Inverse Bremsstrahlung Absorption

→ laser propagates in a plasma and causes the electrons to oscillate: $E = E_0 \sin(\omega t)$ – oscillating field

← peak E-field of the laser

$$\ddot{x} = -\frac{eE_0}{m}\sin(\omega t)$$ ← oscillatory motion / force

acceleration electron mass

($F = m.a$) and $F = q.V$
$= q.\nabla E$

→ integrate: $\dot{x} = \frac{eE_0}{m\omega}\cos(\omega t)$ $a = \frac{F}{m} = \frac{eE}{m} = \frac{eE_0}{m}\sin(\omega t)$

velocity

⇒ thus time-averaged kinetic energy of the electron oscillating in the electron field of the laser:

$$\overline{\frac{1}{2}m\dot{x}^2} = \frac{e^2E_0^2}{4m\omega^2}$$ ← average kinetic energy (oscillatory energy per electron)

→ define the absorption coefficient for light intensity I:

$$I = I_0 \exp(-kx)$$ ← along x-axis

$$\Rightarrow k = -\frac{1}{I}\frac{dI}{dx}$$ //

→ total extra oscillatory energy of the electrons within volume $A \cdot dx$ in plasma with electron density n_e:

$$U_e = n_e\left(\frac{e^2E_0^2}{4m\omega^2}\right)A \cdot dx$$

→ this energy gets converted to thermal energy by collisions between the particles

→ the time-scale for the collisions is given by the electron-ion collision frequency ~ 10s of picoseconds

$\Rightarrow \tau_{ei}$ derived in lecture 3

→ in time t, we expect the fraction of oscillatory energy converted to thermal energy to be:

$$\approx t/\tau_{ei}$$

⇒ change in oscillatory energy dU_e in time t is equal to the change in laser field energy dU_L:

$$dU_e = dU_L = -\frac{t}{\tau_{ei}} \cdot n_e \left(\frac{e^2 E_0^2}{4 m \omega^2}\right) A . dx$$

and energy loss in laser field depends on laser energy fed into the system:

$$U_L = \underset{\uparrow \text{ intensity } (W/cm^2)}{I} A \cdot t$$

as Poynting vector: $S = \frac{1}{\mu_0} \underline{E} \times \underline{B} = \frac{1}{2} \epsilon_0 c E^2 x_0 \hat{z}$

(plane EM wave along $\underline{\hat{z}}$)

$$= c U_{EB} \underline{\hat{z}}$$

S = energy flux = $W/m^2 \sim I$ $\Rightarrow \langle S \rangle = I = \frac{\epsilon_0 c}{2} E_0^2$

$$B_0 = \frac{E_0}{c}$$

and $c = \underset{\sim 1}{\frac{1}{\sqrt{\mu_r \epsilon_r}}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ ← vacuum

⇒ absorption coefficient:

$$K = -\frac{1}{I}\frac{dI}{dx}$$

$$= -\frac{1}{U_L}\cdot\frac{dU_L}{dx} \qquad \text{since} \quad U_L = I A t$$

$$dU_L = dI \cdot A \cdot t$$

$$= \frac{1}{I A \cancel{t}}\cdot\frac{\cancel{t}}{\tau_{ei}} n_e \left(\frac{e^2 E_0^2}{4 m \omega^2}\right) A \cdot \frac{\cancel{dx}}{\cancel{dx}}$$ substituting

$$= \frac{1}{I}\cdot\frac{1}{\tau_{ei}} n_e \left(\frac{e^2 E_0^2}{4 m\omega^2}\right)$$

as $$dU_L = -\frac{t}{\tau_{ei}} \cdot n_e \left(\frac{e^2 E_0^2}{4 m\omega^2}\right)\cdot A \cdot dx$$

→ substitute for the Poynting vector: $I = N = \frac{1}{2}\sqrt{\varepsilon_r}\, \varepsilon_0 E_0^2 c$ ← ref. index

$$\Rightarrow K = \frac{1}{\frac{1}{\cancel{2}}\sqrt{\varepsilon_r}\,\varepsilon_0 \cancel{E_0^2} c}\cdot\frac{1}{\tau_{ei}}\cdot n_e\left(\frac{e^2 \cancel{E_0^2}}{\underset{2}{\cancel{4}} m\omega^2}\right)$$

$$= \frac{1}{2\tau_{ei}}\cdot\left(\frac{n_e e^2}{\varepsilon_0 m \omega^2}\right)\cdot\frac{1}{c\sqrt{\varepsilon_r}}$$

$$= \frac{\omega_p^2}{\omega^2} \quad \text{and} \quad \sqrt{\varepsilon_r} = \sqrt{\left(1 - \frac{\omega_p^2}{\omega^2}\right)}$$

$$\Rightarrow K = \frac{1}{2c\tau_{ei}}\cdot\left(\frac{\omega_p^2}{\omega^2}\right)\left(1 - \frac{\omega_p^3}{\omega^2}\right)^{-1/2} \quad //$$

and $\tau_{ei} \propto T_e^{-3/2}$ ⇒ hotter plasmas less absorbing
(lecture 3)

→ absorption more efficient for short laser λ or lower plasma density

Laser ablation model - simplified

→ steady-state situation, mass flow constant at any point:

$$\rho_a u_a = \rho_c u_c = \text{flux of mass}$$

ablation surface   critical surface

→ similarly, momentum flux constant:

$$P_a + \rho_a u_a^2 = P_c + \rho_c u_c^2$$

flux of momentum: flow of momentum per area $\frac{mu}{tA} = F/A = P$

→ we also assume energy flux is constant:

→ note expansion into vacuum, u, P and ρ change with x, but constant in a given point (steady state)

⇒ isenthalpic throttling process: $H = U + PV$

→ assume enthalpy flux is constant

→ add kinetic energy - also constant

→ also account for laser energy absorption (at ablation surface) ⇒ W

$$\Rightarrow \tfrac{1}{2}\rho_a u_a^3 + H_a \rho_a u_a + W = \tfrac{1}{2}\rho_c u_c^3 + H_c \rho_c u_c = \frac{\text{energy}}{\text{area}}$$

$\frac{Nm}{V} = \rho$

kinetic energy at ablation   enthalpy   absorbed laser energy at ablation   kinetic E   enthalpy

critical surface

→ assume ideal gas equation of state: $PV = Nk_BT$

$$\Rightarrow H = \left(\frac{\gamma}{\gamma - 1}\right)\frac{P}{\rho}$$

where γ normal ratio of heat capacities (simplification)

$H = U + PV = \frac{nd}{2} Nk_BT + Nk_BT$

$\gamma = \frac{5}{3}$

$\rightarrow$ <u>assume</u> $\rightarrow \rho_a \gg \rho_c$ (reasonable)

and $\rightarrow W \gg \frac{1}{2}\rho_a u_a^3 + H_a \rho_a u_a$

(i.e. the velocity of the plasma at the ablation surface, where the energy is being 'converted' into plasma creation, is very small)

i.e. $u_a \rightarrow 0$ at ablation surface
thus the main source of energy flux is the thermal flux W

and $\rightarrow$ the velocity of plasma at the critical surface is the adiabatic sound velocity of the ions (Information cannot travel faster, not a shock)

in lecture 6 we derived: $u_c = \left(\frac{\gamma P_c}{\rho_c}\right)^{1/2}$

$\Rightarrow$ thus: $\frac{1}{2}\rho_a u_a^3 + H_a \rho_a u_a + W = \frac{1}{2}\rho_c u_c^3 + H_c \rho_c u_c$

becomes: $W = \frac{1}{2}\rho_c u_c^3 + H_c \rho_c u_c$

$\rightarrow$ substitute for u_c and H_c:

$$W = \frac{\rho_c}{2}\left(\frac{\gamma P_c}{\rho_c}\right)^{3/2} + \left(\frac{\gamma}{\gamma-1}\right)\frac{P_c}{\cancel{\rho_c}}\cancel{\rho_c}\left(\frac{\gamma P_c}{\rho_c}\right)^{1/2}$$

$$= \frac{1}{\sqrt{\rho_c}} P_c^{3/2}\sqrt{\gamma}\left(\frac{\gamma}{2} + \frac{\gamma}{\gamma-1}\right)$$

$\Rightarrow$ for $\gamma = \frac{5}{3}$ get: $P_c \approx 4\rho_c^{1/3} W^{2/3}$

$\rightarrow$ at the ablation surface $v_a \rightarrow 0$, thus:

$$P_a + \cancel{\rho_a u_a^2}_{=0} = P_c + \rho_c u_c^2 \Rightarrow P_a = P_c + \rho_c u_c^2$$

$\rightarrow$ substitute for $u_c = \left(\frac{\gamma P_c}{\rho_c}\right)^{1/2}$:

$$P_a = P_c + \cancel{\rho_c} \frac{\gamma P_c}{\cancel{\rho_c}} = (1+\gamma) P_c = \frac{8}{3} P_c$$

$\rightarrow$ substituting for P_c:

$$P_a \approx \frac{32}{3} \rho_c^{1/3} W^{2/3}$$

$\rightarrow$ critical density ρ_c given by the laser wavelength, frequency ω:

$$n_c = \frac{\varepsilon_0 m_e \omega^2}{e^2} \quad \text{as} \quad \omega^2 = \frac{n_c e^2}{\varepsilon_0 m_e} \quad \left(\begin{array}{c}\text{no propagation} \\ \text{for } \omega < \omega_p\end{array}\right)$$

$\rightarrow$ for fully ionized plasma $n_i = n_e / Z$ and for nuclei with the same no. of protons and neutrons: $M = 2Z m_p$

$$\Rightarrow \rho_c = 2 n_c m_p = 2 m_p \left(\frac{\varepsilon_0 m_e \omega^2}{e^2}\right)$$

$\Rightarrow$ substitute and get P_a (ablation pressure):

$$P_a \approx \frac{32}{3} \left(\frac{2\varepsilon_0 m_e m_p \omega^2}{e^2}\right)^{1/3} W^{2/3} \quad [\text{Mbar}]$$

in $\quad W\,cm^{-2}$

λ in μm