

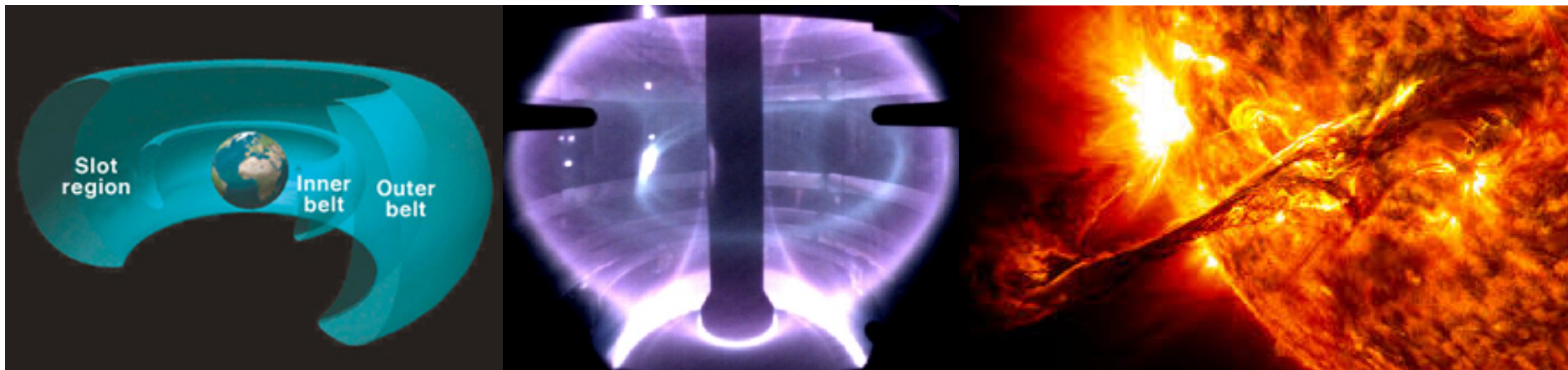
Plasma Physics

TU Dresden

Lecturer: Dr. Katerina Falk



Lecture 9: Laser plasmas & ICF

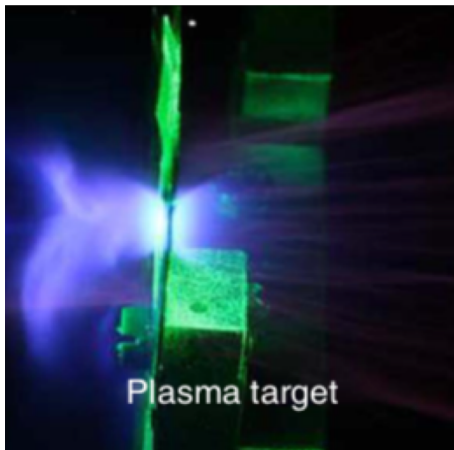


Plasma Physics: lecture 9

- Inverse Bremsstrahlung
- Laser Ablation
- The Lawson criterion revisited
- Inertial Confinement Fusion (ICF)
- Different technological approaches to ICF
- Big laser ZOO

The critical surface

- When a laser hits a solid target, the surface is heated, and a plasma is formed.
- This plasma expands into the vacuum, and its density drops.



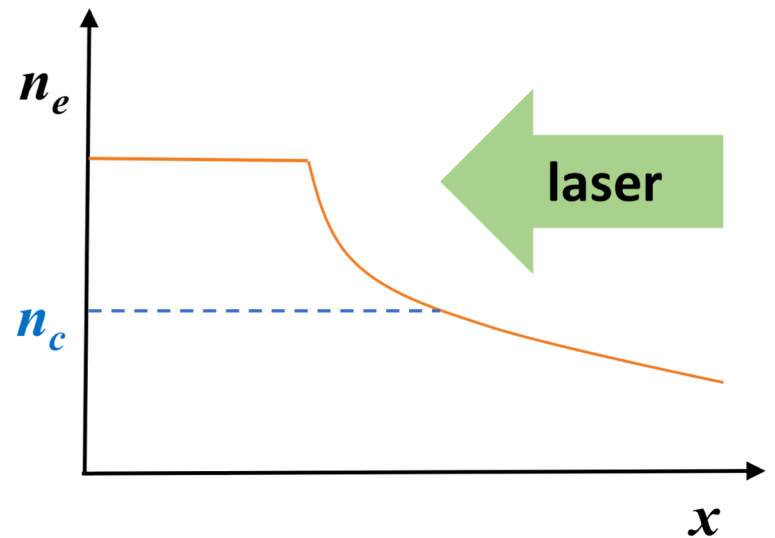
The critical surface

- In lecture 3 we derived the relationships for the propagation of electromagnetic radiation in plasma.
- We have derived the refractive index in plasma:

$$\mu = \sqrt{\varepsilon(\omega)} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

- And the critical density:

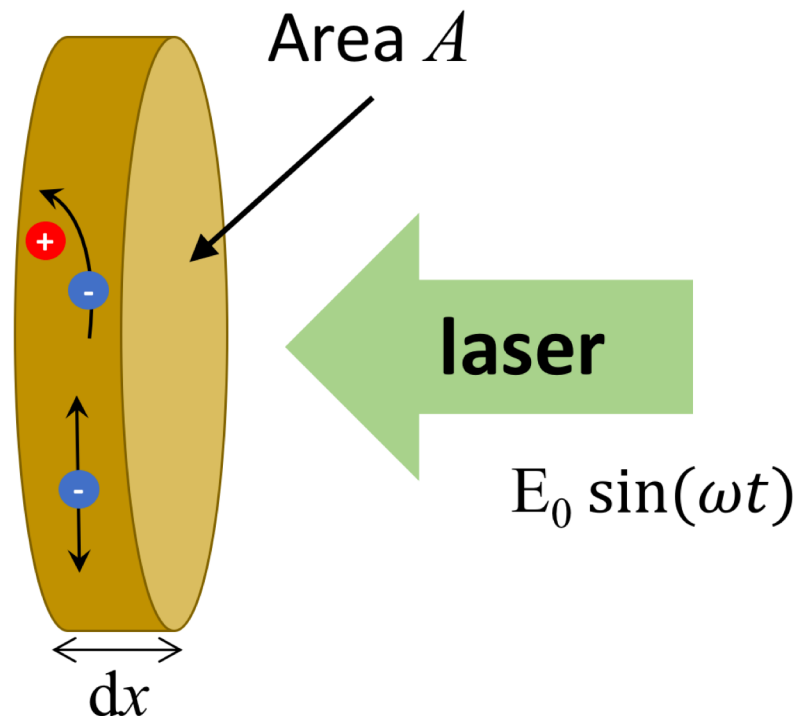
$$n_c = \frac{\omega^2 \varepsilon_0 m}{e^2}$$



- Laser light is absorbed at lower densities and reflected at the critical density n_c .

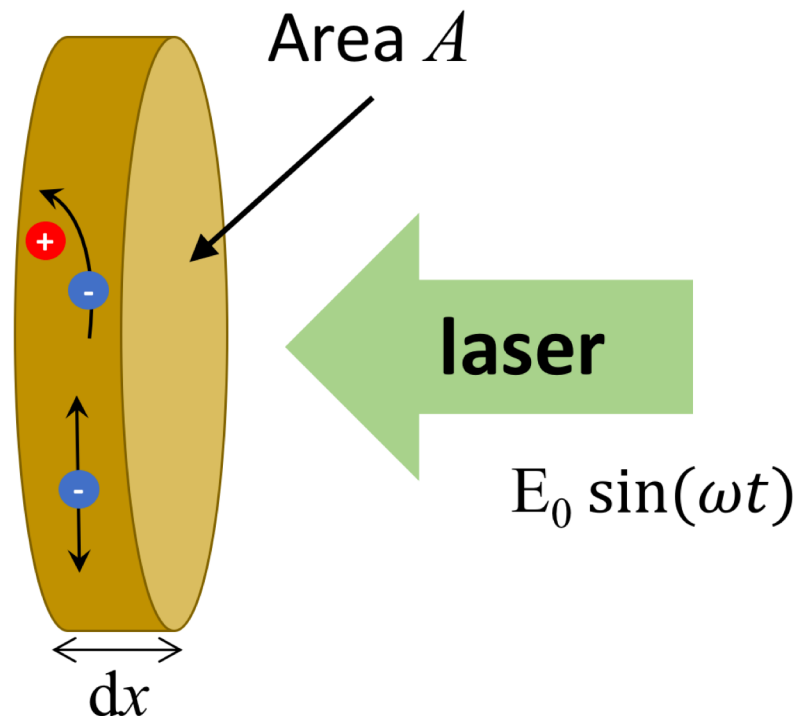
Inverse Bremsstrahlung

- The incident laser light (oscillating EM field) causes the electrons to oscillate.
- In the absence of collisions, these electrons re-radiate at the original laser frequency, so no energy is lost from the light.



Inverse Bremsstrahlung

- However, the electrons also have some random thermal velocity, and in reality they collide with ions. Thus the oscillatory energy is converted to thermal energy.
- We will estimate the absorption coefficient.

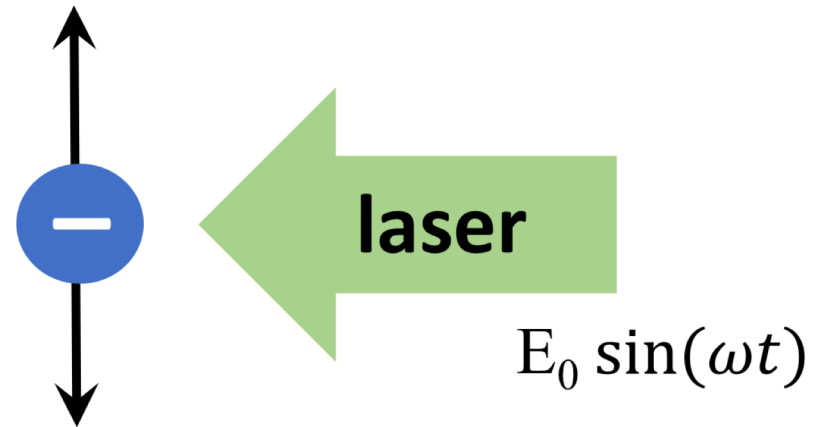


Oscillatory energy

- Incident laser with frequency ω , intensity I and peak electric field E_0 :
- Equation of motion for electron:

$$m\ddot{x} = -eE_0 \sin(\omega t)$$

$$\dot{x} = \frac{eE_0}{\omega m} \cos(\omega t)$$



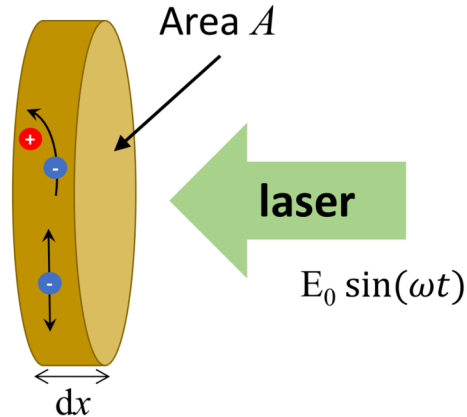
- Time averaged kinetic energy:

$$\overline{\frac{1}{2}m\dot{x}^2} = \frac{e^2 E_0^2}{4m\omega^2}$$

Total oscillatory energy

- For electron density n_e in a given volume, the total oscillatory energy:

$$U_e = n_e \left(\frac{e^2 E_0^2}{4m\omega^2} \right) A dx$$



$$U_L = I A t$$

- The collision time between ions and electrons is τ_{ei} and fraction of the oscillatory energy that is converted to thermal energy to be $\approx t/\tau_{ei}$
- Oscillatory energy gained in time t is equivalent to change in laser field energy:

$$dU_e = dU_L = -\frac{t}{\tau_{ei}} n_e \left(\frac{e^2 E_0^2}{4m\omega^2} \right) A dx$$

Absorption coefficient

- The absorption coefficient κ is defined such that for light of intensity I :

$$I = I_0 \exp(-\kappa x)$$

$$dI = -\kappa I_0 \exp(-\kappa x) dx$$

- Thus:

$$\begin{aligned}\kappa &= -\frac{1}{I} \frac{dI}{dx} \\ &= -\frac{1}{U_L} \frac{dU_L}{dx} \\ &= \frac{1}{IA t} \frac{t}{\tau_{ei}} n_e \left(\frac{e^2 E_0^2}{4m\omega^2} \right) A \\ &= \frac{1}{I} \frac{1}{\tau_{ei}} n_e \left(\frac{e^2 E_0^2}{4m\omega^2} \right)\end{aligned}$$

Absorption coefficient

- The electric field of the laser E_0 , is related to the intensity I via the Poynting vector:

$$I = N = \frac{1}{2} \sqrt{\epsilon_r} \epsilon_0 E_0^2 c$$

- By substituting for intensity:

$$\kappa = \frac{1}{2\tau_{ei}} \left(\frac{n_e e^2}{\epsilon_0 m \omega^2} \right) \frac{1}{c \sqrt{\epsilon_r}}$$

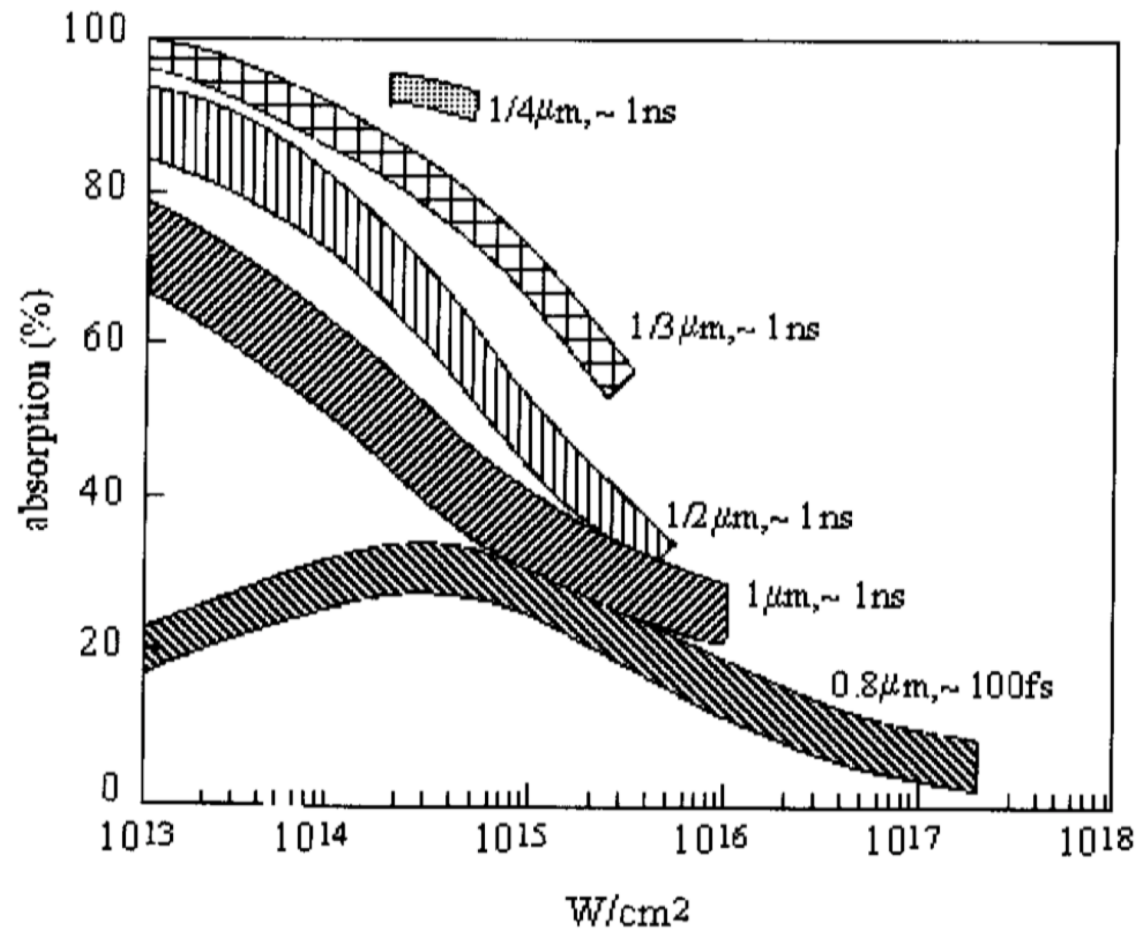
- Substituting for plasma frequency:

$$\kappa = \frac{1}{2c\tau_{ei}} \left(\frac{\omega_p^2}{\omega^2} \right) \left(1 - \frac{\omega_p^2}{\omega^2} \right)^{-1/2}$$

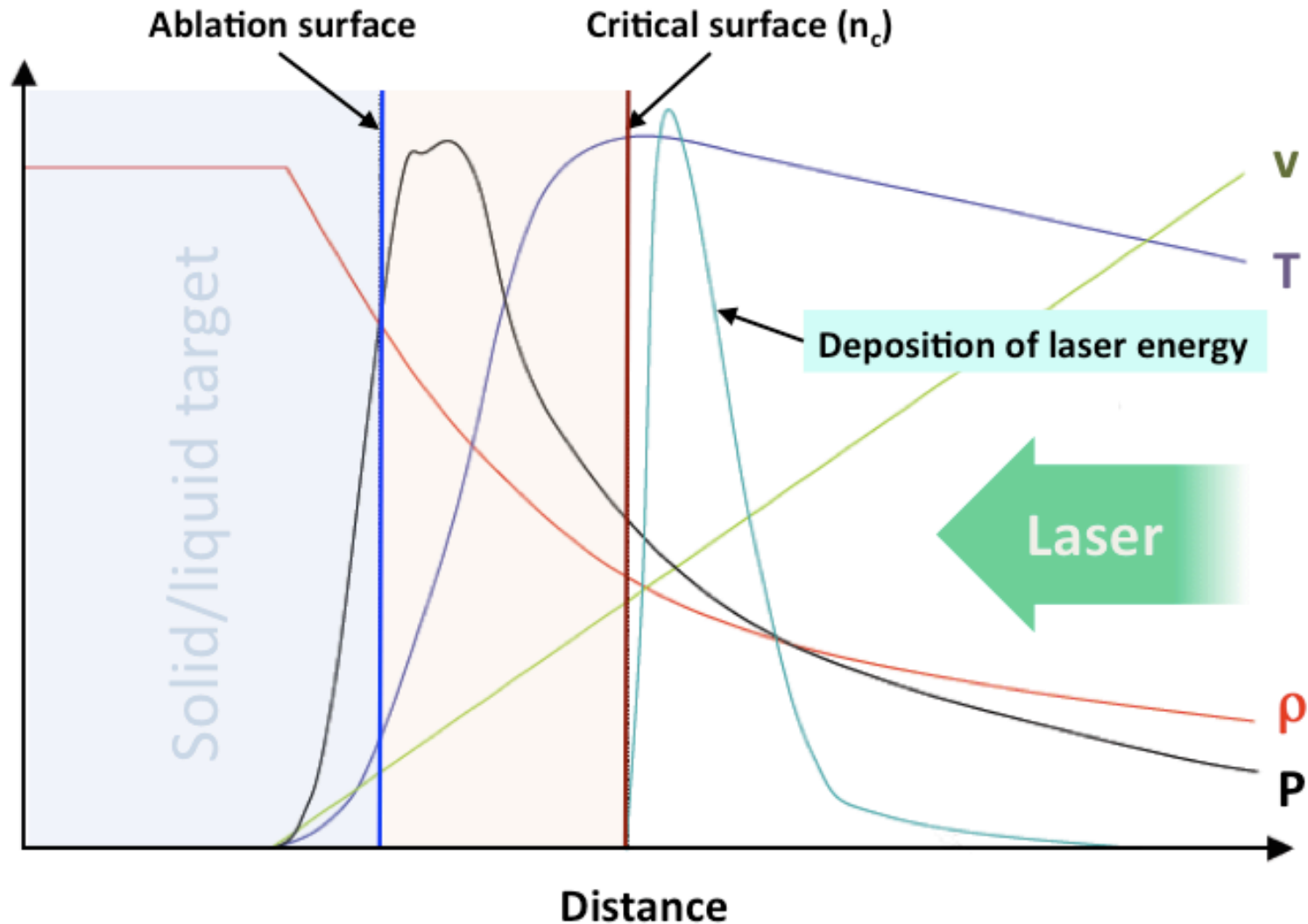
Absorption coefficient

- Thermalisation of the laser-induced oscillatory energy by electron-ion collisions that gives rise to the absorption mechanism, referred to as “inverse Bremsstrahlung”.
- We expect denser plasmas to absorb more efficiently.
- Shorter wavelength can propagate to higher density, thus should be better absorbed.
- At high irradiances, the plasma has higher temperature during the pulse, and thus the absorption drops.
- Note: a more rigorous method to derive the absorption coefficient would provide a factor of 2 greater value for κ , but the general approach is correct.

Absorption coefficient



Laser ablation model



Laser ablation model

- Laser energy is deposited up to critical surface.
- Region outside critical surface is known as the corona, and is approximately isothermal.
- Laser energy flows down the temperature gradient to the target. Material is ablated away at the ablation surface.
- Pressure increases towards the ablation surface, as temperature is dropping, the density increases.
- As the plasma expands away from the target, the velocity increases (mass flow approximately constant).

Laser ablation model

- Assuming a steady-state situation, the mass flow at the ablation and critical surfaces must be constant:

$$\rho_a u_a = \rho_c u_c$$

- The momentum flux is also constant:

$$P_a + \rho_a u_a^2 = P_c + \rho_c u_c^2$$

- Plasma expansion looks like isenthalphic throttling process. We need to account for the kinetic energy flow in the plasma as well as the heat flow W between critical and ablation surfaces:


$$\frac{1}{2}\rho_a u_a^3 + H_a \rho_a u_a + W = \frac{1}{2}\rho_c u_c^3 + H_c \rho_c u_c$$

Enthalpy: $H = U + PV$



Laser ablation model assumptions

- For simplicity we assume the ideal gas equation of state, thus enthalpy is given by:

$$H = \left(\frac{\gamma}{\gamma - 1} \right) \frac{P}{\rho}$$


- Plasma at ablation surface is denser: $\rho_a \gg \rho_c$
- Assume that the thermal flux W is dominant:

$$W \gg \frac{1}{2} \rho_a u_a^3 + H_a \rho_a u_a$$

- Assume plasma flow velocity at the critical surface is Mach ~ 1 , i.e. sound velocity:

$$u_c = \left(\frac{\gamma P_c}{\rho_c} \right)^{1/2}$$

Laser ablation model

- We can now construct the ablation model:

$$W = \frac{1}{2}\rho_c u_c^3 + H_c \rho_c u_c$$

- Substituting the previous expressions:

$$W = \frac{\rho_c}{2} \left(\frac{\gamma P_c}{\rho_c} \right)^{3/2} + \left(\frac{\gamma}{\gamma - 1} \right) P_c \left(\frac{\gamma P_c}{\rho_c} \right)^{1/2}$$

$$W = \frac{1}{\sqrt{\rho_c}} P_c^{3/2} \sqrt{\gamma} \left(\frac{\gamma}{2} + \frac{\gamma}{\gamma - 1} \right)$$

- For $\gamma = 5/3$, rearrange: $P_c \approx 4\rho_c^{1/3} W^{2/3}$

 *predicted pressure at the critical surface*

Laser ablation model

- At the ablation surface the velocity tends to zero, and thus:

$$P_a = P_c + \rho_c u_c^2$$

- Substitute for u_c :

$$P_a = (1 + \gamma)P_c = \frac{8}{3}P_c$$

- Therefore the ablation pressure:

$$P_a \approx \frac{32}{3} \rho_c^{1/3} W^{2/3}$$

- There is a critical density for a specific laser wavelength:

$$\omega^2 = \frac{n_c e^2}{\epsilon_0 m_e}$$

Laser ablation model

- Assume fully ionised atoms of mass $M = 2Zm_p$ (nuclei with roughly equal numbers of protons and neutrons):

$$\rho_c = 2n_cm_p = 2m_p \left(\frac{\epsilon_0 m_e \omega^2}{e^2} \right)$$

- Substitute into the expression for P_a :

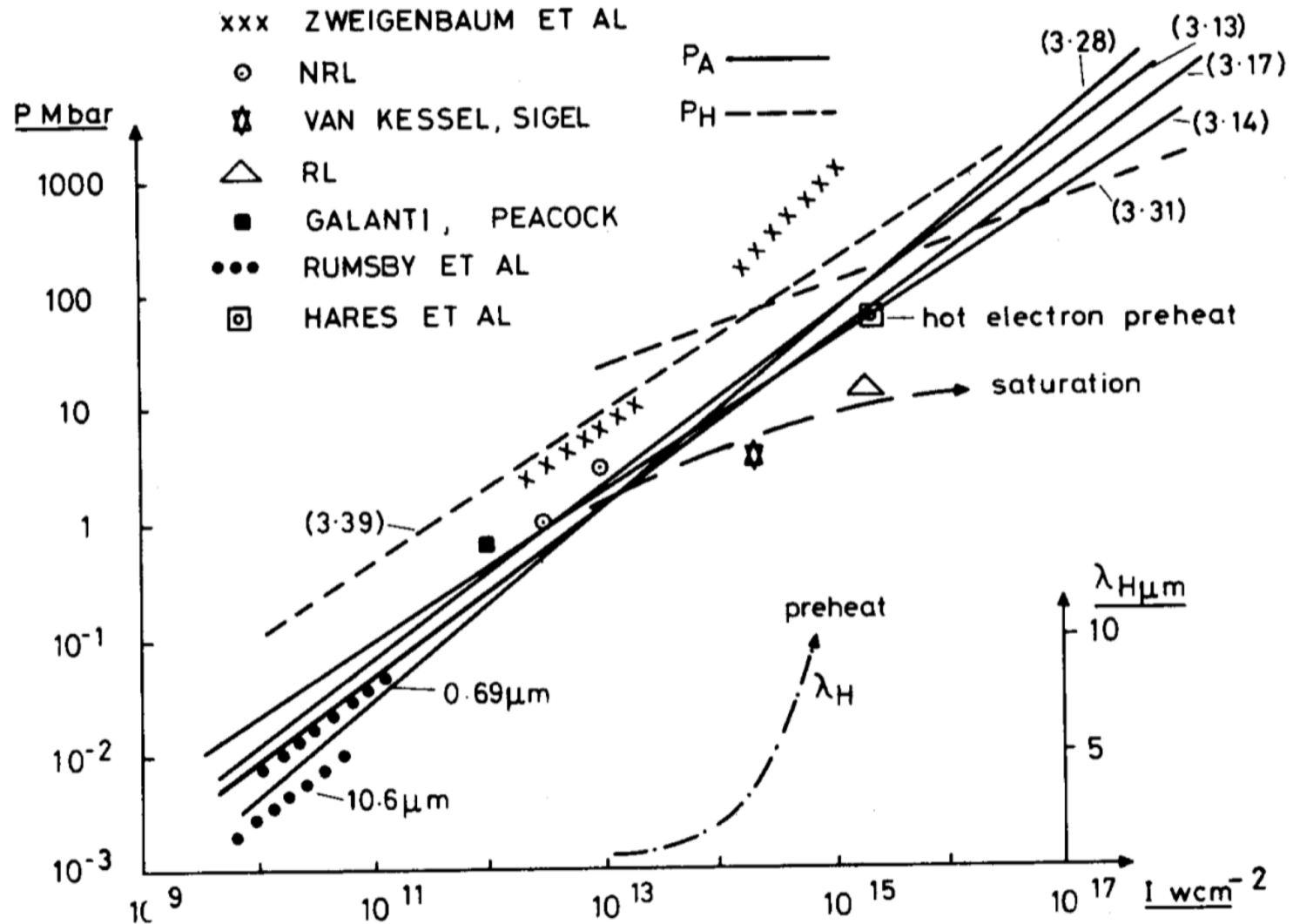
$$P_a \approx \frac{32}{3} \left(\frac{2\epsilon_0 m_e m_p \omega^2}{e^2} \right)^{1/3} W^{2/3}$$

Laser intensity in W/cm²

- For laser intensity in W/cm² we have:

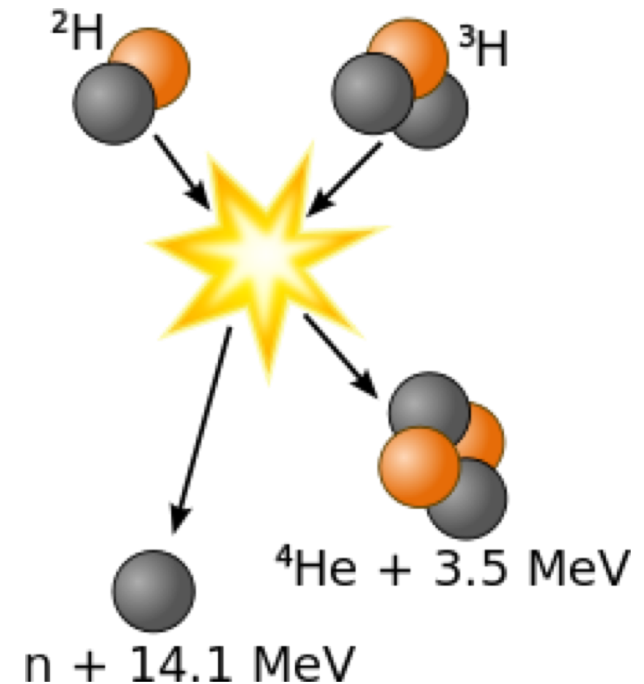
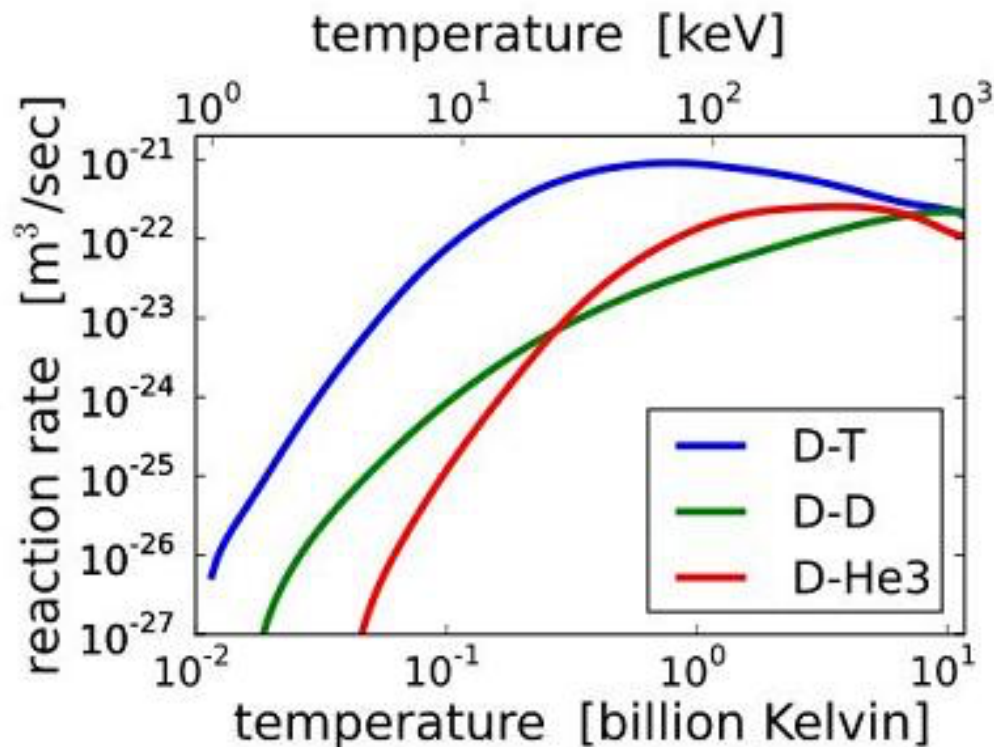
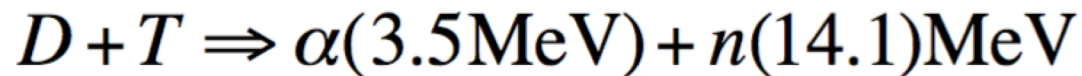
$$P_a \approx 2.8 \times 10^{-9} \times W^{2/3} \text{ Mbar}$$

Laser ablation model

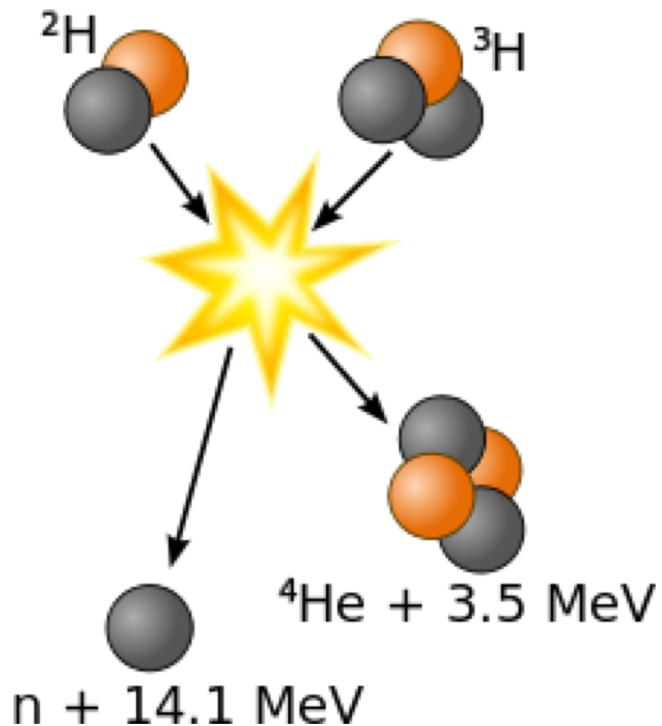


Thermonuclear fusion

- D-T reaction rate is $\sim 10^{-22} \text{ m}^3\text{s}^{-1}$ at a temperature of 10 keV.



Approaches to fusion research



Magnetic
confinement

Inertial confinement
(lasers, Z-pinch)



The Lawson criterion

- For D-T fusion energy gain, the energy out must exceed the energy in. The energy out per volume is given by:

$$E_f = n_D n_T \sigma W \tau = \frac{n^2 \sigma W \tau}{4}$$

Energy per reaction (arrow pointing to W)
Reaction rate (arrow pointing to $n_D n_T \sigma$)
Plasma confinement time (arrow pointing to τ)

Assuming: $n_D = n_T = \frac{n}{2}$

- This must exceed the energy required to heat the plasma to the temperature required for fusion:

$$E_t = 2 \times \frac{3}{2} n k_B T$$

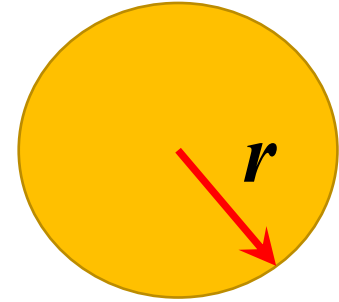
- Thus:

$$\frac{n^2 \sigma W \tau}{4} > 3 n k_B T$$

The Lawson criterion

- We define the Lawson criterion:

$$n\tau > \frac{12k_B T}{\sigma W}$$



- For a $T = 10$ keV, $\sigma \sim 10^{-22} \text{ m}^3\text{s}^{-1}$, $W \sim 17$ MeV:

$$n\tau > \frac{12 \times 10^4}{10^{-22} \times 17 \times 10^6} \sim 10^{20} \text{ sm}^{-3}$$

- Plasma of radius r and temperature T is confined by its own inertia for a time of order its initial radius divided by the sound speed.

$$\tau \approx \frac{r}{4C_s}$$

The Lawson criterion

- Derive the Lawson criterion for ICF:

$$n\tau = \frac{\rho r}{4C_s M} > 10^{20} \text{ s m}^{-3}$$

- Where the sound speed is:

$$C_s = \sqrt{\frac{k_B T}{M_{ion}}}$$

- If we use the mass of a deuterium atom and $T \sim 10$ keV:

$$\rho r > 0.6 \text{ g/cm}^2$$

How much laser energy is needed?

- Combining:

$$\frac{n^2 \sigma W \tau}{4} > 3k_B T \quad \text{and} \quad \tau \approx \frac{r}{4C_s}$$

- We get: $\frac{n^2 \sigma W r}{16C_s} > 3nk_B T$

- Thus radius needed to satisfy the Lawson criterion:

$$r = \frac{48k_B T C_s}{n\sigma W}$$

How much laser energy is needed?

- The fusion energy released per unit volume E_f in a D-T reaction can be written as:

$$E_f = \frac{n^2 \sigma W r}{16 C_s} \approx E_t = 3 n k_B T$$

Energy per unit volume



- Therefore total energy U required to heat a sphere of radius r to fusion conditions is:

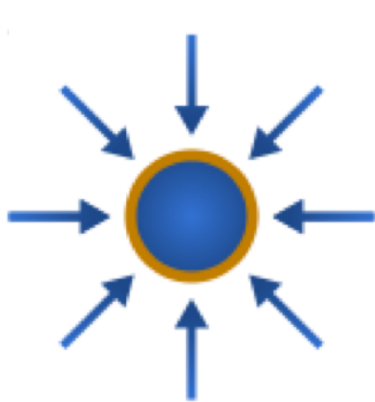
$$U = \frac{4\pi r^3}{3} E_f = 4\pi \left(\frac{48 k_B T C_s}{n \sigma W} \right)^3 n k_B T$$

- At 10 keV this is:

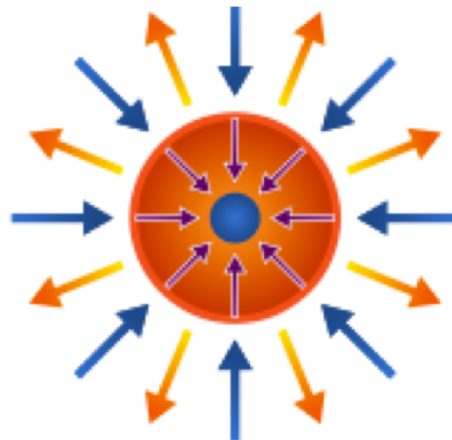
$$U \approx 4 \times 10^{68} \frac{1}{n^2} \text{ J}$$

Inertial confinement fusion (ICF)

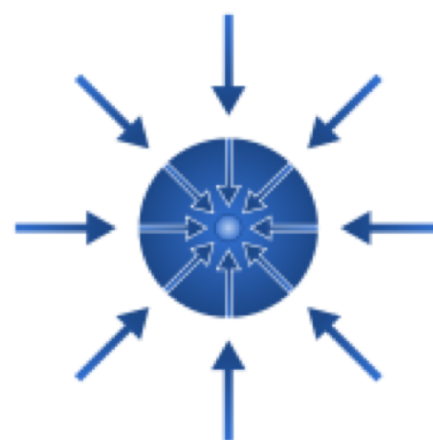
- The idea of laser fusion is to compress a small frozen (solid) hollow D-T pellet into a very high density up to $1000 \text{ g/cm}^3 \approx 10^{26} \text{ cm}^{-3}$ and heat it to extreme temperatures reaching 10 keV (10^7 K) using laser ablation.
- There are several approaches to laser fusion, next slides.
- An alternative approach to ICF is using Z-pinches (discussed in the previous lecture).



Atmosphere formation



Compression



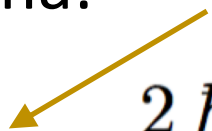
Ignition



Burn

Compression requirements

- If we could compress the D-T by, say a factor of 100, the heating laser would only need to contain ~ 100 kJ, (detailed calculations show we need a factor of ten more). However, to produce such high densities a large pressure is required. Assuming electrons are degenerate in the compressed plasma:

$$P_F = \frac{2}{5} n E_F = \frac{2}{5} \frac{\hbar^2 n}{2m} (3\pi^2 n)^{2/3}$$


Fermi energy

- For $n = 100 \times 6 \times 10^{29}$, we find $E_F \sim 550$ eV, $P \sim 2 \times 10^4$ Mbar. Thus we need pressures of order 200 times typical ablation pressures (more detailed calculations show factors of ~ 50).

Pressure Amplification

- Spherical compression amplifies the pressure.
- Initial acceleration of the imploding shell gives:

$$F = P_a 4\pi r_0^2 = \rho_0 4\pi r_0^2 \Delta r a \quad \rho_0 = \text{initial density}$$

- Assuming constant acceleration, the final velocity is:

$$v^2 = 2as \quad s = \text{distance travelled}$$

- The final radius is small, so assume $s = r_0$ and estimate the final kinetic energy:

$$\frac{1}{2}mv^2 = mas$$

$$= m \frac{P_a}{\rho_0 \Delta r} r_0$$

$$= V P_a \left(\frac{r_0}{\Delta r} \right)$$

Pressure Amplification

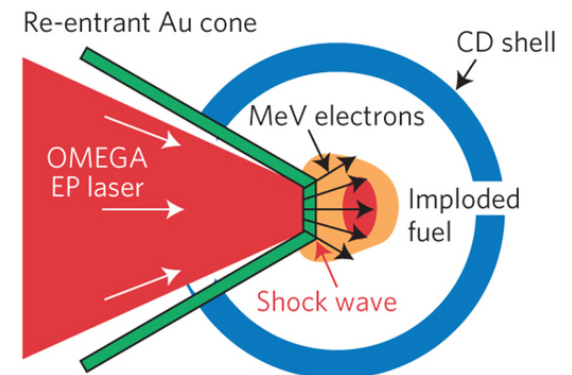
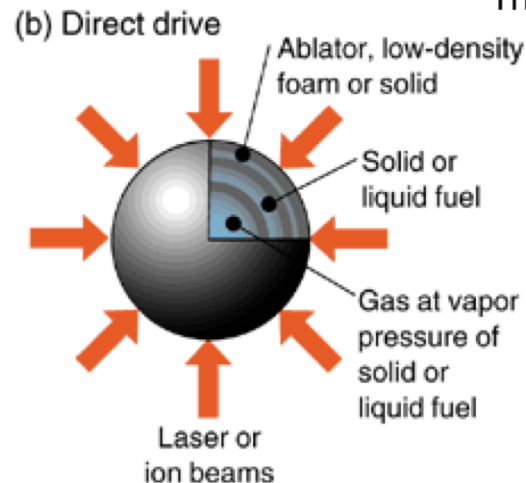
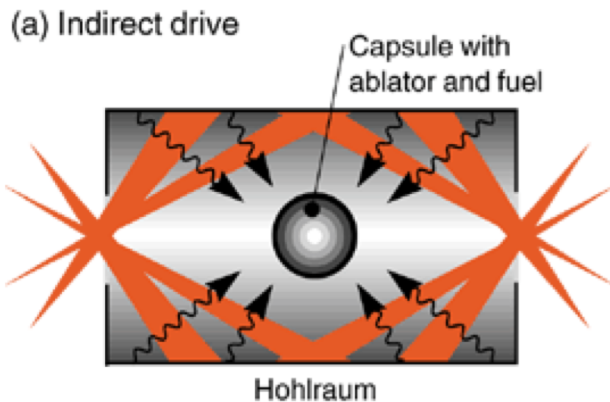
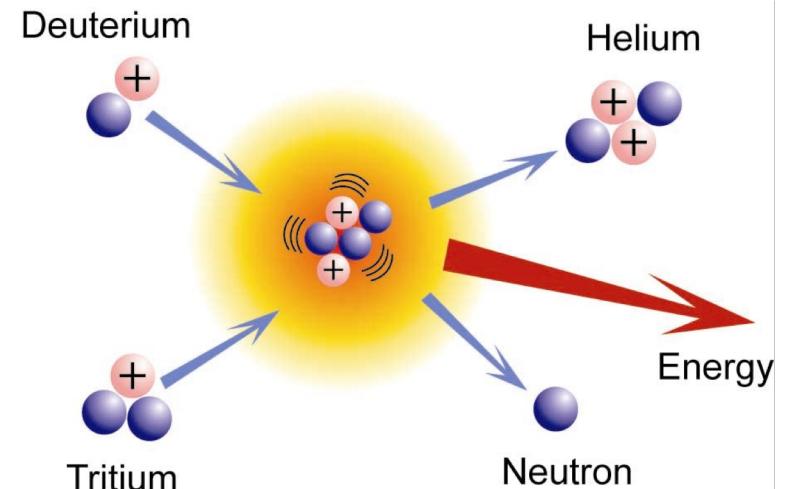
- As pressure is energy per unit volume, if this kinetic energy is converted into energy of compression adiabatically, then the stagnation pressure P_s is:

$$P_s = \frac{1}{V} \frac{1}{2} m v^2 = P_a \left(\frac{r_0}{\Delta r} \right)$$

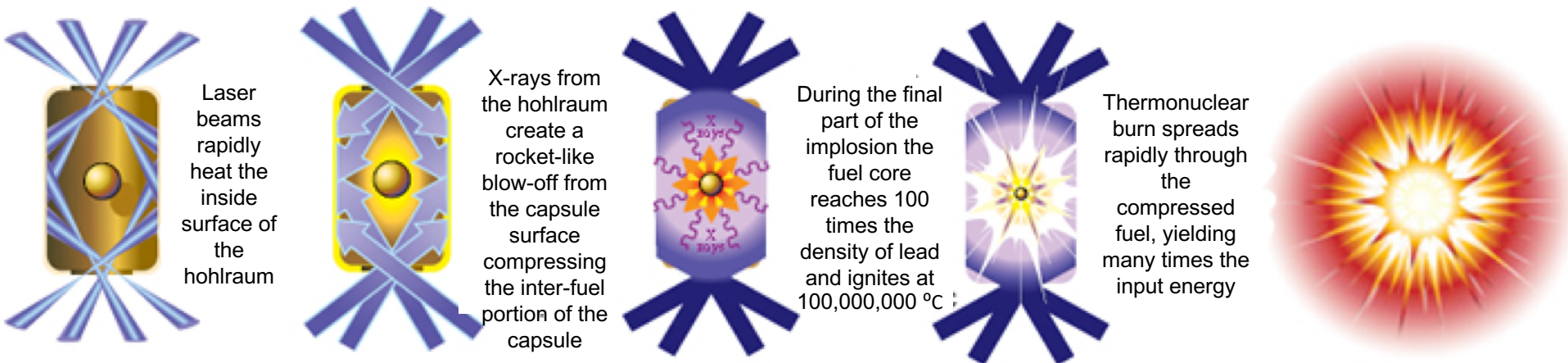
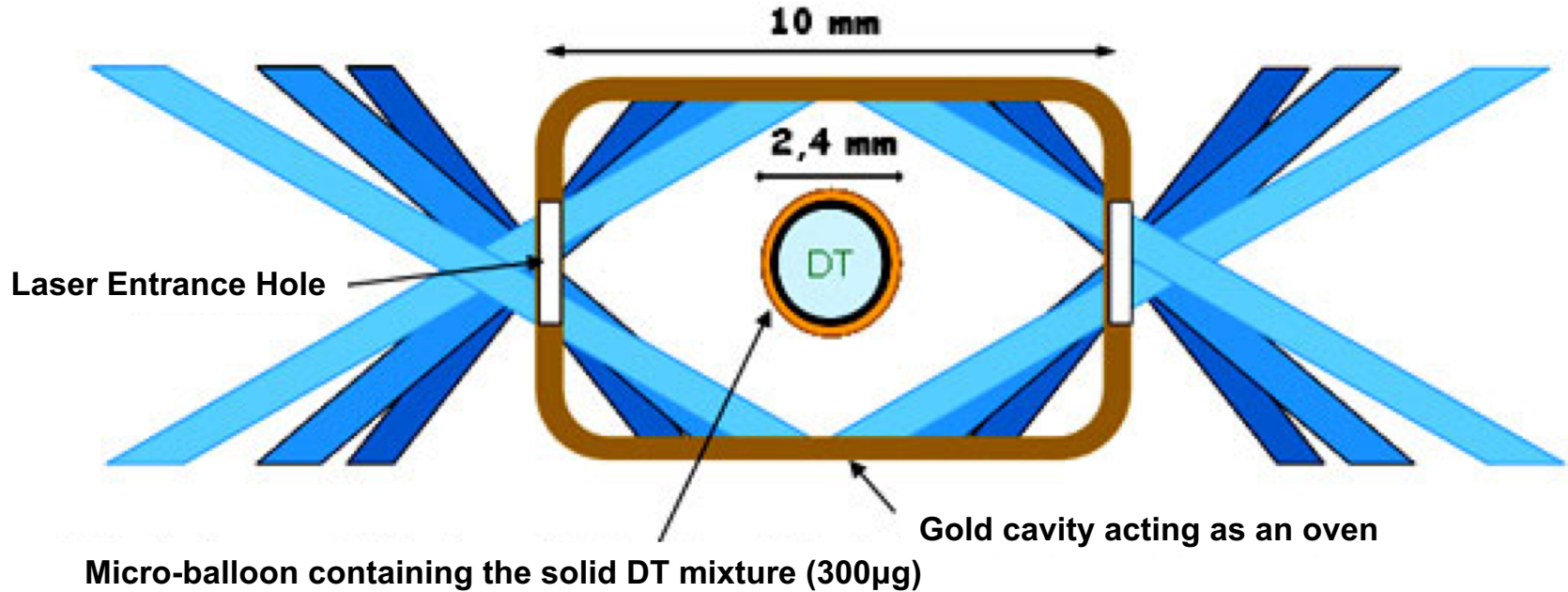
- In spherical compression, the ablation pressure is magnified by the aspect ratio.
- Note: these are just rough order of magnitude estimates. Further we must consider losses and the impact of plasma instabilities (to be discussed in the following lecture).

Approaches to laser ICF

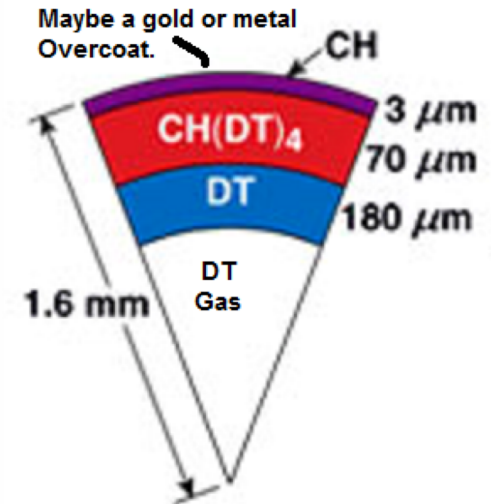
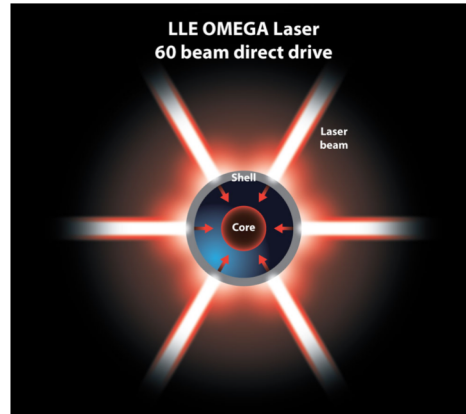
- 60 – 200 laser beams reaching total power: 10^{12} W (TW)
- Reach high densities and temperatures
- 3 standard approaches:
 - Indirect drive
 - Direct drive
 - Fast ignition



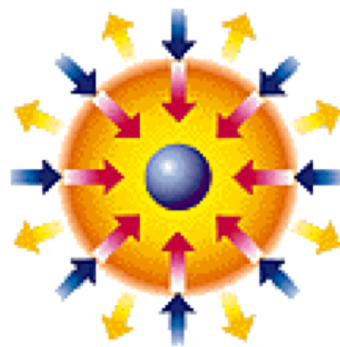
Indirect drive



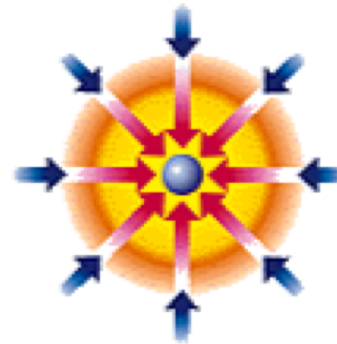
Direct drive



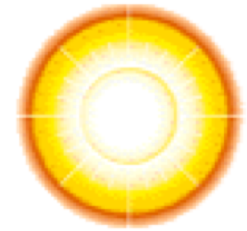
1) Atmosphere formation: Laser beams rapidly heat the surface of the fusion target forming a surrounding plasma envelope.



2) Compression: Fuel is compressed by the rocket-like blow-off of the hot surface material.

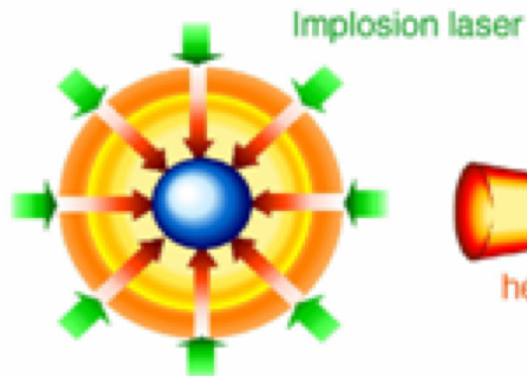
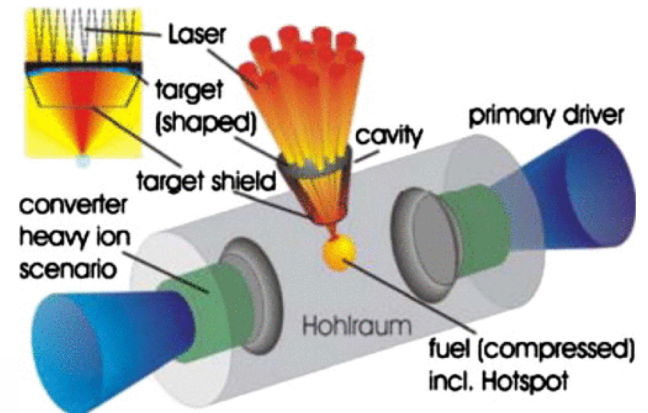
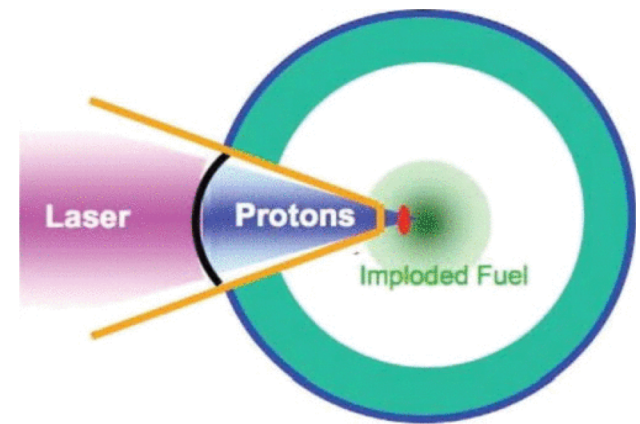
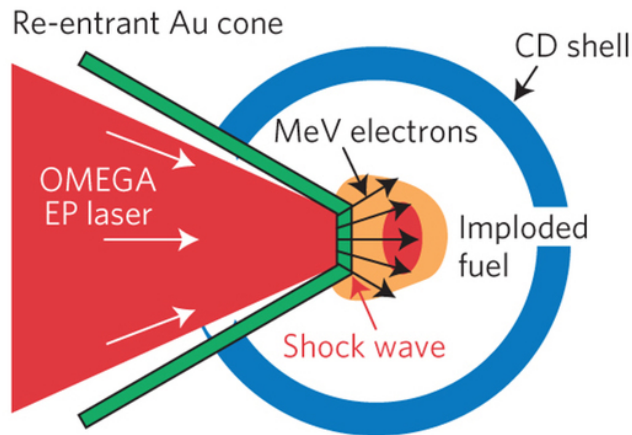


3) Ignition: During the final part of the laser pulse, the fuel core reaches 20 times the density of lead and ignites at 100,000,000 degrees Celsius.

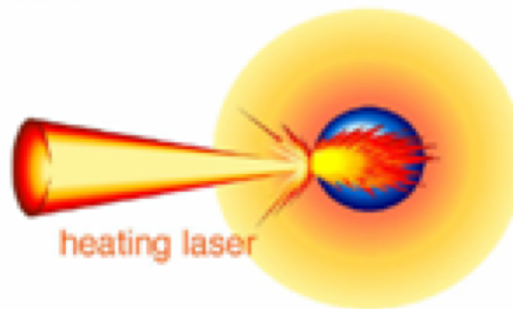


4) Burn: Thermonuclear burn spreads rapidly through the compressed fuel, yielding many times the input energy.

Fast ignition



compression



ignition



burning

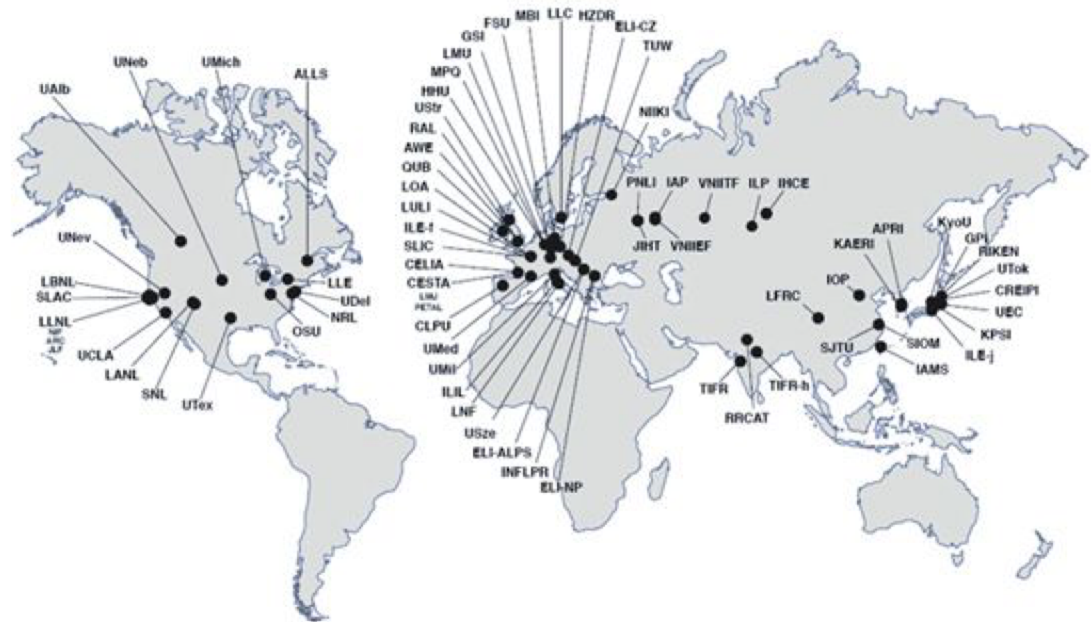
The worlds biggest lasers

**Integrated initiative
Laserlab-Europe**



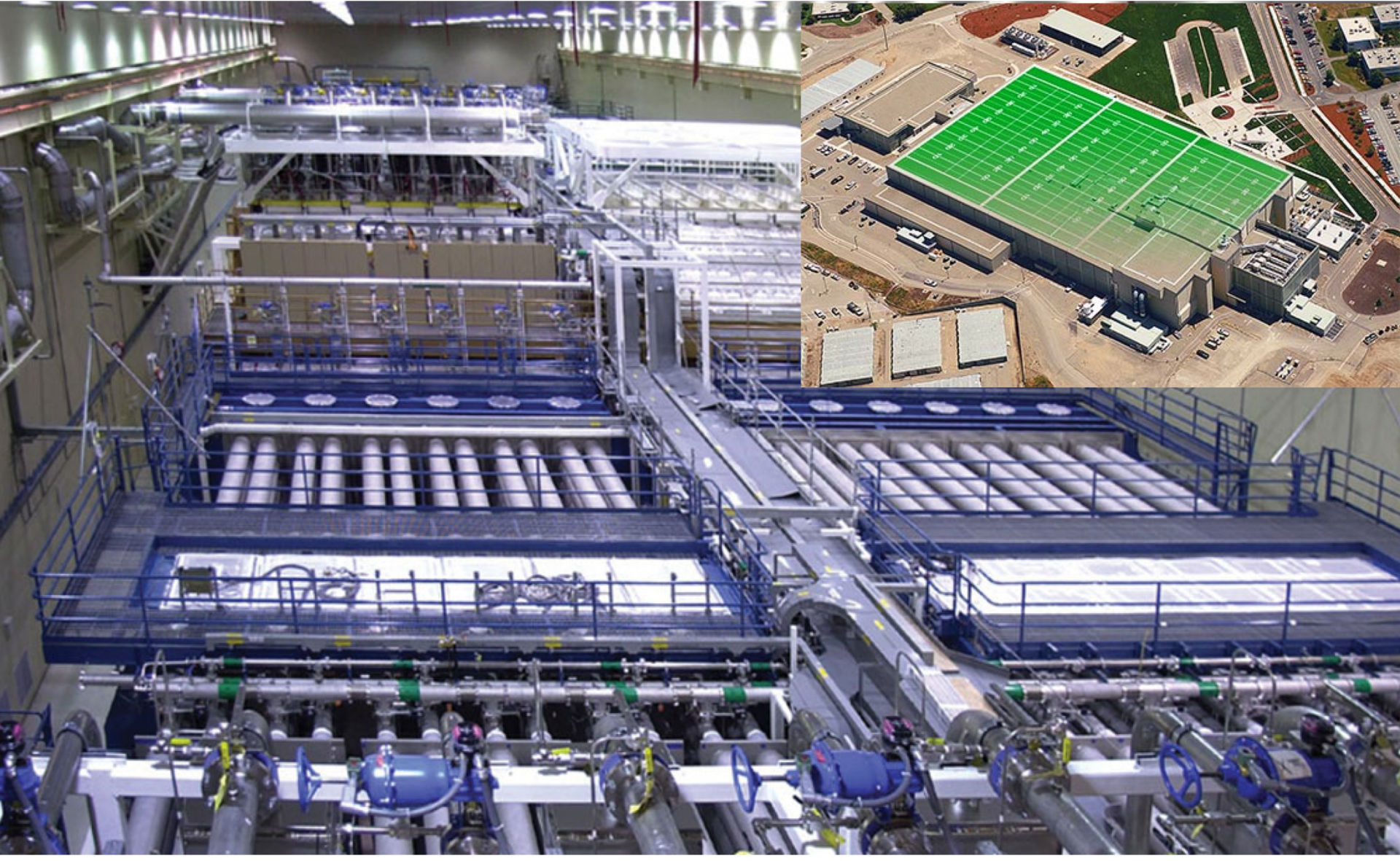
30 national laser facilities
from 16 European
countries

National high-power laser facilities world-wide

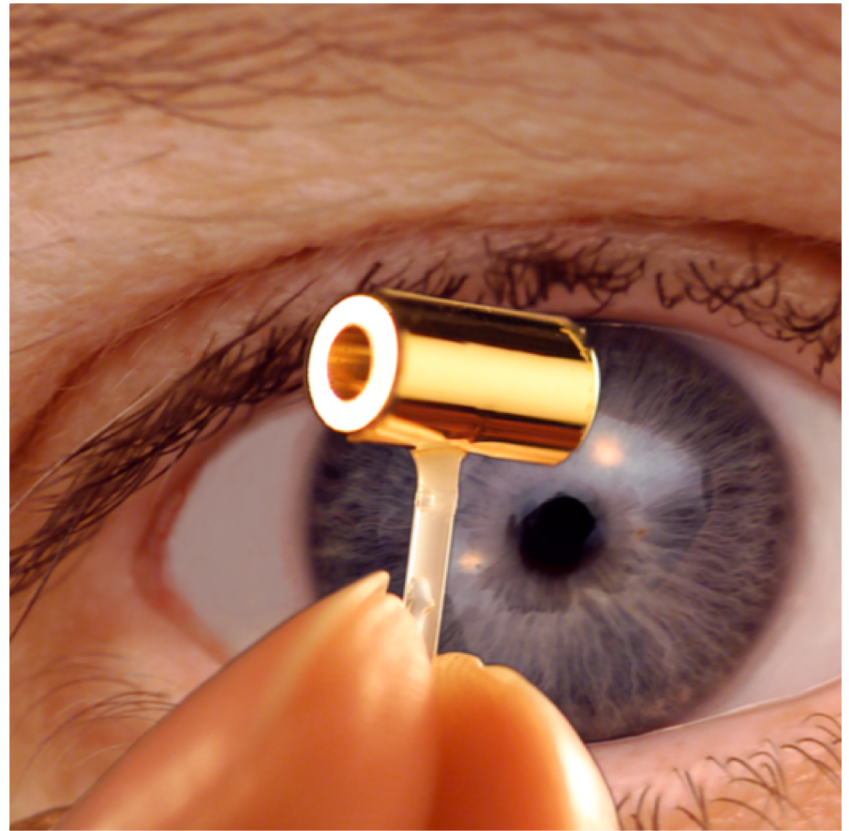
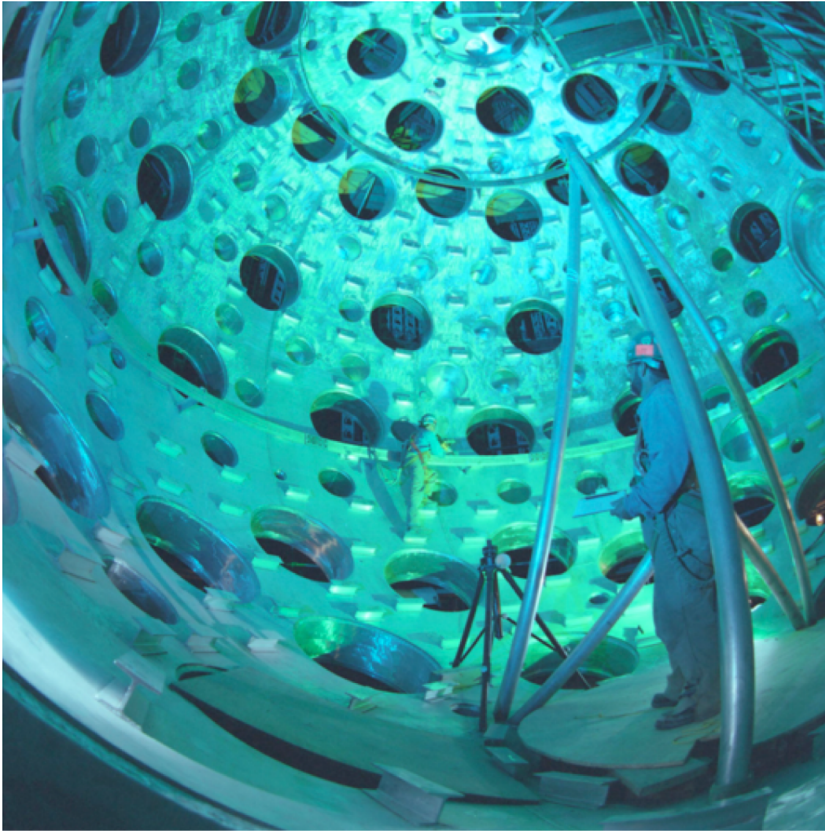


Ultra-high intensity laser systems
worldwide in 2010 (ICUIL)

National Ignition Facility

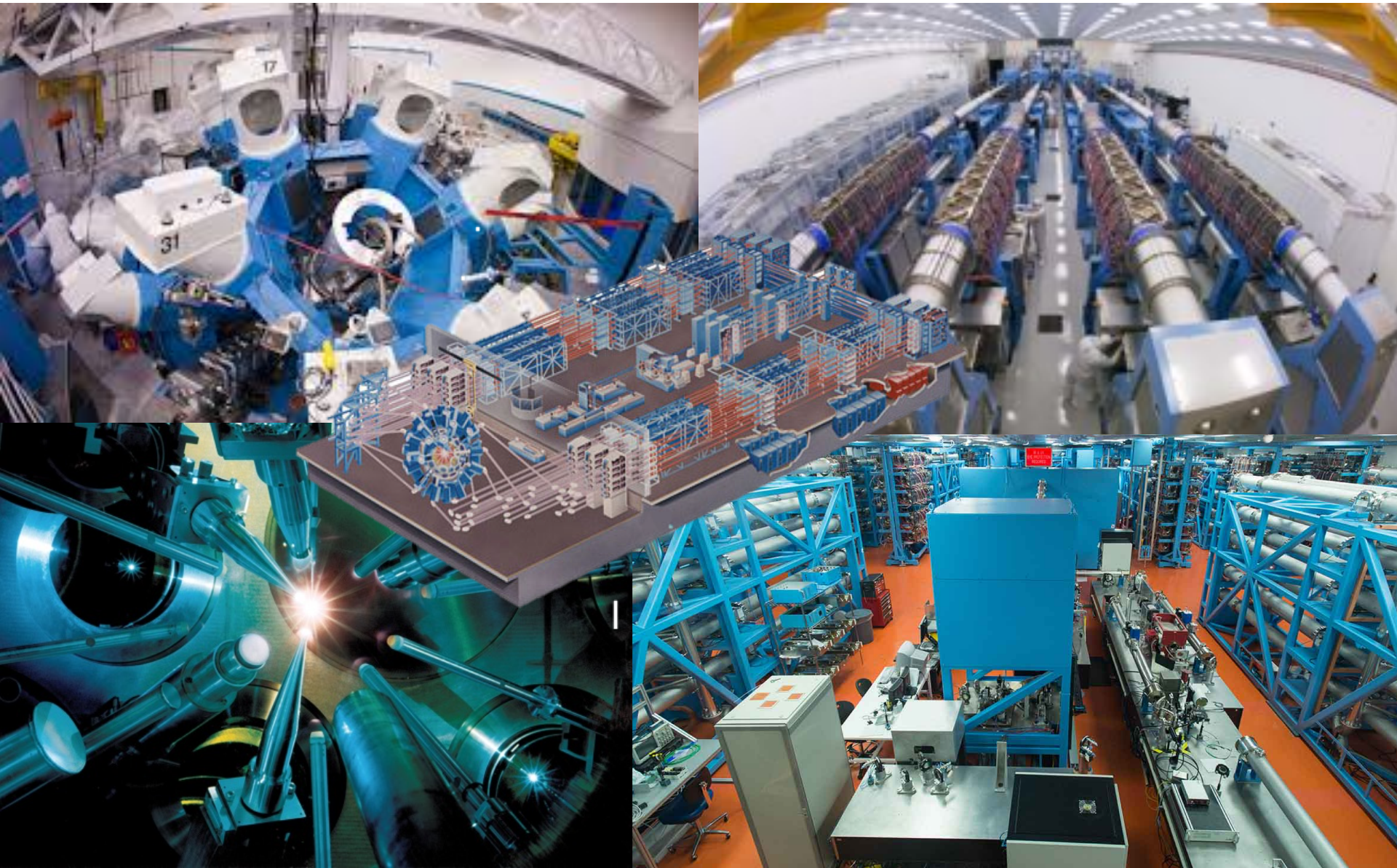


National Ignition Facility



192 laser beams, each nearly a metre in diameter, are focussed with lenses through the holes in the chamber to spots in the centre only a few 100 microns.

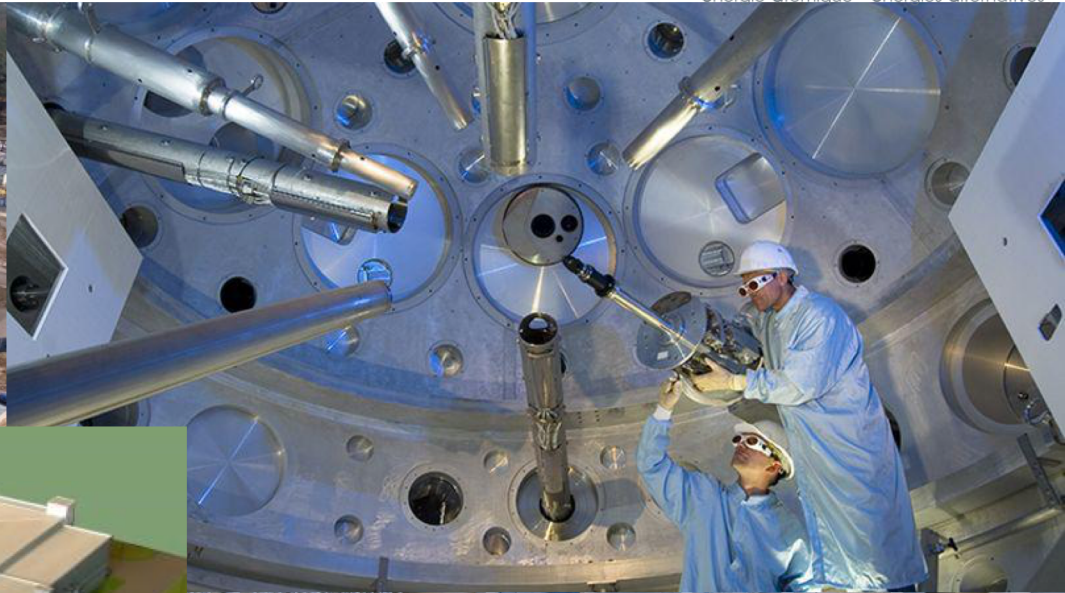
Omega laser facility



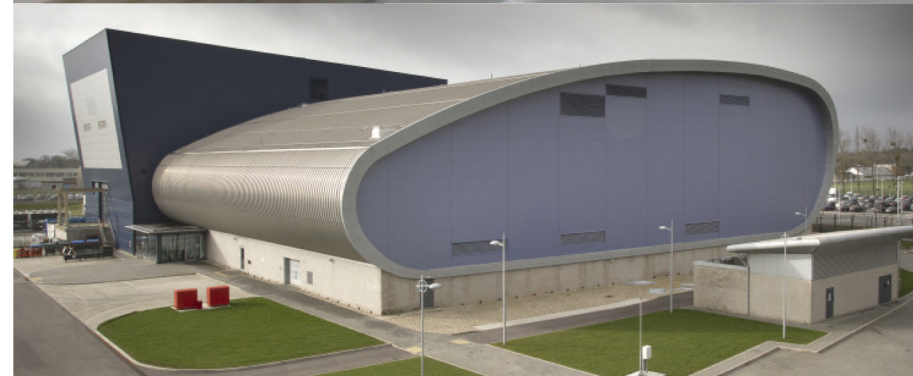
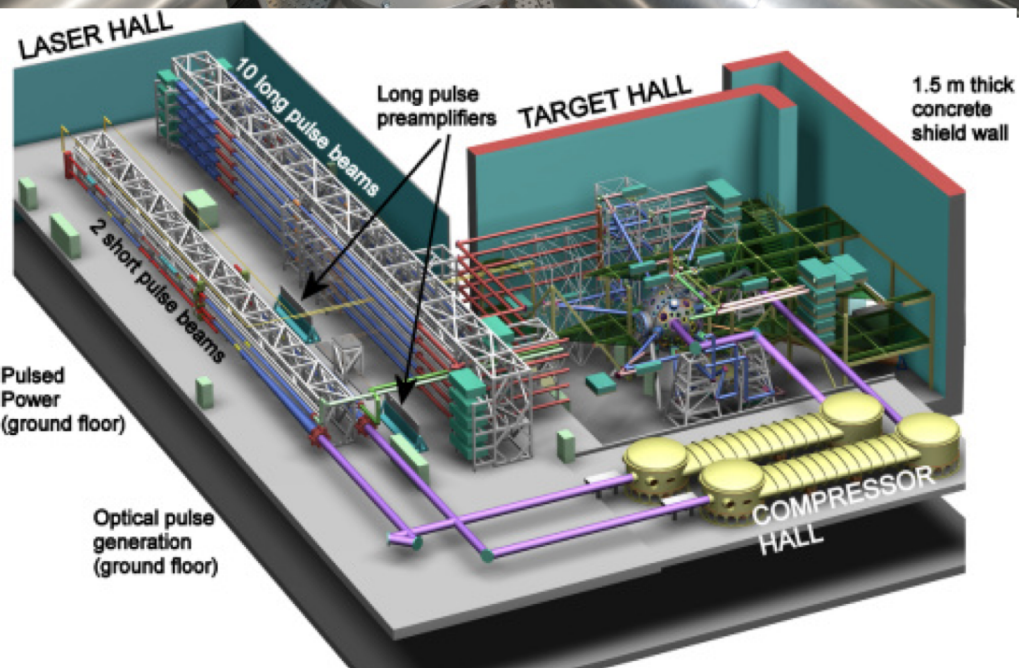
Laser Megajoule (LMJ)



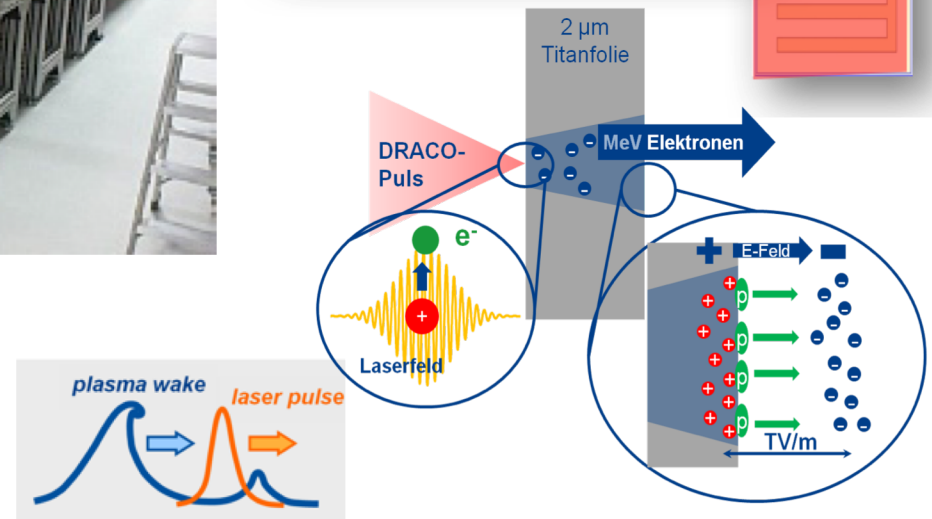
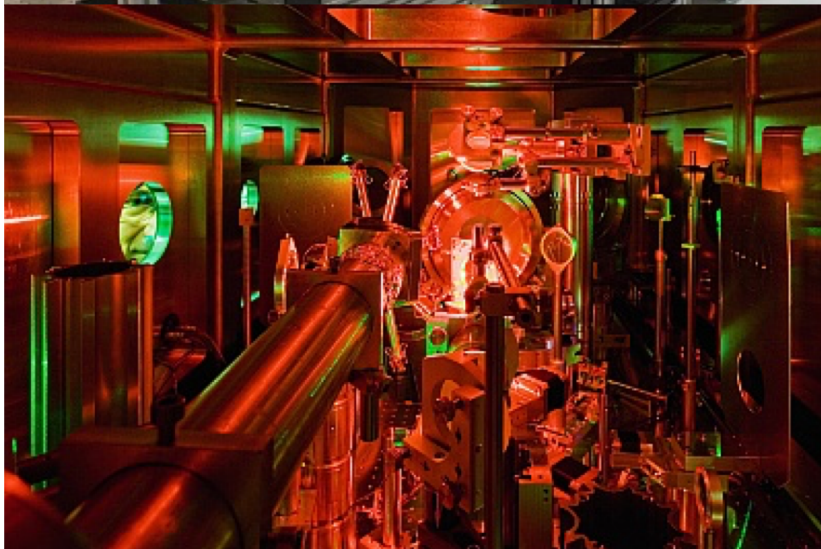
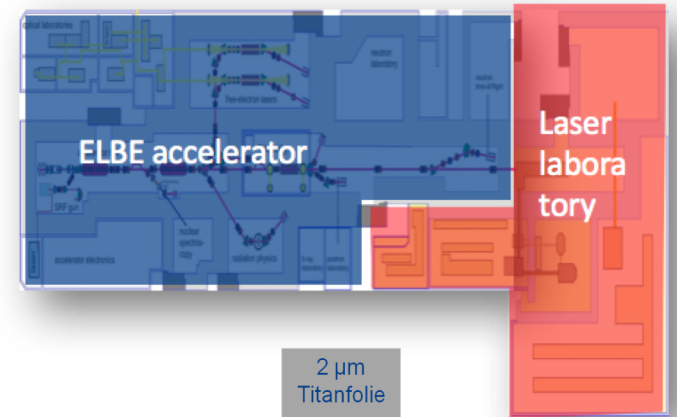
énergie atomique • énergies alternatives



Orion laser



DRACO/PENELOPE laser (HZDR)



- Commercial system, in operation since 2008
- 150 TW & 1 PW dual-beam operation
- Additional 1PW system (PENELOPE) under development

Summary of lecture 9

- Inverse bremsstrahlung absorption favours the use of short wavelength lasers.
- Simple models show that the ablation pressure scales as $W^{2/3}$, giving Mbar pressures at irradiances of 10^{14} Wcm^{-2}
- The Lawson criterion:
$$n\tau > \frac{12k_B T}{\sigma W}$$
- Plasmas confined by their own inertia require large energies to undergo fusion - but this energy scales inversely with the square of the compression.
- Compression of the fusion fuel can take place by the implosion of spherical targets - the pressure amplification factor being of order the aspect ratio, but we must consider losses and plasma instabilities.