum_{\mu k} a_{ijk\mu} / (\omega - E_{i'} - E_k - \Omega_{\mu})$$

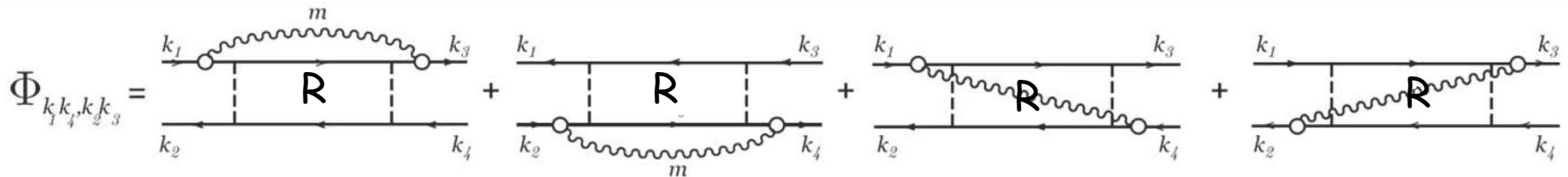


'2 phonon' response:

$$\Phi_{ij'j'}(\omega) \sim \sum_{\mu\nu} a_{ij'j'} / (\omega - \Omega_{\nu} - \Omega_{\mu})$$

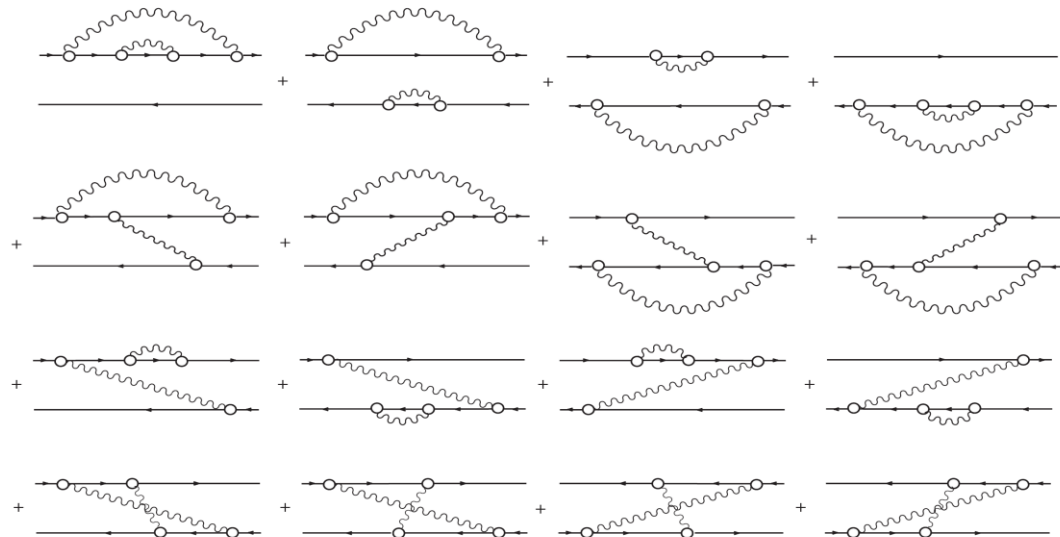
Next-order RQTBA for 3p-3h configurations & iterative procedure for multiphonon response

$$R^{(n+1)} = GG + GG [V + \Phi(R^{(n)})] R^{(n+1)}$$



Nested configurations

γ^4 terms in Φ -amplitude:



3p3h:
two-fish approximation, ...

Response function in the neutral channel

response

$$R(\omega) = \tilde{R}^0(\omega) + \tilde{R}^0(\omega)W(\omega)R(\omega)$$

interaction

$$W(\omega) = V_\sigma + V_\omega + V_\rho + V_e + \underbrace{\Phi(\omega)}_{\text{Subtraction to avoid double counting}} - \underbrace{\Phi(0)}_{\text{Subtraction to avoid double counting}}$$

Static:
RQRPA

$$\left\{ \begin{array}{l} v_\sigma(1, 2) = -g_\sigma^2 \gamma_1^0 D_\sigma(1, 2) \gamma_2^0 \\ v_\omega(1, 2) = +g_\omega^2 (\gamma^0 \gamma_\mu)_1 D_\omega^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu)_2 \\ v_\rho^V(1, 2) = +g_\rho^2 (\gamma^0 \gamma_\mu \vec{\tau})_1 \vec{\tau}_1 \cdot \vec{\tau}_2 D_\rho^{\mu\nu}(1, 2) (\gamma^0 \gamma_\nu \vec{\tau})_2 \end{array} \right.$$

Subtraction
to avoid double
counting

Dynamic:

particle-
vibration
coupling

in time blocking
approximation

$$\begin{aligned} \Phi_{k_1 k_4, k_2 k_3}^\eta(\omega) = & \\ = \sum_{\mu\xi} \delta_{\eta\xi} \left[\delta_{k_1 k_3} \sum_{k_6} \frac{\gamma_{\mu; k_6 k_2}^{\eta; -\xi} \gamma_{\mu; k_6 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_1} - E_{k_6} - \Omega_\mu} + \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma_{\mu; k_1 k_5}^{\eta; \xi} \gamma_{\mu; k_3 k_5}^{\eta; \xi*}}{\eta\omega - E_{k_5} - E_{k_2} - \Omega_\mu} \right. \\ & \left. - \left(\frac{\gamma_{\mu; k_1 k_3}^{\eta; \xi} \gamma_{\mu; k_2 k_4}^{\eta; -\xi*}}{\eta\omega - E_{k_3} - E_{k_2} - \Omega_\mu} + \frac{\gamma_{\mu; k_3 k_1}^{\eta; \xi*} \gamma_{\mu; k_4 k_2}^{\eta; -\xi}}{\eta\omega - E_{k_1} - E_{k_4} - \Omega_\mu} \right) \right] \end{aligned}$$