Plasma Physics

TU Dresden

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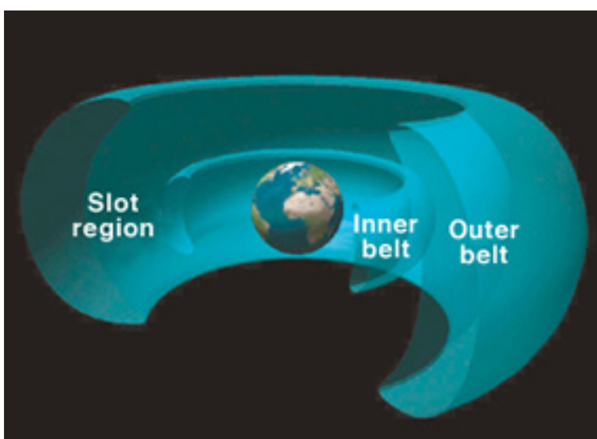
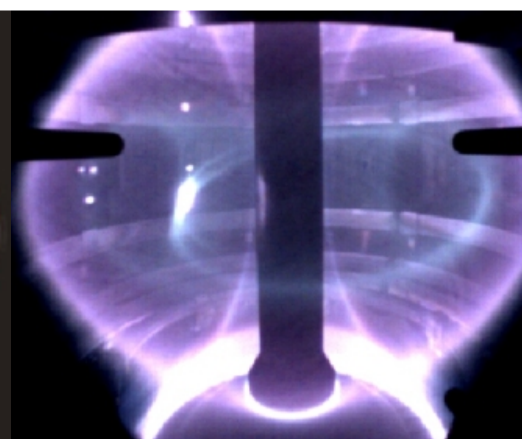
Lecture 1: Introduction, Saha equation, Debye length, plasma frequency





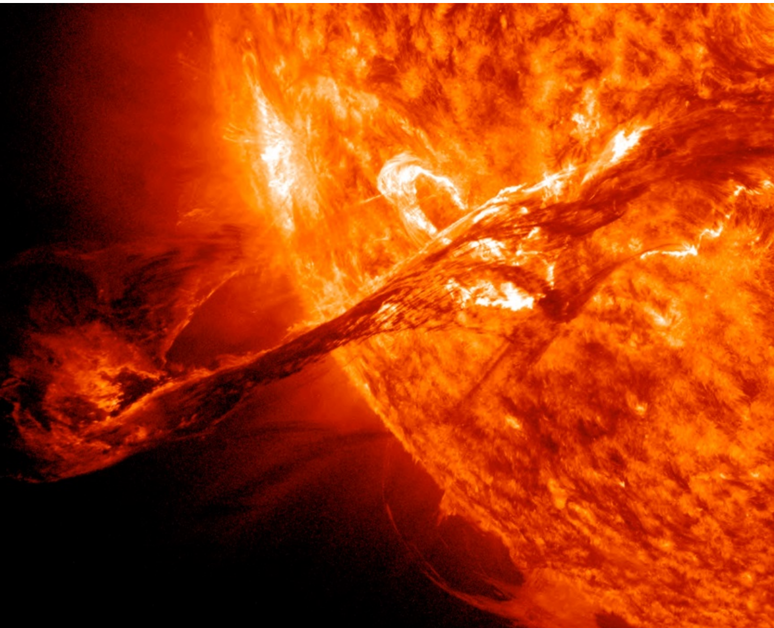

Plasma Physics: lecture 1

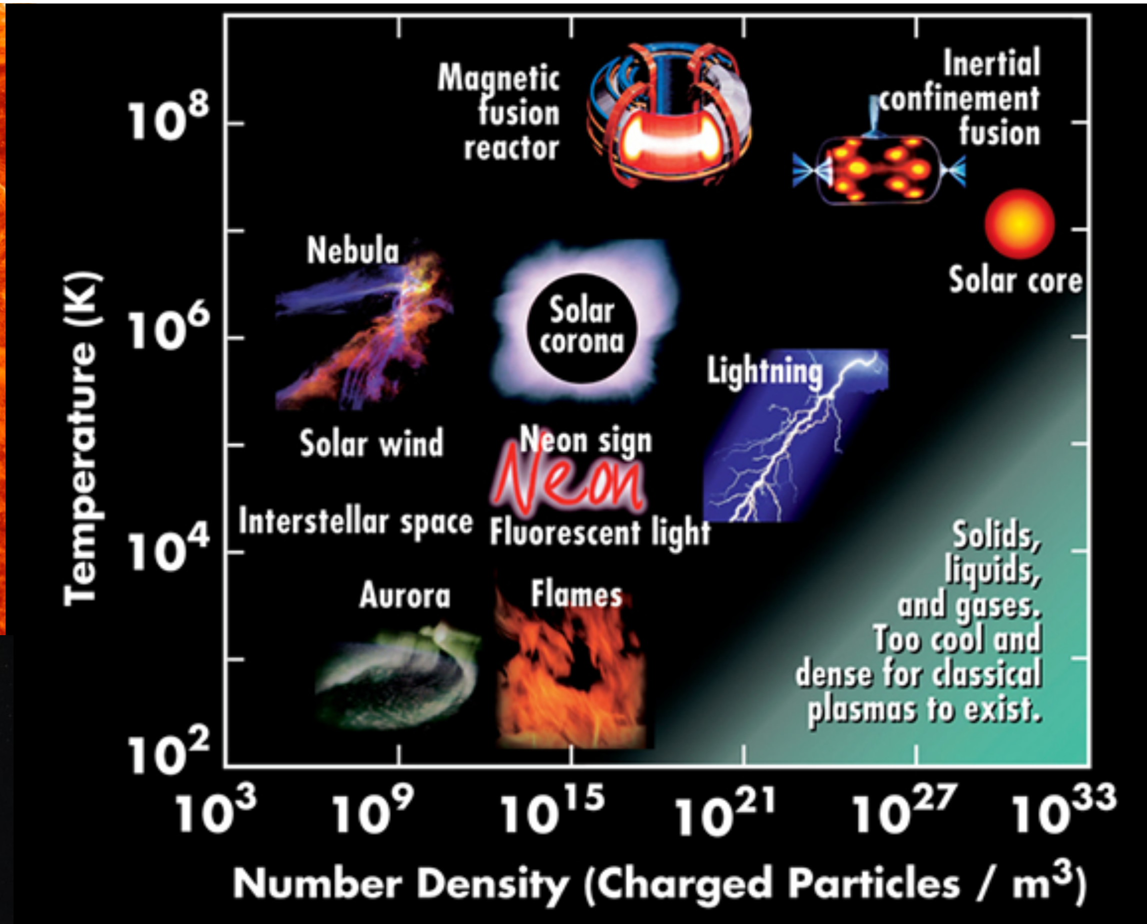
- Definition of the plasma “state”
- Examples of plasmas and motivation
- Saha equation and ionization
- Debye length
- Plasma parameter
- Plasma frequency

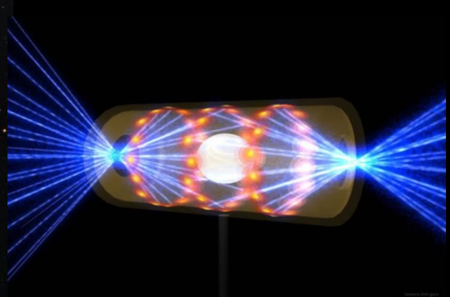
Plasma definitions

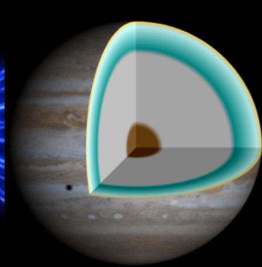
- **Gaseous mixture of positive ions and electrons**
  - Can be partially or fully ionized and reach high densities
- **Quasi-neutral**
  - Balance of n_e and n_i always reached
- **Exhibits collective behavior**
  - Particles in plasma feel electrostatic forces from other particles and move collectively as a fluid
- **Susceptible to electric and magnetic fields**
  - And the fields are modified by the charged particles motion in plasma

Examples of plasmas

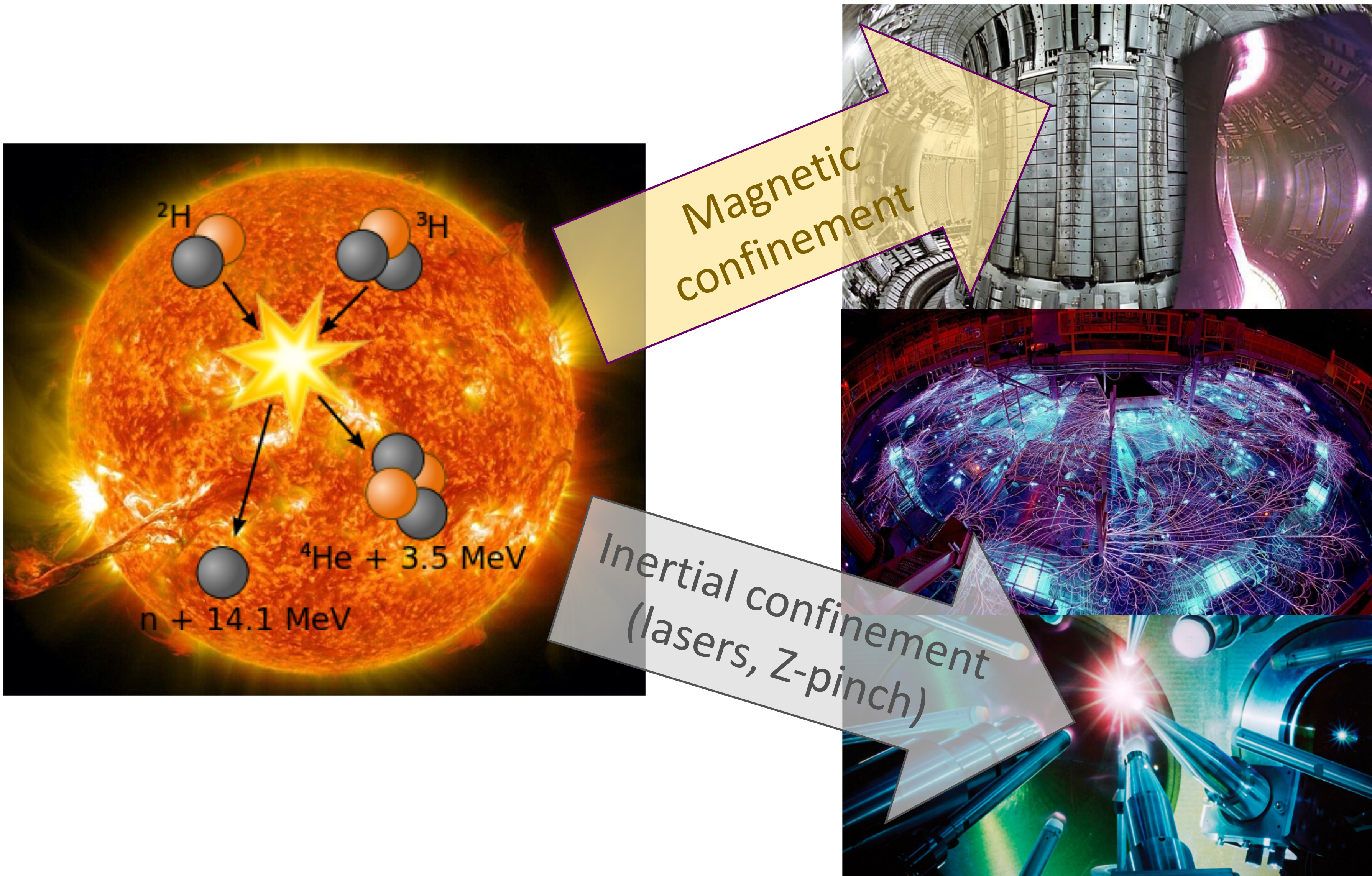






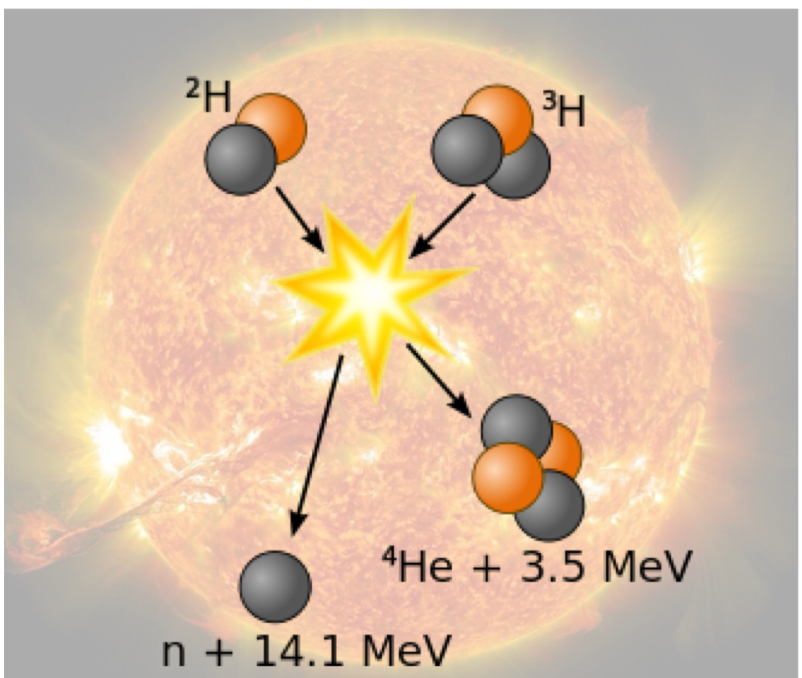


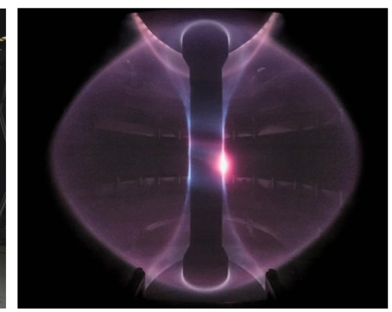
Thermonuclear fusion

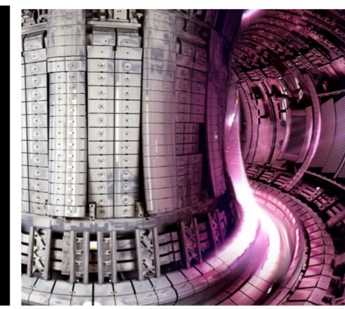


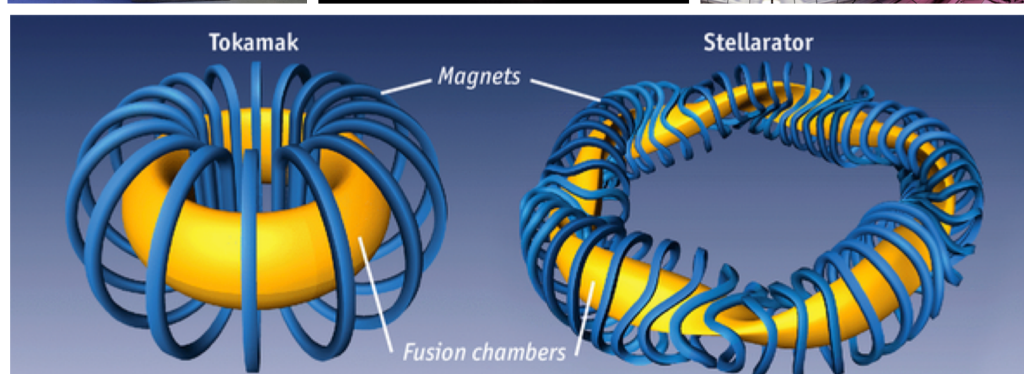
Magnetic confinement (MCF)

- **Magnetic confinement fusion concept to fusion relies on the susceptibility of plasmas to external magnetic fields**
- Powerful magnetic fields are used to confine plasmas to allow long reaction times
- Fusion reaction of hydrogen atoms to form Helium at extreme temperatures
- Two main confinement approaches:
  - Tokamaks
  - Stellarators



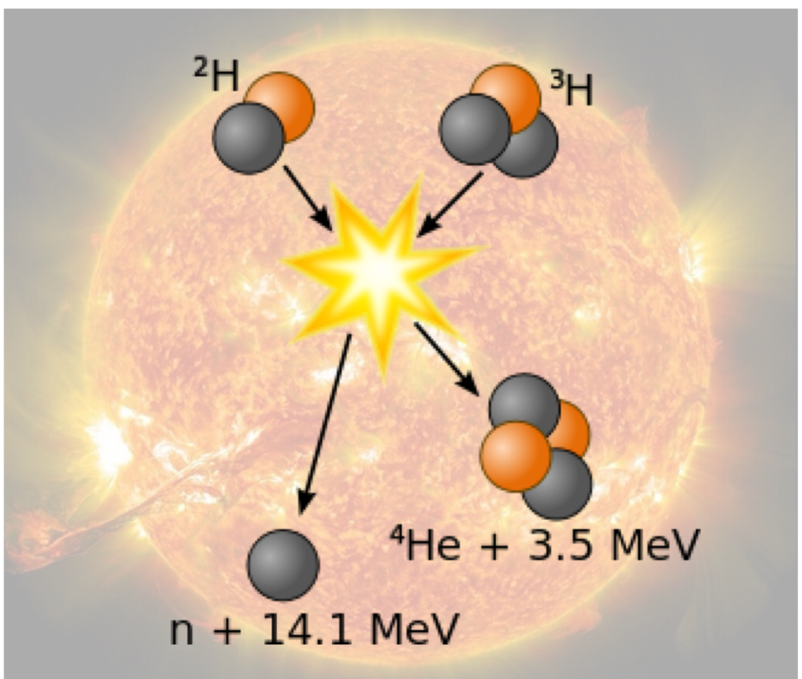


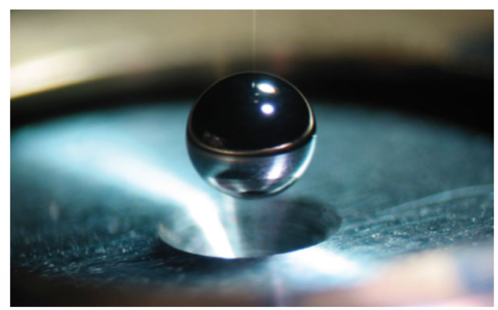


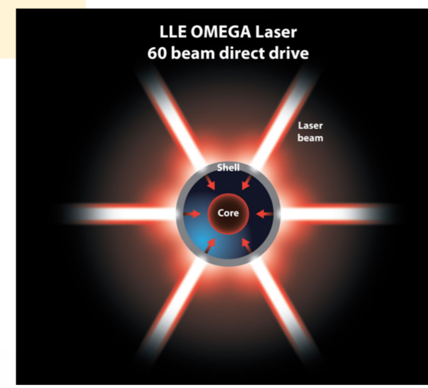


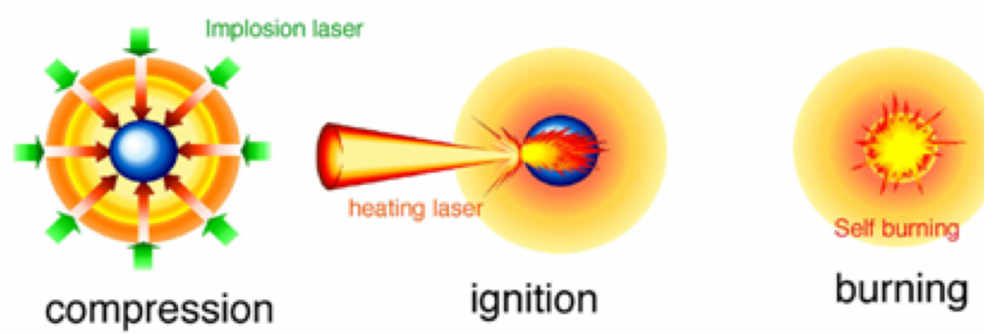
Fusion technology with lasers (ICF)

- **Inertial confinement fusion concept to fusion relies heavily on the knowledge of equation of state and transport properties of WDM**
- 60 – 200 laser beams of combined power of 10^{12} W (TW)
- Fusion reaction of hydrogen atoms to form Helium at extreme temperatures and densities
- Three main laser approaches:
  - Direct drive compression
  - Indirect drive with x-rays
  - Fast ignition fusion





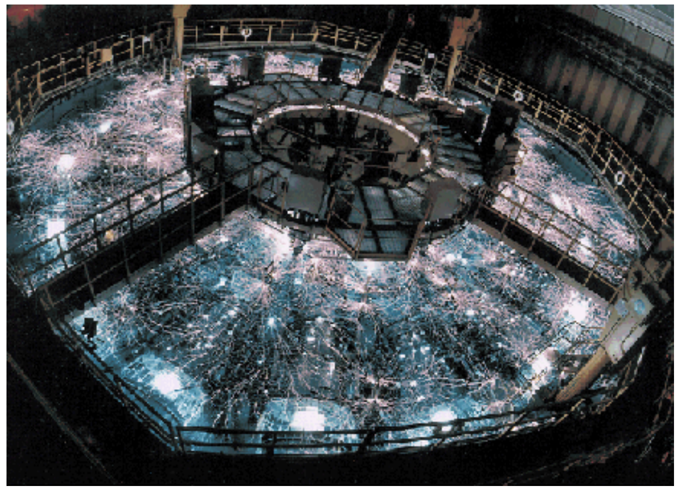
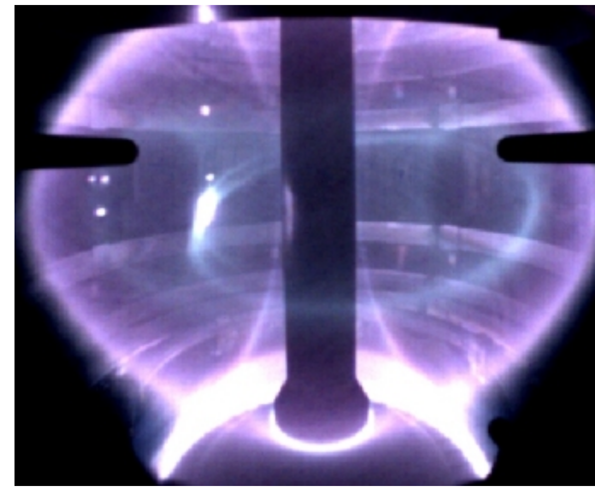
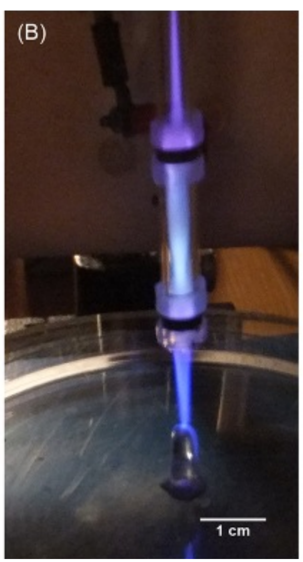


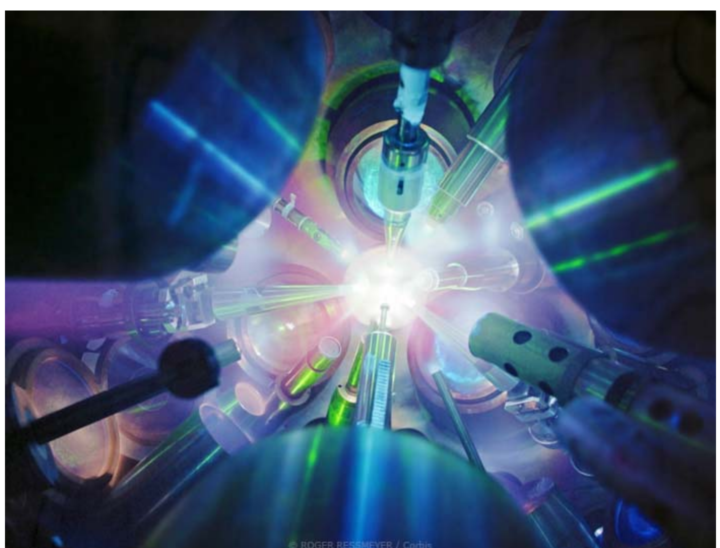


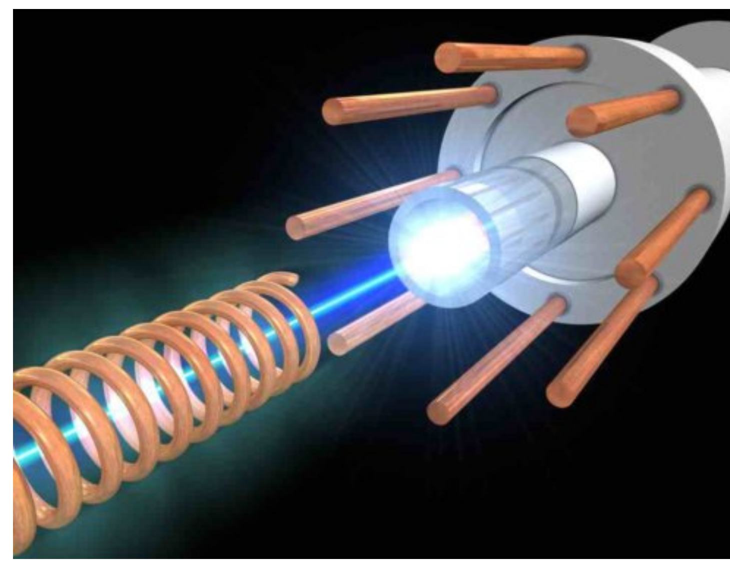
Laboratory Plasmas

How we can create plasmas in a laboratory:

- Tokamak
- Lasers
- Z-pinches
- Plasma focus
- Accelerators

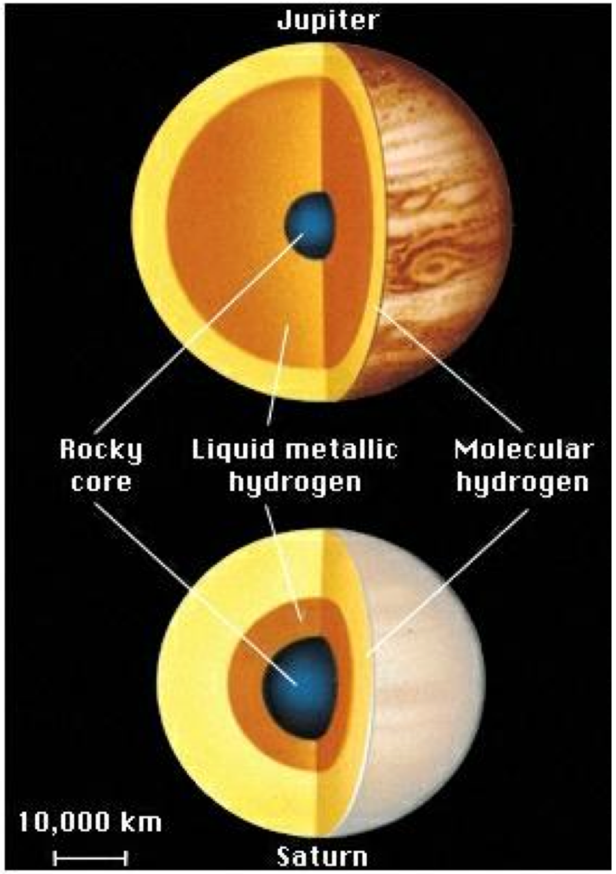


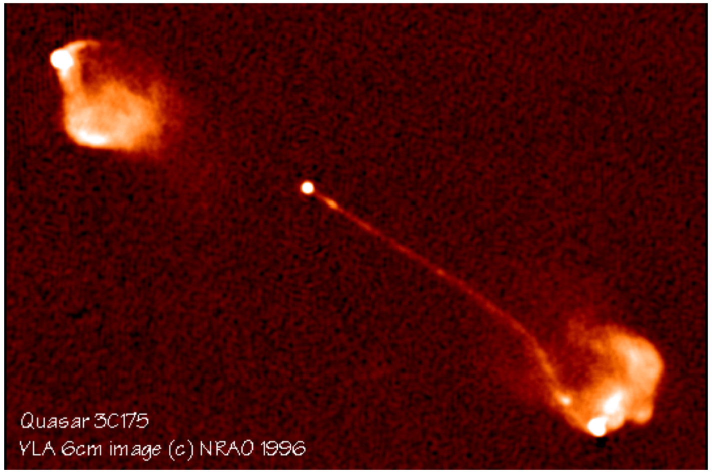


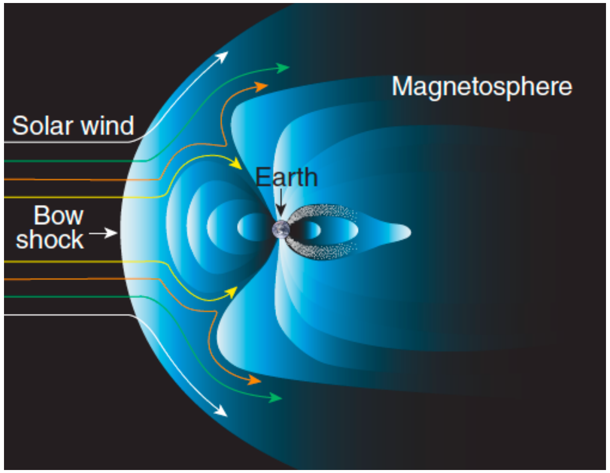
Laboratory Astrophysics

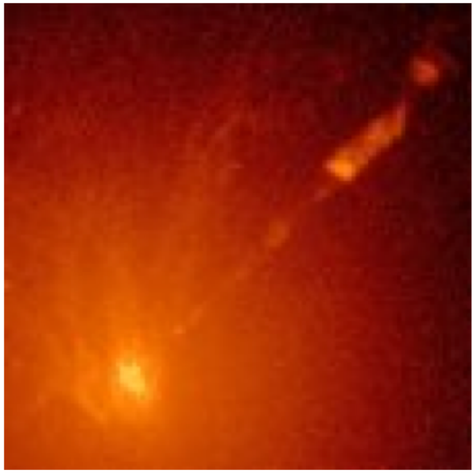
... that we can simulate in a laboratory:

- Astrophysical jets (Z-pinches, lasers)
- Supernova explosions (Z-pinches, lasers)
- Solar/stellar dynamics & corona (lasers)
- Structure of planets (lasers, diamond anvil cells)
- Accretion disks & MHD instabilities (lasers)
- Nebuale/molecular clouds (lasers, Z-pinches)
- Cosmic rays (particle accelerators, lasers)
- Black holes (particle accelerators)
- The Big Bang (LHC ???)





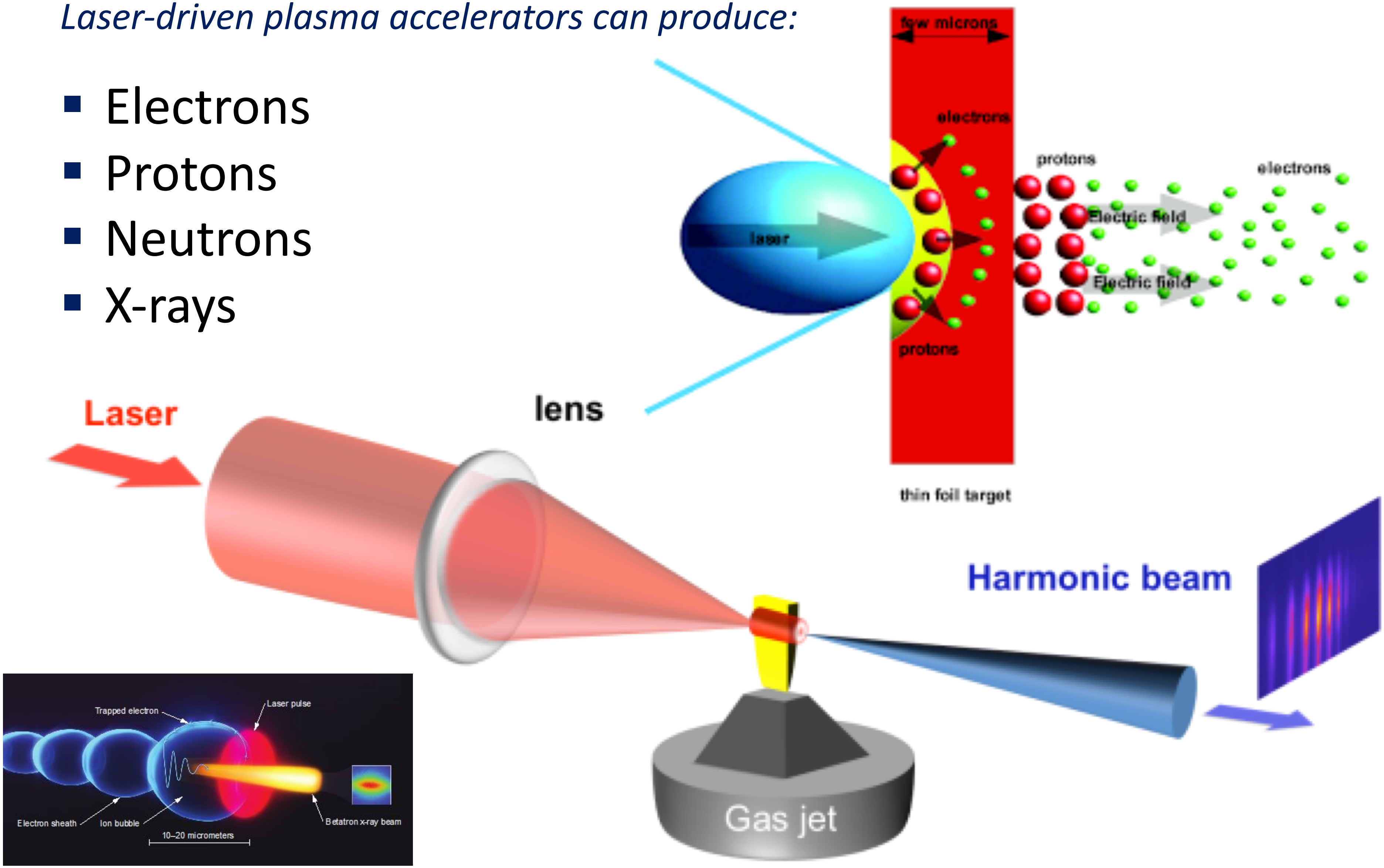




Plasma accelerators

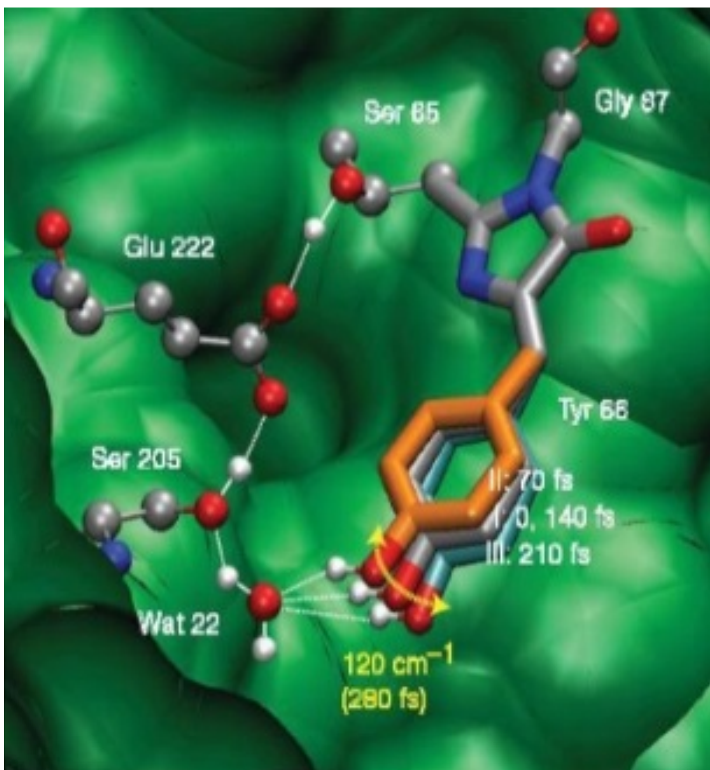
Laser-driven plasma accelerators can produce:

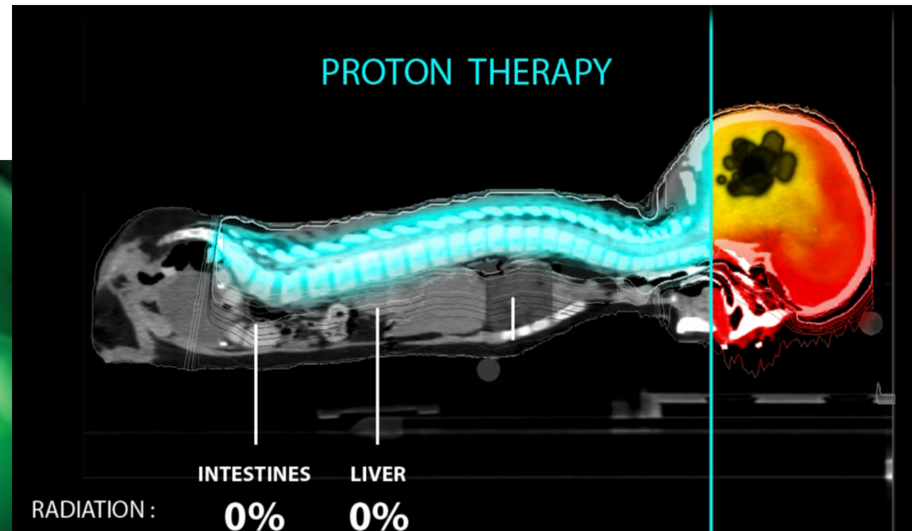
- Electrons
- Protons
- Neutrons
- X-rays

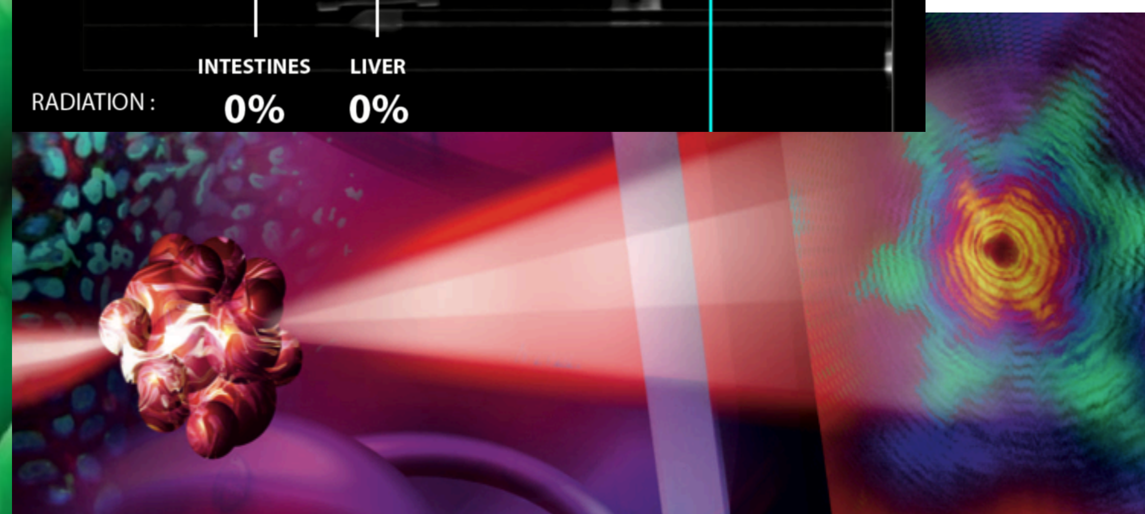


Broad applications

- Study of biological and technical material structure
- Proton therapy for cancer treatment
- Basic science



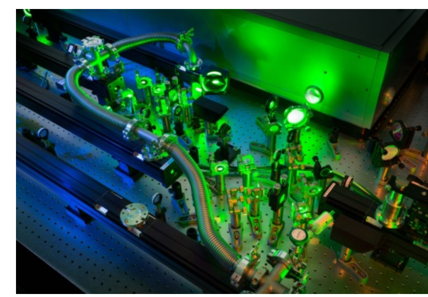




Innovations

- Unexpected products and innovation coming from fusion research are everywhere (e.g. plasma TV, manufacturing, medicine …)

- Power stations are the big goal, but we see applications and startups already in:
    - Novel diagnostic methods
    - Laser technology
    - Microfabrication
    - Materials
    - Aerospace, defense



Start with some chemistry

- Starting from an example of a simple chemical reaction (water vapor):

$$2H_2 + O_2 \Leftrightarrow 2H_2O$$
$$-2H_2 - O_2 + 2H_2O = 0$$

- During the chemical reaction the number of molecules of a particular type can change, but the number of atoms is conserved
- Each species B_i has a coefficient b_i, giving general chemical reaction form:

$$\boxed{\sum_{i=1}^{m} b_i B_i = 0}$$

i.e. $b_1 = -2$
and $B_1 = H_2$

Remembering thermodynamics

- The molecules must convert from one type to another
- Let there be N_i molecules of type B_i, in order to conserve atoms the change in number of atoms must be proportional to b_i:

$$\boxed{dN_i = \gamma b_i}$$

- Constant γ is the same for all molecules
- From thermodynamics, internal energy U:

$$dU = TdS - PdV + \sum_i \mu_i dN_i$$

Remembering thermodynamics

- For chemical equilibrium, the number of particles is constant → constant chemical potential μ, no flow
- System with m components has condition for equilibrium:

$$\sum_{i}^{m} \mu_i dN_i = \sum_{i}^{m} b_i \mu_i = 0 \qquad \boxed{dN_i = \gamma b_i}$$

- Helmholtz free energy F linked to partition function Z_N:

$$\boxed{F = -k_B T \ln Z_N} \quad \text{and} \quad \boxed{\mu_i = \left(\frac{\partial F}{\partial N_i}\right)_{V,T,N_{j \neq i}}}$$

Note: T and V constant

Now some statistical physics

- Condition for chemical equilibrium:

$$b_i \sum_i \left(\frac{\partial F}{\partial N_i} \right)_{T,V,N_{j \neq i}} = \sum_i^m b_i \mu_i = 0$$

- Partition function for a system of weakly interacting indistinguishable particles (like ideal gases):

$$Z_N = \frac{\left(Z_{sp}\right)^N}{N!} = \frac{\prod_i \left(Z_{sp}\right)^{N_i}}{\prod_i N_i!}$$

sp = single particle

- Helmholtz free energy:

$$F = -k_\mathrm{B} T \ln \left[\frac{(Z_{\mathrm{sp}_i})^{N_i}}{N_i!} \right]$$

Now some statistical physics

- Obtain the chemical potential using the Stirling's theorem:

$$
\begin{aligned}
\mu_i &= \left(\frac{\partial F}{\partial N_i}\right)_{T,V,N_{j\neq i}} \\
&= -k_{\mathrm{B}}T\frac{\partial}{\partial N_i}\left(N_i \ln Z_{\mathrm{sp}_i} - \ln N_i!\right) \\
&= -k_{\mathrm{B}}T\frac{\partial}{\partial N_i}\left(N_i \ln Z_{\mathrm{sp}_i} - N_i \ln N_i + N_i\right) \\
&= -k_{\mathrm{B}}T(\ln Z_{sp_i} - \ln N_i)
\end{aligned}
$$

Law of mass action

- Chemical equilibrium condition thus:

$$\sum_i b_i k_\mathrm{B} T (\ln Z_{\mathrm{sp}_i} - \ln N_i) = 0$$

- Rewrite: $$\sum_i b_i \ln Z_{\mathrm{sp}_i} = \sum_i b_i \ln N_i$$

- Compact form: $$\boxed{\prod_i (Z_{\mathrm{sp}_i})^{b_i} = \prod_i (N_i)^{b_i}}$$

Moving to plasma physics

- The ionization of atoms looks similar to a chemical reaction
- Consider ionization of a hydrogen atom:

$$H \Leftrightarrow p^{+} + e^{-}$$

$$p^{+} + e^{-} - H = 0$$

- Assume that the density is low enough, and the temperature high enough, that the electrostatic energy can be ignored compared with the thermal energy. Thus, we assume a very weakly interacting particles (ideal gas).

Deriving the Saha equation

- Apply the law of mass action:

$$\boxed{\prod_i (Z_{\mathrm{sp}_i})^{b_i} = \prod_i (N_i)^{b_i}} \quad \rightarrow \quad \frac{N_{e^-} N_{p^+}}{N_H} = \frac{Z_{sp_{e^-}} Z_{sp_{p^+}}}{Z_H}$$

$$b_{e^-} = b_{p^+} = 1 \quad , \quad b_H = -1$$

- For hydrogen $N_{e^-} = N_{p^+}$:

$$\frac{N_{e^-}^2}{N_H} = \frac{Z_{\mathrm{sp}_{e^-}} Z_{\mathrm{sp}_{p^+}}}{Z_{\mathrm{sp}_H}}$$

Partition functions

- The electrons and protons look like free gas atoms. The partition function is just the total volume divided by the volume of the particle

spin

$$Z_{sp_{e^-}} = 2 \times V\left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2}$$

$$Z_{sp_{p^+}} = 2 \times V\left(\frac{2\pi m_p k_B T}{h^2}\right)^{3/2}$$

- The volume occupied by a particle is given by the **deBroglie wavelength**
- Take zero energy to be free electrons/protons at rest

Partition functions

- Note that a static recombined atom in the ground state of a stationary H atom has an energy of -13.6 eV (Rydberg):

$$Z_{\mathrm{sp}_H} = 4 \times \exp\left(\frac{-[-Ry]}{k_\mathrm{B}T}\right) \times V\left(\frac{2\pi m_H k_\mathrm{B} T}{h^2}\right)^{3/2}$$

- Using the general partition function form:

$$\sum_i g_i \exp(-\varepsilon_i / k_\mathrm{B} T)$$

- Now, substitute:

$$\frac{N_{e^-}^2}{N_H} = \frac{2V\left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} 2V\left(\frac{2\pi m_p k_B T}{h^2}\right)^{3/2}}{4\exp\left(\frac{Ry}{k_B T}\right) V\left(\frac{2\pi m_H k_B T}{h^2}\right)^{3/2}}$$

The Saha equation

- Make (reasonable) approximation $m_p = m_H$ and simplify:

$$\frac{N_{e^-}^2}{N_H} = \exp\left(\frac{-Ry}{k_\mathrm{B}T}\right) V \left(\frac{2\pi m_e k_\mathrm{B} T}{h^2}\right)^{3/2}$$

- Or for $n = N/V$:

$$\boxed{\frac{n_{e^-}^2}{n_H} = \exp\left(\frac{-Ry}{k_\mathrm{B}T}\right) \left(\frac{2\pi m_e k_\mathrm{B} T}{h^2}\right)^{3/2}}$$

Ionization in plasma

- Define the degree of ionization ξ in terms of the total number of atoms (ionized +unionized) N_0:

$$N_e = N_p = \xi N_0$$

$$N_H = (1-\xi)N_0$$

- And rewrite the Saha equation:

$$\boxed{\frac{\xi^2}{(1-\xi)} = \frac{V}{N_0}\exp\left(\frac{-Ry}{k_B T}\right)\left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} = f(N_0, V, T)}$$

Note: Take positive root of the quadratic equation: $\zeta = \dfrac{\sqrt{f^2+4f}-f}{2}$

Typical numbers for H plasma

As an example, take a density of atoms/ions of 10^{20} m^{-3} (typical Tokamak plasma)

T (eV)	$f(N_0,V,T)$	ξ
0.6	2x10E-3	4.4E-02
0.8	0.9	0.6
1.0	37.4	0.975

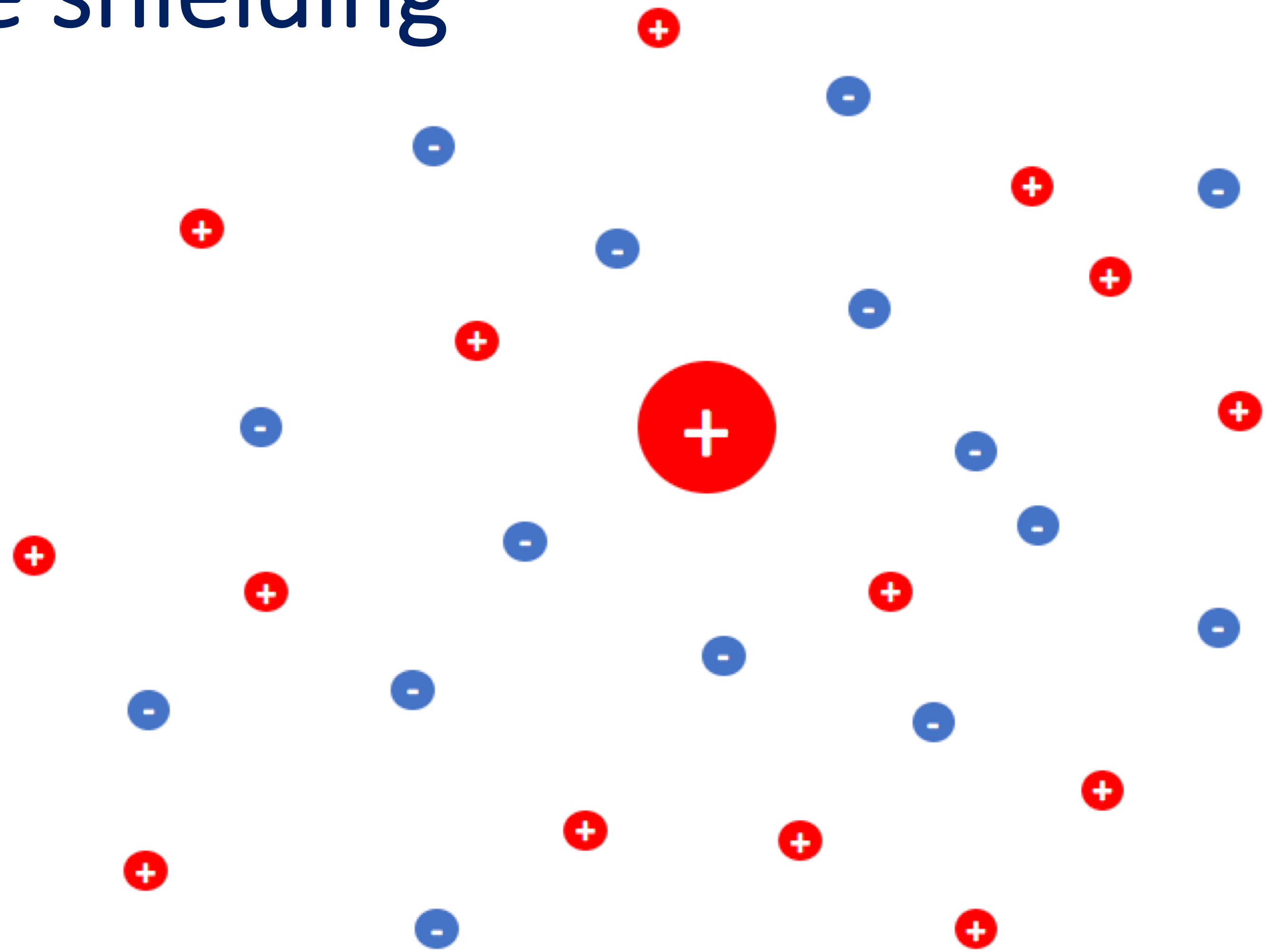
Plasmas are “easy” to make

- Notice that the hydrogen is 97% ionized at a temperature of 1.0 eV
- But the ionization energy was 13.6 eV (Rydberg E)
- The plasma is created at a ‘low’ temperature. Why?
- The answer is in the statistical weight. I.e. ionization occurs by those few high energy electrons on the tail of the distribution function
- The continuum is ‘heavy’ compared with the recombined atoms – explains why it is slightly easier to ionize as the density decreases

Ionisation processes in plasma

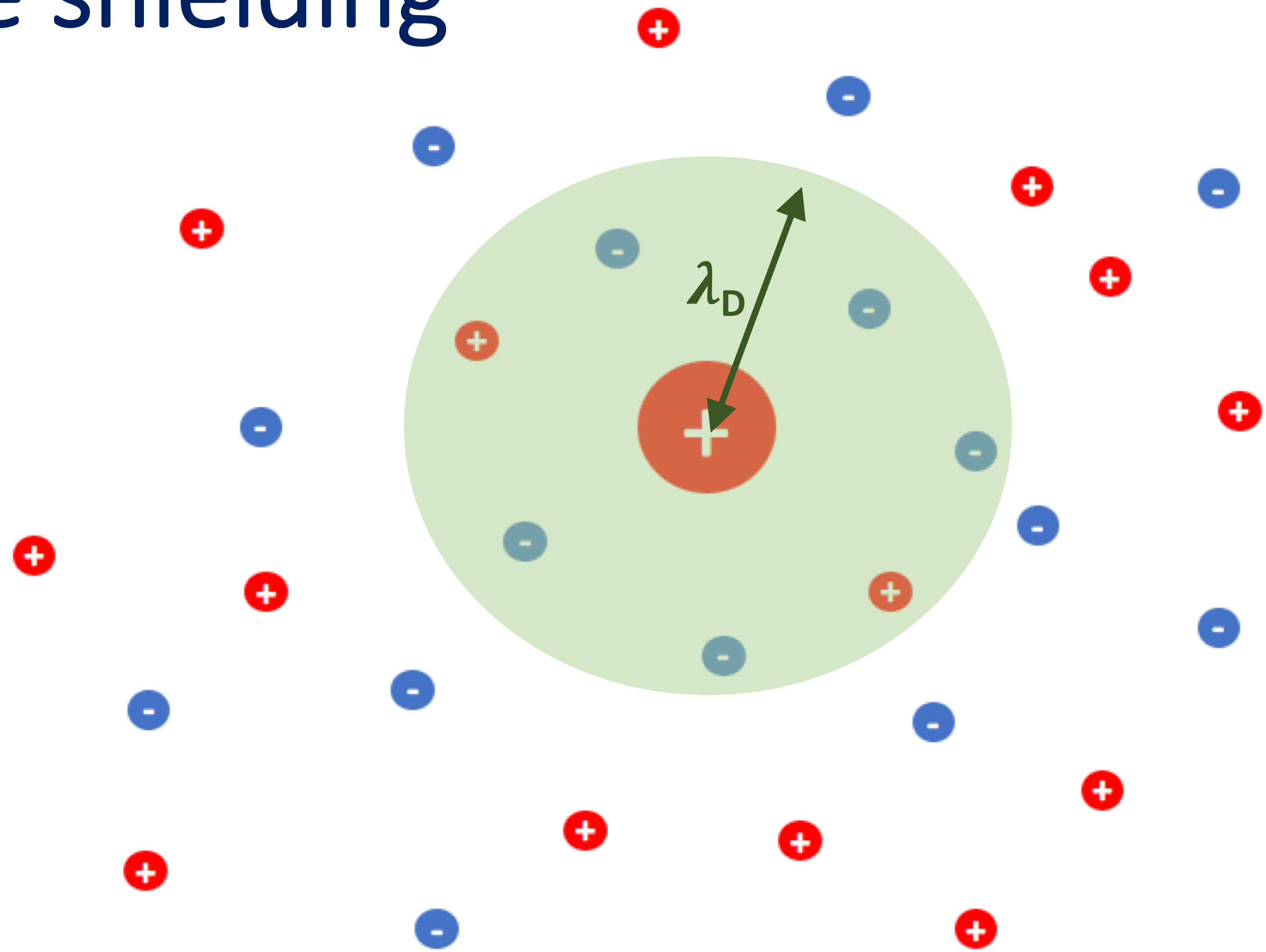
- **Photoionization**
  - Low density plasmas, astrophysical plasmas
- **Collisional/impact ionization**
  - Dense plasmas, shocks, electric discharges (energetic electrons)
  - Particle accelerators, astrophysics (energetic ions)
- Reciprocal processes:
  - Two body recombination $\leftrightarrow$ photoionization
  - Three-body recombination $\leftrightarrow$ impact ionization
  - Balanced out in thermodynamic equilibrium/steady state

Debye shielding



- Excess of electrons close to the test charge
- Deficit of ions close to the test charge

Debye shielding



- Mobile electrons attracted to positive charge
- Cloud of electrons around positive ion shields its charge

Debye shielding

- Starting from Boltzmann distribution for electrons:

$$n_e = n_0 e^{-E/k_B T} = n_0 e^{e\phi/k_B T}$$

- As energy due to electric potential:

$$E = -e\phi(r)$$

- n_0 = average electron number density = Zn_i
- Assume potential at a distance r from the test charge is $\phi(r)$:

$$n_e(r) = n_0 \exp\left(\frac{e\phi(r)}{kT}\right) \approx n_0\left(1 + \frac{e\phi(r)}{kT}\right)$$

Debye shielding

- Therefore, there is an excess of electrons of order:

$$n_e^+(r) \approx n_0 \frac{e\phi(r)}{kT}$$

- By similar reasoning there is a deficit of ions at the same position:

$$n_i^-(r) \approx -n_0 \frac{e\phi(r)}{kT} Z$$

- Apply Gauss' law:

$$\nabla E = -\frac{\rho}{\varepsilon_0} \qquad \text{where} \qquad \rho = n_i \mathrm{Ze} - n_e e$$

→ **Poisson's equation:** $\nabla^2 \phi = \frac{\rho}{\varepsilon_0}$

Debye shielding

- Therefore the excess charge density at the point r is given by:

$$\rho(r) = e(n_i^-(r) - n_e^+(r)) \approx -2n_0 \frac{e\phi(r)}{kT}$$

- For self consistency the potential itself is related to the charge density by Poisson's equation:

$$\nabla^2\phi(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial\phi}{\partial r}\right) = -\frac{\rho(r)}{\varepsilon_0} = \left(\frac{2n_0e^2}{\varepsilon_0 kT}\right)\phi(r)$$

Debye shielding

- Thus:

$$\phi(r) = \frac{A}{r} \exp\left(-\frac{\sqrt{2}r}{\lambda_D}\right)$$

- Where the Debye length is given by:

$$\boxed{\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{ne^2}}}$$

- And:

$$\phi(r) = \frac{Q}{4\pi\varepsilon_0 r} \exp\left(-\frac{\sqrt{2}r}{\lambda_D}\right)$$

Plasma parameter

- Indicates the number of particles inside the Debye sphere
- A 'good' plasma is one with a large number of particles within the Debye sphere:

$$\boxed{N_D = n_0 \frac{4}{3}\pi\lambda_D^3 >> 1}$$

- Determines if system remains ionized and if it is quazi-neutral
- Plasma scale length L:
  - L < λ_D → electrostatic forces between particles dominate
  - L > λ_D → electrostatic forces screened, plasma quasi neutral, long range E and B fields dominate

Ionization threshold

- To ionize atom, we need: $k_B T > e\phi$
- Potential seen by electron near ion: $\phi = \frac{Q}{4\pi\varepsilon_0 r}$, where $Q = Ze$ and average separation of electrons: $r = n_e^{-1/3}$
- Thus: $k_B T > \frac{Ze^2 n_0^{1/3}}{4\pi\varepsilon_0}$

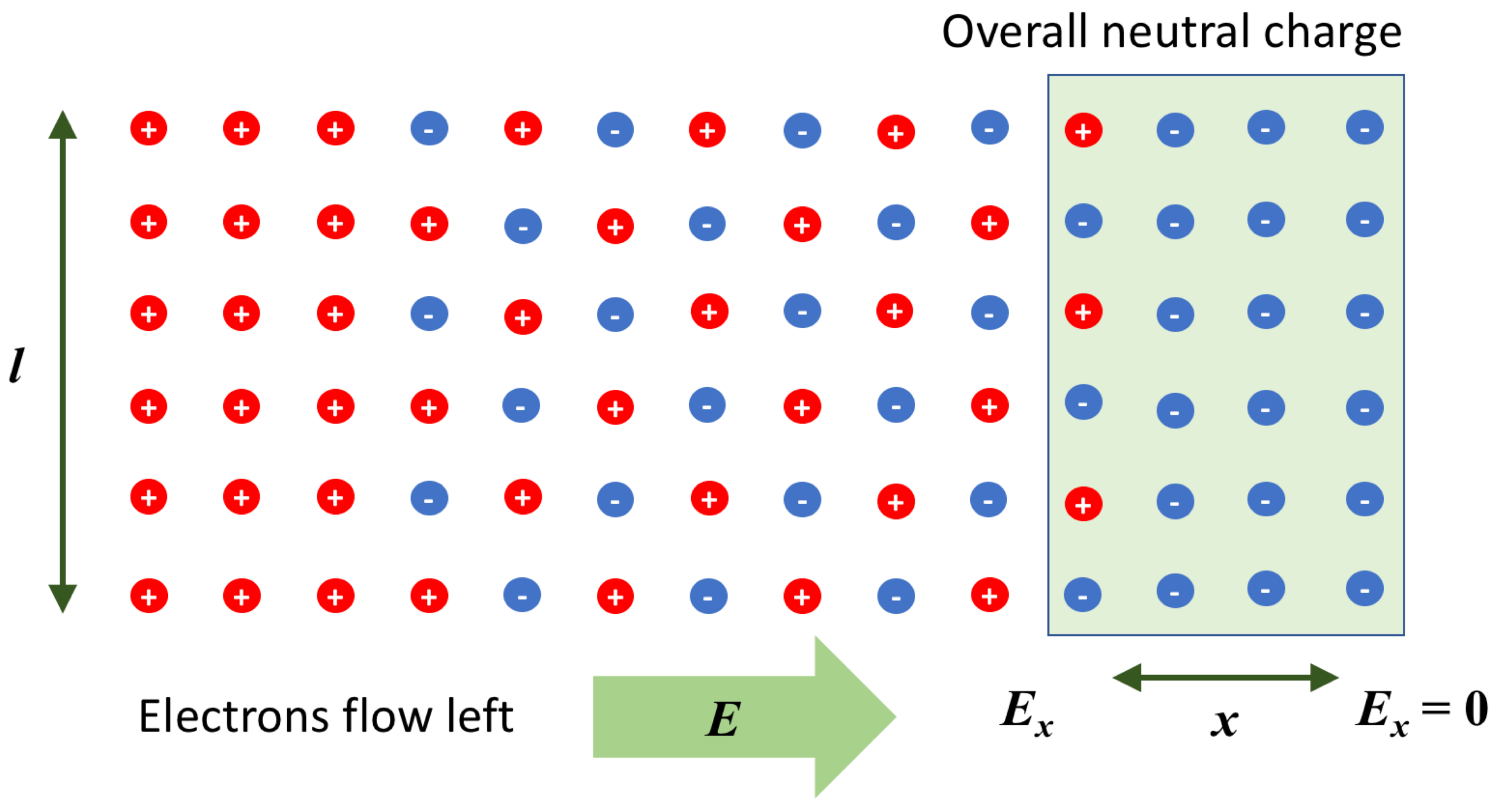
  $\rightarrow \frac{4\pi\varepsilon_0 k_B T}{Ze^2 n_e^{1/3}} > 1$ and for neutral plasma $n_0 = n_e$
- And: $N_D = n_0 \frac{4}{3}\pi\lambda_D = \frac{4}{3}\pi\left(\frac{\varepsilon_0 k_B T}{e^2 n_0^{1/3}}\right)^{3/2}$
- Condition for ionization: $N_\mathrm{D} > 1$

Oscillations in plasma - plasmons

- Ions are significantly heavier compared to electrons → remain stationary while electrons move
- Electrons can be displaced by EM waves/fields
- Assume no collisions
- We ignore thermal motion → electrons are cold, their motion is only due to the electrostatic restoring force

Oscillations in plasma - plasmons

- Displace electrons with the electrostatic force and let go:

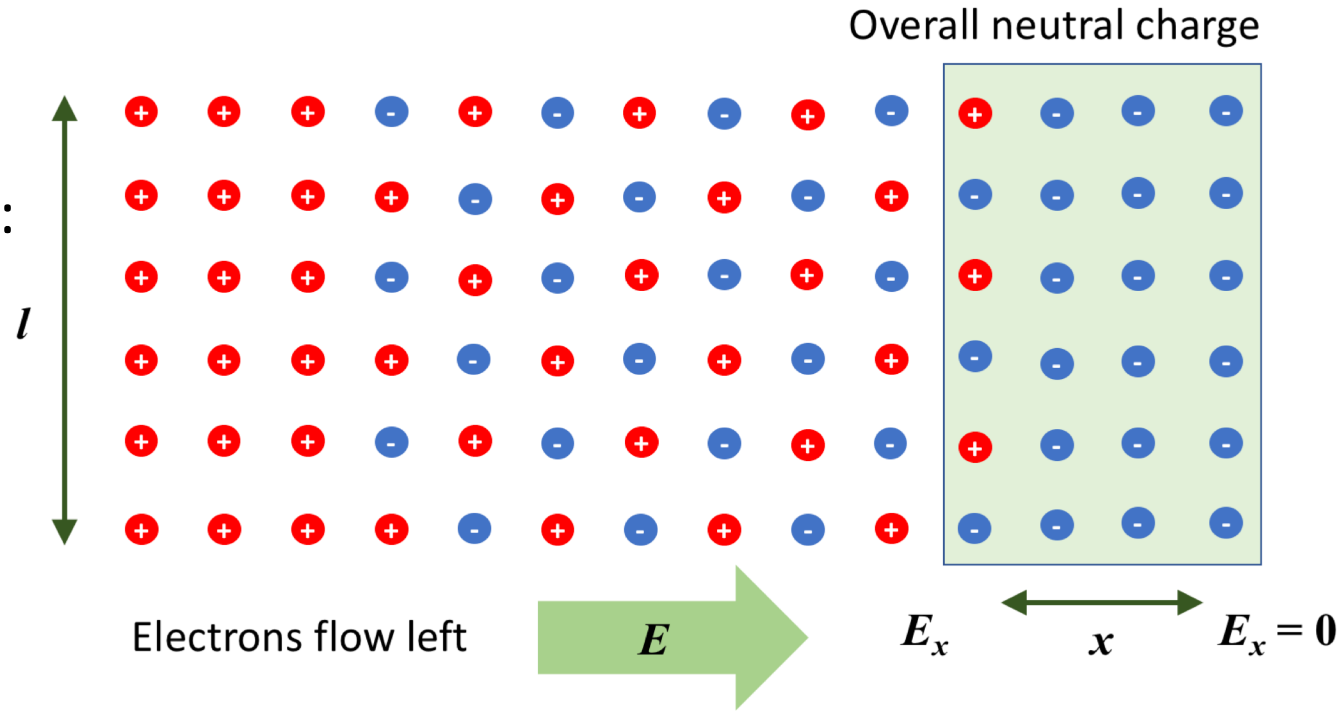


Plasma frequency

Apply Gauss' law:

$$El^2 = \frac{nel^2x}{\varepsilon_0}$$

$$\rightarrow \quad E = \frac{nex}{\varepsilon_0}$$



Equation of motion: $$F = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -eE = -\frac{ne^2x}{\varepsilon_0}$$

Standard solution of differential equation gives the plasma oscillation frequency:

$$\boxed{\omega_p^2 = \frac{ne^2}{\varepsilon_0 m}}$$

Typical plasma frequencies

- Ionosphere: $n \sim 10^4$ cm^{-3}, $f_p \sim 1$ MHz
- Tokamak: $n \sim 10^{12}$ cm^{-3}, $f_p \sim 10$ GHz
- Laser plasma: $n \sim 10^{21}$ cm^{-3}, $f_p \sim 1$ THz
- Useful relation: $f_p \sim 9000\, n^{1/2}$ (n in cm^{-3})

Plasma frequency linked to λ_D

- Debye length represents the distance travelled by a typical thermal electron during the oscillation period of one plasma wave:

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{ne^2}} = \sqrt{\frac{kT}{m}}\sqrt{\frac{\varepsilon_0 m}{ne^2}} = \frac{v_{th}}{\omega_p}$$

Summary of Lecture 1

- Saha equation:

$$\frac{n_{e^-}^2}{n_H} = \exp\left(\frac{-Ry}{k_\mathrm{B}T}\right)\left(\frac{2\pi m_e k_\mathrm{B} T}{h^2}\right)^{3/2}$$

- Charges in a plasma are shielded over a distance of order the Debye length:

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{ne^2}}$$

- Electrostatic waves occur in plasmas and oscillate with a frequency given by the plasma frequency:

$$\omega_p^2 = \frac{ne^2}{\varepsilon_0 m}$$

- Thermal motion linked to plasma oscillations and charge screening in plasma:

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{ne^2}} = \sqrt{\frac{kT}{m}}\sqrt{\frac{\varepsilon_0 m}{ne^2}} = \frac{v_{th}}{\omega_p}$$