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Neutral ρ Meson Properties in an Isospin-Asymmetric Pion Medium

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Abstract

We evaluate the ρ meson self energy at finite temperature T and charged-pion chemical potential μ_Q as well by utilizing a conventional π-ρ effective Lagrangian and functional integral representation of the partition function in the one-pion loop order (i.e., second order in the $\rho\pi\pi$ coupling constant). We find an increase of both the ρ meson mass and the width with increasing temperature and chemical potential μ_Q. At large value of μ_Q this increases may be about two times larger as compared with the pure temperature shift of Gale and Kapusta at vanishing μ_Q.

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I. INTRODUCTION

One of the challenging problems in heavy-ion collisions is to understand the behavior of hadrons in hot and dense nuclear matter. In particular, one focuses on the rho dynamics. This is intimately related to the fundamental question of how the ρ meson properties are modified due to the approach to a chiral symmetry restoration phase transition at large temperature and density. An intriguing question is whether it is possible to extract this effect from relativistic heavy-ion collision data. In answering the second question one can explore the dilepton production in $\pi^+\pi^-$ annihilation [1], because the pion electromagnetic form factor is almost completely dominated by the ρ meson below an invariant mass of about 1 GeV [2], which strengthen the well known and widely used vector dominance model [3].

The answer to the first question is ambitious, especially in the region below the chiral phase transition. Above the critical temperature, which probably coincides with the deconfinement temperature, the ρ meson should disappear from the hadronic spectrum of excitations as predicted by both the chiral mean field models [1] and lattice calculations [4]. The ρ properties below the chiral phase transition depend on the physical picture of the "matter" constituents and their interactions with the rho meson [5]. Those models based on quark degrees of freedom, such as QCD sum rules [6], effective Lagrangians of the Nambu-Jona-Lasinio type [7], predict quite small in-medium modifications of the rho properties up to the critical temperature. Otherwise, the models based on the conventional hadronic degrees of freedom show a strong effect [8–10]. For example, taking into account the coupling of the ρ meson to two pions as well as the strong mixing of pions and delta–nucleon-hole states in nuclear matter, as in Ref. [10], shows a dramatic density dependence of the rho meson width.

For a deeper understanding of the role of the conventional hadronic interactions to the ρ property modification at extreme conditions, which should be considered as background for more exotic interactions, it seems to be important to study the simplest system - a dense and hot pion gas with small baryon density, which is often expected to be produced in the central region in relativistic heavy-ion collisions. Gale and Kapusta [11] analyze the temperature modification of the ρ self energy in the one-loop order (order g_ρ^2) at vanishing pion chemical potential. They find a modest increase of the ρ width and mass with temperature. This result means that if a high energy experiment shows substantial modification of the dilepton spectrum with an invariant mass in the ρ region, it may be some indication of a more exotic interaction, which is beyond the conventional π-ρ interaction mechanism.

The model of Gale and Kapusta is extended in some sense by Koch [12] who considers the pion system in a chemical non-equilibrium state, described by a positive chemical potential μ_π. The chemical potential is associated with the total pion density of the pion gas, and it is supposed that μ_π has the same value for all charge states. Previously, this idea has been put forward by Kataja and Ruuskanen [13] for an explanation of the observed enhancement of pions at low transverse momentum in relativistic heavy ion collisions [14] as a consequence of the Bose-Einstein statistics. In Ref. [12] it is found that the incorporation of the pion chemical potential μ_π gives a strong enhancement of the muon pair yield in the low invariant mass region, provided the lepton pairs are produced predominantly via pion annihilation. This might serve as explanation of the so-called dilepton excess [15] observed in present CERN-SPS heavy-ion experiments [16].

In principal, there is an additional degree of freedom in the conventional ρ-π dynamics, namely a possible non-zero total electric or isospin charge of the pionic system. There is no restriction for the production of a pionic fireball with a finite, negative charge in the first deep-inelastic stage in a relativistic heavy-ion collision. Moreover, some experimental data [17] and theoretical speculations [18] point towards such a possibility. This may be a consequence of the proton-neutron asymmetry of the colliding heavy ions, and the asymmetry increases with increasing of the atomic weight of the colliding ions. The electric charge of a pionic system is controlled by the "charge" chemical potential μ_Q which should not be confused with the chemical potential used by Koch $\mu_\pi = \mu_\pi^0$, which is a measure of the chemical equilibrium breaking. Generally, the chemical potentials for positive and negative pions are $\mu_{\pi^\pm} = \mu_\pi^0 \pm \mu_Q$.

Here we explore this additional degree of freedom. Our work may be viewed as extension of the results of Gale and Kapusta [8], and Koch [12] on the ρ meson self energy at finite temperature to finite values of the chemical potential μ_Q. We evaluate the ρ meson self energy by using as starting point the conventional π-ρ effective Lagrangian and the functional integral representation for the partition function, which is familiar for the relativistic quantum field theory at finite temperature and charge chemical potential, and evaluate in the medium the ρ mass and width. From the methodical point of view our paper is closely related to Ref. [11] but differs from Ref. [12], where the chemical potential μ_π is incorporated by hand in the final expressions via the Bose distributions.

Our paper is organized as follows. In Sec. 2 we evaluate the ρ meson self energy in one-loop order at finite temperature and chemical potential μ_Q. This section presents the very details of the analytical calculations. In Sec. 3 we analyze the dependence of the ρ meson pole position both on μ_Q and μ_π^0 and discuss the possibility of their experimental manifestation. The summary can be found in section 4.

II. THE ρ PROPAGATOR

Our starting point is the effective Lagrangian $\mathcal{L}$ which describes a system of charged pions and neutral vector ρ mesons

$$\mathcal{L} = \frac{1}{2}(D^\nu \phi)^* D_\nu \phi - \frac{1}{2} m_\pi^2 \phi \phi^* - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^2, \tag{2.1}$$

where ϕ is the complex charged pion field, ρ stands for the vector field with the strength $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$, and $D_\nu = \partial_\nu - i g_\rho \rho_\nu$ is the covariant derivative; μ, ν are Lorentz indices. The Hamiltonian of the system is related to the Lagrangian of Eq.(2.1) in the usual way

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi)} \partial_0 \varphi - \mathcal{L} \tag{2.2}$$

with $\varphi = (\phi, \phi^*, \rho)$. The reference for what follows, at finite temperature $T \neq 0$ and $\mu_{\pi^\pm} = 0$, is the paper of Gale and Kapusta [11].

Let us consider the case when the system admits some conserved electric or isospin charge. In the first step we consider the case $\mu_\pi^0 = 0$ and concentrate on the incorporation of μ_Q. We then discuss the role of both μ_Q and μ_π^0 in Sec.3.

The incorporation of the chemical potential μ_Q leads to a transformation of the Hamiltonian which we use for the calculation of the partition function

$$\mathcal{H} \to \mathcal{H} - \mu_Q J_0, \tag{2.3}$$

where J_0 is the time component of Noether's current

$$J_\nu \equiv i\frac{1}{2}(\phi^* D_\nu \phi - \phi(D_\nu\phi)^*). \tag{2.4}$$

The ρ meson propagator in a medium is related to the self energy

$$\left(D^{-1}\right)^{\mu\nu} = \left(D_0^{-1}\right)^{\mu\nu} + \Pi^{\mu\nu}, \tag{2.5}$$

where $D_0^{\mu\nu}$ is the free propagator.

In Euclidean space, the rho meson self energy may be obtain with help of the partition function, having a functional integral representation of the form [19]

$$\mathcal{Z} = \int \mathcal{D}\pi_\varphi \int_{periodic} \mathcal{D}\varphi \exp\left\{\int_0^\beta d\tau \int_V d\mathrm{x}\left(i\pi_\varphi \frac{\partial\varphi}{\partial\tau} - \mathcal{H} + \mu_Q J_0\right)\right\},$$

where again $\varphi = (\phi, \phi^*, \rho)$, and $\pi_\varphi = \partial\mathcal{L}/\partial(\partial_0\varphi)$ are the respective conjugate momenta. The integration over π_φ gives

$$\mathcal{Z} = \int_{periodic} \tilde{\mathcal{D}}\rho\mathcal{D}\phi\mathcal{D}\phi^* e^{S_0+S_{int}}, \tag{2.6}$$

where $S_0 = S_{0\pi} + S_{0\rho}$ describes non-interaction part of the total effective action, and S_{int} corresponds to the interaction part, i,e.,

$$\begin{aligned}
S_{0\pi} &= \int_0^\beta d\tau \int_V d\mathrm{x}\Big(\frac{1}{2}|\partial\phi|^2 - \frac{1}{2}(m_\pi^2 - \mu_Q) - \mu_Q j_0\Big), \\
S_{0\rho} &= \int_0^\beta d\tau \int_V d\mathrm{x}\Big(-\frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho^2 - \frac{1}{2\alpha}(\partial_\mu\rho^\mu)^2\Big); \\
S_{int} &= \int_0^\beta d\tau \int_V d\mathrm{x}\Big(\frac{1}{2}g_\rho^2\rho^2|\phi|^2 + g_\rho(\rho_\mu j^\mu + \mu_Q\rho_0|\phi|^2)\Big)
\end{aligned} \tag{2.7}$$

where $\tilde{\mathcal{D}}\rho = \mathcal{D}\rho \cdot \det(\partial_4)$ ($\det\partial_4 \equiv \det\left(\frac{\partial\,\partial_\mu\rho^\mu}{\partial\rho_4}\right)$) and $j_\mu = i/2(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)$, $i\partial_0 = \partial_\tau$, $\rho_0 = i\rho_4$, etc. $S_{0\rho}$ includes the gauge fixing term. We use the Landau gauge with $\alpha \to 0$.

Expanding Eq.(2.6) in power series in S_{int} and taking the logarithm of both sides, we get in the second order of g_ρ

$$\begin{aligned}
\ln\mathcal{Z} &= \ln\mathcal{Z}_0 + \ln\mathcal{Z}_{int}, \\
\ln\mathcal{Z}_{int} &\simeq \frac{1}{2}g_\rho^2\Big(< \int d\tau d\mathrm{x}\rho^2|\phi|^2 >_0 + < (\int d\tau d\mathrm{x}(\rho_\nu j_\nu + \mu\rho_0|\phi|^2))^2 >_0\Big),
\end{aligned} \tag{2.8}$$

where

$$\mathcal{Z}_0 = \int \mathcal{D}\varphi e_0^S;\ < R >_0 \equiv \mathcal{Z}_0^{-1}\int \mathcal{D}\varphi\, R\, e^{S_0}. \tag{2.9}$$

The calculation of $\ln \mathcal{Z}_{int}$ may be performed by utilizing the methods of Ref. [19] and textbook recipes [20]. After some tedious algebraic exercises, and taking into account relation between the polarization operator $\Pi_{\mu\nu}$ and the partition function

$$\Pi_{\mu\nu} = -2\frac{\delta \, ln\mathcal{Z}_{int}}{\delta D_0^{\mu\nu}}, \tag{2.10}$$

we get the following expression for $\Pi_{\mu\nu}$

$$\Pi^{\mu\nu}(\mu_Q,p) = \tilde{\Pi}^{\mu\nu}(\mu_Q,p) + \mu_Q^2 \Delta\Pi^{\mu\nu}(\mu_Q,p), \tag{2.11}$$

where

$$\begin{aligned}\tilde{\Pi}^{\mu\nu}(\mu_Q,p) &= g_\rho^2 T\Big(\sum_{k_4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\delta^{\mu\nu}}{A(k)}\Big(1+\frac{A(k)^2-B(k)^2}{A(k)^2+B(k)^2}\Big)\\ &-\frac{1}{4}\sum_{k_4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{(2k-p)^\mu(2k-p)^\nu}{A(k)A(k-p)}\mathcal{F}(\mu_Q,k,p)\Big),\end{aligned} \tag{2.12}$$

$$\begin{aligned}\Delta\Pi^{\mu\nu}(\mu_Q,p) &= g_\rho^2 T\Big(\delta_4^\nu\delta_4^\mu\sum_{k_4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{A(k)A(k-p)}\mathcal{F}(\mu_Q,k,p)\\ &+2\sum_{k_4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{(\delta_4^\nu(2k-p)^\mu+\delta_4^\mu(2k-p)^\nu)(A(k-p)B(k)+A(k)B(k-p))}{\mu_Q(A(k-p)^2+B(k-p)^2)(A(k)^2+B(k)^2)}\Big).\end{aligned} \tag{2.13}$$

In the above, the fourth component of the momentum four-vectors is the Matsubara frequency, i.e., k_4 or $p_4 = 2\pi T\times$integer. The functions $A(q)$ and $B(q)$ depend on the chemical potential as

$$A(q) = q_4^2 + \mathbf{q}^2 + m_\pi^2 - \mu_Q^2 \equiv q^2 + m_\pi^2 - \mu_Q^2, \quad B(q) \equiv -2\mu_Q q_4.$$

The function $\mathcal{F}(\mu_Q,k,p)$ is a combination of A and B

$$\begin{aligned}\mathcal{F}(\mu_Q,k,p) &\equiv 1+\frac{A^2(k)-B^2(k)}{A^2(k)+B^2(k)}+\frac{A^2(p-k)-B^2(p-k)}{A^2(p-k)+B^2(p-k)}\\ &+\frac{\big(A^2(p-k)-B^2(p-k)\big)\big(A^2(k)-B^2(k)\big)+A(k)A(p-k)B(k)B(p-k)}{\big(A^2(p-k)+B^2(p-k)\big)\big(A^2(k)+B^2(k)\big)}.\end{aligned} \tag{2.14}$$

The last term in Eq.(2.11) comes from $\mu_Q J_0$ in Eq.(2.3), which in one-loop (i.e., order g_ρ^2) approximation generates the non-covariant interaction proportional to μ_Q^2.

In the limit of $\mu_Q = 0$, we have $A(q) = q^2 + m_\pi^2$, $B(q) = 0$, $\mathcal{F}(\mu_Q,k,p) = 4$, and Eq.(2.11) reduces to the self energy of Ref. [11], obtained within the finite-temperature Feynman rules as

$$\begin{aligned}\Pi^{\mu\nu}(p)_{\mu_Q=0} &= 2g_\rho^2 T\sum_{k_4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\delta^{\mu\nu}}{k^2+m_\pi^2}\\ &-g_\rho^2 T\sum_{k_4}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{(2k-p)^\mu(2k-p)^\nu}{(k^2+m_\pi^2)((k-p)^2+m_\pi^2)}.\end{aligned} \tag{2.15}$$

In this paper we restrict ourselves to the simplest case where the vector field is taken in the it's rest frame with $\mathbf{p} = 0$. This relatively simple case gives the physical picture and the order of magnitude of the medium corrections. If we find that the effect is considerable, then the corresponding generalizations may be performed straightforwardly.

We calculate the self energy of Eqs.(2.12), (2.13) by making use of the standard technique [20,21], i.e., the discrete summation is replaced by the contour integral as

$$T\sum_{n=-\infty}^{\infty} f(k_0 = ik_4) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dk_0 \frac{1}{2}[f(k_0) + f(-k_0)] + \frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon} dk_0 [f(k_0) + f(-k_0)]\frac{1}{e^{\beta k_0} - 1}, \tag{2.16}$$

with $\beta = 1/T$. The first term in Eq. (2.16) does not depend on temperature T, while the second one does. In our case, however, the first term may depend on μ_Q. This means that it cannot be interpreted as exact vacuum part. Nevertheless, for convenience we call it as a "vacuum" part, assuming a possible μ_Q dependence, while the second part we denote as a "matter" part.

Consider the first term in Eq. (2.12):

$$I_1^{\mu\nu} \equiv T\sum_{k_4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\delta^{\mu\nu}}{A(k)}\Big(1 + \frac{A^2(k) - B^2(k)}{A^2(k) + B^2(k)}\Big) \equiv \delta^{\mu\nu}(I_{1,vac} + I_{1,mat}), \tag{2.17}$$

where $I_{1,vac}$ and $I_{1,mat}$ correspond to the first and second terms of Eq. (2.16) respectively. The analysis of $I_{1,vac}$ shows that it does not depend only on the temperature but also on the chemical potential as well; so it really may be interpreted as the vacuum part.

The matter part of I_1 is of physical interest. The application of the contour closing integration method gives the following expression

$$I_{1,mat} = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{4}{2\pi i}\oint dk_0 \frac{-k_0^2 + \omega^2 - \mu_Q^2}{(k_0^2 - \omega^2 + \mu_Q^2)^2 - 4k_0^2\mu_Q^2}(e^{\beta k_0} - 1)^{-1}, \tag{2.18}$$

which via straightforward calculation results in

$$I_{1,mat} = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2\omega}N(\omega), \tag{2.19}$$

where $\omega = \mathbf{k}^2 + m_\pi^2$.

$$N(\omega) = (e^{\beta(\omega-\mu_Q)} - 1)^{-1} + (e^{\beta(\omega+\mu_Q)} - 1)^{-1} \tag{2.20}$$

is the pion Bose distribution for particles and antiparticles. One can see that all the dependence on μ_Q in I_1 is absorbed into the Bose factor.

We denote the second term in Eq.(2.12) as $-g_\rho^2 I_2^{\mu\nu}(p)$. In the frame where $\mathbf{p} = 0$, all the non-diagonal terms with $\mu \neq \nu$ vanish, and both the matter and the vacuum time component with $\mu = \nu = 0$ depend on μ_Q. Let us consider first the spatial components I_2^{ij}

$$I_2^{ij}(p_4) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k^i k^j \cdot T \sum_{k_4} \frac{1}{A(k-p/2)A(k+p/2)} \mathcal{F}(\mu, k, p),$$

where for simplicity we use the variable transformation $k \to k + p/2$, assuming that $p_4/2$ is the Matsubara frequency. Using Eq.(2.16) we get the medium contribution of I_2 in the form

$$I_{2,mat}^{ij}(p_4) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k^i k^j \cdot 2I_{2,mat}(\mathbf{k}, p_4);$$

$$I_{2,mat}(\mathbf{k}, p_0) = \oint \frac{dk_0}{2\pi i} \frac{1}{A_+ A_-} (e^{\beta k_0} - 1)^{-1}$$
$$\left\{ 1 + \frac{A_+^2 + B_+^2}{A_+^2 - B_+^2} + \frac{A_-^2 + B_-^2}{A_-^2 - B_-^2} + \frac{(A_+^2 + B_+^2)(A_-^2 + B_-^2) + 4A_+A_-B_+B_-}{(A_+^2 - B_+^2)(A_-^2 - B_-^2)} \right\}, \quad (2.21)$$

where $A_\pm = (k_0 \pm ip_4/2)^2 - \omega^2 + \mu_Q^2$, $B_\pm = 2\mu_Q(k_0 \pm ip_4/2)$. A close inspection of the integrand shows that only the poles of the bracketed term give a contribution into the contour integral. The final result in Minkowski space is

$$I_{2,mat}^{ij}(p_0) = -\frac{4}{3}\delta^{ij} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\omega} \frac{\omega^2 - m_\pi^2}{4\omega^2 - p_0^2 - i\epsilon} N(\omega). \quad (2.22)$$

Here and elsewhere we analytically continue the Matsubara frequency p_4 to $ip_4 \to p_0 + i\epsilon$, where $\epsilon \to 0^+$. The matter part of Eq.(2.13) reads

$$\tilde{\Pi}_{mat}^{ij}(\mu, p) = g_\rho^2 \delta^{ij} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\omega} \left\{ 1 - \frac{4}{3} \cdot \frac{\omega^2 - m_\pi^2}{4\omega^2 - p_0^2 - i\epsilon} \right\} N(\omega). \quad (2.23)$$

A direct calculation of the time component of Eq.(2.12) gives a value proportional to μ_Q^2

$$\tilde{\Pi}^{00} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\omega} \big(1 + N(\omega)\big) - \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\omega} \Big(1 - \frac{4\mu_Q^2}{4\omega^2 - p_0^2 - i\epsilon}\Big) \big(1 + N(\omega)\big). \quad (2.24)$$

This term is equal, but with opposite sign, to the non-covariant term of Eq.(2.13)

$$\Delta\Pi^{\mu\nu}(p, \mu_Q) = -4\delta_4^\nu \delta_4^\mu \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\omega} \frac{1}{4\omega^2 - p_0^2 - i\epsilon} \big(1 + N(\omega)\big). \quad (2.25)$$

So, we find that the total $\Pi_{mat}^{44}(\mathbf{p} = 0)$ in Eq.(2.11) is equal zero, which confirms the current conservation or, the transversality of $\Pi^{\mu\nu}$ with respect to the external momentum. We see also that $\Pi_{mat}^{ij} = \tilde{\Pi}_{mat}^{ij}$. The transversality of the vacuum part at $\mu_Q = 0$ is demonstrated in several works, see e.g., the textbook [22].

In Minkowski space the self energy $\Pi^{\mu\nu}$ may be expressed in the form

$$\Pi^{\mu\nu} = FP_L^{\mu\nu} + GP_T^{\mu\nu}, \quad (2.26)$$

where G and F are the so-called "longitudinal" and "transverse" masses, and $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ are the longitudinal and transverse projection tensors:

$$P_T^{00} = P_T^{0i} = P_T^{i0} = 0,\ P_T^{ij} = \delta^{ij} - p^i p^j/\mathrm{p}^2,$$
$$P_L^{\mu\nu} = p^\mu p^\nu/p^2 - g^{\mu\nu} - P_T^{\mu\nu}. \tag{2.27}$$

In the limit of $\mathbf{p} \to 0$ we get

$$\Pi^{00} = \Pi^{0i} = \Pi^{i0} = 0$$
$$\Pi^{ij} = \delta^{ij} G + p^i p^j \Big((\frac{1}{p^2} + \frac{1}{\mathrm{p}^2}) F - \frac{1}{\mathrm{p}^2} G \Big) |_{\mathrm{p}\to 0}. \tag{2.28}$$

At $\mathbf{p} = 0$ the tensor structure of Π^{ij} is trivial: $\Pi^{ij} \sim \delta^{ij}$. This is satisfied only if in Eq.(2.28) $F = G$. The final expression for the ρ propagator reads

$$D^{\mu\nu} = \frac{g^{\mu\nu} - p^\mu p^\nu/p^2}{p_0^2 - m_\rho^2 - F(p)_{vac} - F_{mat}(p)}, \tag{2.29}$$

where $F_{vac,mat}(p) = -(1/3) g_{\mu\nu} \Pi^{\mu\nu}_{vac,mat}$.

The self energy F_{vac} is divergent. We evaluate it by using dimensional regularization and renormalize with the counterterms C_{inf} and C_f for divergent and finite parts of F_{vac}, respectively. C_{inf} cancels the divergent part of F_{vac}, while C_f is found from the natural condition for the ρ propagator pole in vacuum at the physical point $p_0^2 = m_\rho^2$

$$p_0^2 - m_\rho^2 - F_{vac}(p_0)|_{p_0^2 \to m_\rho^2} \to -i\, m_\rho \gamma_\rho, \tag{2.30}$$

where m_ρ and γ_ρ is the ρ mass and width in the vacuum. Equation (2.30) gives explicit expressions for C_f using the known formulae for $Re\, F_{vac}$ and $Im\, F_{vac}$:

$$Re\, F_{vac}(p_0) = \frac{g_\rho^2}{48\pi^2} \Big(p_0^2 (1 - 4m_\pi^2/p_0^2)^{3/2} \left\{ ln \frac{1 + (1 - 4m_\pi^2/p_0^2)^{1/2}}{1 - (1 - 4m_\pi^2/p_0^2)^{1/2}} \right\} + 8m_\pi^2 \Big), \tag{2.31}$$

$$Im\, F_{vac}(p_0) = -\frac{g_\rho^2}{48\pi^2} p_0^2 (1 - 4m_\pi^2/p_0^2)^{3/2} \Theta(p_0 - 2m_\pi), \tag{2.32}$$

$$C_f = -\frac{g_\rho^2}{48\pi^2} \Big(m_\rho^2 (1 - 4m_\pi^2/m_\rho^2)^{3/2} \left\{ ln \frac{1 + (1 - 4m_\pi^2/m_\rho^2)^{1/2}}{1 - (1 - 4m_\pi^2/m_\rho^2)^{1/2}} \right\} + 8m_\pi^2 \Big). \tag{2.33}$$

In an arbitrary point $p_0 = M$, the quantity $\tilde{F}(M) \equiv Re F_{vac}(M) + C_f$ is finite.

Now we can write the renormalized ρ propagator in matter

$$D^{\mu\nu} = \frac{g^{\mu\nu} - p^\mu p^\nu/p_0^2}{p_0^2 - m_\rho^2 - (\tilde{F}(p_0) + Re\, F_{mat}(p_0)) - i(Im\, F_{vac}(p_0) + Im\, F_{mat}(p_0)}. \tag{2.34}$$

For completeness we also display the matter part of self energy

$$Re\, F_{mat}(M) = g_\rho^2 Re \int \frac{d^3\mathrm{k}}{(2\pi)^3} \frac{1}{\omega} \Big(1 - \frac{4}{3} \frac{\omega^2 - m_\pi^2}{4\omega^2 - M^2 - i\epsilon} \Big) N(\omega),$$
$$Im\, F_{mat}(M) = -\frac{g_\rho^2}{48\pi} M^2 (1 - 4m_\pi^2)^{3/2} N(M/2) \Theta(M - 2m_\pi). \tag{2.35}$$

The mass m_ρ^* and the width γ_ρ^* in matter can be found similarly to the vacuum case using condition

$$p_0^2 - m_\rho^2 - (\tilde{F}(p_0) - Re\, F_{mat}(p_0)) - i(Im\, F_{vac}(p_0) + Im\, F_{mat}(p_0)|_{p_0^2 \to m_\rho^{*2}} \to -i\, m_\rho^* \gamma_\rho^*, \quad (2.36)$$

which leads to final equations for the pole position:

$$m_\rho^{*2} = m_\rho^2 + \tilde{F}(m_\rho^{*2}) + Re\ F_{mat}(m_\rho^{*2}), \quad (2.37)$$

$$\gamma_\rho^* = \frac{1}{m_\rho^*}\left\{Im\ F_{vac}(m_\rho^{*2}) + Im\ F_{mat}(m_\rho^{*2})\right\}. \quad (2.38)$$

III. RESULTS

In Figs. 1 - 4 we display results of numerical calculations of the shift of the ρ pole position in a pion gas medium with respect to the vacuum. In Figs. 1 and 2 we show the μ_Q dependence of $\Delta m_\rho = m_\rho^* - m_\rho$ and $\Delta\gamma_\rho = \gamma_\rho^* - \gamma_\rho$ respectively, at fixed values of the temperature $T = 50, 100, 150$ and 200 MeV. One observes a strong increase of Δm_ρ and $\Delta\gamma_\rho$ with increasing chemical potential at large values of the temperature, in particular near the condensation instability where $\mu_Q \to m_\pi$. At values of $\mu_Q \leq 70$ - 90 MeV this increase is rather moderate, but at very large values of $\mu_Q \simeq 130$ MeV and temperatures $T \sim 200$MeV the shifts of Δm_ρ and $\Delta\gamma_\rho$ may be as much as ~ 140 and ~ 100 MeV, respectively. As rule, the μ_Q dependence gives approximately a factor 2 in the total increase of Δm_ρ and $\Delta\gamma_\rho$ as compared with the $\mu_Q = 0$ case. This is confirmed in Fig. 3 where we show the temperature dependence of Δm_ρ at fixed values of $\mu_Q = 0$, 60, and 130 MeV.

One should notice that, as a rule, the temperature dependence is stronger than the dependence on the chemical potential (except near to the condensation instability). This has been observed also in a quite different context [23]. The origin of this observation is that the chemical potential enters logarithmical, while the temperature as a power [23].

Eq.(2.38) allows to take into account qualitatively the total pion chemical potentials $\mu_{\pi^\pm} = \mu_\pi^0 \pm \mu_Q$, where μ_π^0 is the same as used in Ref. [12] when μ_Q is equal zero. The chemical potentials $\mu_{\pi^\pm}$ are associated with the total $\pi^\pm$ density, while μ_Q is measure of the π^+ and π^- asymmetry. If we neglect the weak μ_π dependence of $m_\rho^*(\mu_\pi)$ in the intermediate region of the chemical potential, then we find that all the dependence of γ_ρ^* on the chemical potential can be included in the Bose distribution function $N(\omega)$. That is,

$$N(\mu_\pi, \omega) = (e^{\beta(\omega - \mu_Q - \mu_\pi^0)} - 1)^{-1} + (e^{\beta(\omega + \mu_Q - \mu_\pi^0)} - 1)^{-1} \quad (3.1)$$

determines the dependence of the width on the both chemical potentials.

To quantify this assertion, in Fig. 4 we display the temperature dependence of γ_ρ^* at fixed values of μ_Q, μ_π^0, namely μ_Q=40, μ_π^0=90 and μ_Q=90, μ_π^0=40 MeV, respectively. One can see, indeed, that dependence of $\Delta\gamma_\rho$ on μ_π^0 results in a shift which is of the same order of magnitude as the shift which comes from the μ_Q dependence.

It is worth estimating the typical mean value of the charge chemical potential to be expected in intermediate energy heavy-ion collisions. For this aim we use the method of

Ref. [18] and assume that the pions in a fireball are the product of the baryon and isobar interactions during the ion-ion collision, i.e., they keep the information about the isotopic asymmetry in the initial state. From the chemical equilibrium in the whole system one can find relations for the chemical potentials $\mu_p = \mu_B + \mu_Q$, $\mu_n = \mu_B$, $\mu_{\pi^+} = \mu_Q$, $\mu_{\pi^-} = -\mu_Q$, where μ_B is the baryon chemical potential. Using the ratio of the electric charge Q to the baryonic number B, $Q/B = (N_p + N_{\pi^+} - N_{\pi^-})/(N_p + N_n)$, and Boltzmann distribution for the particles, one can find an equation for the charge chemical potentials as function of B, Q, temperature and the relative pion multiplicity $\alpha \equiv N_{\pi^-}/(N_p + N_n)$

$$\frac{Q}{B} \simeq \frac{z}{z+1} + \alpha(z^2 - 1), \tag{3.2}$$

with $z \equiv e^{\beta\mu_Q}$. We find an approximate solution of the above equation in the form

$$\frac{\mu_Q}{T} = -a + b\alpha - c\alpha^2, \tag{3.3}$$

where the coefficients a, b, and c depend on the ratio Q/B. For U+U and Pb+Pb collisions they are: $a \simeq 0.455$ and 0.420, $b \simeq 2.18$ and 2.17, and $c \simeq 4.44$ and 5.04, respectively. For a typical multiplicity in the range of $\alpha \simeq 0.07$ - 0.1 we find the ratio $-\mu_Q/T$ in the range of 0.32 - 0.28 and 0.25 - 0.29 for U+U and Pb+Pb collisions respectively. This means that for a temperature $T \sim 200$ MeV the absolute value of the negative charge chemical potential may be as large as $|\mu_Q| \sim 60$ MeV. The sign of μ_Q should be seen in the relative yields of the π^+, π^0, π^- meson , as is predicted in Ref. [18] (there named π^- enhancement), but it is not essential for the ρ meson pole because of the symmetry of equations (2.37), (2.38) to this sign. Larger values of $|\mu_Q|$ may be expected in charged-pion fluctuations.

For these estimated values of μ_Q our calculation predicts a rather modest shift of the rho pole position. However, the value of $\mu_Q \sim 0.3T$ can modify the predicted strong enhancement of the muon pairs in the low invariant mass region, $2m_\pi \leq M_{\mu^+\mu^-} \leq 600$ MeV, for lepton pairs which are produced via the annihilation of pions in the hot and dense collision zone [12]. The physical reason of this is the following fact. The expected enhancement is connected with the statistical weight of the ρ meson in matter

$$f_\rho \simeq \exp(-\frac{E_\rho - \mu_\rho}{T}), \tag{3.4}$$

where the ρ meson chemical potential is the sum $\mu_\rho = \mu_{\pi^+} + \mu_{\pi^-} = 2\mu_\pi^0$. It clear that $\mu_\pi^0 + |\mu_Q| \leq m_\pi$, and $\mu_{\pi,max}^0 < m_\pi - |\mu_Q|$. This means that the maximum enhancement would be reduced by the factor $\simeq e^{\frac{2|\mu_Q|}{T}}$, which according to our estimation is $\sim e^{0.6} \simeq 1.82$ and needs to be taken into account in further estimations.

We should emphasize however, that the above estimates of T and μ_Q apply to not too high bombarding energies. In this case our model system is also affected by the baryons, which must be taken into account in more complete investigations. Otherwise, at much higher bombarding energies, say at CERN-SPS energy or above, a baryon-less mesonic fireball might be formed, supposed the nuclear transparency is large (as might be for not too heavy ions). Then in some sections of the rapidity space, regions may be formed where the chemical potentials are large and the in-medium modifications on m_ρ^* and γ_ρ^* might become stronger than estimated conservatively above. Quantitative estimates need much more detailed studies, as also the relationship to a disoriented chiral condensate.

IV. SUMMARY

In summary, we have calculated the ρ meson self energy in a pion medium at finite temperature and charge chemical potential which is responsible for the difference of π^+ and π^- densities in matter. The calculation is performed within the functional integral representation for the partition function in order g_ρ^2. We find that the ρ mass and the width increase with the chemical potential. This increase may be about two times larger as compared with the temperature shift at zero μ_Q at very large values of $\mu_Q \sim m_\pi$ and may be realized in charge fluctuations of the pion gas produced in the central region in relativistic heavy-ion collisions. The predicted effect is small for the mean value of μ_Q, which is expected from the proton-neutron asymmetry in intermediate energy heavy-ion collisions. In this region of μ_Q the found dependence is smaller than the temperature effect of Gale and Kapusta. But one has to be careful in calculating the dilepton production in a charge pion matter. The charge chemical potential leads to the relative decrease of the dilepton rate in the low invariant mass region, which is essential for the interpretation of the dilepton spectra in heavy ion collisions.

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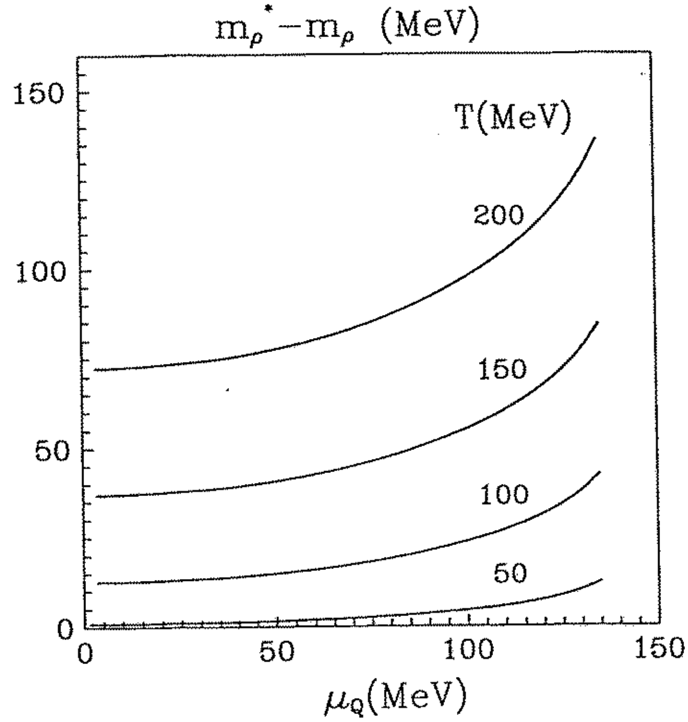

FIG. 1. The difference $m_\rho^* - m_\rho$ as function of μ_Q at several temperatures.

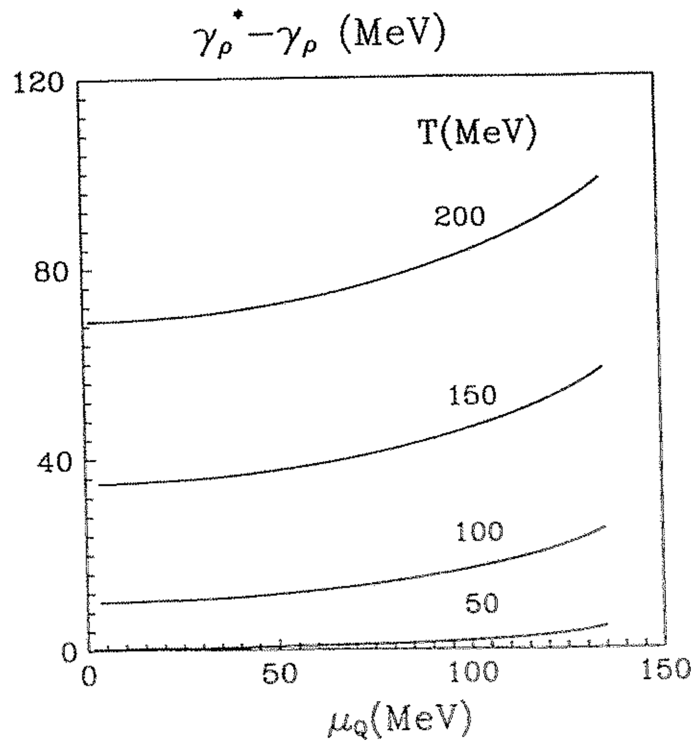

FIG. 2. The difference $\gamma_\rho^* - \gamma_\rho$ as function of μ_Q at several temperatures.

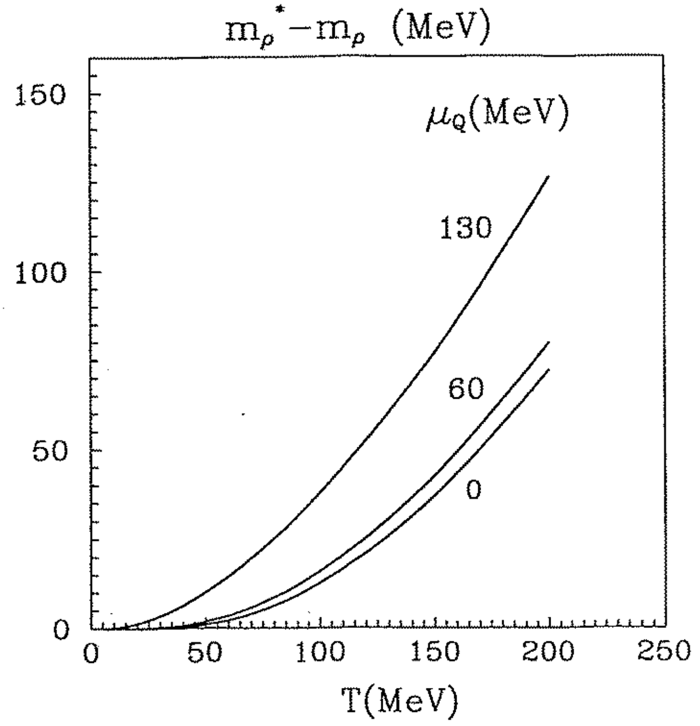

FIG. 3. The difference $m_\rho^* - m_\rho$ as function of temperature at several values of the chemical potential μ_Q.

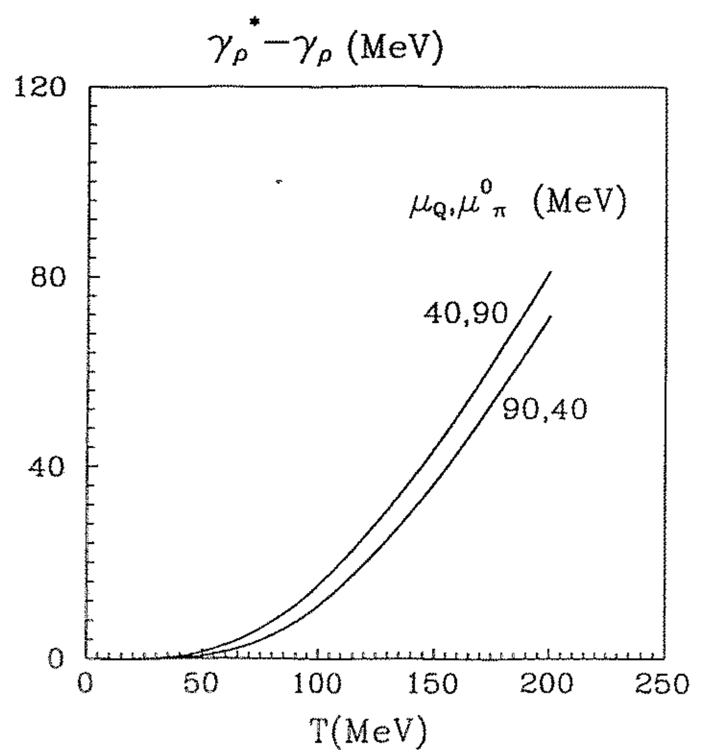

FIG. 4. The difference $\gamma_\rho^* - \gamma_\rho$ as function of temperature at different values of μ_Q, μ_π^0.