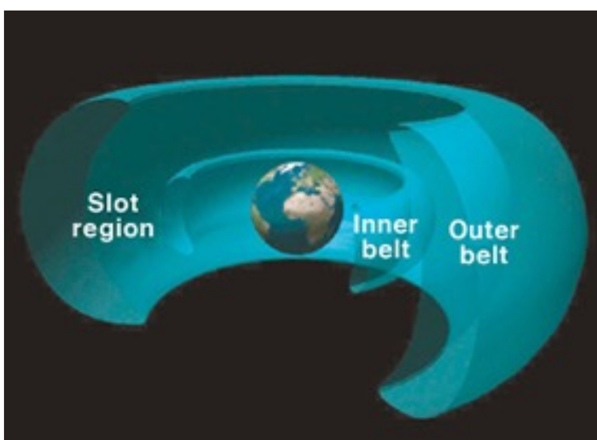
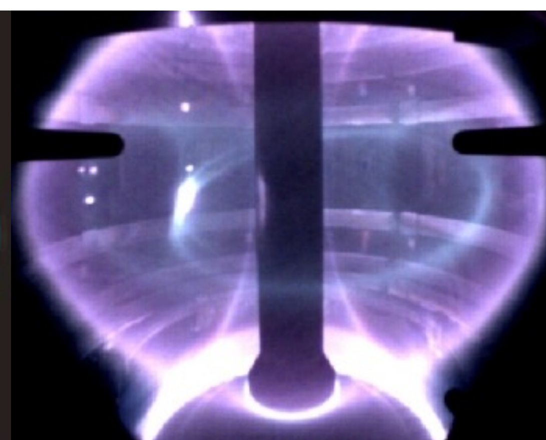
Plasma Physics

TU Dresden

Lecturer: Dr. Katerina Falk

Lecture 8: Magnetic confinement fusion





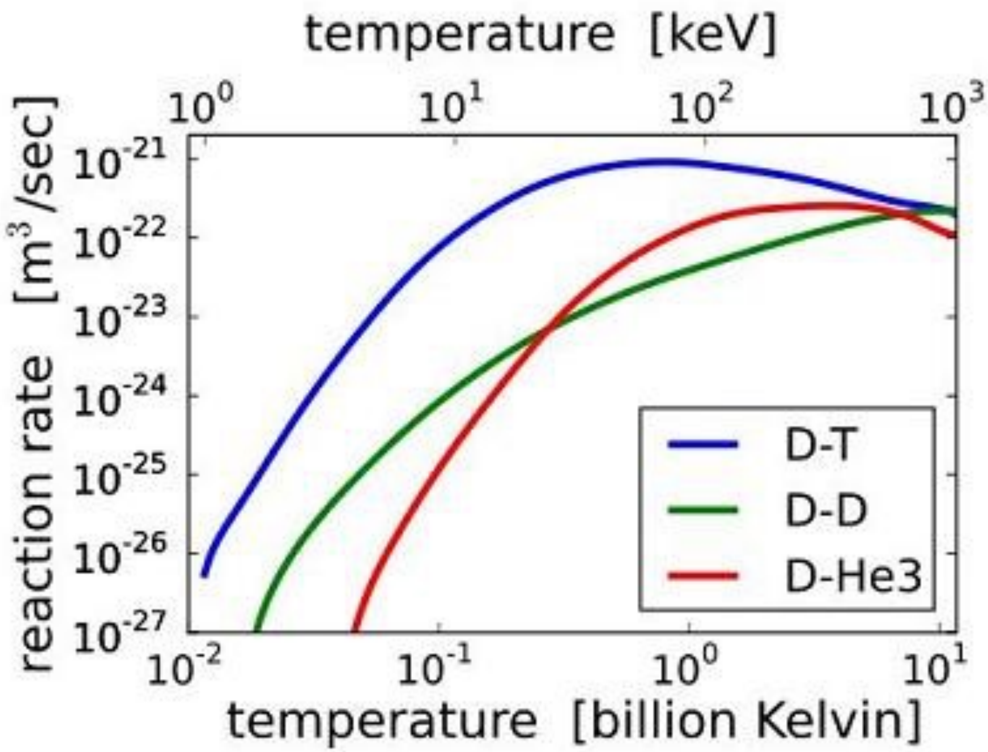
Plasma Physics: lecture 8

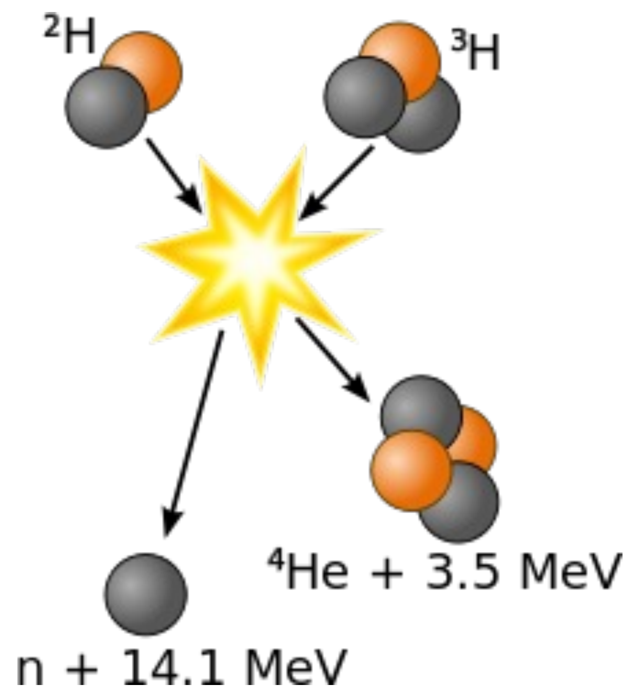
- The Lawson criterion
- The magnetic confinement concept for fusion
- Magnetic mirror revisited
- Tokamaks
- Stellarators
- Z-pinches
- Bennett relation
- MagLIF

Thermonuclear fusion

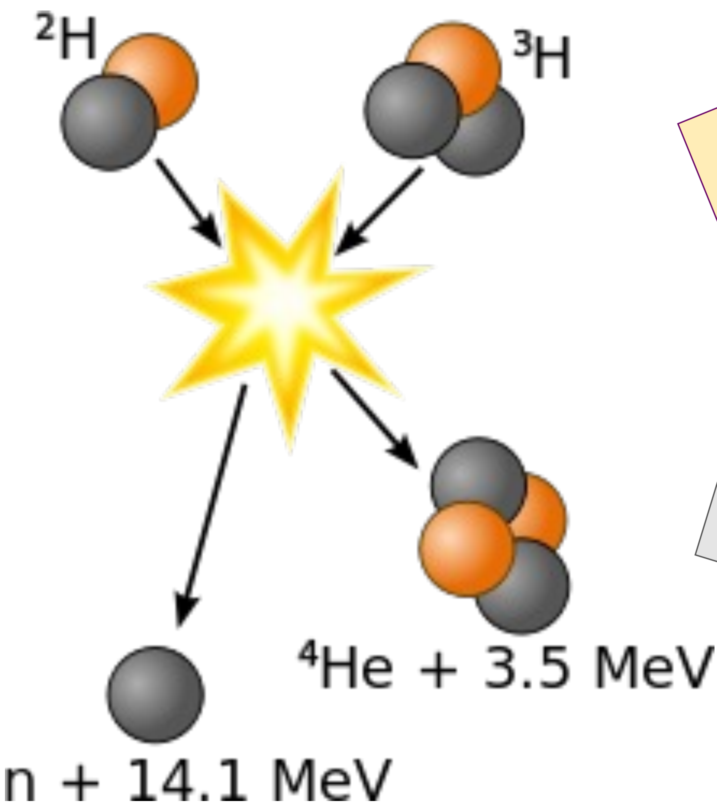
- D-T reaction rate is ~ 10^{-22} m^3s^{-1} at a temperature of 10 keV.

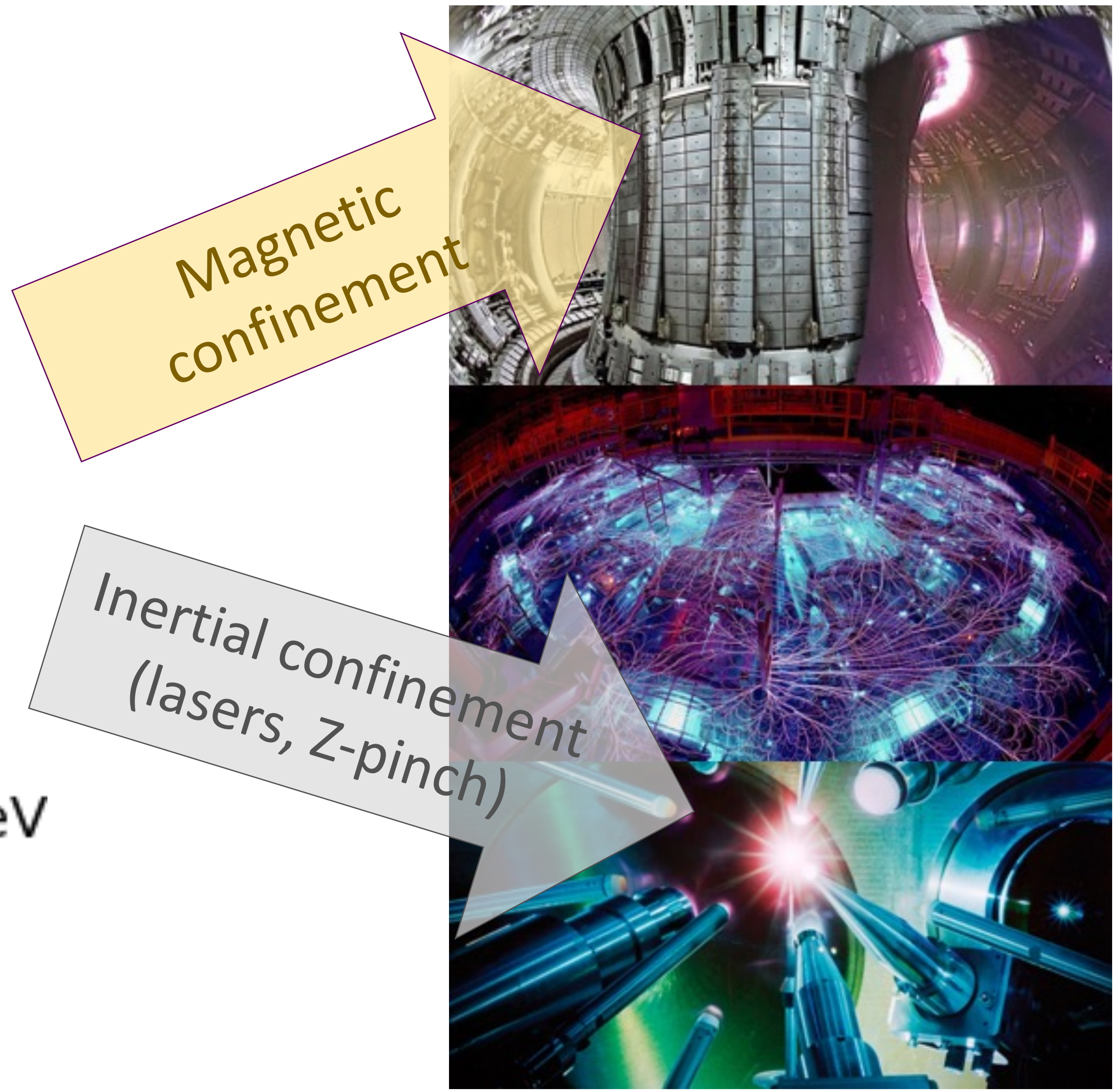
$$D + T \Rightarrow \alpha(3.5\text{MeV}) + n(14.1)\text{MeV}$$





Approaches to fusion research





The Lawson criterion

- For D-T fusion energy gain, the energy out must exceed the energy in. The energy out per volume is given by:

Energy per reaction

$$E_f = n_D n_T \sigma W \tau$$

Assuming: $n_D = n_T = \frac{n}{2}$

Reaction rate

Plasma confinement time

- This must exceed the energy required to heat the plasma to the temperature required for fusion:

$$E_t = 2 \times \frac{3}{2} n k_B T$$

- Thus:

$$\frac{n^2 \sigma W \tau}{4} > 3 n k_B T$$

The Lawson criterion

- We define the Lawson criterion:

$$n\tau > \frac{12k_BT}{\sigma W}$$

- For a $T = 10$ keV, $\sigma \sim 10^{-22}$ m^{3}s^{-1}, $W \sim 17$ MeV:

$$n\tau > \frac{12 \times 10^4}{10^{-22} \times 17 \times 10^6} \sim 10^{20} \text{ sm}^{-3}$$

- The criterion can be fulfilled by long reaction times.
- So we need, for example, to confine a plasma with number density 10^{20} m^{-3} for of order seconds - we can try to do this with magnetic fields!

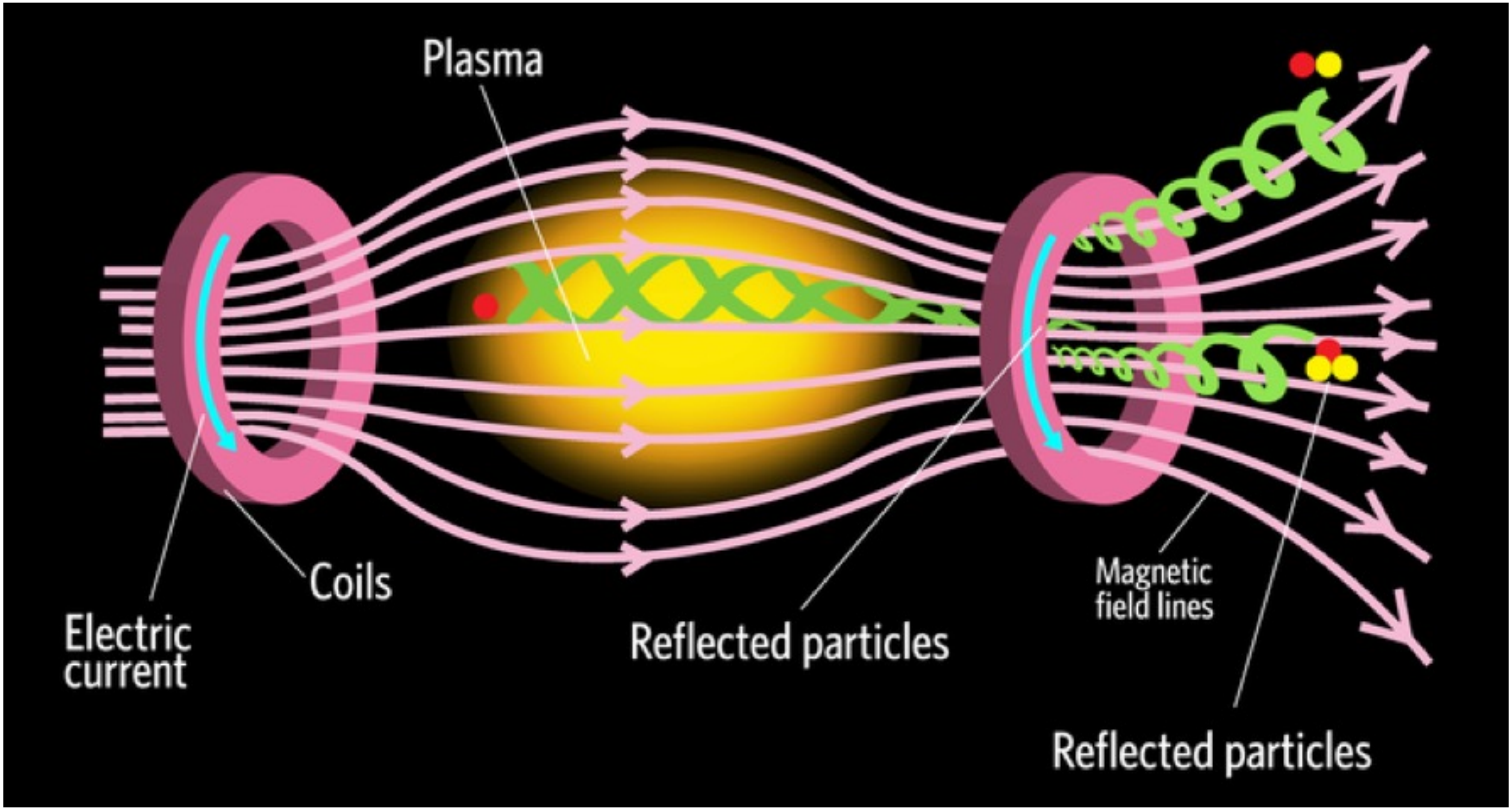
Magnetic confinement

- The energy density (pressure) in a magnetic field is given by:

$$P_B = \frac{B^2}{2\mu_0}$$

- To confine a plasma, this will need to exceed the thermal pressure $P_{th} = nk_BT$.
- We can create magnetic fields $\sim$ Tesla in the laboratory.
- The pressure of the plasma is only about 1 atmosphere (= 100 000 Pa).
- For temperatures of $\sim 10^8$ K, this means that we can indeed confine a plasma of number density $\sim 10^{20}$ m^{-3}.

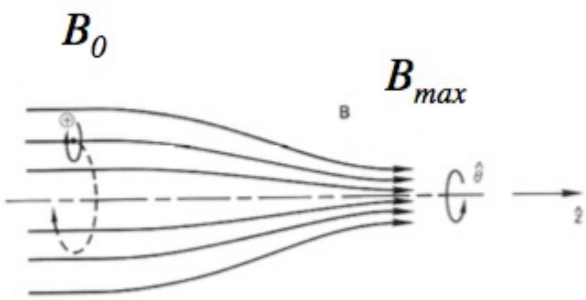
Magnetic mirrors



Magnetic mirrors

- As **B** increases:
  - $v_\perp$ must increase to keep the magnetic moment μ constant
  - $v_\parallel$ must decrease to keep the kinetic energy constant

- At field maximum B_m:
  - Particles with $v_\parallel^2(0) < v_\perp^2(0)\left(\frac{B_m}{B_0} - 1\right)$ are reflected.
  - Particles with $v_\parallel^2(0) > v_\perp^2(0)\left(\frac{B_m}{B_0} - 1\right)$ are lost.



- Trapped particles oscillate between two reflection points

Loss cone

- Particle velocity components:

$$\sin^2\theta = \frac{v_{\perp 0}^2}{v_0^2} = \frac{B_0}{B}$$

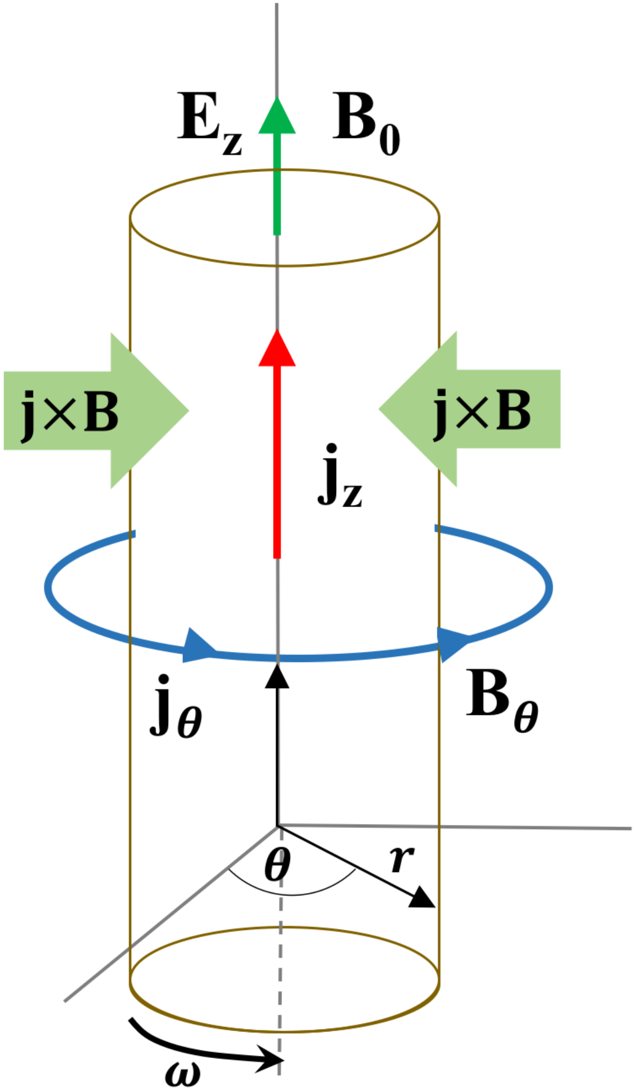
- Thus, particles with the pitch angle of the orbit θ smaller than θ_m will escape from the confinement:

$$\sin^2\theta_{\mathrm{m}} = \frac{B_0}{B_{\mathrm{m}}}$$

- The angle can be changed by particle collisions, thus many particles can be lost → **magnetic mirror does not provide good plasma confinement. What else?**

The Z-pinch

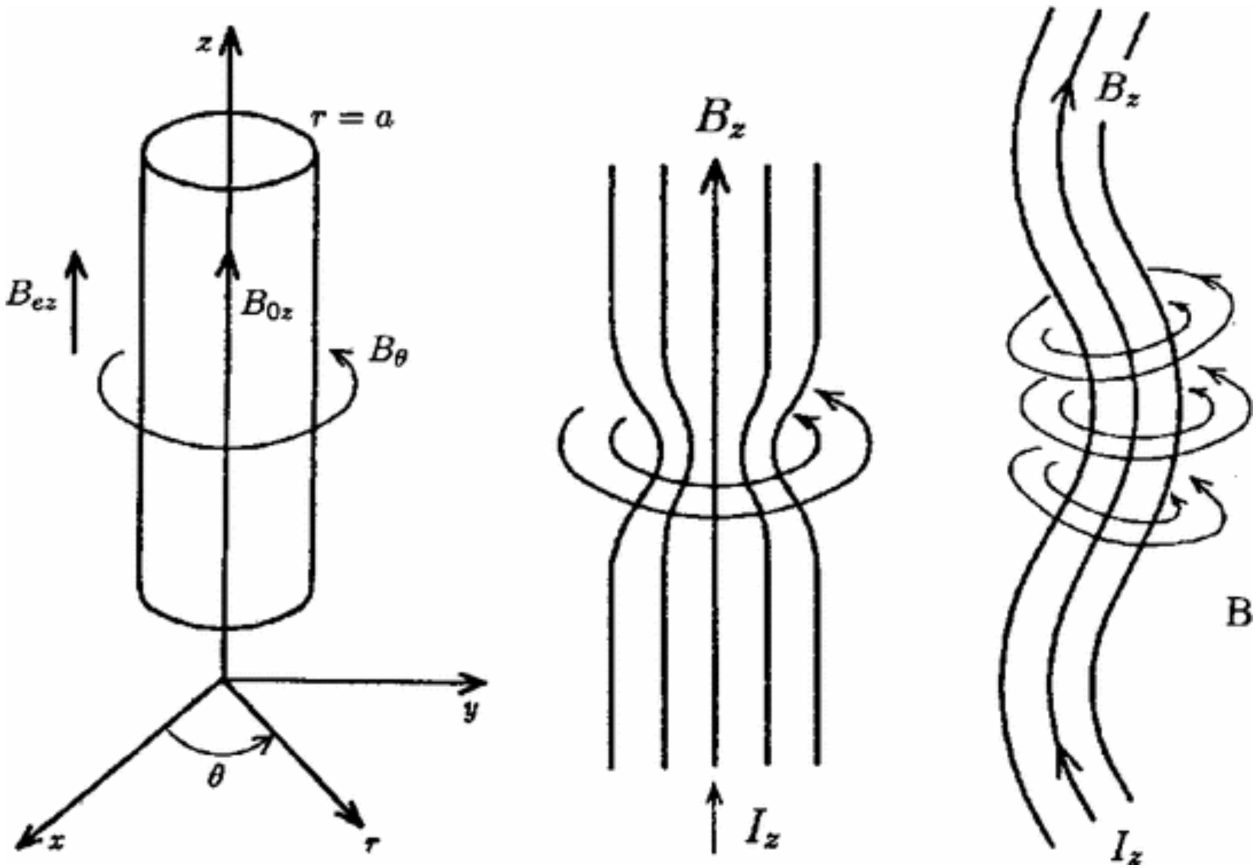
- A strong current (MA) passes through a cylindrical plasma a magnetic field is produced around it (just like the field around a wire).
- The $\mathbf{j}\times\mathbf{B}$ force on both electrons and ions forces them to the centre and increases the plasma density.
- The current heats the plasma (Ohmic heating).



$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

Magnetic instabilities

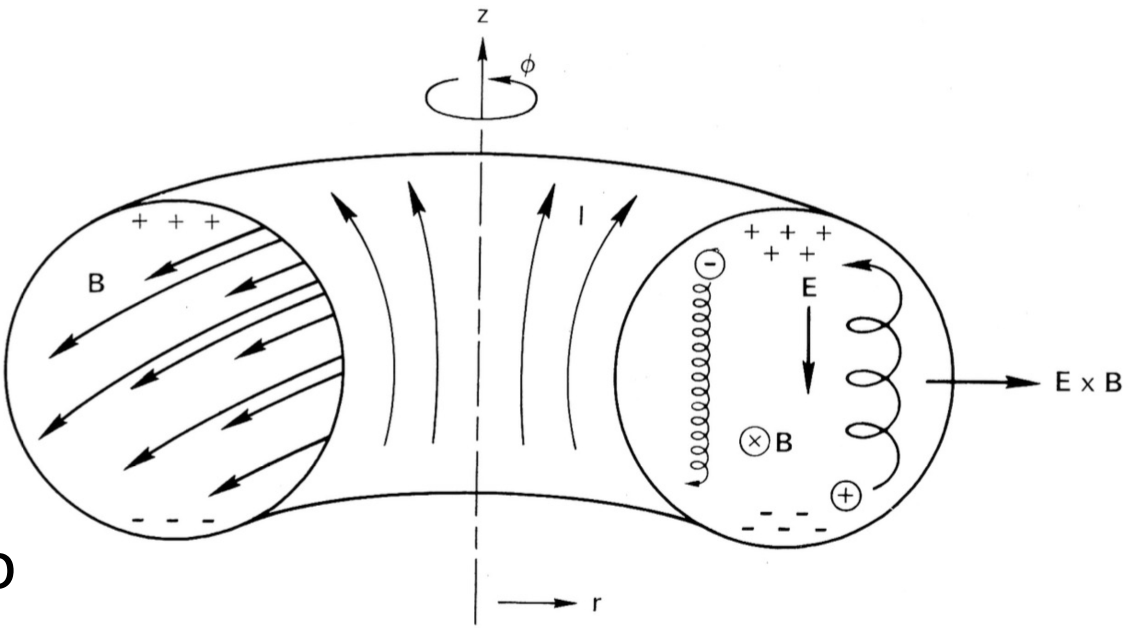
- Magnetized plasmas are subject to sausage and kink instabilities:



- More on plasma instabilities in lecture 10

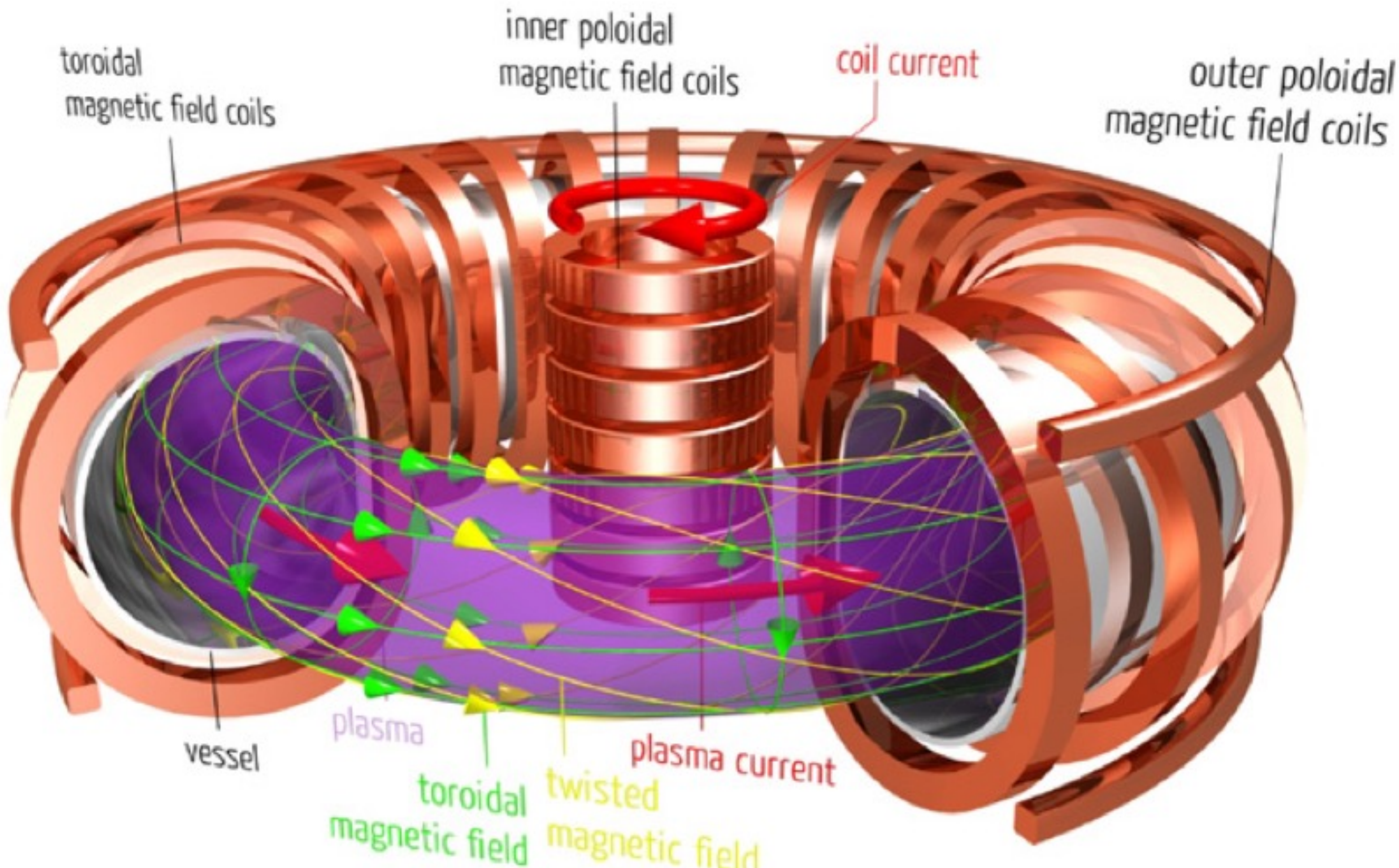
The torus

- A simple torus ‘joins the ends’ of a Z-pinch. There is no particle loss along the toroidal direction.
- The problem with sausage and kink instabilities remains.
- Curvature drift (lecture 2) causes electrons and ions to separate.



- This creates an electric field. Ions and electrons then both drift in the E field towards larger radii, and the plasma touches the wall of any container.

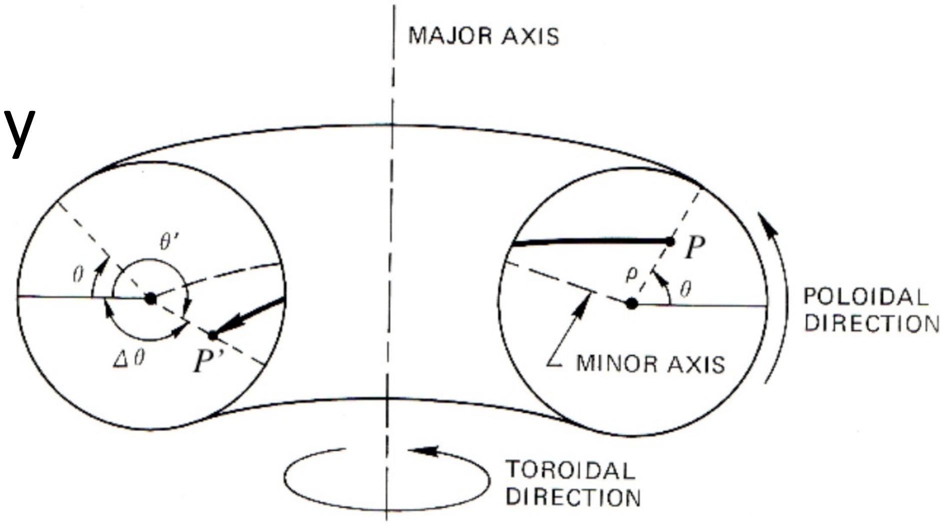
The tokamak



- Add a poloidal field along the current carrying direction, so we have both toroidal and poloidal fields.

Tokamak prevents drift

- The field lines in tokamak link regions at the top and bottom of the torus, effectively shorting the electric field due to drift.
- From a single particle point of view, a particle drifting upward at P is moving away from the centre of the plasma.
- At point P' it is moving towards the centre.



- A tokamak also helps stabilise against magnetic plasma instabilities (next lecture).

Stability in tokamaks

- Tokamak provides some degree of stability against the sausage instability.
- Have both B_z and B_θ (azimuthal) components: $B_\theta = \frac{\mu_0 I_z}{2\pi r}$
- Conservation of magnetic flux in plasma of radius a:

$$\delta(B_z \pi a^2) = 0$$

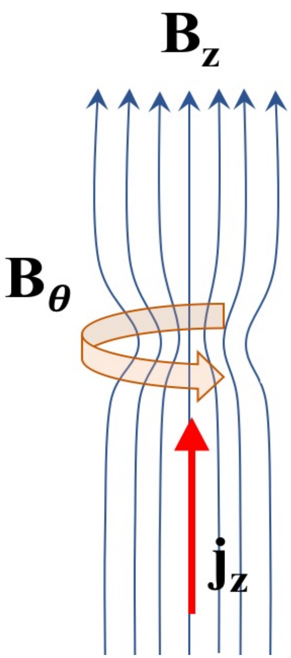
$$\delta B_z \pi a^2 + B_z \pi 2a\delta a = 0$$

- Thus:

$$\delta B_z = -B_z \frac{2\delta a}{a}$$

- By conservation of current:

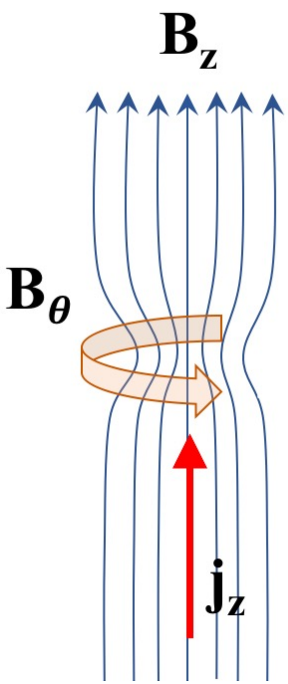
$$\delta B_\theta = -B_\theta \frac{\delta a}{a}$$



Stability in tokamaks

- The difference of the two mag. pressures:

$$\delta p_{\mathrm{m}} = \delta\left(\frac{B_z^2}{2\mu_0} - \frac{B_\theta^2}{2\mu_0}\right) = \frac{B_z \delta B_z}{\mu_0} - \frac{B_\theta \delta B_\theta}{\mu_0}$$



- Substitute:

$$\delta p_{\mathrm{m}} = -\frac{B_z^2}{\mu_0}\frac{2\delta a}{a} + \frac{B_\theta^2}{\mu_0}\frac{\delta a}{a}$$

- Thus plasma is stable against sausage instability if:

$$\boxed{B_z^2 > \frac{B_\theta^2}{2}}$$

The longitudinal B-field inside the plasma acts to suppress the instability as contraction of the plasma increases its component of magnetic pressure.

The “safety” q-factor in tokamaks

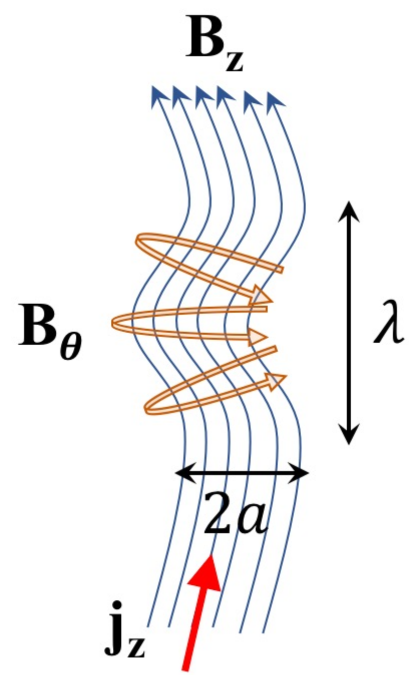
- The tokamak configuration also provides stability against the kink instability (complicated derivation).
- The poloidal field provides a degree of ‘tension’ in the plasma, which can act as a restoring force.
- The Kruskal-Shavranov limit can be written as:

$$\left|\frac{B_\theta}{B_z}\right| < \frac{2\pi a}{\lambda}$$

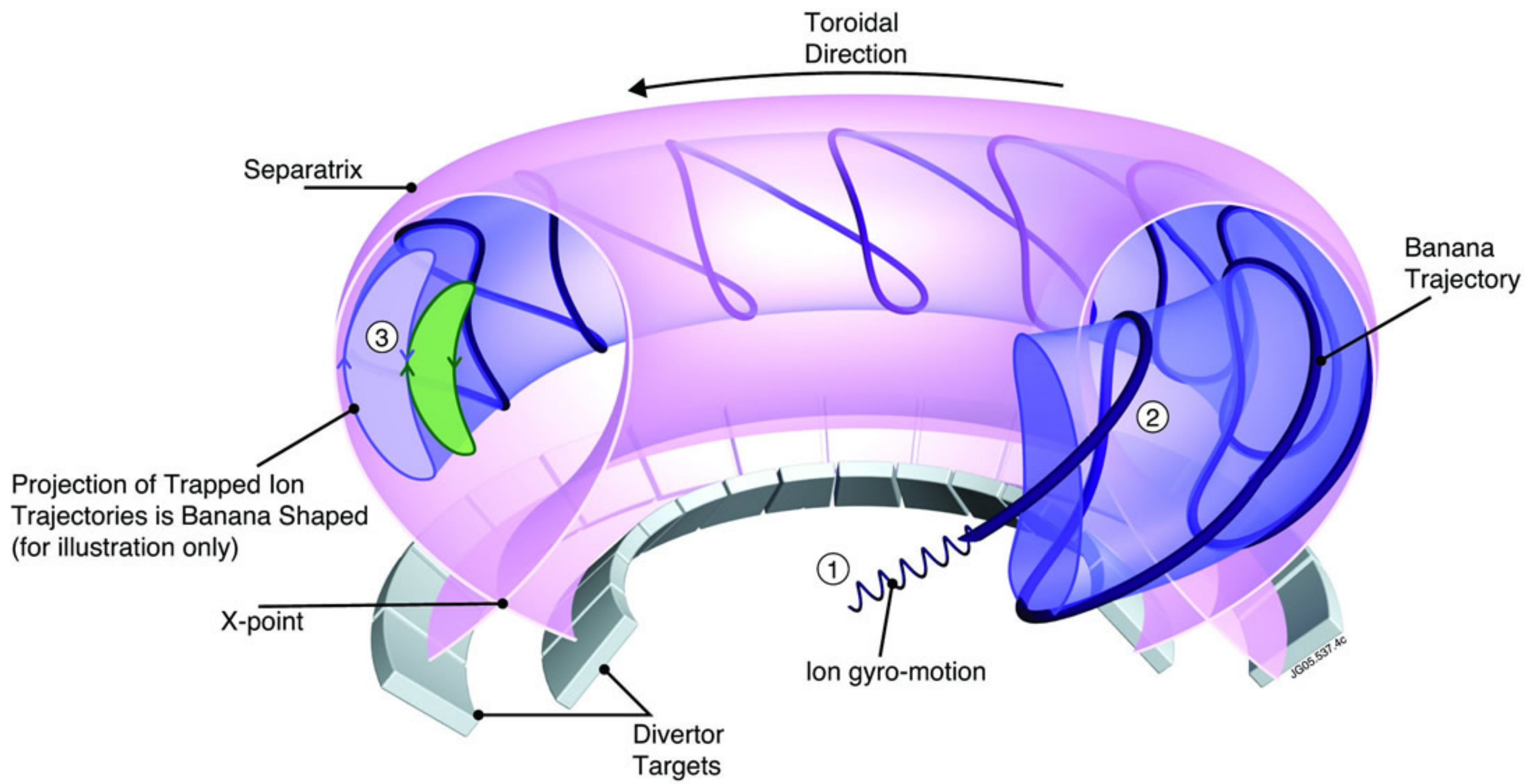
- For tokamak with the longest mode of $\lambda = 2\pi R$:

Radius of the ring

$$q \equiv \frac{B_\mathrm{t}}{B_\mathrm{p}}\frac{a}{R} > 1$$



Plasma flows in tokamaks



- Finite size of ”bananas” due to small finite drift velocity
- X-point with zero poloidal field is introduced allowing online removal of waste material/impurities through the divertor device.

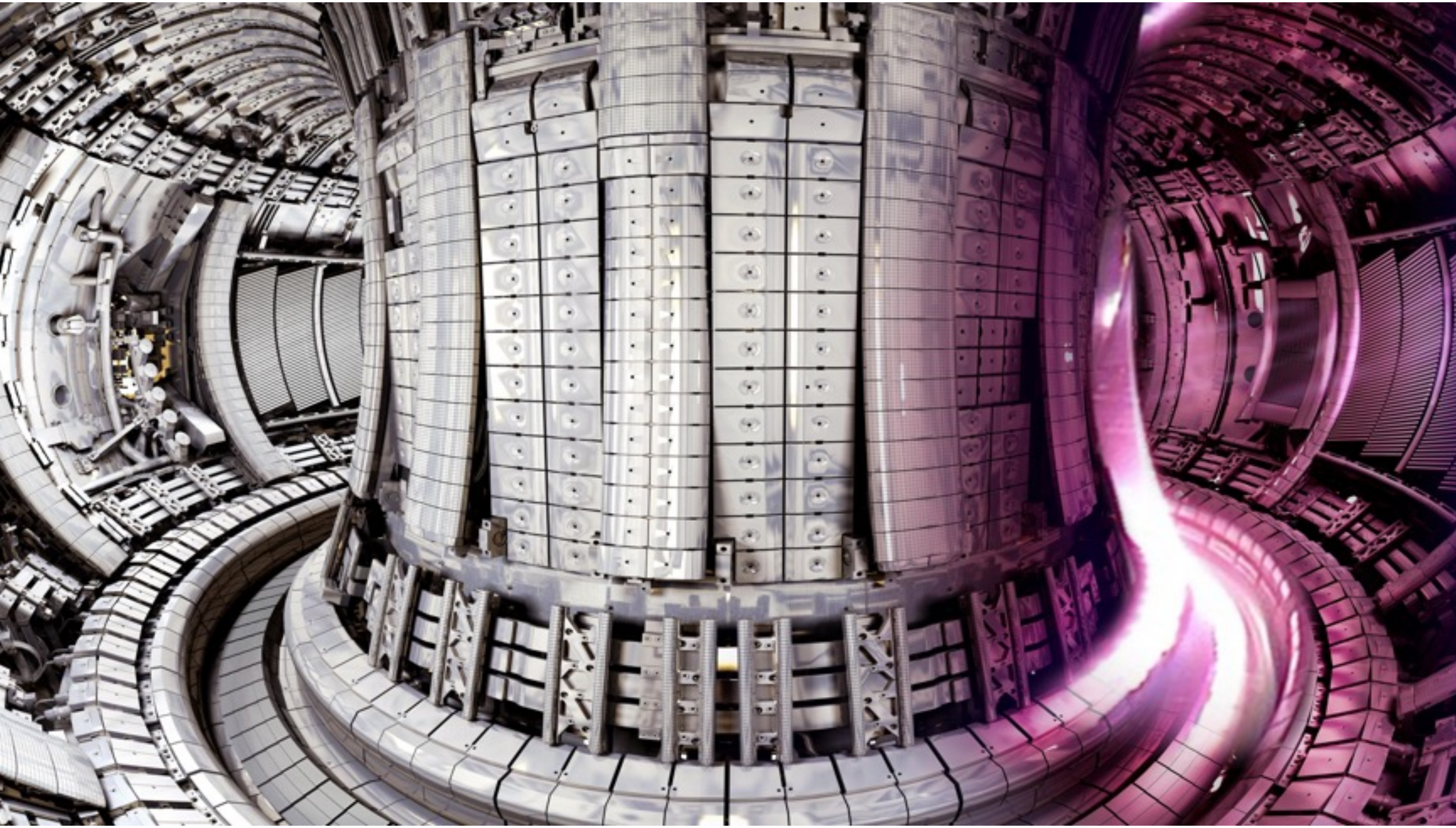
Notes about tokamaks

- Presence of heavier elements (impurities, He ash) leads to cooling of the plasma (divertors are needed).
- When the plasma touches the vessel walls it undergoes rapid cooling or "thermal quenching" causing confinement break up, i.e. X-point ensures that the plasma edge is uncoupled from the vessel walls
- Tokamaks are primarily heated by Ohmic heating due to I^2R (lecture 3), which becomes less effective as the plasma heats up, as τ_{ei} scales as $T^{-3/2}$.
- High current produces a high B_p, and requires a low value of a. Therefore we cannot use too high a current.
- Fusion temperatures cannot be achieved by Ohmic heating only, thus additional heating is needed.

Plasma heating in tokamaks

- **Ohmic heating**: by current induced by continually increasing the current through an electromagnetic winding on the plasma torus → pulsed process → *short confinement periods*
- **Magnetic compression**: gas heated by sudden compression, not widely used
- **Neutral-beam injection**: introduction of high energy (rapidly moving) atoms or molecules into an ohmically heated, magnetically confined plasma, under development for ITER
- **Radio-frequency heating**: by high-frequency electromagnetic waves (microwaves) generated by oscillators (gyrotrons or klystrons) outside. Various techniques exist including electron cyclotron resonance heating (ECRH) and ion cyclotron resonance heating.

JET (Joint European Torus)

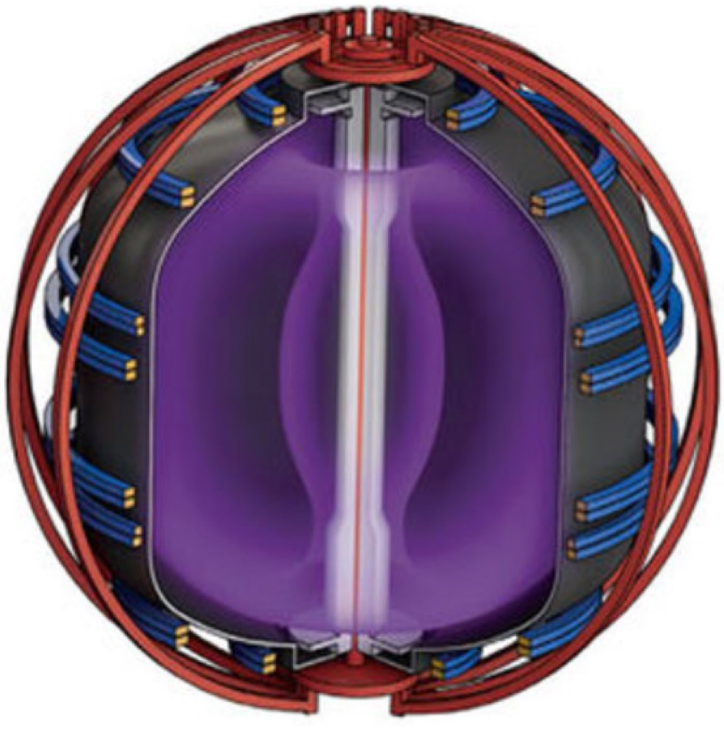


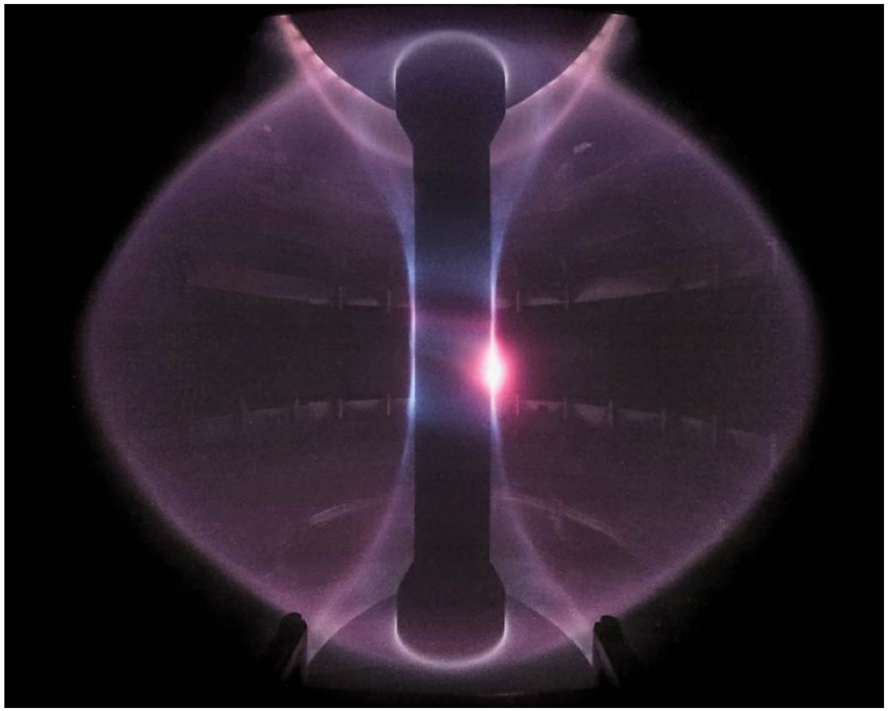
EUROfusion

ITER under construction

Spherical tokamak

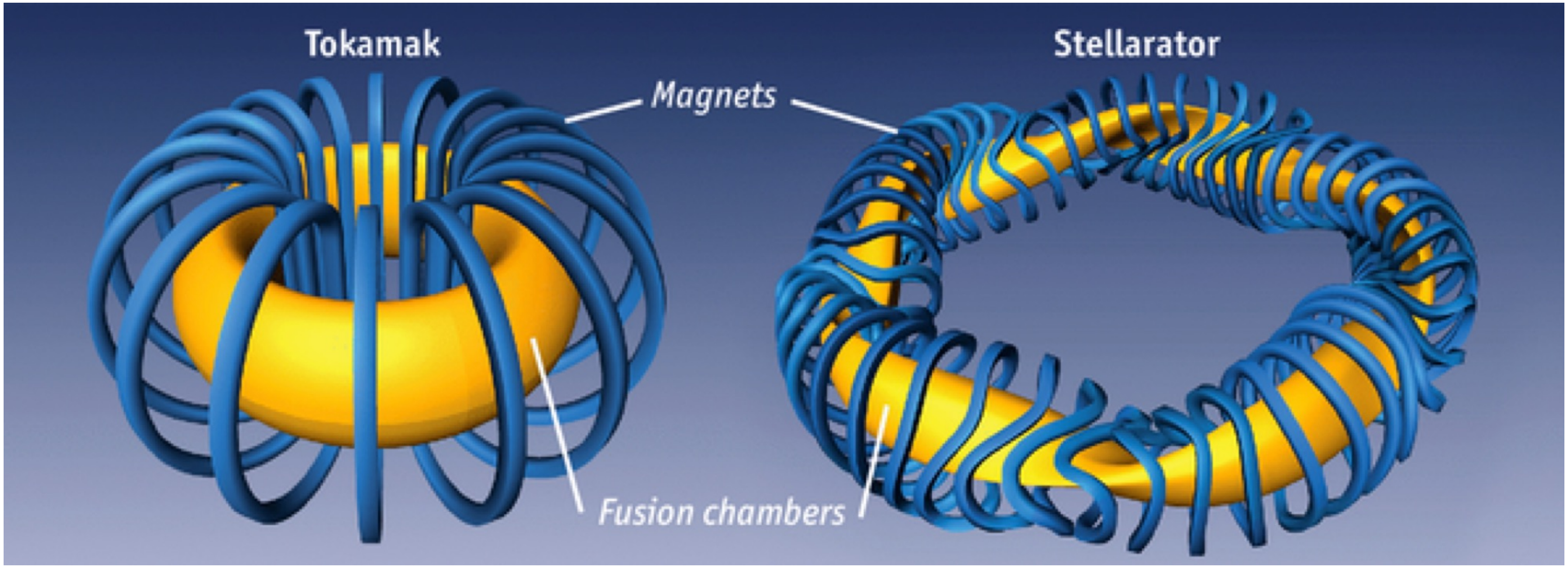
- Arrangement of the magnet coils greatly reduces the aspect ratio while avoiding the erosion issues of the compact tokamak.
- Plasma has very narrow plasma profile.



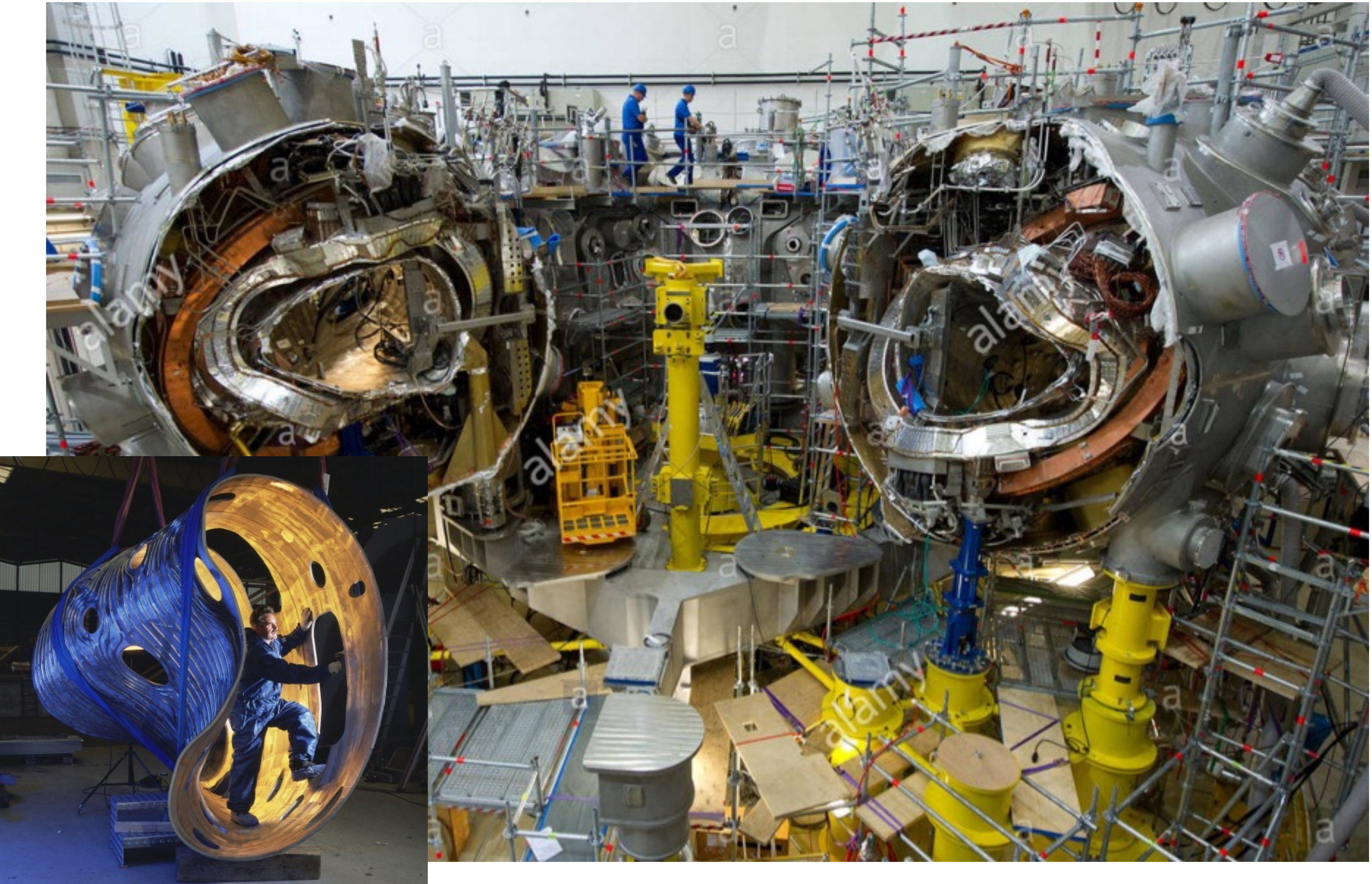


Stellarator

- No induced electrical current within the plasma at a macroscopic level, the plasma is neutral and unmoving
- Plasma heating by 10 MW of microwaves for electron cyclotron resonance heating (ECRH) giving 80 MJ
- Several different geometric configurations available

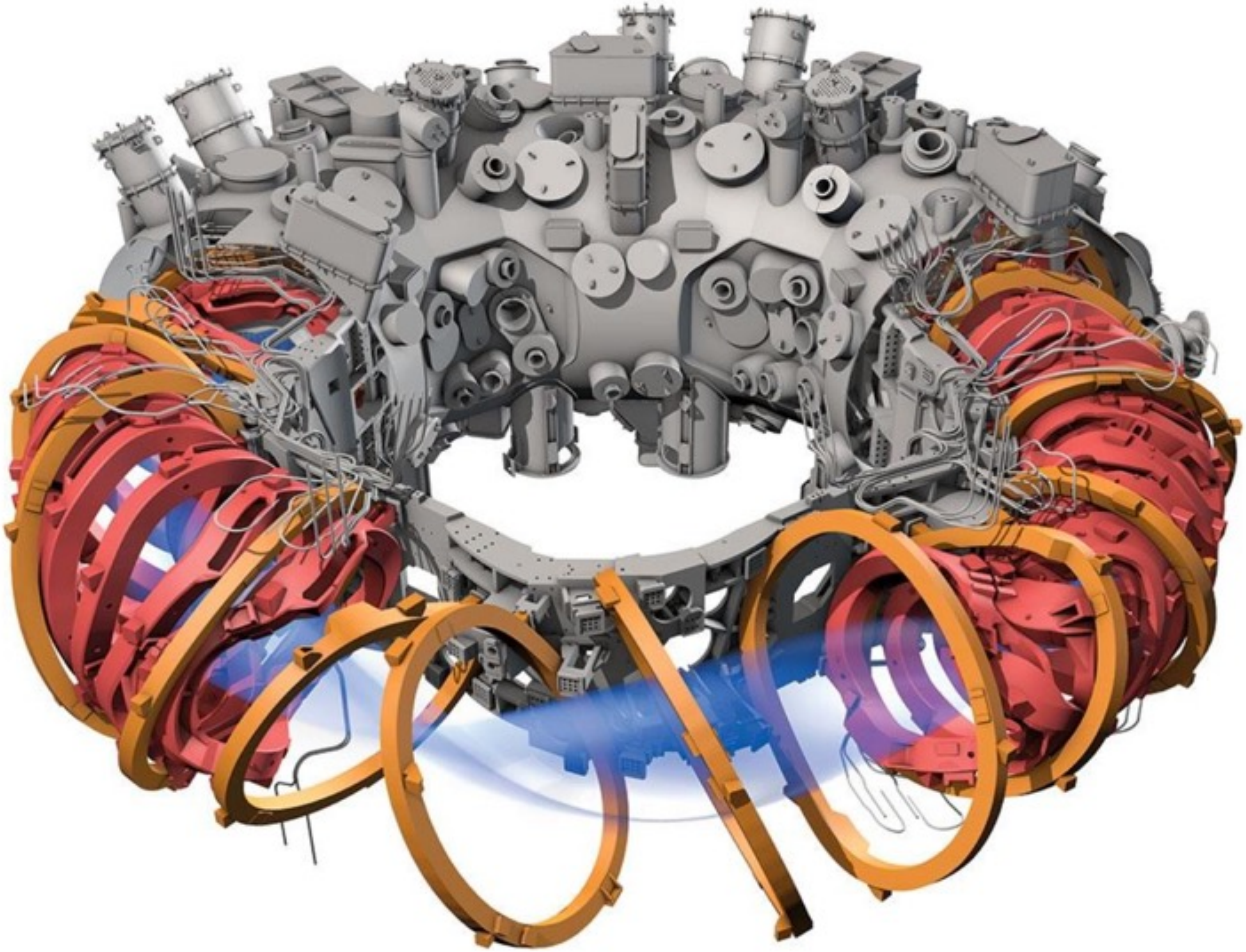


Wendelstein 7-X



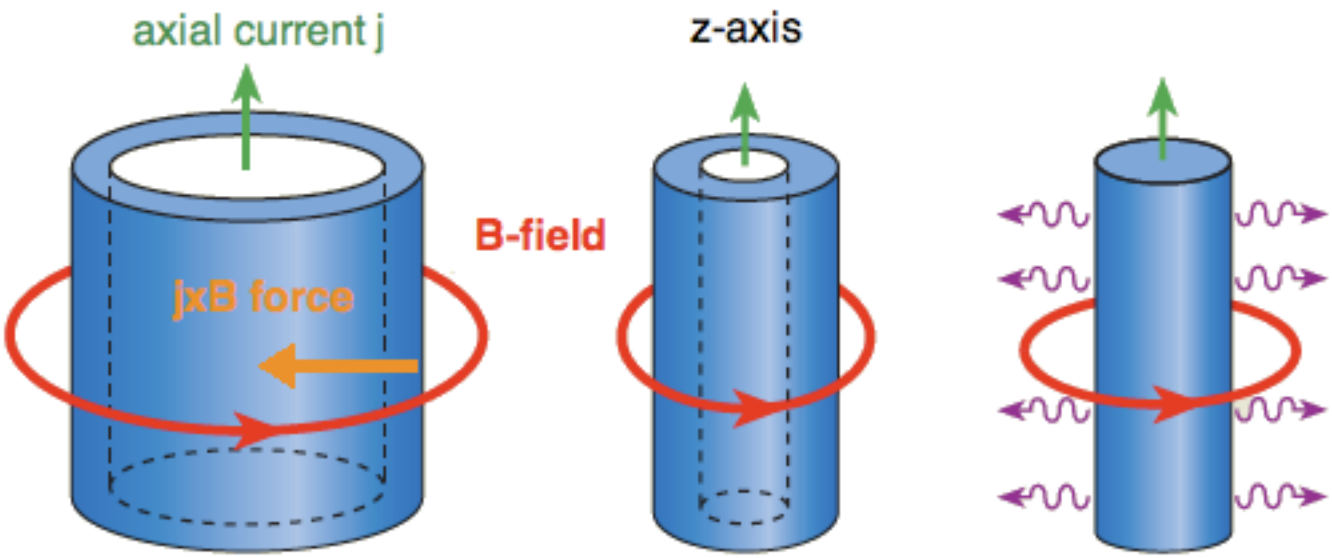
Wendelstein 7-X

MAX-PLANCK-GESELLSCHAFT



Z-pinches for fusion

- Cylindrical wire array or metallic plate
- The implosion is caused by the high axial current I_z (or current density j_z)
- High conductance of the plasma does not allow the B-field to penetrate the shell → $P_B \gg nk_BT$ → **confinement and compression**



The Bennett relation

- Starting from momentum equation:

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathbf{j}\times\mathbf{B} - \nabla P \quad \text{and for equilibrium:} \quad \rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = 0$$

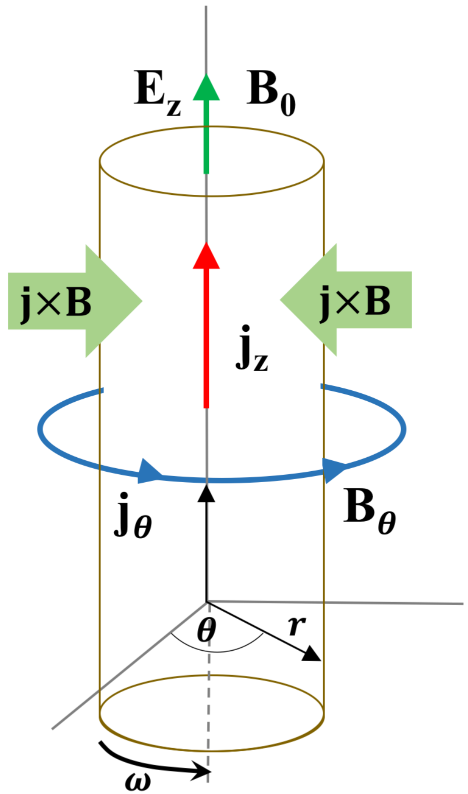
- In cylindrical geometry:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -j_z B_\theta \quad \text{and} \quad \nabla\times\mathbf{B} = \mu_0 \mathbf{j}$$

- For ideal gas: $P = (Z+1) n_i k_B T$

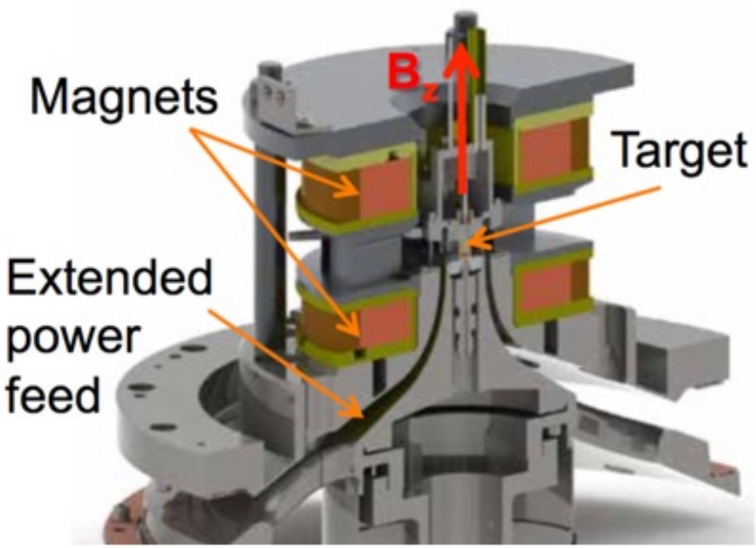
- The Bennett relation:

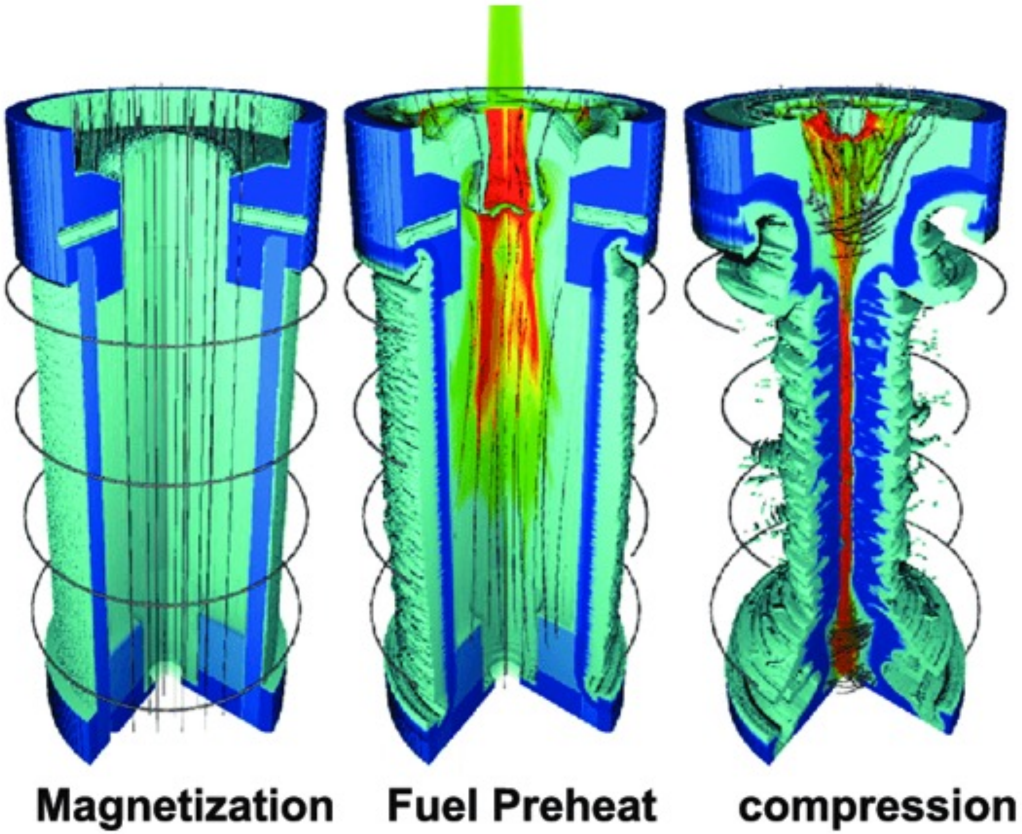
$$\boxed{\mu_0 I^2 = 8\pi (Z+1) N k_B T}$$



MagLIF

- Magnetized linear inertial fusion is a novel concept to ICF that was recently proposed and tested (2015)
- MagLIF consists of 3 stages:
  - Axial magnetization
  - Laser heating
  - Linear compression



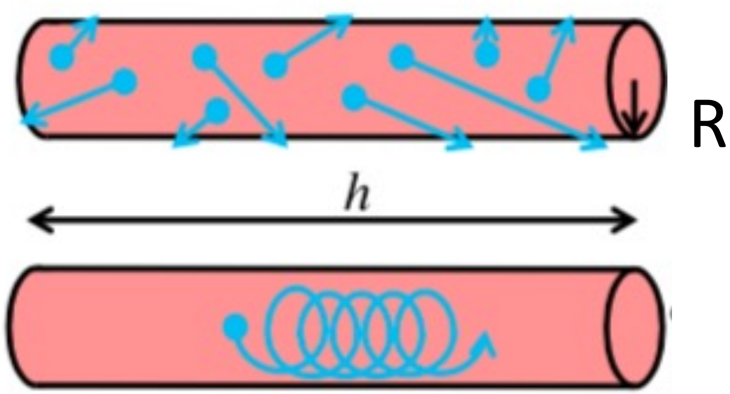


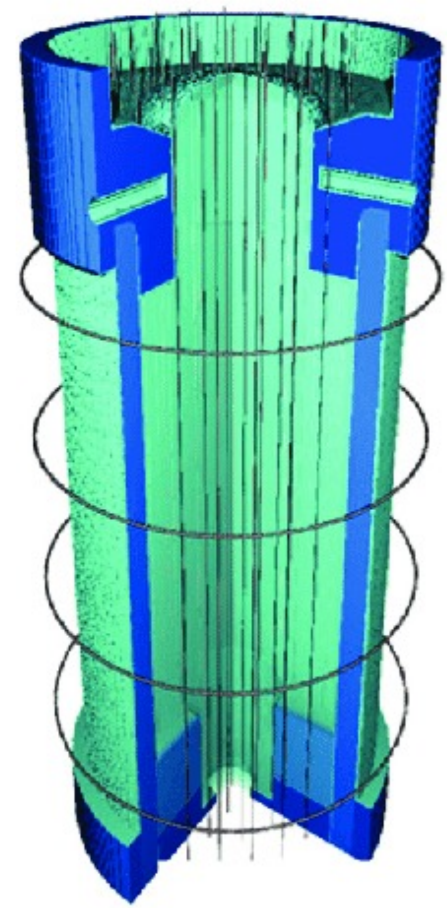
MagLIF – phase 1

- Axial magnetization of fuel
- Fields reaching: $B_\theta = 10 - 30$ Tesla
- Inhibits thermal conduction losses
- Possible stabilization of liner compression

Low B-field:

High B-field:



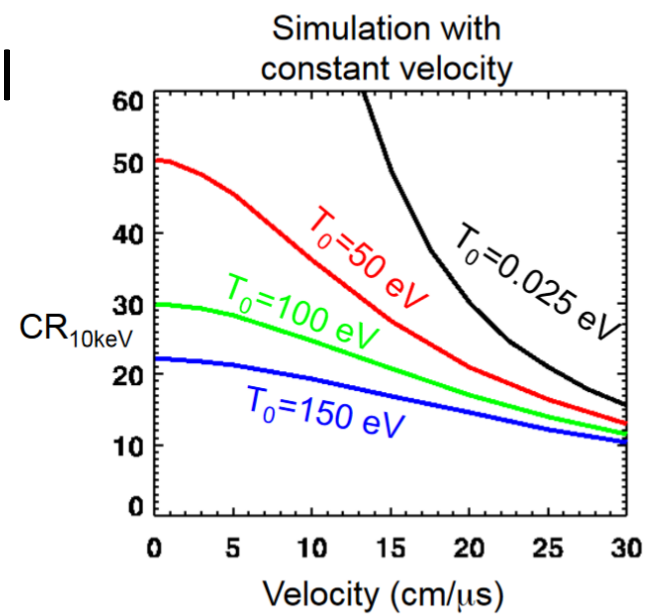


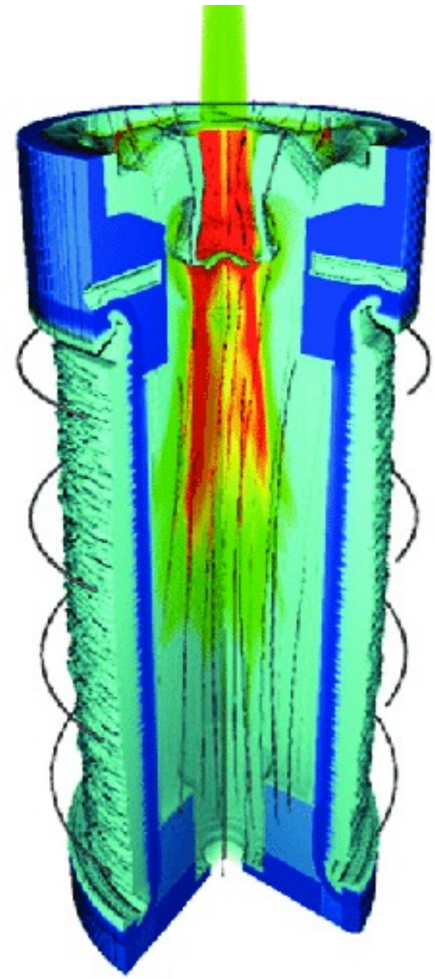
Magnetization

MagLIF – phase 2

- Laser heating of fuel
- $E_{laser} = 2 - 10$ kJ
- Reduces amount of radial fuel compression needed to reach fusion temperatures
- Preheats the fuel

to ~100 – 250 eV



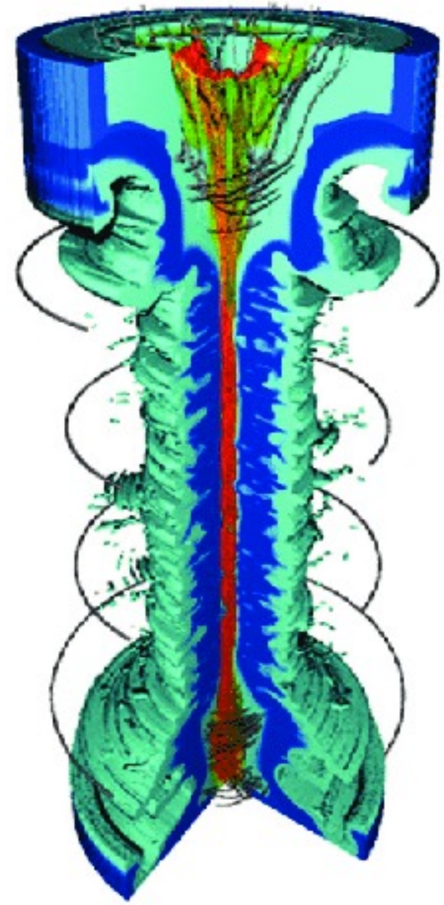


Fuel Preheat

MagLIF – phase 3

- Linear "z-pinch" compression driven by a current (~24 MA)
- "Slow" => quasi-adiabatic compression of fuel
- Shock velocity reaching 70 – 100 km/s in ~100 ns
- Thick liners that are robust to instabilities

→ **Plasma is confined inertially**

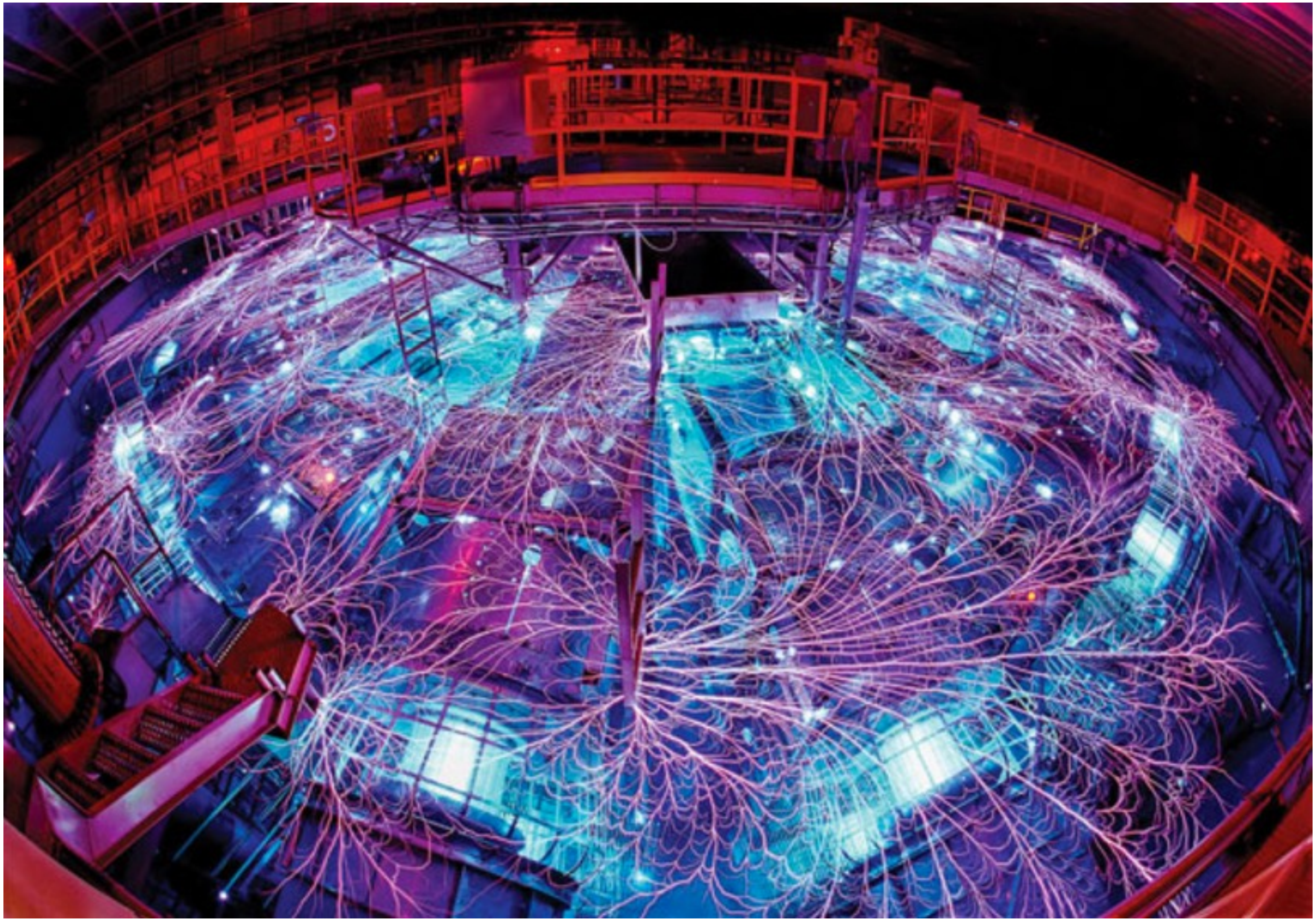


compression

MagLIF

- Magnetic fields can also make laboratory fusion easier, e.g., strong fields can affect charged particles (electrons, alphas) and thus plasma heat transport and confinement properties
- The ρr needed for ignition can be significantly reduced by the presence of a strong magnetic field largely through inhibiting electron conduction
- This means the stagnation plasma pressure at ignition temperatures is significantly reduced
- Lower density requirement than standard ICF

The Z-Machine (Sandia Nat. Lab.)



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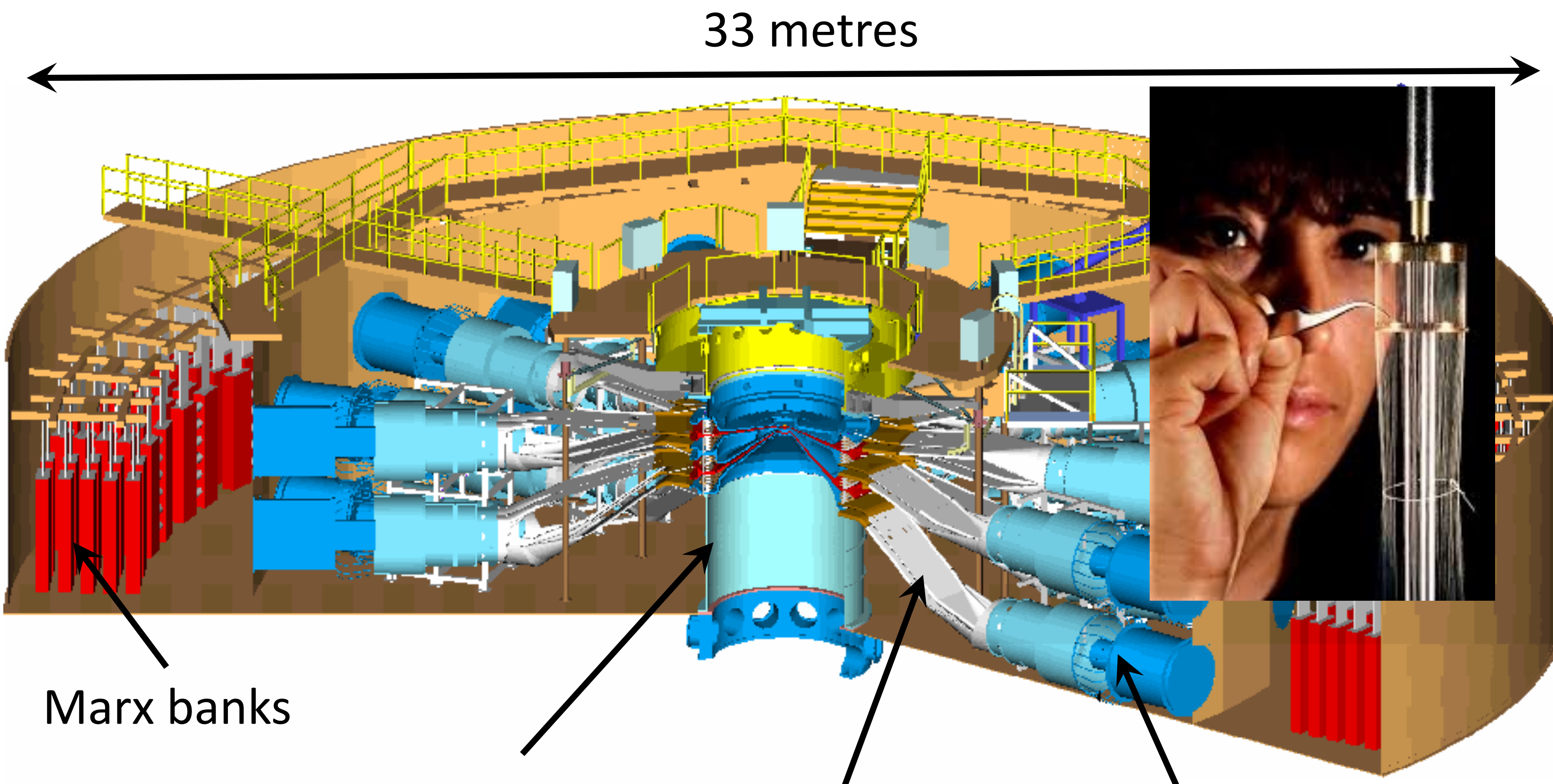

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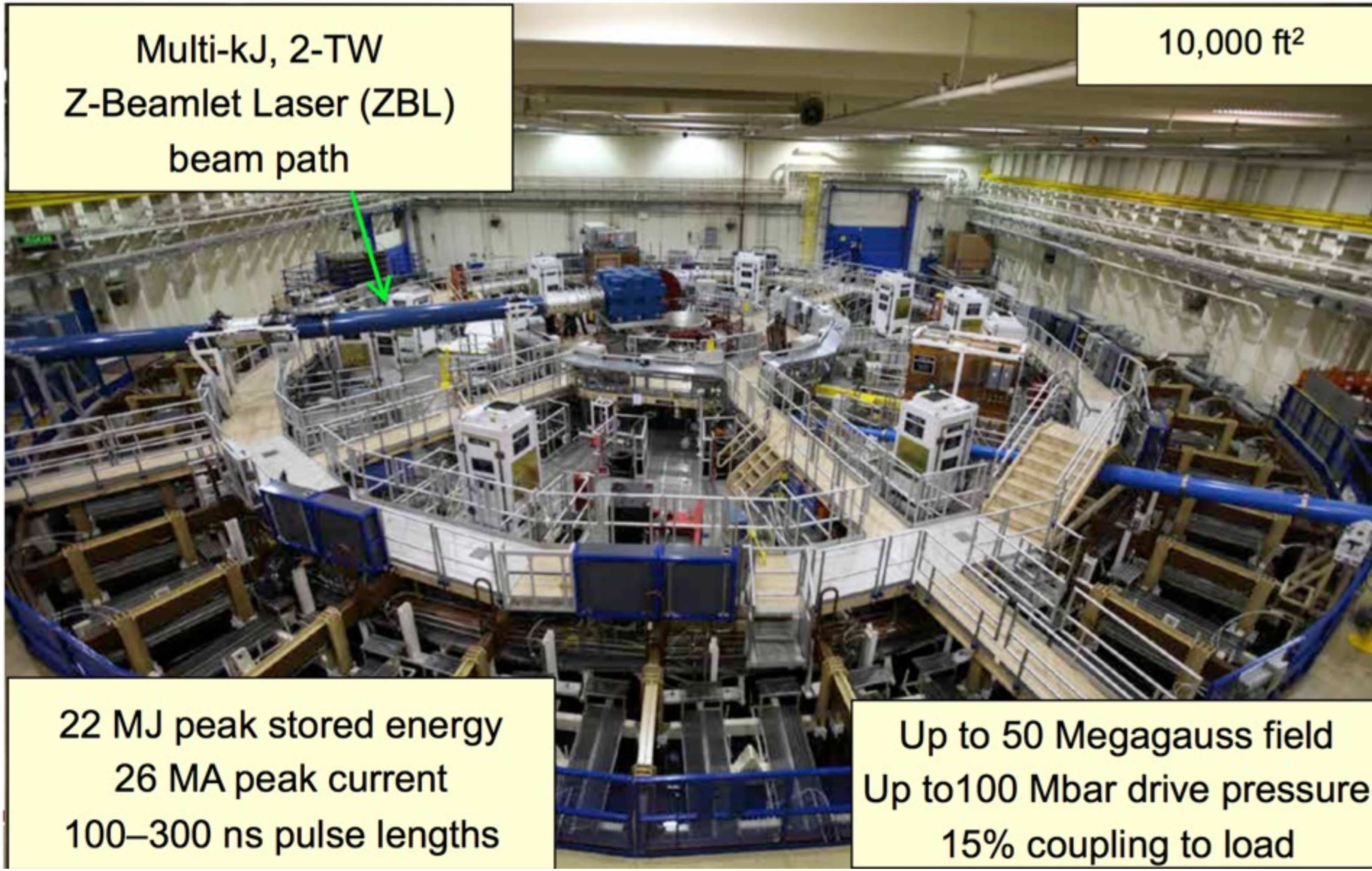

Summary of lecture 8

- Fusion of light nuclei has the potential to solve the energy crisis - but high temperatures are needed to overcome electrostatic repulsion of the nuclei.
- To get more energy out than put in, the plasma must satisfy the Lawson criterion.
- Mirrors lose particles from the ends, as do Z-pinches.
- The Z- pinch is unstable to the sausage and kink instability. The torus is subject to these instabilities and has problems with particle drift.
- Many of these problems can be overcome with a tokamak. A field along the long direction (toroidal) as well as poloidal, stabilises against instabilities and stops the drift problem.
- The tokamak needs to operate at high q, which restricts the current - other modes of heating are required.

Summary of lecture 8

- The Kruskal-Shavranov limit:

$$q \equiv \frac{B_\mathrm{t}}{B_\mathrm{p}} \frac{a}{R} > 1$$

- Alternative advanced fusion schemes with magnetic confinement include: sterallators and z-pinches combined with laser heating (MagLIF)
- Bennet relation for Z-pinches and tokamaks:

$$\mu_0 I^2 = 8\pi(Z+1)Nk_B T$$