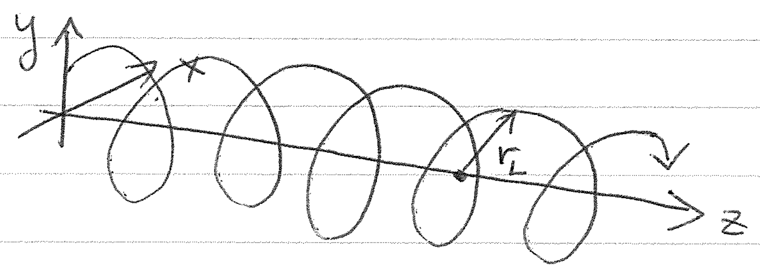
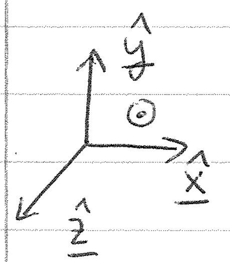
Charged particle in uniform B-field:



overall helical motion



$\rightarrow \underline{B} = B\hat{z} \Rightarrow m\dfrac{dv_z}{dt} = 0$ as v_z constant

$$m\frac{dv_z}{dt} = -q v_y B$$

$$m\frac{dv_y}{dt} = q v_x B$$

$$\rightarrow m\frac{dv}{dt} = q\begin{bmatrix} \hat{x} & v_x & 0 \\ \hat{y} & v_y & 0 \\ \hat{z} & v_z & B \end{bmatrix}$$

$\rightarrow$ taking derivatives:

$$\frac{d^2 v_y}{dt^2} = \frac{qB}{m}\cdot\frac{dv_y}{dt} = -\left(\frac{qB}{m}\right)v_x$$

$$\frac{d^2 v_x}{dt^2} = -\frac{qB}{m}\cdot\frac{dv_x}{dt} = -\left(\frac{qB}{m}\right)^2 v_y$$

$\Rightarrow$ $v_x = v_\perp \cos(\omega_c t + \phi)$

$v_y = v_\perp \sin(\omega_c t + \phi)$

$\rightarrow$ integrate: $x = x_0 + \dfrac{v_\perp}{\omega_c}\sin(\omega_c t + \phi)$

$y = y_0 - \dfrac{v_\perp}{\omega_c}\cos(\omega_c t + \phi)$

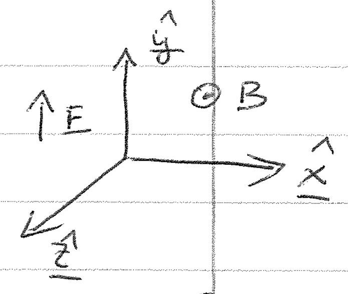
$z = z_0 + v_z t \rightarrow$ const.

at $t = 0$ (before B field) particle @ (x_0, y_0, z_0)

⇒ radius of a circle : $r_L = \frac{V_\perp}{\omega_c} = \sqrt{x^2 + y^2}$

⇒ $r_L = \frac{V_\perp}{\omega_c} = \frac{m V_\perp}{q B} = \frac{m}{q B} \sqrt{\frac{2 k_B T}{m}} = \sqrt{\frac{2 k_B T m}{q B}}$

⇒ Larmor radius



General constant force drift

⇒ vector identity : $\nabla \times \nabla \times A = \nabla (\nabla \cdot A) - \nabla^2 A$

dot product of 2 orthogonal vectors = 0
i.e. $\vec{\omega_c} \cdot \vec{V} = 0$

⇒ the extra drift velocity V_D:

$$V_d = \frac{\vec{\omega_c} \times F}{m \omega_c^2} = \frac{\vec{\omega_c} \times F}{q^2 B^2} \cdot \frac{m^2}{m} \quad \text{as } \omega_c = \frac{qB}{m}$$

and $\vec{\omega_c} = \frac{q B}{m}$

$$= \frac{q B \times F \cdot \cancel{m}}{\cancel{m} \, q^2 B^2}$$

$$= \frac{1}{q} \cdot \frac{\underline{F} \times \underline{B}}{B^2}$$

⇒ add $\underline{E}$ perpendicular to $\underline{B}$

⇒ additional force $\underline{F} = q \, \underline{E}$

Gradient drift in non-uniform B field

→ oscillating force (no gradient component).

$$| -q\, V_\perp \cos(\omega_c t) \cdot B_0 | = 0$$

→ time average of $(\cos \omega_c t)^2 = 1/2$

Curvature drift in non-uniform B field

$$V_d = \frac{1}{q} \cdot \frac{F \times B}{B^2} \quad \text{and} \quad F_{\nabla B} = \frac{\frac{1}{2} m V_\perp^2}{B} \cdot \nabla B \cdot \frac{R}{R^2}$$

(from electromagnetism)

$$\Rightarrow V_{\nabla B} = \frac{1}{2} m v_\perp^2 \cdot \frac{1}{q} \cdot \frac{R \times B}{R^2 B^2}$$

Magnetic mirrors

→ B-field symmetric along z-axis:

$$B_\theta = 0 \quad \text{and} \quad \frac{\partial B}{\partial \theta} = 0$$

remember: $\omega_c = -\dfrac{qB}{m}$ and $r_L = \dfrac{V_\perp}{\omega_c} = -\dfrac{V_\perp m}{qB}$

$$F_z = -q\, V_\perp B_r$$

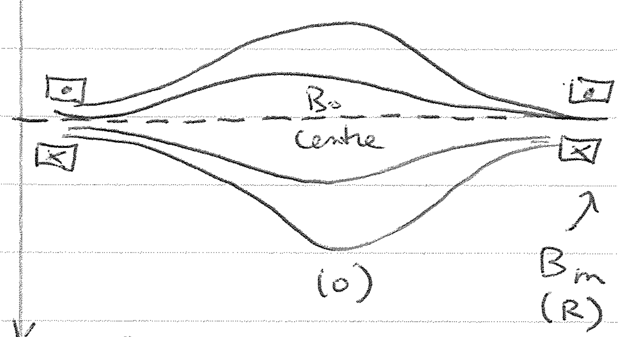
$$\Rightarrow q V_\perp \cdot -\frac{1}{2} r_L = \frac{\cancel{q} V_\perp}{2} \cdot \frac{V_\perp m}{\cancel{q} B} = \frac{1}{2} \frac{m V_\perp^2}{B} = \mu$$ //

$$\Rightarrow F_z = -\mu \frac{\partial B_z}{\partial z}$$

$$\Rightarrow \boxed{F_{\parallel} = -\mu \nabla_{\parallel} \underline{B}} \rightarrow 1^{st} \text{ adiabatic invariant}$$

$\rightarrow$ no transfer of heat and matter

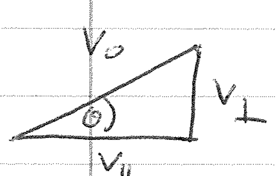
magnetic moment: $\mu_0 = \frac{1}{2} m \frac{V_\perp^2}{B}$



$$\Rightarrow \frac{1}{2} m \frac{V_{\perp 0}^2}{B_0} = \frac{1}{2} m \frac{V_{\perp R}^2}{B}$$

$$\frac{1}{2} m \left(V_\perp^2(0) + V_\parallel^2(0) \right) = \frac{1}{2} m V_\perp^2(R)$$

no $V_\parallel$ at R !



- as particle moves to higher $\underline{B}$ $\Rightarrow V_\perp \nearrow$

$\Rightarrow V_\parallel \searrow$

$$V_\perp^2(R) = \left(V_\perp^2(0) + V_\parallel^2(0) \right) \text{ and } \frac{V_{\perp 0}^2}{B_0} = \frac{V_\perp^2}{B}$$

$$\Rightarrow \frac{V_\perp^2(0)}{B_0} = \left(V_{\perp 0}^2 + V_{\parallel 0}^2 \right) / B$$

$$\Downarrow$$

$$\frac{V_\perp^2}{V_{\perp 0}^2} = \frac{B}{B_0}$$

$$\Rightarrow B_R = B_0 \left(1 + \frac{V_{\parallel 0}^2}{V_{\perp 0}^2} \right)$$

$$\Rightarrow V_\parallel^2(0) = V_{\perp 0}^2 \left(\frac{B_m}{B_0} - 1 \right) \Rightarrow \text{stopping condition !}$$

$V_\parallel(R) = 0$

$$\Rightarrow V_R^2 = V_{\perp R}^2 + \underbrace{V_{\parallel R}^2}_{=0} = V_0^2$$

proof that $\mu = $ const.: $\boxed{\mu = \frac{1}{2} m \frac{V^2}{B}}$

if $\underline{E} = 0 \Rightarrow$ total K.E. (W) is constant $\Rightarrow \frac{dW}{dt} = 0$

and $W = \frac{1}{2} m \left(v_\perp^2 + v_{\parallel}^2\right)$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2} m v_\perp^2\right) = -\frac{d}{dt}\left(\frac{1}{2} m v_{\parallel}^2\right) \qquad \Big/ \; \mu B = \frac{1}{2} m v_\perp^2$$

$$\Rightarrow \frac{d}{dt}(\mu B) = -V_z \underbrace{m \frac{d v_z}{dt}}_{F_z} \qquad \text{as} \quad V_{\parallel} = V_z$$

$B = B_z$ as $\frac{du}{dt} = \frac{du}{dv} \cdot \frac{dv}{dt} = v \cdot m \frac{dv}{dt}$

letting $u = \frac{1}{2} m v^2$

But B_z is time independent:

$$\mu \frac{d B_z}{dt} + B_z \frac{d\mu}{dt} = -F_z V_z = \mu V_z \frac{d B_z}{dz}$$

as $F_z = -\mu \frac{\partial B_z}{\partial z}$

(or $F_{\parallel} = -\mu \nabla_{\parallel} B$)

$$\Rightarrow \frac{d B_z}{dt} = \frac{dz}{dt} \cdot \frac{d B_z}{dz} = V_z \frac{d B_z}{\partial z} \Rightarrow V_z \frac{\partial B_z}{\partial z} = \frac{\partial B_z}{\partial t}$$

$$\Rightarrow \cancel{\mu \frac{d B_z}{dt}} + B_z \frac{\partial \mu}{dt} = \cancel{\mu \frac{\partial B_z}{dt}} \quad \therefore \quad \frac{d\mu}{dt} = 0 \;\; //$$