Plasma Physics

TU Dresden

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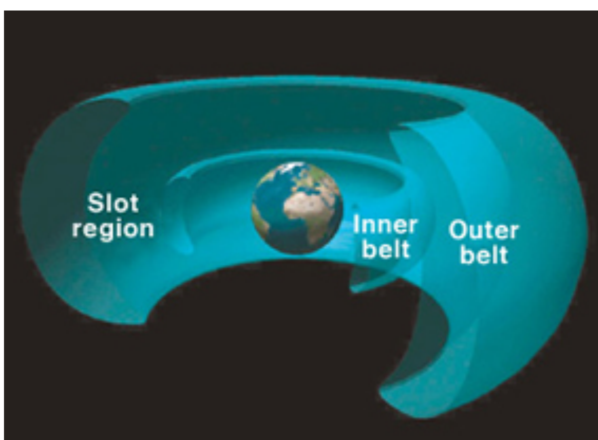
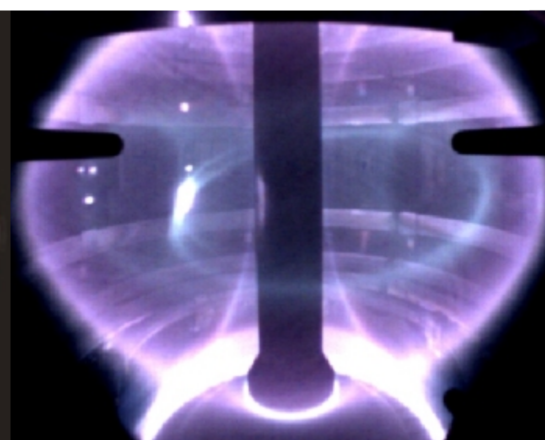
TECHNISCHE UNIVERSITÄT DRESDEN

Lecture 11: Plasma shocks



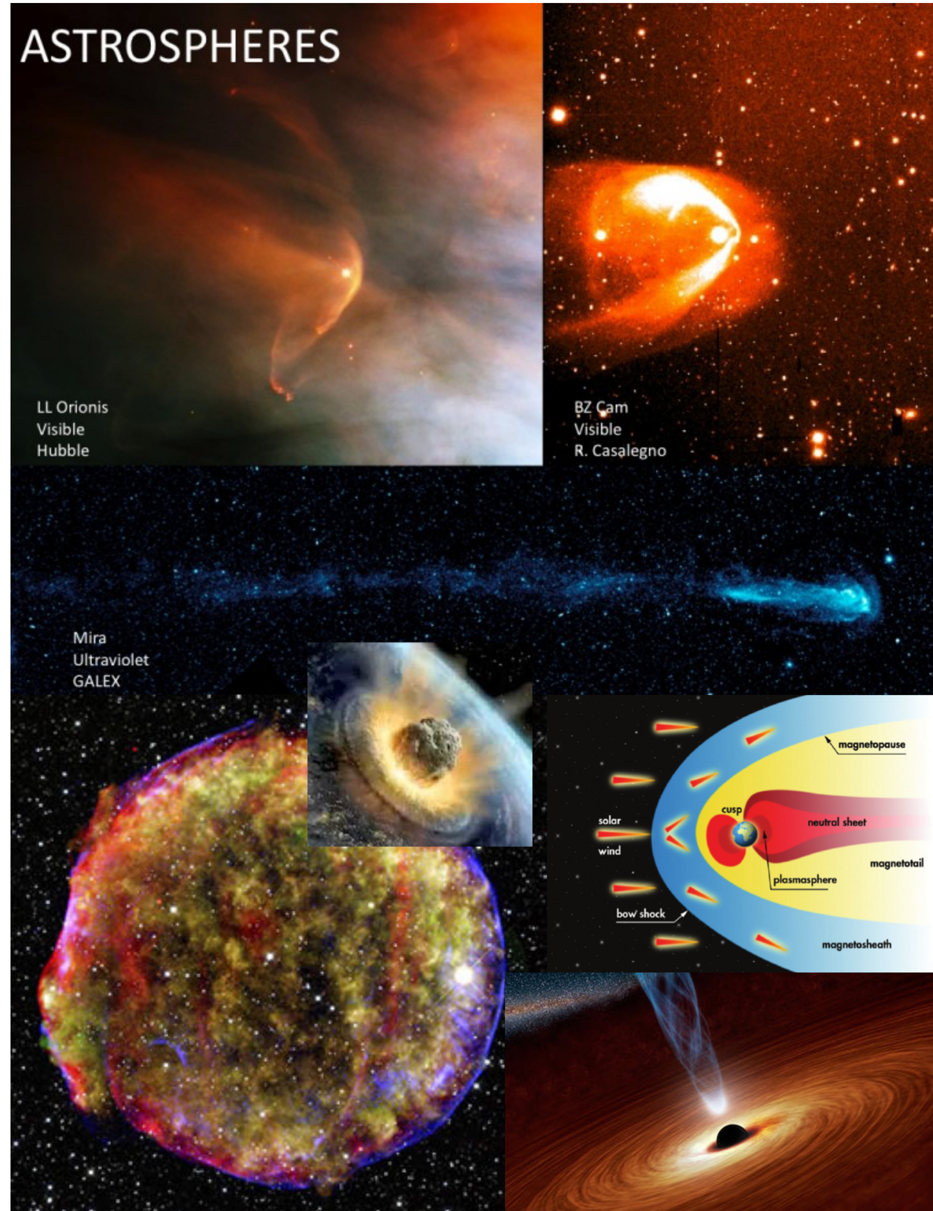


Plasma Physics: lecture 11

- Reaching extreme pressures/conditions
- Introduction to shock physics
- Rankine-Hugoniot relations
- Collisionless shocks
- Radiative shocks
- Applications for astrophysics

(Plasma) shocks all around us

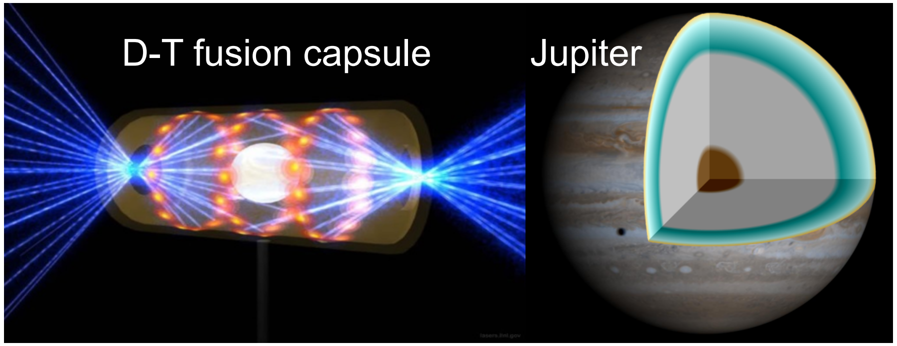
- Lightning strikes (thunder)
- Solar wind
- Supernovae
- Astrophysical jets
- Accretion discs
- Celestial collisions (craters)
- Active galaxies
- Pulsars
- Protoplanetary formations
- Sonic boom (jet planes)
- Earthquakes
- etc.

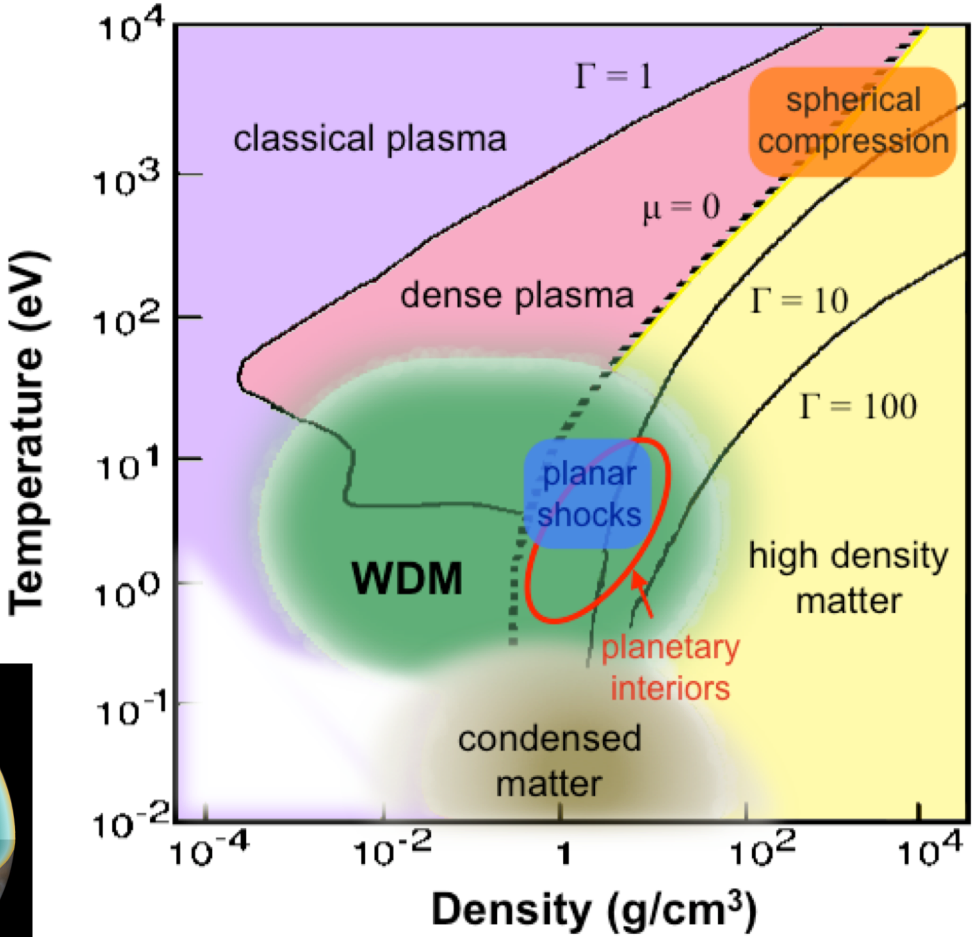


Dense plasmas and warm dense matter

The equation of state of light elements is essential to understanding the structure of Jovian planets and inertial confinement fusion (ICF) experiments. The equation of state (EOS) in the WDM regime is largely unknown.

- WDM = intermediate state between solids and plasmas
  - Temperature: 1 – 100 eV
  - Density: ~ 1g/cm^3 (solid densities)
- Ions strongly coupled and fluid-like, exhibit long-range order
- Electrons fully or partially degenerate => quantum effects become important





Generating dense plasma - in a laboratory

Typical methods to generate WDM under controlled conditions:

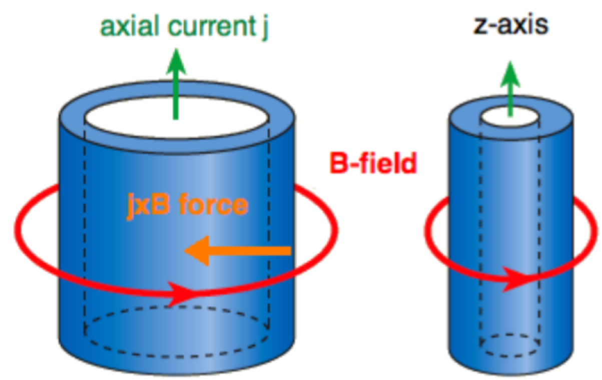
- Static compression with diamond anvil cells
- Dynamic compression with lasers or z-pinches
- Isochoric heating (x-rays, protons or electrons)

Main challenges:

- Producing large enough samples with homogeneous conditions
- Non-equilibrium WDM states on sub-nanosecond timescales

Dynamic compression

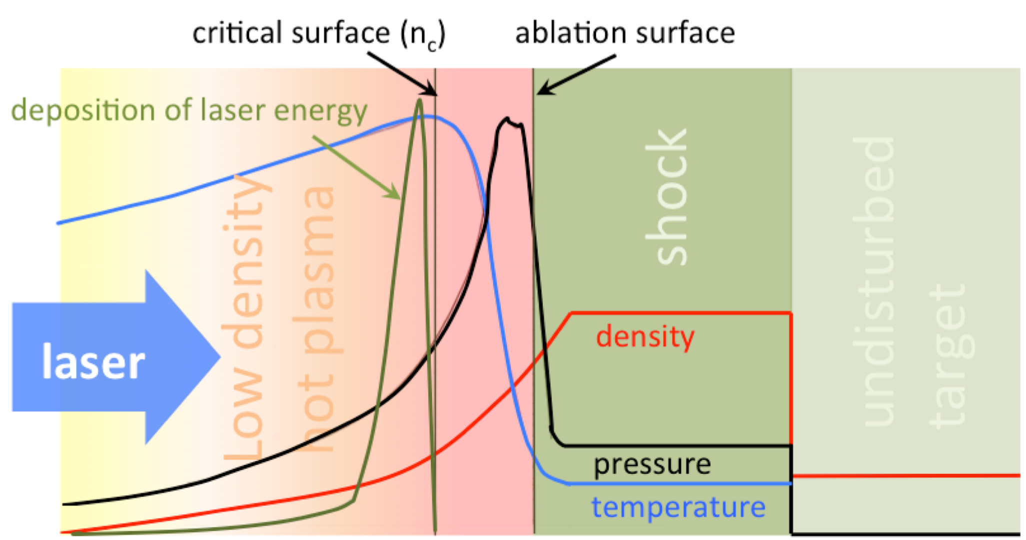
- **Experimental platforms for dynamic compression:**
  - High energy laser facilities
  - Z-pinches



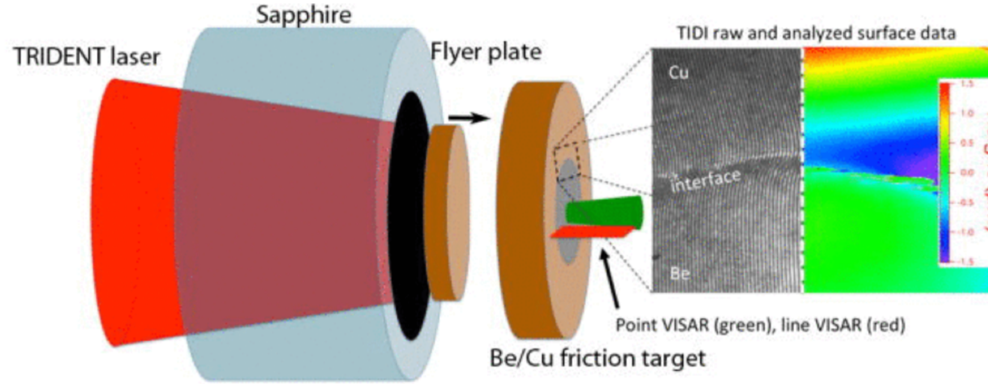
- **Typical solid/liquid target compression techniques:**
  - Shocks driven directly by laser ablation
  - Flyer plates (usually on z-pinches, driven by jxB force)
  - By laser-irradiated hohlraum x-ray emission (similar to ICF)
    ➔ achieve states on the principal Hugoniot
  - Shock-and-release ➔ achieve states away from the Hugoniot (few Mbar, 1–100 eV)

- **Hugoniot relations:**
  - Derived from the conservation of energy, momentum and mass across the shock front

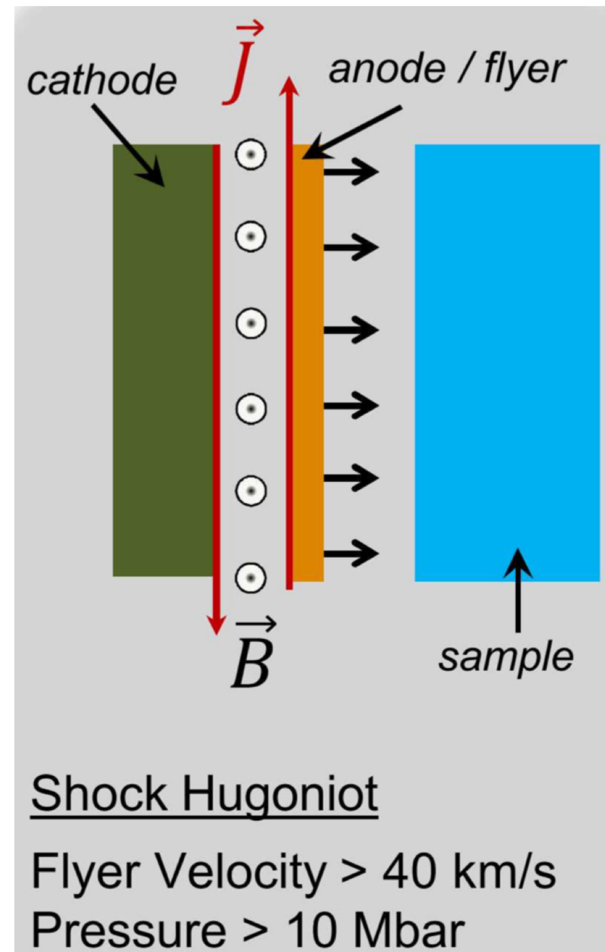
- **Main challenges:**
  - Preheat of material before the shock changing initial conditions
  - Shock stability and reproducibility (mainly issue on lasers)



Flyer plates



- **Ideal flyer plate:**
  - Cold, planar material moving at very high velocity, which cleanly strikes a surface of a target material.
  - Acts as a piston to drive a shock in the material. It's velocity can be directly measured.
  - Can be driven by lasers or other means.
- **Magnetically driven flyer plates:**
  - Driven by the jxB force at Z-pinches.
  - Provides a very clean way to accelerate plates without problematic ablation issues.
  - Great for equation of state measurements in WDM.



Shock physics overview

- **Normal shock:**
  - A type of propagating disturbance that moves faster than the local speed of sound in the medium. The flow direction is normal to the shock front.
- **Oblique shock:**
  - A shock wave that, unlike a normal shock, is inclined with respect to the incident upstream flow direction. It will occur when a supersonic flow encounters a corner that effectively turns the flow into itself and compresses. Like a tilted piston.
- **Collisionless shock:**
  - When the mean free path of the particles is longer than the shock width, MHD effects must be relied upon to carry the shock.

Shock physics overview

- **Non-radiative shock:**
  - The radiation seen from a shock depends on the populations of atoms and molecules in the post-shock medium. If the temperature is too high, these transitions may not occur with great frequency, hence, a non-radiative shock. These are inefficient at cooling.
- **Radiative shock:**
  - Shocks with conditions affected by radiation from the shock-heated matter.
- **Fast/slow shocks:** more or less just what it sounds like.
- **Truncated/incomplete shocks:**
  - Young undeveloped shocks which don't display expected spectra of radiative shocks. Relates to cooling time.

Shock physics overview

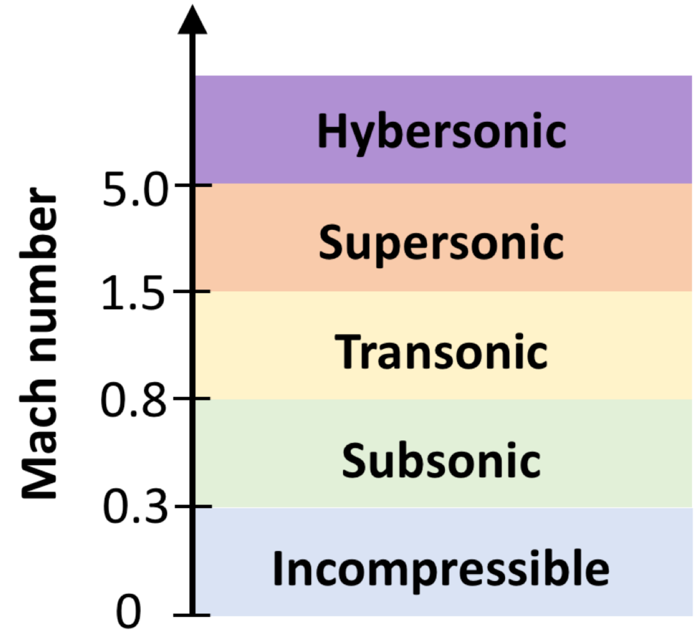
- **Rarefraction wave:** also relief/release/unloading/Taylor wave
  - Decrease in density/pressure caused by expanding material. The rarefaction wave is the progression of particles being accelerated away from a compressed or shocked zone, in opposite direction from the shock.
- **Rarefraction shock:**
  - An abrupt transition in which matter was cooled and rarefied.
- **Blastwave:**
  - Supersonic wave developed from a shock after the source of pressure ends allowing the rarefraction wave to propagate and overtake the shock (explosives).
- **Compression wave:**
  - A longitudinal wave (such as a sound wave) propagated by the elastic compression of the medium – not supersonic.

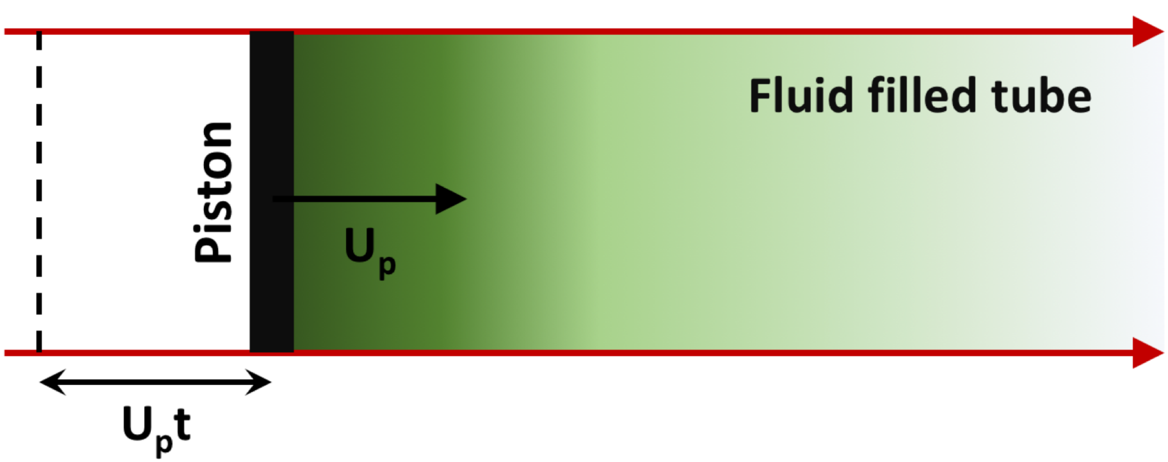
Physics of shocks

- A shock is a discontinuity in thermodynamic conditions in a moving fluid (flow)
- Mach number: $M = \dfrac{u_u}{c_s}$
- In plasma physics we use: $M = \dfrac{u_u}{v_A}$

Alfvén speed

- A steepening pressure build up in front of the piston if the piston speed U_p is greater than the sound speed → **shock wave**

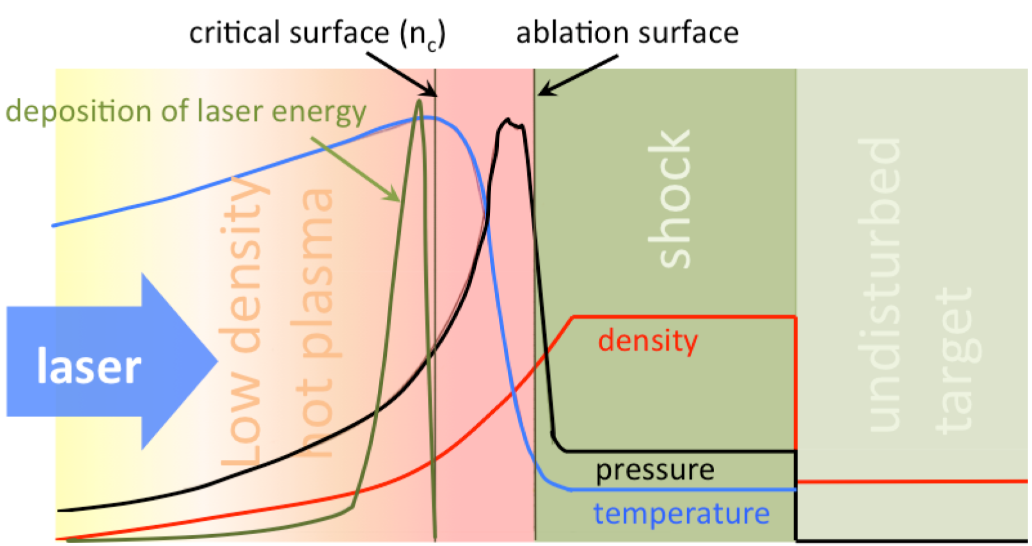


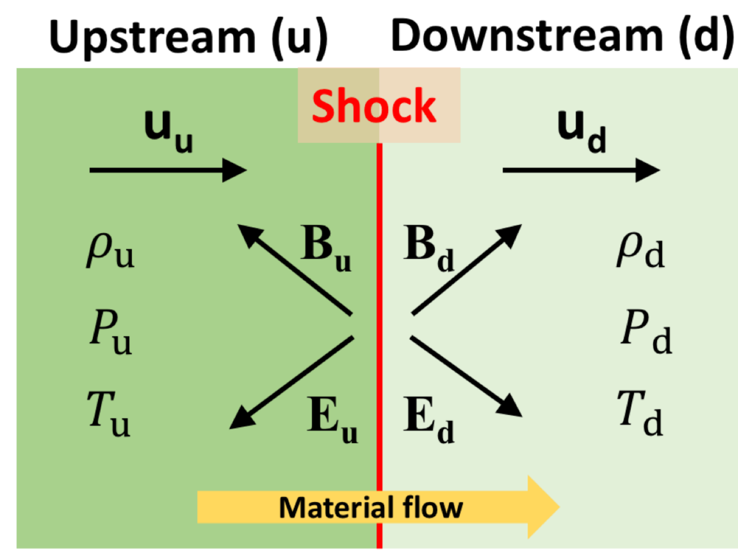


$$c_{s_{adiabatic}} = \sqrt{\gamma \frac{P}{\rho}}$$

Physics of shocks

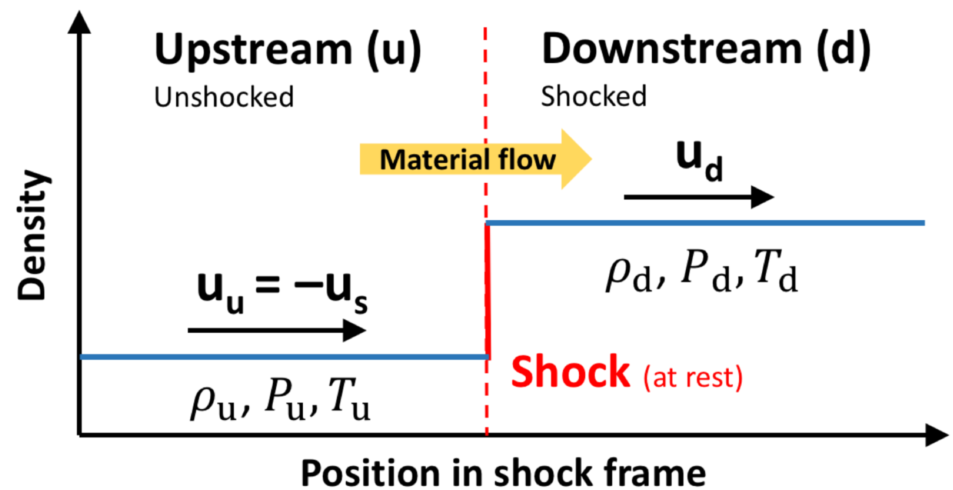
- Non-linear kinetic phenomenon (fluid description is problematic)
- Assume: a shock is a thin transition, i.e. infinite planar shock (thus, we allow the use of MHD equations)
- Work in the shock frame (i.e. shock is stationary)
- The jump conditions (Rankine-Hugoniot relations) are characterized by conserved quantities across the shock:
  - Mass, momentum flux, specific total energy, magnetic and electric field





The Rankine-Hugoniot relations

- First, consider jump conditions with no B-field
- We work in the rest frame of the shock.
- Assume adiabatic shock.
- Conservation laws:



Continuity eq.:
$$\frac{\partial \rho}{\partial t} = -\nabla(\rho \mathbf{u})$$

Momentum eq.:
$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla(\rho \mathbf{u}\mathbf{u}) - \nabla P$$

Specific internal energy

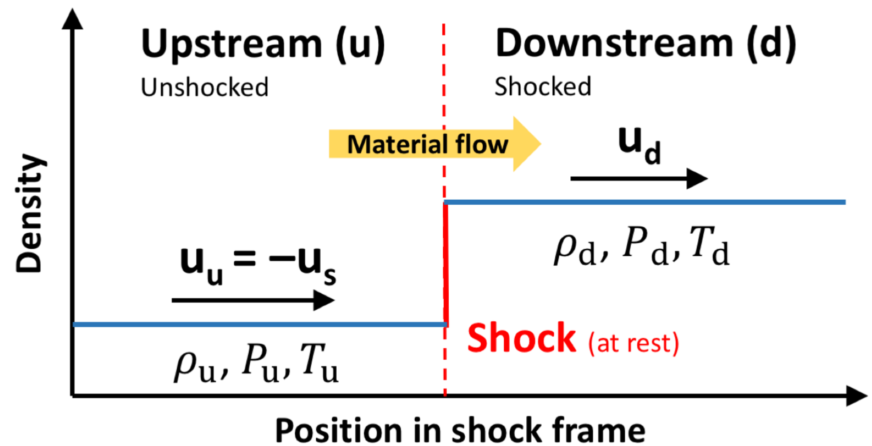
Energy eq.:
$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2} + \rho\epsilon\right) = -\nabla\left[\rho\mathbf{u}\left(\epsilon + \frac{u^2}{2}\right) + P\mathbf{u}\right]$$

The Rankine-Hugoniot relations

- In the rest frame of the shock, the conservation across infinitely small spatial increment gives general form:

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t} \rho_Q dx = -\int_{x_1}^{x_2} \Gamma_Q(x) dx = \Gamma_Q(x_2) - \Gamma_Q(x_1)$$

- Thus we get:



$$\boxed{\rho_u u_u = \rho_d u_d}$$

$$\boxed{\rho_u u_u^2 + P_u = \rho_d u_d^2 + P_d}$$

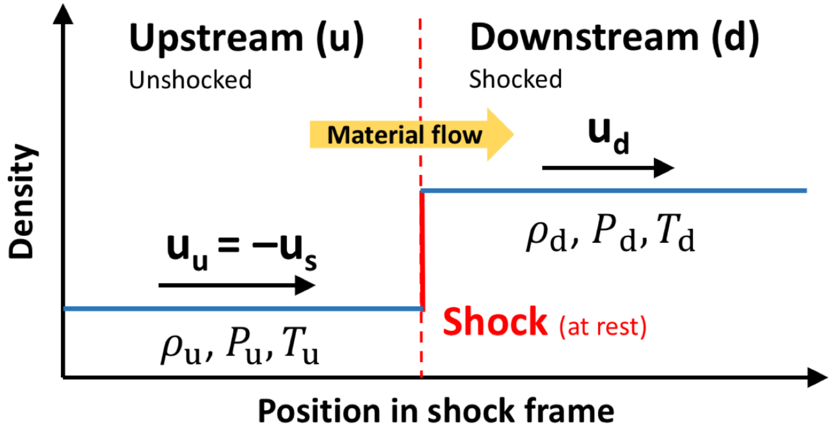
$$\boxed{\left[\rho_u u_u \left(\epsilon_u + \frac{u_u^2}{2}\right) + P_u u_u\right] = \left[\rho_d u_d \left(\epsilon_d + \frac{u_d^2}{2}\right) + P_d u_d\right]}$$

- For ideal gas, the specific internal energy: $\epsilon = \frac{1}{\gamma - 1} \frac{P}{\rho}$

Equation of state measurements

- Shocks can create new states in materials.
- Initial conditions can be varied.
- The post-shock fluid (or particle) velocity u_p is the difference between incoming and outgoing fluid velocities in the shock frame:

$$u_p = u_u - u_d$$



- Thus we get:

$$\rho_u u_u = \rho_d (u_u - u_p)$$

$$P_d - P_u = \rho_u u_u (u_u - u_d) = \rho_u u_u u_p$$

Equation of state measurements

- For convenience we simplify the relations denoting the initial conditions as ρ_0, P_0, E_0 :
- For small initial pressure $P_0 \to 0$, we then obtain a practical form for the Rankine-Hugoniot relations:

$$\boxed{\rho_0 u_s = \rho\,(u_s - u_p)}$$

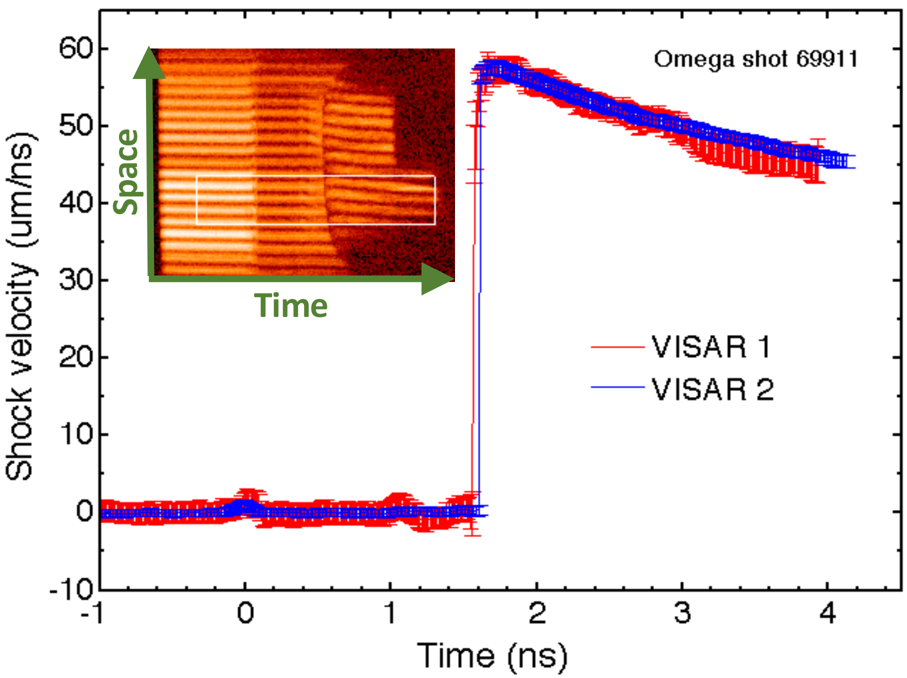
$$\boxed{P - P_0 = \rho_0 u_s u_p}$$

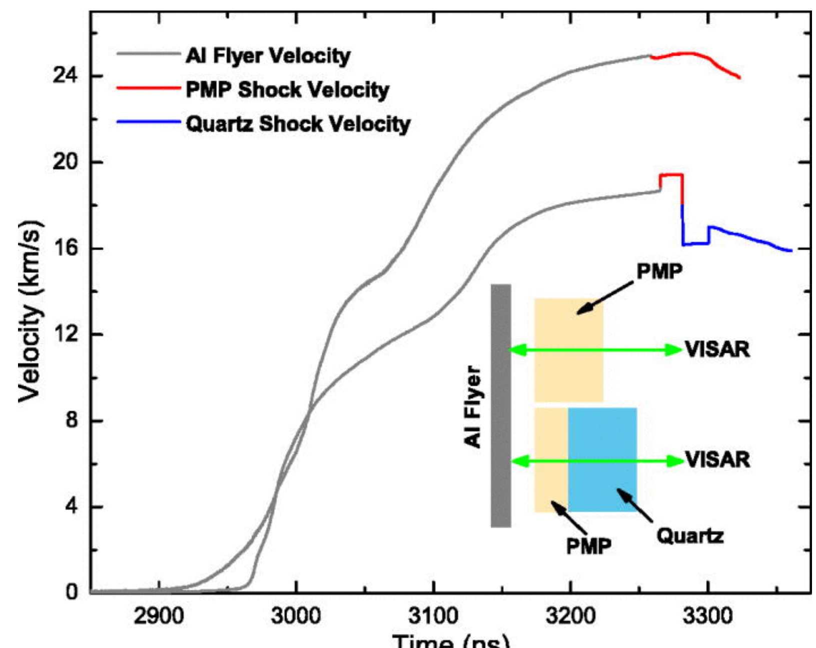
$$\boxed{P u_p = \frac{1}{2}\rho_0 u_s u_p^2 + \rho_0 u_s(\epsilon - \epsilon_0)}$$

VISAR measurement

- Shock and particle velocities can be measured.
- For know initial conditions, the pressure and density of shocked material inferred by:

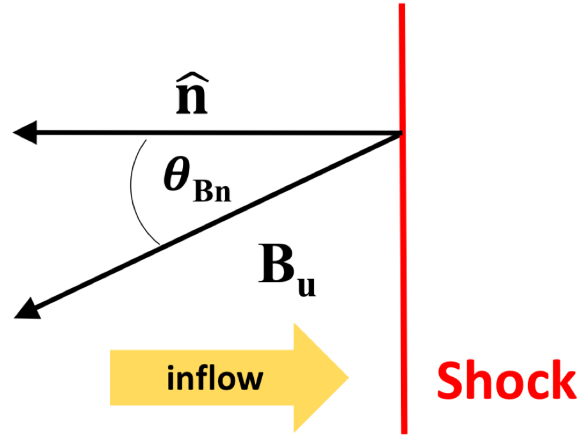
$$P = \rho_0 u_s u_p \qquad \text{and} \qquad \rho = {\rho_0 u_s}/{(u_s - u_p)}$$



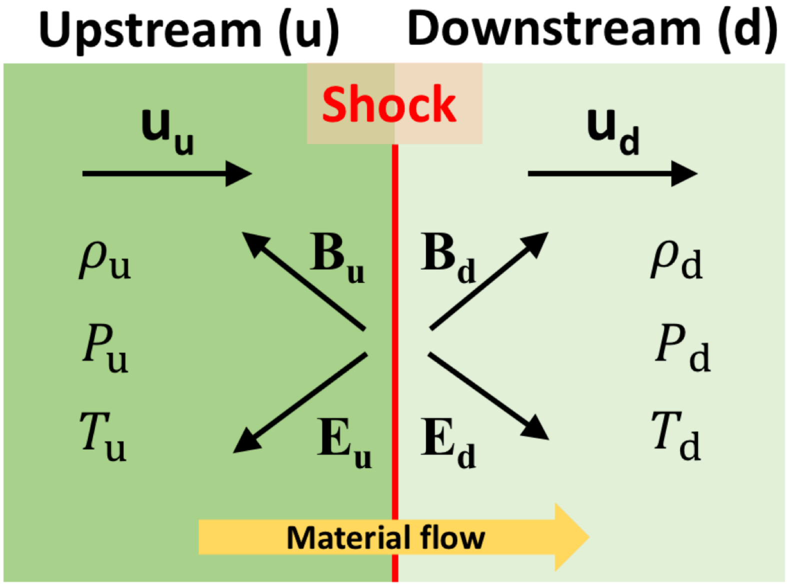


R-H relations with B-field

- The geometry of the shock is now very important
- Define θ_{Bn}: the angle of the upstream B-field to the shock normal
- We then have main types of magnetized shock geometries:
  - Parallel shock: $\theta_{Bn} = 0°$
  - Perpendicular shock: $\theta_{Bn} = 90°$
  - Oblique shock: $0° < \theta_{Bn} < 90°$

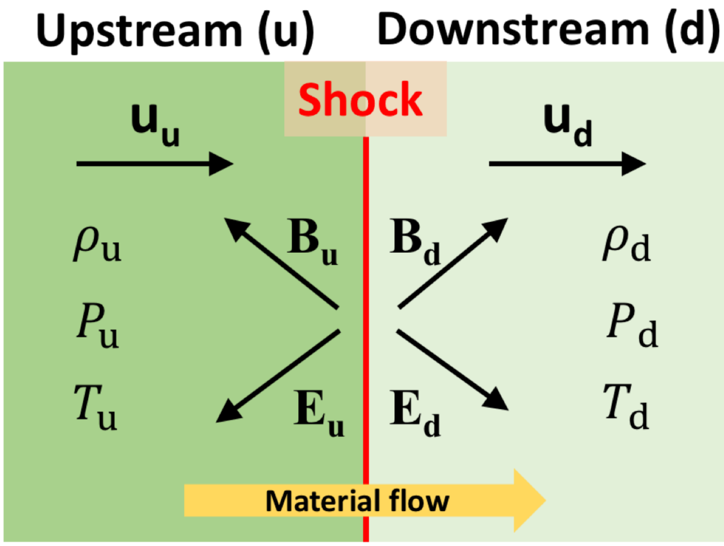


$$[X] = X_u - X_d = X_1 - X_2$$



Mass conservation with B-field

- Assume an adiabatic shock condition: $c_{s_{adiabatic}} = \sqrt{\gamma \frac{P}{\rho}}$.
- Use MHD to describe the shock dynamics, must pay attention to spatial directions.
- Starting with **mass conservation**, i.e. use the continuity equation, in the shock normal direction (n):



$$\rho_u u_{u_n} = \rho_d u_{d_n} \rightarrow \boxed{[\rho u_n] = 0}$$

- Integrating the Faraday's law across the shock transition and using the Ohm's law implies that $\hat{\mathbf{n}} \times \boldsymbol{E} = \hat{\mathbf{n}} \times (\boldsymbol{u} \times \boldsymbol{B})$ is constant across the shock boundary, thus:

$$\boxed{[u_n \mathbf{B}_t - B_n \mathbf{u}_t] = 0}$$

E and B field conservation

- **Magnetic field** is conserved, i.e. the same amount of flux in/out:
  - The divergence theorem: $\iiint_V \nabla \mathbf{B} \cdot dV = \oint_S \mathbf{B} \cdot dS$

    and $\nabla \cdot \mathbf{B} = 0$

    No magnetic monopoles

→ normal B-field continuous: $\boxed{[\mathbf{B}_n] = 0}$

- **Electric field** is conserved:
  - The Stokes theorem: $\oint \mathbf{E} \cdot d\mathbf{l} = \int \nabla \times \mathbf{E} \cdot d\mathbf{s}$

    and $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0$

    In steady state, Faraday's law

→ transverse E-field component is continuous: $\boxed{[\mathbf{E}_t] = 0}$

Momentum conservation with B-field

|| to the shock normal:

- Momentum conservation across the shock gives:

$$\rho_u u_{u_n} \mathbf{u}_{u_n} - \rho_d u_{d_n} \mathbf{u}_{d_n} = m \cdot \mathbf{a}_n$$

- Kinetic pressure differential: $P_d - P_u$
- Magnetic pressure differential: $\dfrac{B_{tu}^2}{2\mu_0} - \dfrac{B_{td}^2}{2\mu_0}$
- Then using momentum equation, we get conservation of the momentum flux normal to the shock:

$$\boxed{\left[\rho u_n^2 + P + \frac{B_t^2}{2\mu_0}\right] = 0}$$

Momentum conservation with B-field

$\perp$ to the shock normal:

- Momentum conservation across the shock gives:

$$\rho_u u_{u_t} \mathbf{u}_{u_t} - \rho_d u_{d_t} \mathbf{u}_{d_t} = m \cdot \mathbf{a}_t$$

- No pressure differential in the tangential direction
- No change in B pressure in the tangential direction ($\mathbf{B}_n = \text{const.}$)
- Magnetic tension force $\frac{1}{\mu_0}(\mathbf{B} \cdot \nabla)\mathbf{B}$: $\quad \frac{\mathbf{B}_n \mathbf{B}_{tu} - \mathbf{B}_n \mathbf{B}_{td}}{\mu_0}$
- Then using momentum equation, we get conservation of the momentum flux normal to the shock:

$$\boxed{\left[\rho u_t^2 + \frac{B_n B_t}{\mu_0}\right] = 0}$$

Energy conservation with B-field

- Consider different forms of energy conservation
- **Kinetic energy:** $E_K = \frac{1}{2}\rho_u u_n u_n^2$

α = degrees of freedom

- **Internal energy:** $\epsilon = \frac{\alpha}{2} k_B T$ and for ideal gas: $PV = nk_B T$

$$\rightarrow \frac{P}{\rho} = k_B T \rightarrow \epsilon = \frac{\alpha}{2} \cdot \frac{P}{\rho} = \frac{1}{\gamma - 1} \cdot \frac{P}{\rho} \quad \text{and} \quad \gamma = \frac{C_p}{C_v} = \frac{\alpha + 2}{\alpha} = \frac{5}{3}$$

For monatomic ideal gas

- **Work done:** $W = PdV = \left[\frac{P}{\rho}\right]$
- **Enthalpy:** $H = \epsilon + pdV = \epsilon + \frac{P}{\rho} = \frac{\gamma}{\gamma - 1} \cdot \frac{P}{\rho}$
- **Poynting flux:** $\frac{\mathbf{E}\times\mathbf{B}}{\mu_0} = \frac{(\mathbf{u}\times\mathbf{B})\times\mathbf{B}}{\mu_0} = \frac{\mathbf{u}B^2}{\mu_0} - \frac{(\mathbf{u}\cdot\mathbf{B})\mathbf{B}}{\mu_0}$

Energy conservation with B-field

- Thus, the complete expression for energy conservation for an ideal gas plasma shock:

$$\boxed{\left[\rho u_n\left(\frac{1}{2}u^2+\frac{\gamma}{\gamma-1}\cdot\frac{P}{\rho}\right)+u_n\frac{B^2}{\mu_0}-\mathbf{u}\cdot\mathbf{B}\frac{B_n}{\mu_0}\right]=0}$$

- We get the shock compression ratio r: $$r=\frac{\rho_d}{\rho_u}$$
  - We have shock if: $r>1$
  - Maximum compression ratio for ideal gas in perpendicular adiabatic shocks (proof later): $r_m=4$

The MHD Rankine-Hugoniot relations

- Mass conservation: $\boxed{[\rho u_n] = 0}$

- Electromagnetic field conservation:

$$\boxed{[\mathbf{B}_n] = 0} \qquad \boxed{[\mathbf{E}_t] = 0} \qquad \boxed{[u_n \mathbf{B}_t - B_n \mathbf{u}_t] = 0}$$

- Momentum conservation: $\boxed{\left[\rho u_n^2 + P + \frac{B_t^2}{2\mu_0}\right] = 0}$

- Energy conservation:

$$\boxed{\left[\rho u_n \left(\frac{1}{2} u^2 + \frac{\gamma}{\gamma - 1} \cdot \frac{P}{\rho}\right) + u_n \frac{B^2}{\mu_0} - \mathbf{u} \cdot \mathbf{B} \frac{B_n}{\mu_0}\right] = 0}$$

Solution for perpendicular shocks

- Up- and down-stream B-field ∥ in the shock plane
- B-field in tangential direction, $\mathbf{u}$ is normal to shock
- For ⊥ we have: $[\mathbf{B}_n] = 0$, $\mathbf{B}_{nu} = 0$, thus $\mathbf{B}_{nd} = 0$
- Mass conservation: $\rho_u u_{nu} = \rho_d u_{nd}$

→ **compression ratio:** $r = \frac{\rho_d}{\rho_u} = \frac{u_{nu}}{u_{nd}}$

- ⊥ momentum in $u_{tu} = 0$ frame: $u_{nu}\mathbf{B}_{ut} = u_{nd}\mathbf{B}_{dt}$
- Thus, momentum conservation:

$$\rho_u u_u^2 \left(1 - \frac{1}{r}\right) + (P_u - P_d) + \frac{B_u^2}{2\mu_0}(1 - r^2) = 0$$

Solution for perpendicular shocks

- Energy conservation:

$$\frac{1}{2}\rho_u u_u^2\left(1-\frac{1}{r^2}\right)+\frac{\gamma}{\gamma-1}\left(P_u-\frac{1}{r}P_d\right)+\frac{B_u^2}{2\mu_0}(1-r)=0$$

- Eliminate P_d by substituting the momentum equation:

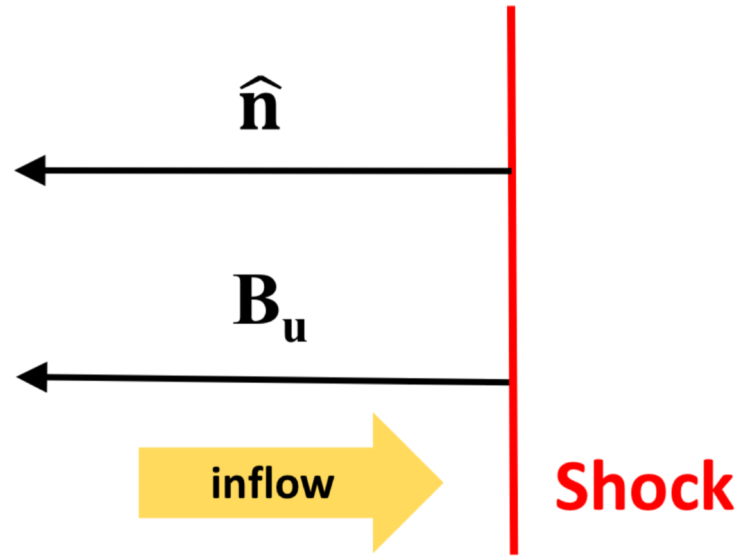
$$(r-1)\left[r^2\frac{(2-\gamma)B_u^2}{u_u^2(\mu_0\rho_u)}+r\left(\frac{\gamma B_u^2}{u_u^2(\mu_0\rho_u)}+\frac{2\gamma P_u}{u_u^2\rho_u}+\gamma-1\right)-(\gamma+1)\right]=0$$

- For high Mach number $M\to\infty$, $u_u\to\infty$, the only shock solution: $[r(\gamma-1)-(\gamma+1)]=0$

→ $r=\dfrac{(\gamma+1)}{(\gamma-1)}$ and for ideal gas $\gamma=\frac{5}{3}$ → $r_{max}=\dfrac{\frac{5}{3}+\frac{3}{3}}{\frac{5}{3}-\frac{3}{3}}=4$

Solution for parallel shocks

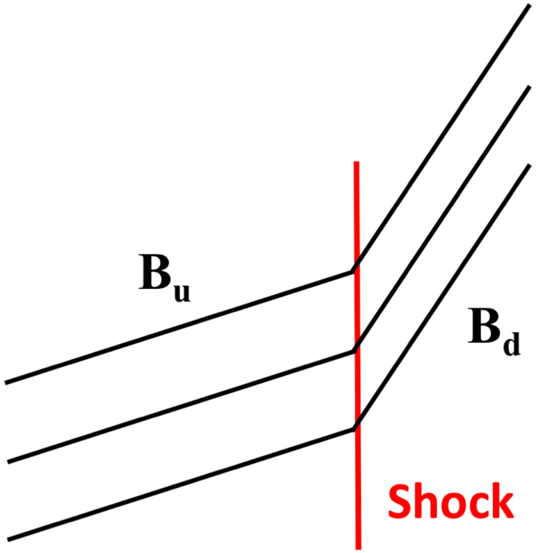
- No ⊥ B-field component, $\mathbf{B}_t = 0$
- Behaves like a simple hydrodynamic shock
- Two main cases should be considered



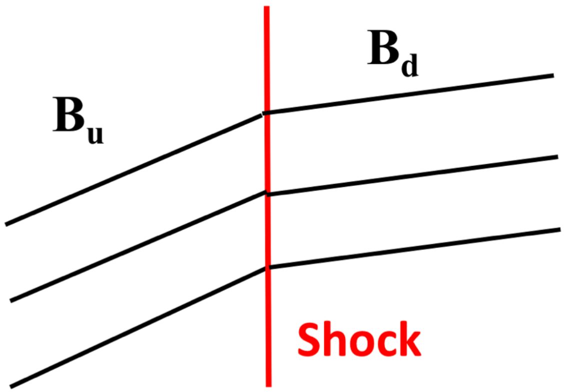
- **Collisional plasma**: simple case, a particle hits a barrier and reflects back from it
- **Collisionless plasma**: the situation becomes very messy, must consider kinetic effects

Solution for oblique shocks

- **Fast shocks**: fast mode waves
  - Most common, bow shocks, supernovae
  - Density rises, **B-field increases**
  - But $\mathbf{B_n}$ remains constant
  - Field turns away from shock normal



- **Slow shocks**: slow mode waves
  - Density rises, B-field decreases
  - Field turns towards shock normal

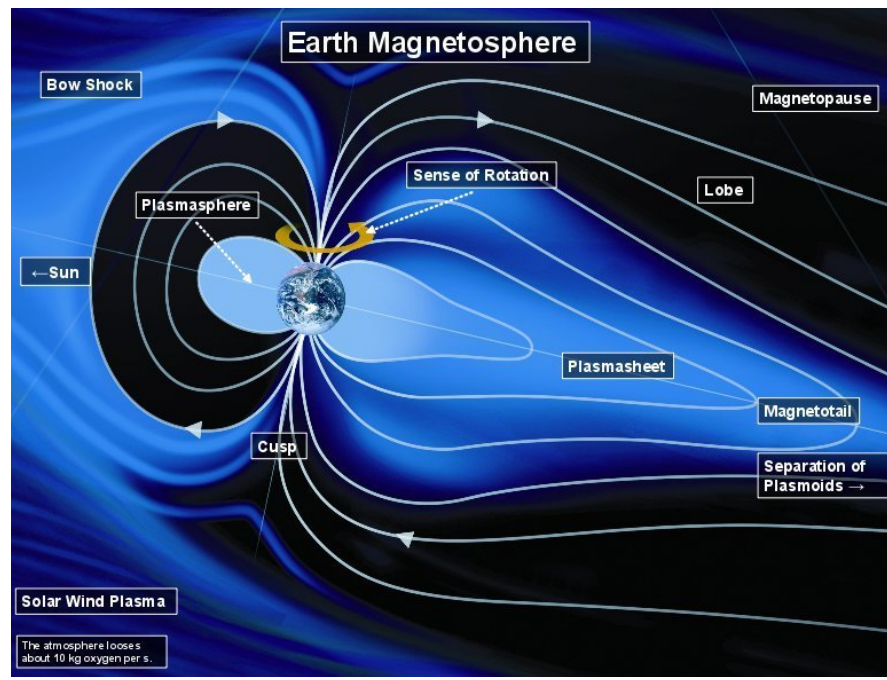


Collisionless shocks

- Occur when the mean free path exceeds the shock width
- Common in low density astrophysical plasmas
- Generates non-Maxwellian distribution of particles
- Unstable to generation of waves
- Waves scatter particles
- Thermalisation of particles occurs within a few gyroradii of shock front → **waves and turbulence downstream**
- Unproven hypothesis states that a pre-existing B-field is needed for the formation of collisionless shock

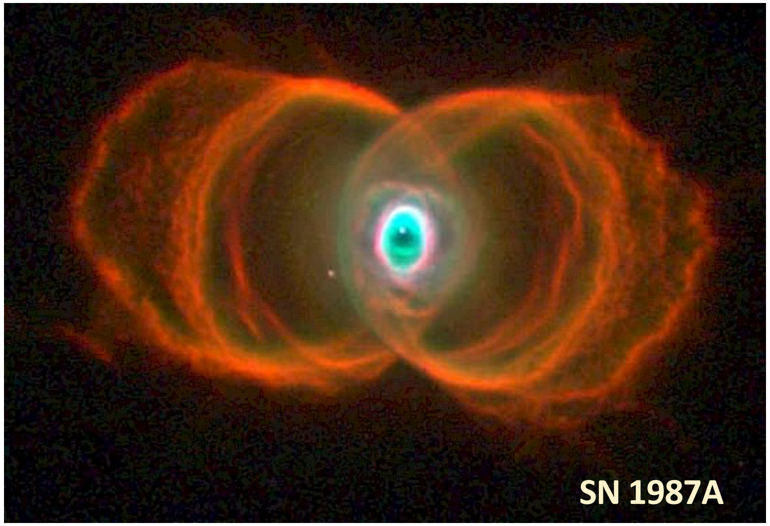
The Bow shock

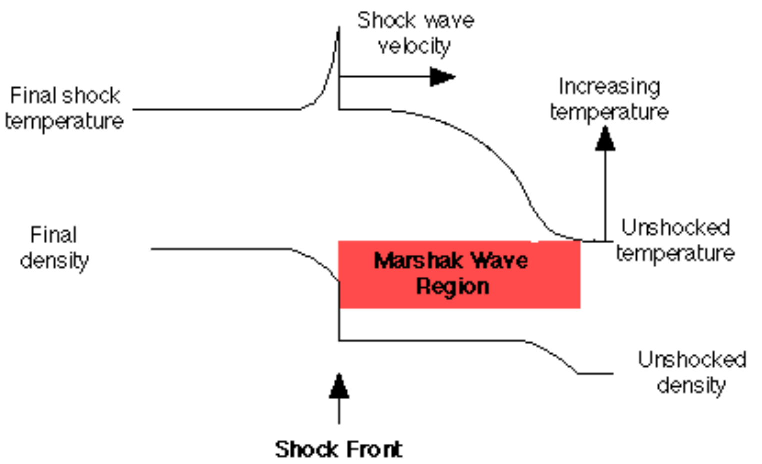
- Bow shocks form the boundary between a magnetosphere and an ambient (or at least surrounding) magnetized medium.
- This occurs when the magnetic field of an astrophysical object interacts with the nearby flowing ambient plasma.
- Typical example is the encounter of the solar wind with Earth's magnetosphere. Magnetopause is formed and Earth's B-field is compressed.
- Astrospheres of stars, where the stellar wind meets the interstellar medium



Radiative shocks

- Shocks can heat plasma into radiative regime
- Fast shocks so that radiative flux ($\propto T^4$ and thus $\propto u_s^8$) exceeds the material energy fluxes ($\propto u_s^3$)
- A steady region of material ahead of the shock front affected by radiation from the shock is formed, i.e. *radiative precursor*:
  - *Transmissive*: lightning, precursor generated by explosion, not by the shock itself
  - *Absorptive*: increases temperature of the material ahead of the shock





Radiative shock types

Thick-thick shocks:

- Both up- and down-stream regions are optically thick.
- The viscous density increase as well as the radiative effects can be treated as a part of a single, extended, shock structure. Such systems are both hot and dense.
- Shocks in stellar interiors, blastwave inside supernova.

Thick-thin shocks:

- Downstream region optically thick, upstream region optically thin.
- There is a cooling layer downstream of the viscous shock transition. An optically thick piston/shocked material drives a radiative into a medium with small depth compared to the steady state precursor. Medium ahead quickly heats up and becomes optically thin.
- Common in laser experiments. Supernova blastwave emerging from the star surface, accretion shocks in binary star systems.

Radiative shock types

Thin-thin shocks:

- Both up- and down-stream regions are optically thin.
- The entire downstream region is a radiative cooling layer and it ends when the temperature equilibrates with the surroundings.
- Most commonly observed shocks in astrophysics. They are easy to see as the radiation can escape. Supernova remnants, astrophysical jets.

Thin-thick shocks:

- Downstream region optically thin, upstream region optically thick.
- Shocked material must become optically thin while remaining optically thick in the upstream region and sustaining a steady precursor. Radiation driven ionization fronts. Low density flow impacting denser material.
- Nontrivial case that is likely to transition to thin-thin shock in a relatively short time.

Isothermal shocks

- A special case of a radiative shock. Given very efficient cooling by radiation, the shock may behave isothermally. The sound speed is lower and the jump conditions change.
- Isothermal sound speed: $c_{iso} = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{k_B T}{m}}$
- Jump conditions: $T_u = T_d$

$$\frac{\rho_d}{\rho_u} = \frac{u_u}{u_d} = \left(\frac{u_u}{c_{iso}}\right)^2 \equiv M_T^2$$

→ compression factor in an isothermal shock can be arbitrarily high.

- Real shocks are a compromise between adiabatic and isothermal shocks. Usually the adiabatic description is however more appropriate.

Summary of lecture 11

- Rankine-Hugoniot relations for unmagnetized adiabatic shocks:

$$\rho_u u_u = \rho_d u_d$$

$$\rho u_u^2 + P_u = \rho u_d^2 + P_d$$

$$\left[\rho_u u_u \left(\epsilon_u + \frac{u_u^2}{2}\right) + P_u u_u\right] = \left[\rho_d u_d \left(\epsilon_d + \frac{u_d^2}{2}\right) + P_d u_d\right]$$

- Collisionless shocks occur when the particle mean free path exceeds the shock spatial profile. Such shocks are common in low density astrophysical plasmas, e.g. bow shocks.
- Radiative shocks occur in radiative plasmas and the conditions of the shock and the material ahead of it are affected by radiation. These occur for high shock velocities.
- Isothermal shocks are a special case of radiative shocks that can occur if efficient cooling by radiation that balances the heating process, resulting in lower sound speed an higher Mach number → arbitrarily high compression.