On the synchronizability of Tayler–Spruit and Babcock–Leighton type dynamos

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Originally published:

January 2018

Solar Physics 293(2018), 12

DOI: https://doi.org/10.1007/s11207-017-1232-y

On the synchronizability of Tayler-Spruit and Babcock-Leighton type dynamos

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Abstract The solar cycle appears to be remarkably synchronized with the gravitational torques exerted by the tidally dominant planets Venus, Earth and Jupiter. Recently, a possible synchronization mechanism was proposed that relies on the intrinsic helicity oscillation of the current-driven Tayler instability which can be stoked by tidal-like perturbations with a period of 11.07 years. Inserted into a simple $\alpha - \Omega$ dynamo model these resonantly excited helicity oscillations led to a 22.14 years dynamo cycle. Here, we assess various alternative mechanisms of synchronization. Specifically we study a simple time-delay model of Babcock-Leighton type dynamos and ask whether periodic changes of either the minimal amplitude for rising toroidal flux tubes or the Ω effect could eventually lead to synchronization. In contrast to the easy and robust synchronizability of Tayler-Spruit dynamo models, our answer for those Babcock-Leighton type models is less propitious.

Keywords: Solar cycle, Models Helicity, Theory

1. Introduction

Despite its long history, which traces back to Wolf (1859), the idea of planetary influence on the solar dynamo is widely considered as marginal, if not "astrological". There are indeed good reasons for skepticism: the gravitational forces exerted by the planets are tiny when compared to the intrinsic buoyancy and Coriolis forces that are believed to govern the solar dynamo (Callebaut, de Jager, and Duhau, 2012). The tidal accelerations of a few 10^{-10} m/s^2, which produce a tidal height of the order of 1 mm at the tachocline (Condon and Schmidt, 1975), seem - at first glance - ridiculous when asking for possible influences on the solar dynamo.

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Yet, there are remarkable correlations of the solar cycle with planetary orbits. This applies, in particular, to the apparent synchronization (Bollinger, 1952; Takahashi, 1968; Wood, 1972; Öpik, 1972; Condon and Schmidt, 1975; Grandpierre, 1996; Hung, 2007; Wilson, 2013; Okhlopkov, 2014, 2016) of the solar cycle with the 11.07 years conjunction cycle of Venus, Earth and Jupiter, which are the three dominant tide producing planets [1]. The average solar cycle duration of $(2008.9 - 1610.8)/36 = 11.06$ years, derived from the data of the last 36 cycles (Richards, 2009; Li, 2017), indicates an astonishing coincidence. Even more remarkable is the recent finding of Okhlopkov (2016) that the synchronization may have lasted for the last 50 cycles. Further to this, fossil records suggest that the solar cycle has been amazingly stable for at least the last 290 million years: the cycle length during the early Permian, e.g., was recently estimated as 10.62 years (Luthardt, 2017). Hence, one might reconsider the tidal height $h_{\rm tidal} \approx 1\,{\rm mm}$ and ask whether this is indeed as irrelevant as it looks like. Given the huge gravitational acceleration at the tachocline of $g \approx 500\,{\rm m/s}^2$ (Wood, 2010), we find an equivalent velocity of $v \sim (2gh_{\rm tidal})^{1/2} \approx 1\,{\rm m/s}$ which is not at all negligible, as already noted by Öpik (1972).

While such a "hard" *synchronization* of the basic Hale cycle with planetary tidal forces was advocated by only a few researchers, much more interest was dedicated to various kinds of "soft" planetary *modulation* of that cycle (whose length is usually believed to be determined by intrinsic solar parameters (Charbonneau, 2010; Cameron and Schüssler, 2017)). Intriguing connections have been found between various periodicities of the solar magnetic field (Suess-de Vries, Hallstadt, Eddy etc.) and corresponding planetary constellations (Jose, 1965; Charvatova, 1997; Abreu *et al.*, 2012; Wolf and Patrone, 2010; Scafetta, 2010, 2014; McCracken, Beer and Steinhilber, 2014; Cionco and Soon, 2015; Scafetta *et al.*, 2016). As an example, Abreu *et al.* (2012) had revealed synchronized cycles in proxies of the solar activity and the planetary torques, with periodicities that remain phase-locked over 9400 years. While still under scrutiny (Cameron and Schüssler, 2013; Poluianov and Usoskin, 2014; Abreu *et al.*, 2014), any such relationship - if confirmed - would have important consequences for the predictability not only of the solar dynamo but, very likely, of the terrestrial climate, too (Hoyt and Schatten, 1997; Gray *et al.*, 2010; Solanki, Krivova and Haigh, 2013; Scafetta, 2013; Ruzmaikin and Feynman , 2015; Soon *et al.*, 2016).

Returning to the problem of "hard" synchronization, we have recently tested a physical mechanism that seems promising for explaining it (Stefani *et al.*, 2016). We set out from a rarely discussed type of stellar dynamo models, in which the poloidal-to-toroidal field transformation is traditionally ensured by the Ω effect, while the toroidal-to-poloidal transformation starts only when the toroidal field itself becomes unstable to a non-axisymmetric current-driven instability. Early versions of such a dynamo mechanism were discussed by Ferriz Mas, Schmitt,

[1] The "generous omission" of Mercury, whose tidal effect is nearly the same as that of Earth, but whose 88 days revolution period is often considered as "so short that its influence appears only as an average, non-fluctuating factor..." Öpik (1972), might be another argument for skeptics. However, it could also be worthwhile to re-analyze the 50-80 years sub-band of the Gleissberg cycle as identified by Ogurtsov *et al.* (2002) in the light of the 66.4 years period of the four-fold co-alignment of Mercury, Venus, Earth and Jupiter (Verma, 1986).

and Schüssler (1994) and Zhang *et al.* (2003), and auspiciously applied to explain grand minima in terms of on-off intermittency (Schmitt, Schüssler, and Ferriz Mas, 1996). The underlying kink-type Tayler instability (TI) had been theoretically treated by many authors (Tayler, 1973; Pitts and Tayler, 1985; Gellert, Rüdiger, and Hollerbach, 2011; Rüdiger, Kitchatinov, and Hollerbach, 2013; Rüdiger *et al.*, 2015; Stefani and Kirillov, 2015), and was recently also observed in a liquid metal experiment (Seilmayer *et al.*, 2012). Based on this TI, a version of a non-linear dynamo mechanism had been proposed (Spruit, 2002) which is now known as the "Tayler–Spruit dynamo". This first version was soon criticized by Zahn, Brun, and Mathis (2007) who argued that the non-axisymmetric ($m = 1$) TI mode would produce the "wrong" poloidal field, being unsuitable for regenerating the dominant axisymmetric ($m = 0$) toroidal field. Fortunately, the same authors offered a possible rescue for the Tayler-Spruit dynamo concept provided that the $m = 1$ TI would produce an α effect with some $m = 0$ component.

The emergence of such a TI-related α effect is far from trivial, though. For comparable large values of the magnetic Prandtl number [Pm], Chatterjee *et al.* (2011); Gellert, Rüdiger, and Hollerbach (2011); Bonanno *et al.* (2012, 2017) found evidence for spontaneous symmetry breaking between left- and right-handed TI modes, leading indeed to a finite value of α. Things are different, however, for low Pm (as typical for the solar tachocline) for which we observed a tendency of the TI to produce oscillations of the helicity and the α-effect related to it (Weber *et al.*, 2013, 2015). The first result of Stefani *et al.* (2016) was that those oscillations between left- and right-handed $m = 1$ TI modes are very susceptible to $m = 2$ perturbations, which could explain their easy synchronizability with tidal forces as exerted by planets.

Appropriately parametrized, this resonant behaviour of α was then implemented into a simple zero-dimensional $\alpha - \Omega$ dynamo model which turned out to undergo oscillations with period doubling. In summary, we found that an 11.07 year tidal-like oscillation may lead to a resonant excitation of a 11.07-year oscillation of the TI-related α-effect, and thereby to a 22.14 year Hale cycle of the entire dynamo.

We note in passing that the notion "Tayler-Spruit dynamo", as used above, is not exactly correct for our modified model which comprises, besides of the resonantly excited oscillatory part of α, also some small, but non-zero constant part (subjected only to some standard type of α-quenching). Interestingly, the product of this constant part of α with Ω had to be positive to make the dynamo working, while a negative product led to decaying solutions. In a slightly extended model (Stefani *et al.*, 2017) we further showed that this positive product can even provide the correct equator-ward direction of the butterfly diagram of sun-spots, in pleasant contrast to what one would naively expect from the Parker-Yoshimura sign rule (Parker, 1955; Yoshimura, 1975; Pipin *et al.*, 2013).

In the light of such promising features of a tidally synchronized solar dynamo model of Tayler-Spruit type (with the semantic caveat noticed above), in this paper we ask for alternative synchronization mechanisms which are closer to the more widely accepted concept of flux-transport dynamos (Charbonneau, 2010). In those models, the Babcock-Leighton mechanism (Babcock, 1961; Leighton,

1964) interprets the generation of poloidal field by the stronger diffusive cancellation of the (closer to the equator) leading sunspots compared with that of the trailing (farther from the equator) spots. This leads to a spatially separated, or flux-transport type of dynamo, which is also known to exhibit correct butterflies if combined with an appropriate meridional circulation (Choudhuri, Schüssler, and Dikpati, 1995).

Specifically, we will investigate two models of putative planetary influences on flux-transport dynamos. The first model relies on periodic tidal perturbations of the adiabaticity in the tachocline region, which is crucial for the storage capacity of magnetic flux tubes before they are set free to erupt (Abreu *et al.*, 2012). This perturbation will be emulated as a periodic change of the minimum magnetic field beyond which magnetic flux tubes are allowed to rise.

The second model traces back to an idea of Zaqarashvili (1997) who had explained the 22 years cycle as an Alfvén wave excited via parametric resonance from a 11 years period change of differential rotation, which - in his view - could rely on the motion of the sun around the Sun-Jupiter common mass center. For this model to work it had to pre-suppose a significant poloidal field of fossil origin. An 11 years oscillation of the differential rotation is indeed known from measurements (Brown *et al.*, 1989; Howe, 2009), although it is usually explained in terms of back-reaction of the dynamo field on the flow, either by the Malkus-Proctor effect (Malkus and Proctor, 1975) or by so-called Λ-quenching (Kitchatinov, Rüdiger and Küker, 1994).

As in Stefani *et al.* (2016), we will refrain from any expensive higher dimensional simulations and restrict ourselves to simple, zero-dimensional dynamo models. As a basis we will utilize the time-delay concept introduced by Wilmot-Smith *et al.* (2006) which appears to be particularly suited for our purposes. It implements two time delays into the dynamo cycle, one representing the rise time of flux-tubes which transports toroidal field from the tachocline to the working site of the α-effect, the other one representing the time needed for the meridional circulation to transport poloidal field back to the working site of the Ω-effect.

Actually, both synchronization mechanisms will be studied for three paradigmatic regimes distinguished by the ordering of the involved time scales, namely a diffusion-dominated, a flux-transport dominated, and an intermediate regime. In either case, we will further distinguish between negative and positive products of α and Ω which typically lead to oscillatory or pulsating behaviour, respectively. In higher-dimensional models this sign is known to determine the direction of the butterfly diagram (Parker, 1955; Yoshimura, 1975; Pipin *et al.*, 2013): this issue will not be discussed here.

However, before entering these new topics, we will recall the synchronization mechanism based on the Tayler-Spruit-type dynamo model and discuss, as a left-over question from Stefani *et al.* (2016), its behaviour with respect to variations of the diffusion time.

2. Synchronization of Tayler-Spruit type dynamos

In this section we will analyze a further aspect of the reduced zero-dimensional α–Ω dynamo model of Stefani *et al.* (2016), consisting of two coupled ordinary differential equations for the toroidal and the poloidal field components,

$$\frac{\mathrm{d}b(t)}{\mathrm{d}t} = \Omega a(t) - \frac{b(t)}{\tau} \tag{1}$$

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = \alpha(t)b(t) - \frac{a(t)}{\tau}, \tag{2}$$

wherein a represents the poloidal field (specifically, its vector potential), and b the toroidal field. While in Stefani *et al.* (2016) the diffusion time had been fixed to $\tau = 1$ year, it will now be considered as variable (for the sake of shortness, we will skip the time unit "year" in all following numerical analyses).

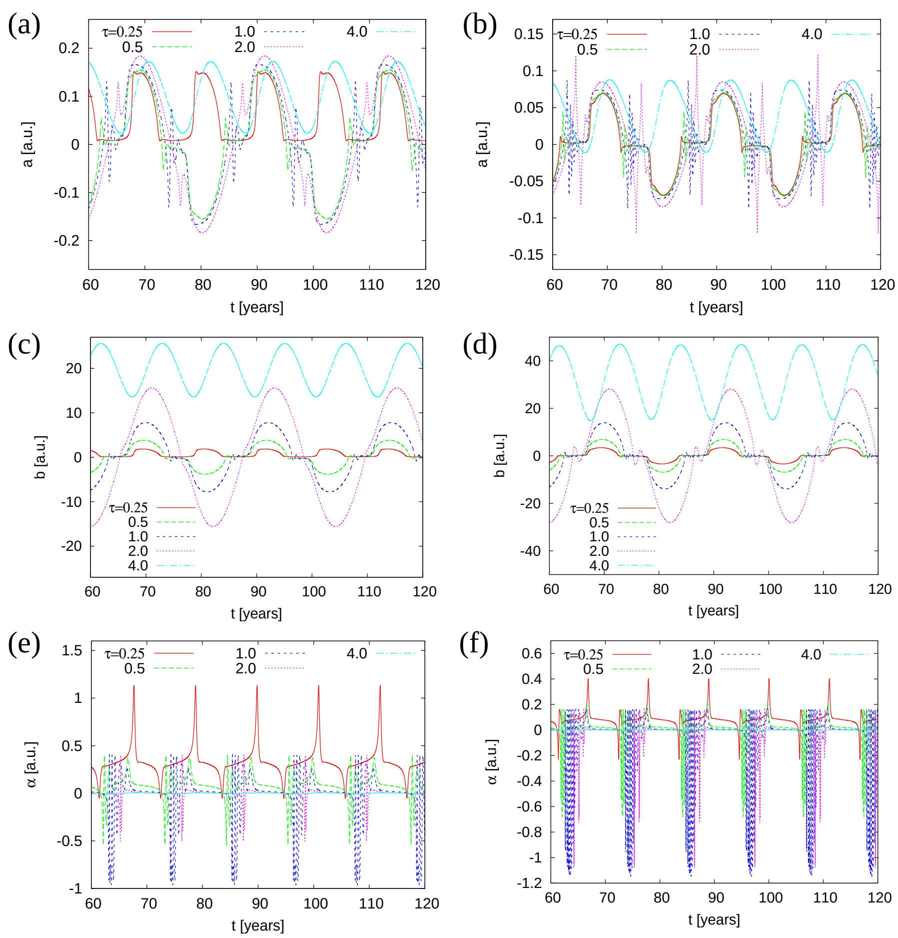

Figure 1. Time dependence of a (a,b), b (c,d) and α (e,f) for the two parameter sets $\Omega = 50$, $c = 0.4$, $p = 8$, and $h = 10$ (a,c,e) and $\Omega = 200$, $c = 0.16$, $p = 8$, and $h = 10$ (b,d,f), and varying values of τ. For the case (a,c,e), pulsations with an 11.07 period occur for a low $\tau = 0.25$ and a large $\tau = 4$, while for intermediate τ we find oscillations with a period of 22.14. For the case (b,d,f), pulsations are found only for the large $\tau = 4$, while for other τ oscillations occur.

In contrast to the constant value of Ω, which represents the induction effect of the differential rotation, α is supposed to depend on the instantaneous toroidal magnetic field:

$$\alpha(t) \;=\; \frac{c}{1+gb^2(t)} + \frac{pb^2(t)}{1+hb^4(t)} \sin\left(2\pi t/T_{\rm tidal}\right) . \tag{3}$$

Equation (3) is motivated as follows: the first term, scaled by c, reflects some *constant* part that is only quenched, in the usual way, by the magnetic-field energy (b^2) in the denominator. The second contribution, scaled by a parameter p, is *periodic* in time and emulates the resonant excitation of the α-oscillation by fixing its period to $T_{\rm tidal}$, but maximizing its amplitude at some particular value of $|b|$ where the tidal excitation happens to be in resonance with the intrinsic helicity oscillation of the TI.

As shown previously (Stefani *et al.*, 2016), this dynamo fails to work when either c is set to zero, or the product of c and Ω is negative. Working dynamos come in two guises: their field might either pulsate (keeping one single sign) with a period equal to $T_{\rm tidal}$, or oscillate with a period of $2T_{\rm tidal}$, which is the "desired" behaviour that was indeed found for a wide range of parameters.

What was left over from Stefani *et al.* (2016) was an assessment of the role of the diffusion time τ which had been set to 1 for the assumed tidal forcing period of 11.07. Figure 1 illustrates now the results for varying τ. For the parameter choice $\Omega = 50$, $c = 0.4$, $p = 8$, and $h = 10$ (a,c,e), pulsations with a 11.07 period occur for a low value $\tau = 0.25$ as well as for a large value $\tau = 4$, while for intermediate τ we find oscillations with a period of 22.14. For the case $\Omega = 200$, $c = 0.16$, $p = 8$, and $h = 10$ (b,d,f), pulsations are found for the largest considered value $\tau = 4$, while for other τ oscillations are observed.

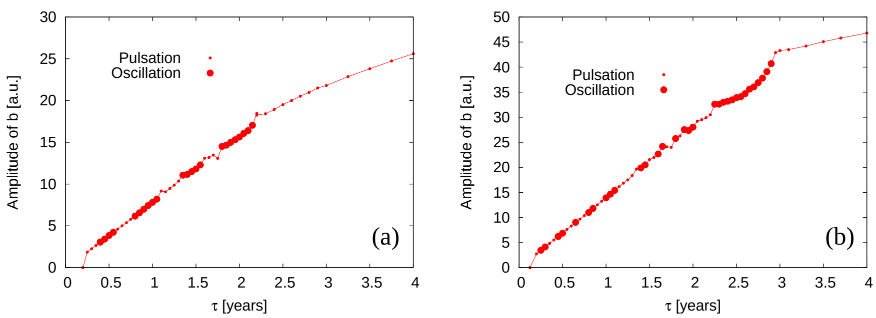

Figure 2. Transitions between oscillations and pulsations for varying the diffusion time τ for $\Omega = 50$, $c = 0.4$, $p = 8$ (a), and $\Omega = 200$, $c = 0.16$, $p = 8$ (b).

A more detailed analysis reveals, however, a more complicated behaviour, as documented in Figure 2. We observe a sequence of transitions between oscillatory and pulsatory behaviour, partly with quite narrow bands, in particular for the higher value $\Omega = 200$ (b).

While the oscillatory behaviour is what we are usually searching for, one might ask whether the pulsations are indeed as unphysical as they look like.

It is tempting here to think about Maunder type grand minima, for which measurements of ^{10}Be had indicated a rather unperturbed 11 years cycle (Beer, 1998), possibly connected, however, with a changed field parity (Sokoloff and Nesme-Ribes, 1994; Weiss and Tobias, 2016; Moss and Sokoloff, 2017). While our zero-dimensional model cannot decide about the parity of 3D-dynamo fields, it might be interesting to learn into what kind of behaviour our pulsations would translate once higher dimensional models were applied. Let us bravely assume for the moment that our pulsation regime would indeed correspond to the regime of grand minima. We might then ask for the details of transitions into, and from, such a minimum.

For $\Omega = 50$, $c = 0.4$, $p = 8$, and $h = 10$, Figure 3 illustrates such a process, by forcing the diffusion time τ to change between 1 and 0.7, and back. As can be expected from Figure 2a, the initial oscillations is replaced, at $t \approx 80$, by a pulsation, but recovers again at $t \approx 170$. It is most remarkable that the periodicity remains "on beat" during these two transitions.

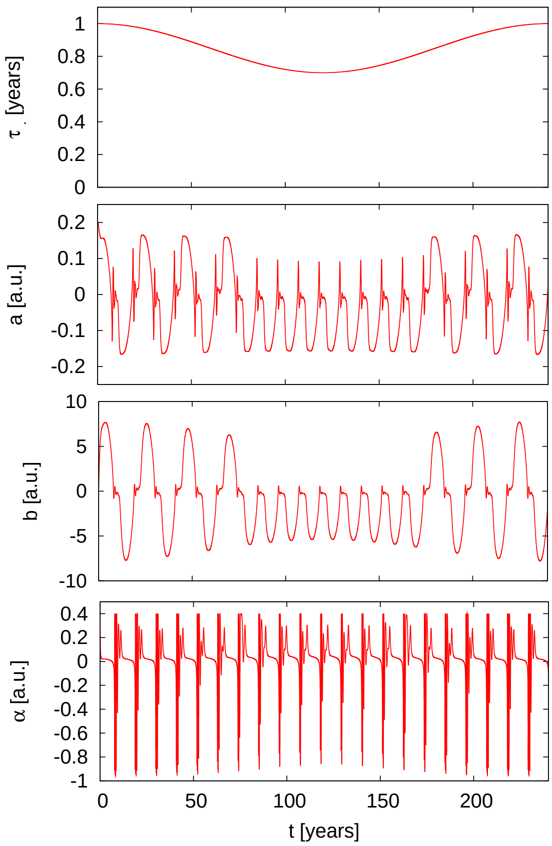

Figure 3. Transitions between oscillations and pulsations, and back, when changing the value of τ. Note the phase coherence throughout the "grand minimum".

3. Synchronization of Babcock-Leighton type dynamos

We turn now to the question whether a similar synchronization effect, as discussed in the previous section for Tayler-Spruit type dynamos, could also result from appropriate periodic changes in a flux-transport (or Babcock-Leighton) dynamo model. After describing the mathematical model, we will present simulations for three regimes which differ in the ordering of the relevant time-scales.

3.1. The model

We modify the model of equation system (1,2,3) according to Wilmot-Smith *et al.* (2006), who had introduced two specific time delays in their dynamo model. Consider the system

$$\frac{\mathrm{d}b(t)}{\mathrm{d}t} = \Omega(t)a(t-\tau_0) - \frac{b(t)}{\tau} \tag{4}$$

$$\frac{\mathrm{d}a(t)}{\mathrm{d}t} = \alpha(t-\tau_1)b(t-\tau_1) - \frac{a(t)}{\tau} \tag{5}$$

with

$$\alpha(t) = \alpha_0 \frac{1}{4}[1+\mathrm{erf}(b^2(t)-b^2_{\mathrm{min}}(t))][1-\mathrm{erf}(b^2(t)-b^2_{\mathrm{max}})]\,. \tag{6}$$

Apparently similar to system (1,2,3), this equation system has a number of peculiarities: First, as suggested by Wilmot-Smith *et al.* (2006), there are two time delays in the system. The first one, τ_0, represents the time needed for meridional circulation to transport the poloidal field, assumed to be produced by some Babcock-Leighton effect close to the solar surface, to the tachocline region, which is supposed to be the site of the Ω-effect. The second delay, τ_1, represents the rise time of flux-tubes which, in turn, transport toroidal field from the tachocline region to the surface. The effective α-effect is supposed to work only between a minimum $|b_{\mathrm{min}}|$ and a maximum value $|b_{\mathrm{max}}|$, where flux-tubes start to rise when they have grown to a minimum strength $|b_{\mathrm{min}}|$, and $|b_{\mathrm{max}}|$ is the field amplitude at which the quenching of α becomes significant. Recently, this model has been extended by Hazra, Passos and Nandy (2014) who added a stochastic fluctuation of α and argued that the Babcock-Leighton mechanism alone cannot recover the solar cycle from a grand minimum.

What is new in our model, compared to that of Wilmot-Smith *et al.* (2006), is the consideration of either b_{min} or Ω as periodically time-dependent. Again, we have in mind a gravitational perturbation with a period of 11.07 years (and some relative amplitude $\mathcal{A}$) as produced by the Venus-Earth-Jupiter system, i.e.

$$b_{\mathrm{min}}(t) = b_{\mathrm{min},0}(1+\mathcal{A}\sin(2\pi t/T_{\mathrm{tidal}})) \tag{7}$$

or

$$\Omega(t) = \Omega_0(1+\mathcal{A}\sin(2\pi t/T_{\mathrm{tidal}}))\,. \tag{8}$$

Why consider $b_{\min}$ as time-dependent? The idea traces back to Abreu *et al.* (2012) who had argued that the overshoot layer, which coincides spatially with the tachocline, is crucial for the storage and amplification of the magnetic flux tubes before they eventually erupt to the solar photosphere. The key factor here is the superadiabaticity, a dimensionless measure of the stratification of the specific entropy. The maximum field strength of a flux tube that can be stored prior to eruption is very sensitive to this superadiabaticity; hence small changes of it (as provoked, e.g., by tidal forces) could decide about the ultimate strength of the rising flux tube. In our model this will be emulated by a periodic time dependence of $b_{\min}$.

While such a periodic variation of the adiabaticity, and therefore of $b_{\min}$, is still speculative, a corresponding 11 years oscillation of Ω has indeed been observed in form of "torsional oscillations" (Brown *et al.*, 1989; Howe, 2009). Although these are commonly discussed in terms of a large-scale feedback of Lorentz forces (Malkus-Proctor effect (Malkus and Proctor, 1975)) or Λ-quenching (Kitchatinov, Rüdiger and Küker, 1994), they might also result directly from planetary influences (Zaqarashvili, 1997).

In the following we will assess if, and under which conditions, the two time periodic variations of $b_{\min}$ or Ω might lead to a synchronization of the dynamo. In all considered regimes we will find dynamos with a positive product of α and Ω to undergo pulsations, while dynamos with negative product of α and Ω usually oscillate.

3.2. Flux-transport dominated regime

We start with a regime in which the flux-transport is fast compared to dissipation effects, i.e. $\tau > \tau_0 + \tau_1$. Both positive and negative products of α and Ω will be considered.

3.2.1. Positive dynamo number

Figure 4 illustrates the results for two specific sets of dynamo parameters. With the constant parameters $\tau = 15$, $\tau_0 = 2$, $\tau_1 = 0.5$, $b_{\min} = 1$, $b_{\max} = 7$, we evaluate the two combinations $\alpha_0 = 0.17$, $\Omega = 0.34$ (a,c,e) and $\alpha_0 = 0.75$, $\Omega = 1.5$ (b,d,f). Panels (a) and (b) show $a(t)$, $b(t)$ and $\alpha(t)$ for the unperturbed system which turns out very similar to the pulsation behaviour shown in Figure 7 of Wilmot-Smith *et al.* (2006)). Panels (c) and (d) show (only) $a(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of $b_{\min}$ with a period of 11.07. In either case, the effect of periodic variation of $b_{\min}$ can be neglected. Correspondingly, panels (e) and (f) show (only) $b(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of Ω. Although some shifting effects are visible, there is no sign of any synchronization. One might guess, however, that this has simply to do with the large gap between the period of the forcing (11.07) and the periods of the unperturbed dynamo, i.e., for $\mathcal{A} = 0$, which are very large (43 for (a,c,e) and 88 for (b,d,f)).

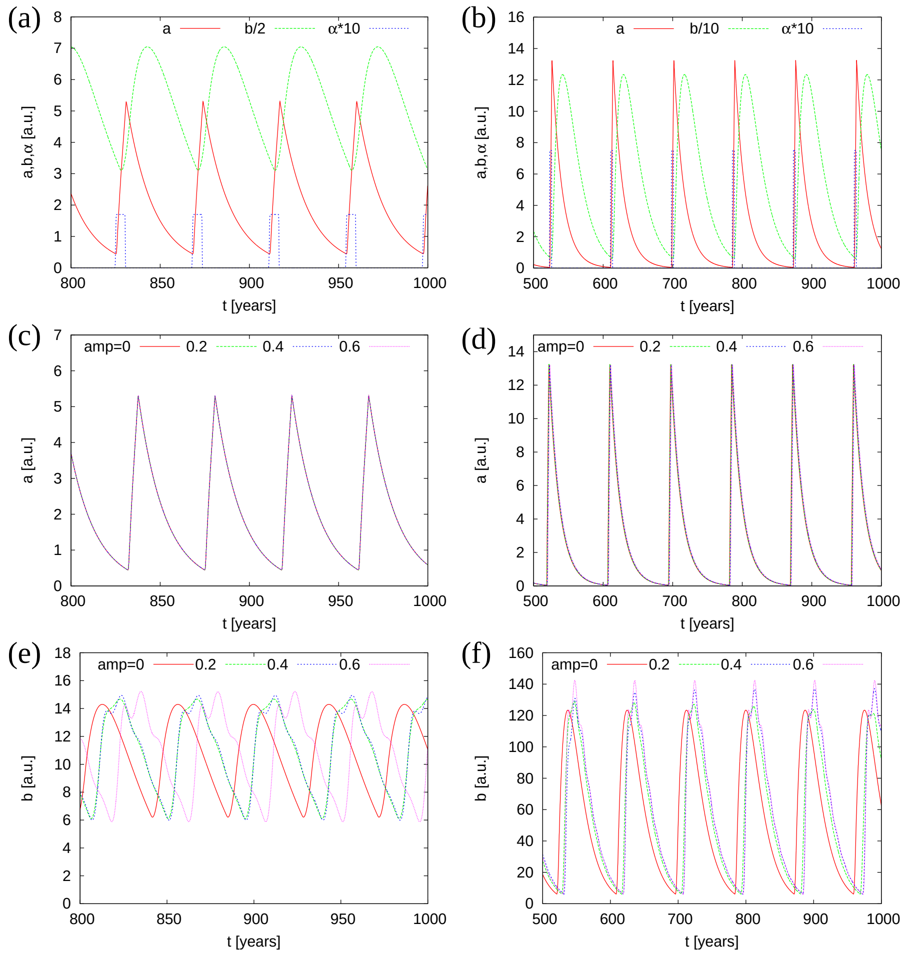

Figure 4. Time series in the flux-transport dominated regime with positive product of α and Ω. The constant parameters are: $\tau = 15$, $\tau_0 = 2$, $\tau_1 = 0.5$, $b_{\min} = 1$, $b_{\max} = 7$. The variable parameters are: $\alpha_0 = 0.17$, $\Omega = 0.34$ (a,c,e), $\alpha_0 = 0.75$, $\Omega = 1.5$ (b,d,f). Panels (a) and (b) show $a(t)$, $b(t)$ and $\alpha(t)$ for the unperturbed system (the behaviour is very similar to that shown in Fig. 7 of Wilmot-Smith *et al.* (2006)). Panels (c) and (d) show only $a(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of $b_{\min}$ with perturbation period 11.07. Evidently, the periodic variation of $b_{\min}$ has a negligible effect. Panels (e) and (f) show only $b(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of Ω. The variation of the amplitude of Ω leads to some shifting effect, but no synchronization is observed.

3.2.2. Negative dynamo number

Keeping all parameters as before, and changing only the sign of Ω, we obtain the time series of Figure 5. Rather than a pulsations we observe now the “desired” oscillation of the dynamo.

Again, periodic variations of $b_{\min}$ have no noticeable effect (c,d). The periodic perturbation of Ω leads to some shifting, but not to any synchronization. This time, synchronization might have been expected for (c,e) since the unperturbed dynamo (a) has a period of 29 which is not far from the double of the excitation period. Interestingly, the dynamo switches off completely for the largest perturbation $\mathcal{A} = 0.6$ (e), where in its weak phase Ω is reduced in such a way that the dynamo cannot work anymore.

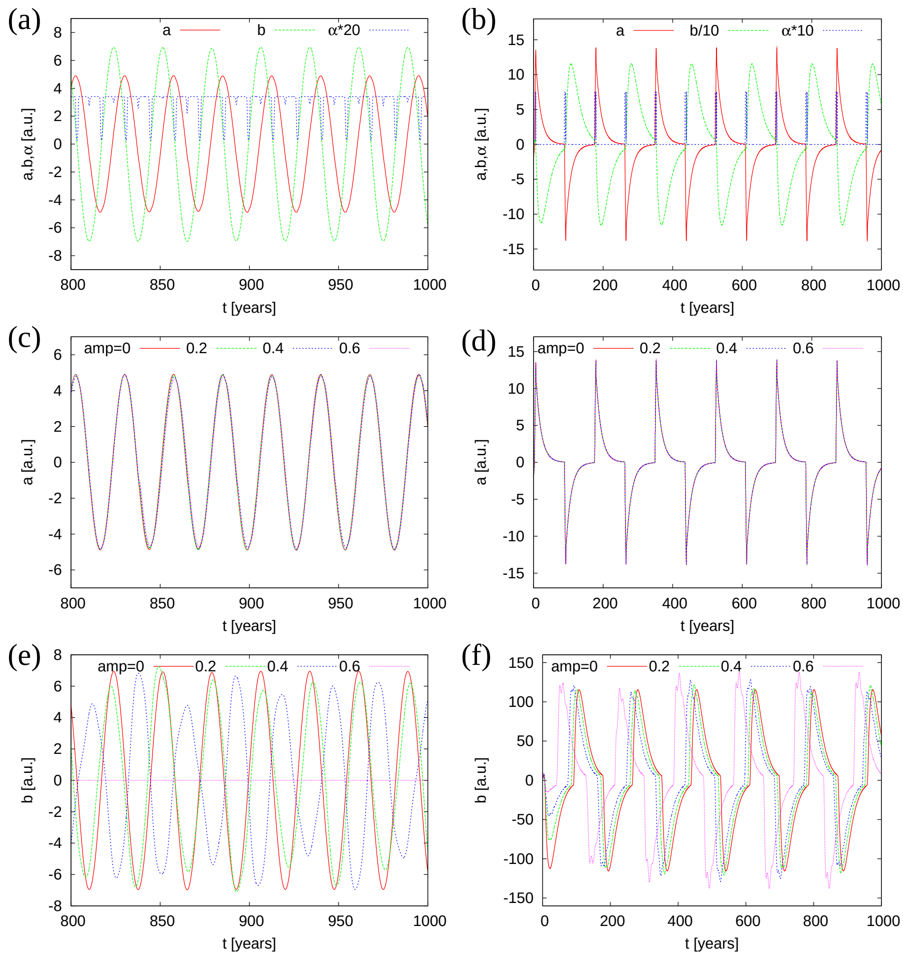

Figure 5. Similar as Figure (4) but with negative product of α and Ω. The constant parameters are: $\tau = 15$, $\tau_0 = 2$, $\tau_1 = 0.5$, $b_{\min} = 1$, $b_{\max} = 7$. The variable parameters are: $\alpha_0 = 0.17$, $\Omega = -0.34$ (a,c,e), $\alpha_0 = 0.75$, $\Omega = -1.5$ (b,d,f). Panel (a) corresponds to Figure 3, (b) to Figure 4 of Wilmot-Smith *et al.* (2006). Again, the periodic variation of $b_{\min}$ (c,d) has a negligible effect. The variation of the amplitude of Ω (e,f) has some shifting effect, but there is no synchronization. Note the switching off of the dynamo for $\mathcal{A} = 0.6$ in (e).

3.3. Diffusion dominated regime

Now we consider the opposite case that the diffusion is faster than the flux transport, i.e $\tau < \tau_0 + \tau_1$.

3.3.1. *Positive dynamo number*

We start again with positive product of α and Ω. Figure 6 shows the results for two specific sets of dynamo parameters. With the constant parameters $\tau = 1$, $\tau_0 = 10$, $\tau_1 = 4$, $b_{\min} = 1$, $b_{\max} = 7$, we consider the two combinations $\alpha_0 = 0.75$, $\Omega = 1.5$ (a,c,e) and $\alpha_0 = 2.5$, $\Omega = 5$ (b,d,f). Panels (a) and (b) show $a(t)$, $b(t)$ and $\alpha(t)$ for the unperturbed system. The erratic behaviour at the beginning of (a) is actually similar to that of Figure 12 of Wilmot-Smith *et al.* (2006) (which has slightly different $\alpha_0 = -1$, $\Omega = -3$, though).

For both dynamo strengths, neither the variation of $b_{\min}$ (c,d) nor that of $\Omega = 5$ (e,f) lead to synchronization, despite some shiftings and deformations of the signals.

3.3.2. *Negative dynamo number*

We continue with the case of a negative product of α and Ω in the diffusive regime. Again, Figure 7 shows the typical oscillations instead of the pulsations which had dominated for positive product. With the constant parameters left unchanged, the variable parameters are now: $\alpha_0 = 0.75$, $\Omega = -1.5$ (a,c,e), $\alpha_0 = 2.5$, $\Omega = -5.0$ (b,d,f). The behaviour in (b) corresponds to that of Figure 8 of Wilmot-Smith *et al.* (2006). The periodic variations of $b_{\min}$ (c,d) and Ω (e,f) have some effect, but provide no real synchronization.

3.4. Intermediate regime

While the previous two regimes were already studied (without the periodic perturbations) in Wilmot-Smith *et al.* (2006), we further consider here an intermediate regime characterized by the time ordering $\tau_1 < \tau < \tau_0$, i.e. the diffusion is slower than the rise of flux-tubes, but faster than the meridional circulation.

3.4.1. *Positive dynamo number*

Let us start again with positive product of α and Ω. With the constant parameters $\tau = 3$, $\tau_0 = 5$, $\tau_1 = 1$, $b_{\min} = 1$ $b_{\max} = 7$, we check the two combinations $\alpha_0 = 0.5$, $\Omega = 1$ (a,c,e) and $\alpha_0 = 2$, $\Omega = 4$ (b,d,f). Again panels (a) and (b) of Figure 8 show $a(t)$, $b(t)$ and $\alpha(t)$ for the unperturbed system which exhibit pulsations, as is usual for positive product of α and Ω. Panels (c) and (d) show $a(t)$ for the perturbed system, with various amplitudes $\mathcal{A}$ of the perturbation of $b_{\min}$. In either case (c) and (d), the effect of periodic variation of $b_{\min}$ leads only to a weak shift of the time series. Correspondingly, panels (e) and (f) show $b(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of Ω. In (e) we observe for the first time a synchronization (compare the systematic shift of the maxima for $\mathcal{A} = 0.6$ compared to those for $\mathcal{A} = 0$).

This synchronization effect is further quantified in Figure 9. For the parameters $\alpha_0 = 0.5$, $\Omega = 1$, and the close-by parameters $\alpha_0 = 0.6$, $\Omega = 1.2$, we demonstrate how the period of the dynamo oscillation approaches the double period 22.14 of the 11.07 period variation of Ω, when the amplitude of the

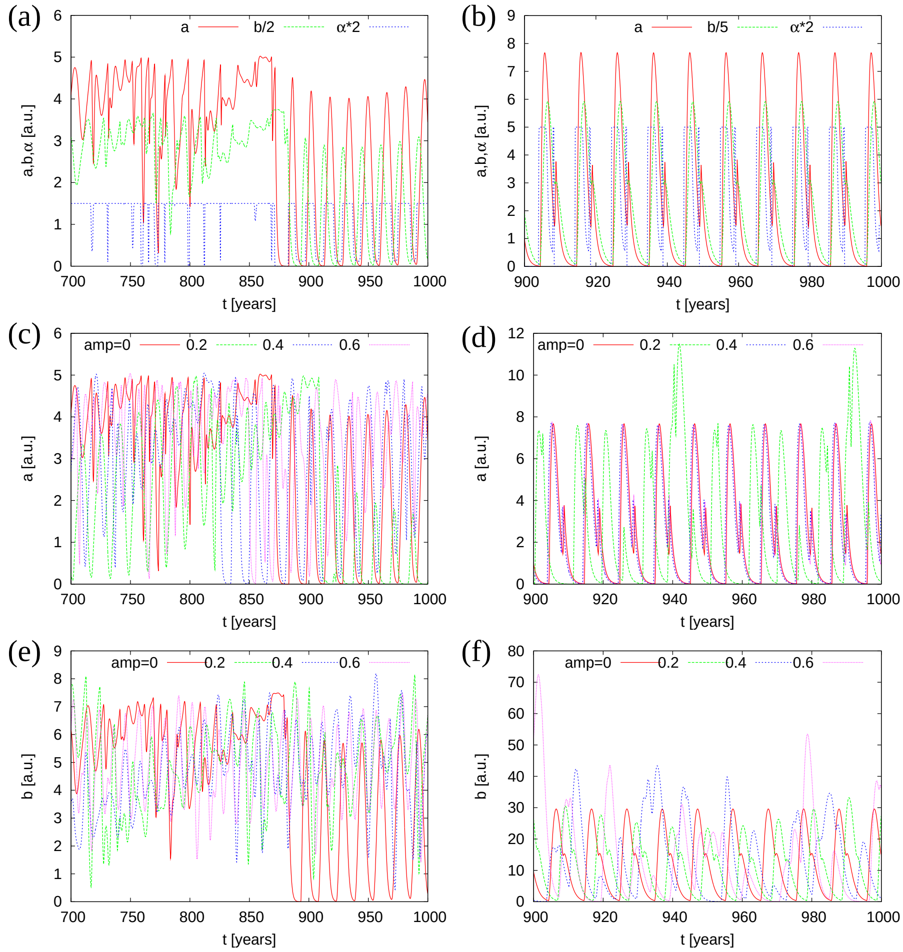

Figure 6. Time series in the diffusion dominated regime with positive product of α and Ω. The constant parameters are: $\tau = 1$, $\tau_0 = 10$, $\tau_1 = 4$, $b_{\rm min} = 1$, $b_{\rm max} = 7$. The variable parameters are: $\alpha_0 = 0.75$, $\Omega = 1.5$ (a,c,e), $\alpha_0 = 2.5$, $\Omega = 5.0$ (b,d,f). Panels (a) and (b) show $a(t)$, $b(t)$ and $\alpha(t)$ for the unperturbed system (the erratic behaviour at the beginning of (a) is similar to that of Figure 12 of Wilmot-Smith *et al.* (2006) which works at $\alpha_0 = -1$, $\Omega = -3$). Panels (c) and (d) show $a(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of $b_{\rm min}$ with perturbation period 11.07. Panels (e) and (f) show $b(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of Ω. The periodic variations of $b_{\rm min}$ and Ω provide some effects, but no synchronization.

perturbation is increased. What is observed here is, therefore, a synchronization of order 1:2 (Pikovsky, Rosenblum, and Kurths, 2001). The typical parabolic shape of the curves is representative of the occurrence of parametric resonance (Giesecke, Stefani, and Burguete, 2012; Giesecke, Stefani, and Herault, 2017). Still, the amplitude of the perturbation must be large (around 0.4) in order to provide synchronization. It is not very likely, that the typical 1 per cent changes of Ω, as observed in the sun Howe (2009), could provide such an entrainment effect.

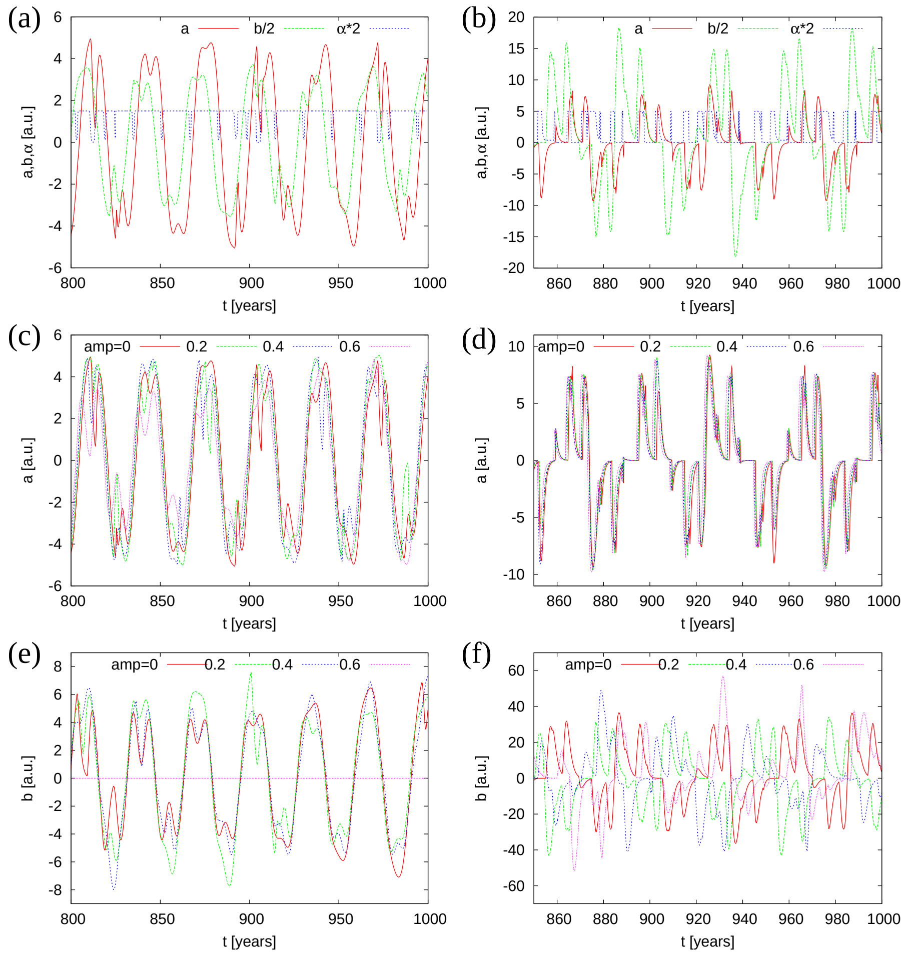

Figure 7. Similar as Fig. (6) but with negative product of α and Ω. The constant parameters are: $\tau = 1$, $\tau_0 = 10$, $\tau_1 = 4$, $b_{\min} = 1$, $b_{\max} = 7$. The variable parameters are: $\alpha_0 = 0.75$, $\Omega = 1.5$ (a,c,e), $\alpha_0 = 2.5$, $\Omega = 5.0$ (b,d,f). The behaviour in (b) corresponds to that of Figure 8 of Wilmot-Smith *et al.* (2006). The perturbation of $b_{\min}$ (c,d) and Ω (e,f) have some effect but do not lead to synchronization.

3.4.2. Negative dynamo number

We switch over now to the case of negative product of α and Ω und use, with all other parameters unchanged, $\alpha_0 = 0.5$, $\Omega = -1$ (Figure 8(a,c,e)), and $\alpha_0 = 2$, $\Omega = -4$ (b,d,f). We see that the stronger dynamo (b) undergoes a clear oscillation, whereas the weaker one (a) shows a behavior somewhere between oscillation and pulsation. While the perturbations of the stronger dynamo (d,f) lead only to minor changes without synchronization, the perturbation of the weaker dynamo is more interesting. From panel (e) we can try to infer the length of the dominant period in more detail when varying $\mathcal{A}$ (Figure 11). Although not

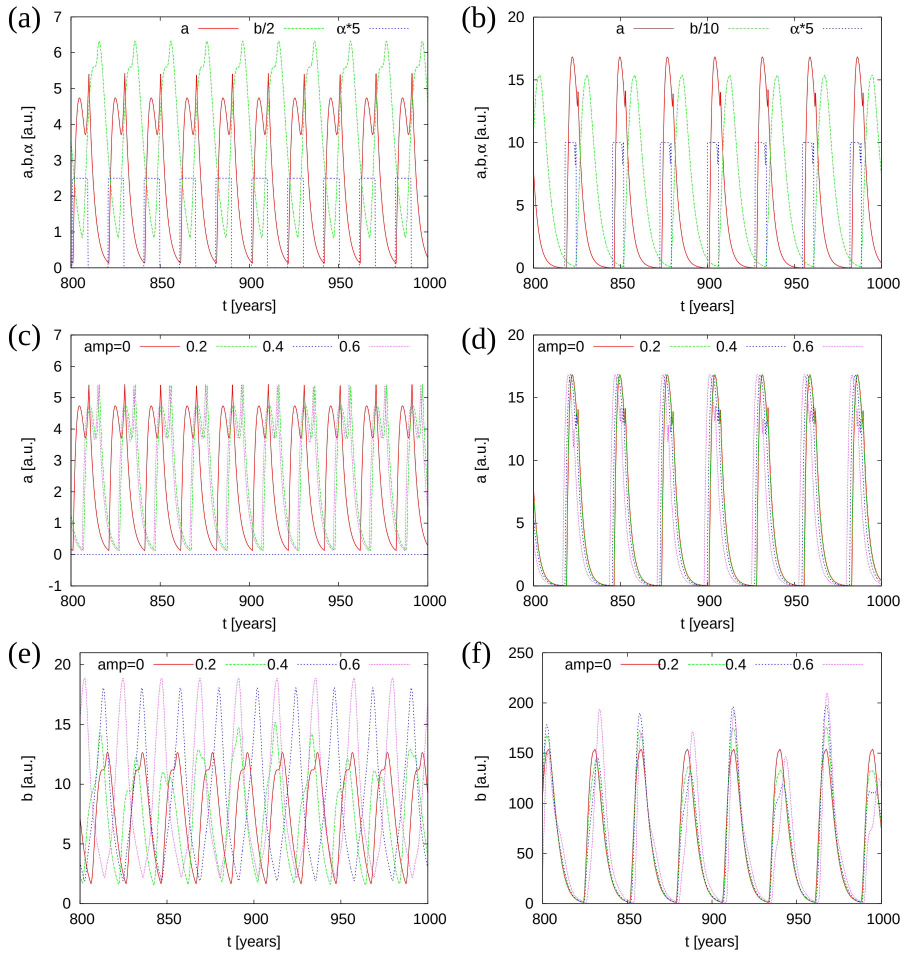

Figure 8. Time series in the intermediate regime with positive product of α and Ω. The constant parameters are: $\tau = 3$, $\tau_0 = 5$, $\tau_1 = 1$, $b_{\rm min} = 1$, $b_{\rm max} = 7$. The variable parameters are: $\alpha_0 = 0.5$, $\Omega = 1$ (a,c,e), $\alpha_0 = 2$, $\Omega = 4.0$ (b,d,f). Panels (a) and (b) show $a(t)$, $b(t)$ and $\alpha(t)$ for the unperturbed system. Panels (c) and (d) show (only) $a(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of $b_{\rm min}$ with perturbation period 11.07. Panels (e) and (f) show $b(t)$ for various amplitudes $\mathcal{A}$ of the perturbation of Ω. The variation of the amplitude of Ω leads to synchronization.

as clear as in the previous Figure 11, we still obtain a sort of synchronization, this time already for smaller values of $\mathcal{A} \sim 0.1$.

4. Discussion and summary

In his statistical analysis, titled ”Is there a chronometer hidden deep in the sun?”, Dicke (1978) had provided remarkable evidence for a positive answer to this fundamental question.

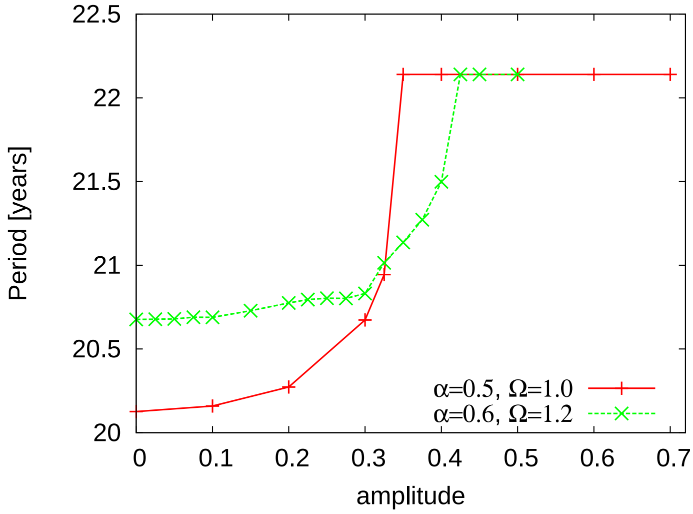

Figure 9. Dominant period of pulsation in dependence on the amplitude $\mathcal{A}$ of the Ω variation for $\alpha_0 = 0.5$, $\Omega = 1.0$ and $\alpha_0 = 0.6$, $\Omega = 1.2$ in the intermediate regime. Synchronization with twice the 11.07 driving period appears at $\mathcal{A} = 0.35$ and $\mathcal{A} = 0.425$, respectively.

Taking this finding seriously, we have assessed and compared the synchronizability of simplified dynamo models of the Tayler-Spruit and the Babcock-Leighton type. The synchronization of the Tayler-Spruit type dynamo is based on the resonant excitation of the $m = 0$ component of the α-effect (which results from the $m = 1$ Tayler instability) by an $m = 2$ tidal-like perturbation. We argued that a typical 11.07 years tidal perturbation, as exerted by the conjunction cycle of Venus, Earth and Jupiter, would end up in a 22.14 years oscillation of the dynamo field, at least for certain bands of the diffusion time τ. For intervening bands, and for larger values of τ, synchronization still occurs, but the 22.14 years oscillation is then replaced by a 11.07 years pulsation. Whether these pulsations are irrelevant for the sun, or whether they could be linked to the behaviour during grand minima (Weiss and Tobias, 2016; Moss and Sokoloff, 2017), remains to be clarified in higher-dimensional models. Remarkably, at any rate, is the phase coherence when passing from oscillations to pulsations, and back, as evidenced in Figure 3.

In contrast to this, a corresponding Babcock-Leighton type model proved rather stubborn to synchronization. Specifically, we have pursued two ideas on how such a synchronization could work. The first one bears on the concept of a sensitive adiabaticity, i.e. flux storage capacity of the tachocline, which might be easily influenced by minor perturbations as provoked by tidal forces. This idea was implemented by periodically changing the value $b_{\rm min}$ which represents the critical threshold of the toroidal field above which flux ropes would become magnetically buoyant.

The second idea took into account the - indeed observable - periodic change of the differential rotation, which is generally believed to be a *result* of the dynamo-

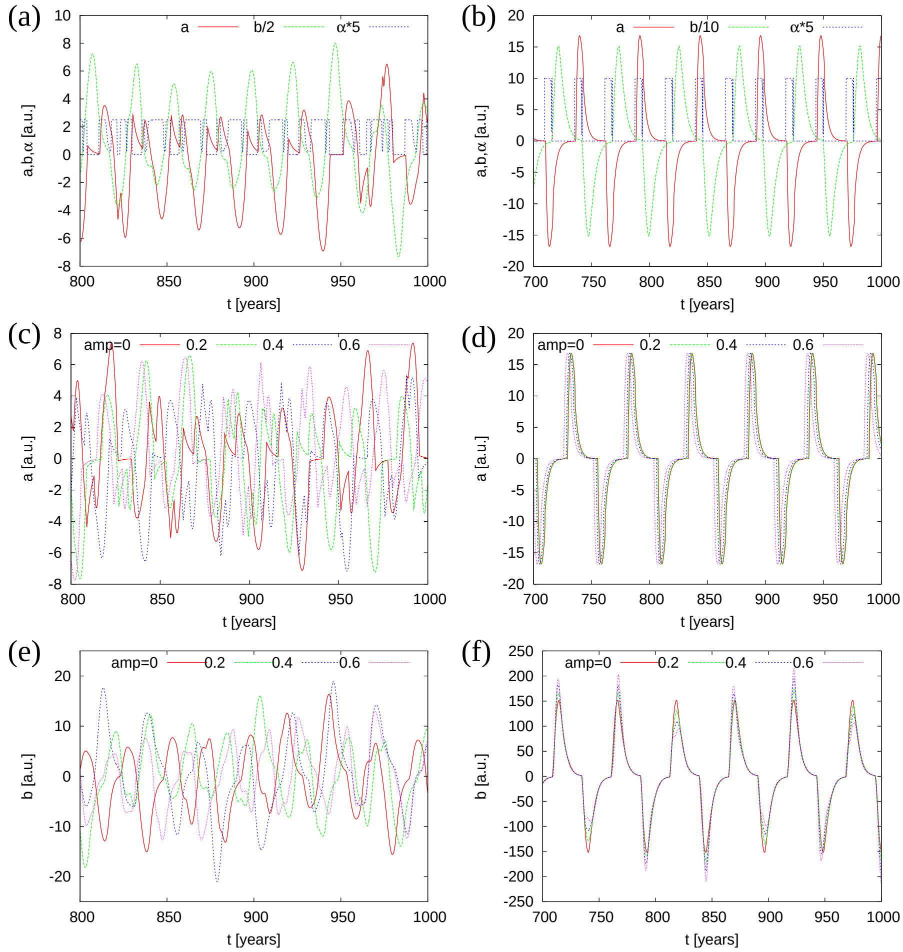

Figure 10. Similar as Fig. (8) but with negative product of α and Ω. The constant parameters are: $\tau = 3$, $\tau_0 = 5$, $\tau_1 = 1$, $b_{\rm min} = 1$, $b_{\rm max} = 7$. The variable parameters are: $\alpha_0 = 0.5$, $\Omega = -1$ (a,c,e), $\alpha_0 = 2$, $\Omega = -4.0$ (b,d,f). This time, the variation of the amplitude of Ω leads to synchronization for the weak dynamo (e), while the strong dynamo (f) is by and large unaffected.

generated magnetic field (Malkus-Proctor effect or Λ-quenching), but for which a direct planetary influence can not be excluded completely.

Neither in the flux-transport dominated nor in the diffusion-dominated regime we have observed any sign of synchronization. Only in the intermediate regime, and here for comparably strong perturbations of Ω, synchronization was obtained. For a positive product of α and Ω it was possible to synchronize the pulsations, for a negative product the oscillation was synchronized. We stress, however, that we have not covered the relevant parameter space exhaustively.

Therefore, our results should in no way be considered as an argument against Babcock-Leighton dynamo models. They only underline the peculiarity of synchronizing such types of dynamos by means of small periodic changes of their

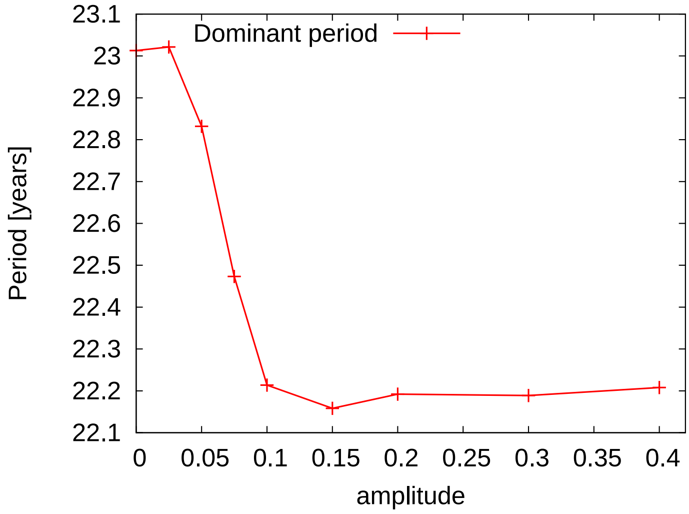

Figure 11. Dominant period of oscillation in dependence on the amplitude of the Ω variation for the intermediate regime with $\alpha_0 = 0.5$, $\Omega = -1$. Synchronization with twice the 11.07 years period appears at $\mathcal{A} = 0.1$. Since the oscillation is slightly irregular, the resonance is not as clearly expressed as in Figure 9.

governing physical parameters. This is in stark contrast to the amazingly simple and robust synchronizability of Tayler-Spruit type dynamos which need only a weak $m = 2$ forcing to stoke the TI-related oscillations of α. We recall here our previous finding (Stefani *et al.*, 2016) that the vacillations between left- and right-handed TI modes do barely change the kinetic energy of the flow, so that not much energy is needed to excite them. The estimated tidal velocities of the order of 1 m/s might serve this purpose well.

At any rate, it goes without saying that much more stringent modeling is required to corroborate - or reject - our TI-based synchronization model of the solar dynamo.

Acknowledgments This work was supported by Helmholtz-Gemeinschaft Deutscher Forschungszentren (HGF) in frame of the Helmholtz alliance LIMTECH.

Disclosure of Potential Conflicts of Interest

The authors declare that they have no conflicts of interest.

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