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Multiplet of skyrmions states on a curvilinear defect: skyrmion lattices as a ground state

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We show that the presence of a localized curvilinear defect drastically changes magnetic properties of a thin perpendicularly magnetized ferromagnetic film. For a large enough defect amplitude a discrete set of equilibrium magnetization states appears forming a ladder of energy levels. Each equilibrium state has either zero or unit topological charge, i.e. topologically trivial and skyrmion multiplets generally appear. Transitions between the levels with the same topological charge are allowed and can be utilized to encode and switch a bit of information. There is a wide range of geometrical and material parameters, where the skyrmion level has the lowest energy. As a result a periodically arranged curvilinear defects generate a skyrmion lattice as the ground state.

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Introduction.—An isolated magnetic chiral skyrmion is a localized topologically nontrivial excitation, which may appear in a perpendicularly magnetized ferromagnetic film, when the Dzyaloshinskii-Moriya interaction (DMI) is present [1–3]. During the last years isolated skyrmions have been widely considered as data carriers in spintronic data storage and logic devices of a racetrack configuration [3–8]. Besides nanotracks [3–5, 9] individual skyrmions were obtained in nanodisks [5, 10, 11]. Due to nonlocal magnetostatic effects in confined magnetic objects, the skyrmion state can have lower energy as compared to the topologically trivial homogeneous state [5, 10].

In contrast to individual skyrmions, their periodic 2D arrays, i.e. skyrmion lattices [12–16] are relevant for electronics relying on topological properties of materials. In this regard, dense lattices of small-sized skyrmions facilitate the signal readout in prospective spintronic devices by enhancing the topological Hall effect [17–20]. Typically, skyrmion lattices are in-field low temperature pocket phases [12–15] which hinder their application potential.

Here we demonstrate that magnetic skyrmion can be pinned on a localized curvilinear defect and can have two or more equilibrium states with very different skyrmion radius, i.e. one deals with a multiplet of skyrmion states. In this context, a doublet of skyrmion states can be used to represent a single bit of information, see Fig. 1(b). This unique feature of a skyrmion on a curvilinear defect paves the way towards a new memory concept which is based on immobile skyrmions.

It is remarkable that when the radii of the skyrmion and the curvilinear defect are comparable, the energy of the skyrmion state can be the lowest one within the class of radially symmetrical solutions. In this way we

demonstrate the possibility to realize the lowest energy skyrmion states on a curvilinear defect relying on local interactions only without the need of any magnetic field or magnetostatic effects. As a consequence, a periodically arranged lattice of the defects can generate a skyrmion lattice as a ground state, see Fig. 1(c). It is important that such a skyrmion lattice exists in zero magnetic field and for a temperature regime, which allows individual skyrmions, e.g. for room temperatures [9, 21]. In contrast to the planar case [22, 23] the proposed zero-field lattice does not require four-spin interactions, it can have an arbitrary symmetry and its length scale can be much larger than atomic one. The proposed static reconfigurable lattice of skyrmions opens new exciting perspective for the manipulation and control of spintronic devices relying on the topological Hall effect [17–20].

Model.—Similarly to the well studied planar case [2, 25–29] the form of a chiral skyrmion is mainly determined by competition of three local interactions: exchange, easy-normal anisotropy and DMI. Thus the energy functional of our model reads

$$E = L \int [A\mathcal{E}_{\text{ex}} + K(1 - m_n^2) + D\mathcal{E}_{\text{D}}] d\mathcal{S}, \quad (1)$$

here L is the film thickness and the integration is performed over the film area. The first term of the integrand is the exchange energy density with $\mathcal{E}_{\text{ex}} = \sum_{i=x,y,z} (\partial_i \mathbf{m})^2$, and A being the exchange constant. Here $\mathbf{m} = \mathbf{M}/M_s$ is the unit magnetization vector with M_s being the saturation magnetization. The second term is the easy-normal anisotropy where $K > 0$ and $m_n = \mathbf{m} \cdot \mathbf{n}$ is the normal magnetization component with \mathbf{n} being the unit normal to the surface. The exchange-anisotropy competition results in the magnetic

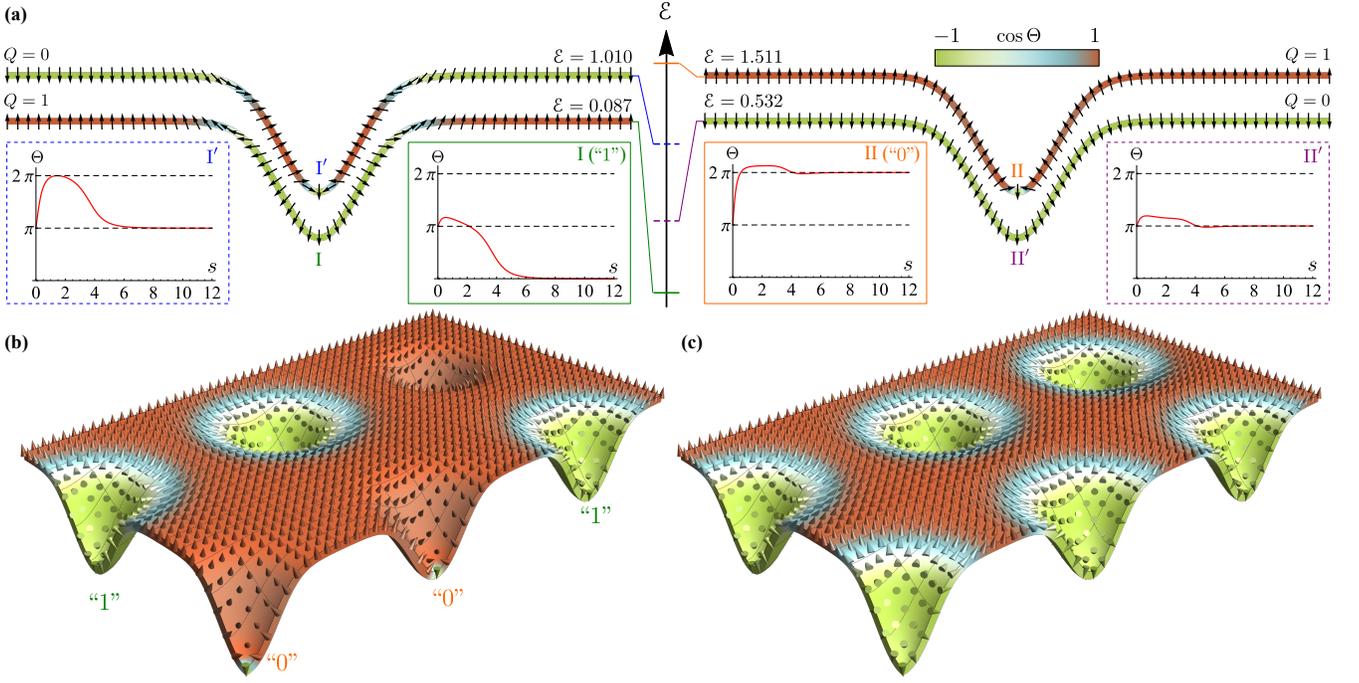


FIG. 1. **Individual skyrmion profiles and skyrmion lattices.** (a): Equilibrium magnetization states of a Gaussian concave bump ($\mathcal{A} = -3$, $r_0 = 1$ and $d = 1$) are shown by means of vertical cross-sections. Arrows show the magnetization distribution and color corresponds to the normal component $m_n = \cos \Theta$. The corresponding solutions $\Theta(s)$ of Eq. (3) are shown in the insets I, II, I' and II'. Vertical axis $\varepsilon = E/E_{BP}$ shows distribution of the corresponding energy levels obtained from (S8) with $E_{BP} = 8\pi AL$ being energy of the Belavin-Polyakov soliton [24]. (b): Two skyrmion states with big (I) and small (II) radii are shown on the same bumps arranged in a square lattice. These skyrmion solutions can be considered as logical states “1” and “0” of an information bit. (c) Skyrmion lattice as a ground state.

length $\ell = \sqrt{A/K}$, which determines a length scale of the system. The last term in (1) represents DMI with $\mathcal{E}_D = m_n \nabla \cdot \mathbf{m} - \mathbf{m} \cdot \nabla m_n$. Such a kind of DMI originates from the inversion symmetry breaking on the film interface; it is typical for ultrathin films [28, 30, 31] or bilayers [32], and it results in so called Néel (hedgehog) skyrmions [5, 33]. For a surface of rotation with a radially symmetrical magnetization distribution the same type of DMI effectively appears in the exchange term due to curvature effects [34–36], thus a direct competition takes place. This results in a skyrmion solution of Néel type. Another types of DMI may lead to a spiral-like skyrmion, which are intermediate ones between Néel and Bloch types. This case would require a more bulk analysis.

In our model we disregard nonlocal magnetostatic effects. Still, in stark contrast to the planar case, this is not required for the realization of a skyrmion lowest energy state [37]. We also assume magnetization homogeneity along the normal direction, which is valid for $L \lesssim \ell$.

We now consider a curvilinear defect of the film, which is formed by a complete revolution of the curve $\gamma = r\mathbf{e}_x + z(r)\mathbf{e}_z$ around z -axis – a bump. The parameter $r \geq 0$ denotes the distance to the axis of rotation. Curvilinear properties of the surface at each point are completely

determined by two principal curvatures k_1 and k_2 , see the explicit forms in Sec. S.I in [38].

The constrain $|\mathbf{m}| = 1$ is utilized by introducing the spherical angular parameterization $\mathbf{m} = \sin \theta \cos \phi \mathbf{e}_s + \sin \theta \sin \phi \mathbf{e}_\chi + \cos \theta \mathbf{n}$ in the local orthonormal basis $\{\mathbf{e}_s, \mathbf{e}_\chi, \mathbf{n}\}$, where \mathbf{e}_s is unit vector tangential to the curve γ , and $\mathbf{e}_\chi = \mathbf{n} \times \mathbf{e}_s$ is the unit vector in azimuthal direction, see Fig. S1. Expressions for \mathcal{E}_{ex} and \mathcal{E}_D for a general case of a local curvilinear basis were previously obtained in Ref. 34 and Ref. 36, respectively. Without edge effects (e.g. for a closed surface or for an infinitely large film) the DMI energy density can be reduced to the form

$$\mathcal{E}_D = \sin^2 \theta [2(\nabla \theta \cdot \boldsymbol{\varepsilon}) + \mathcal{H}], \quad (2)$$

where $\boldsymbol{\varepsilon} = \cos \phi \mathbf{e}_s + \sin \phi \mathbf{e}_\chi$ is normalized projection of the vector \mathbf{m} on the tangential plane and $\mathcal{H} = k_1 + k_2$ is the mean curvature. Expression (2) clearly shows the appearance of an effective DMI-driven uniaxial anisotropy proportional to the mean curvature. It has the same curvilinear origin as the recently obtained exchange-driven anisotropy and DMI [34, 35]. Depending on sign of the product $D\mathcal{H}$ this anisotropy can be of easy-normal ($D\mathcal{H} > 0$) or easy-surface ($D\mathcal{H} < 0$) type [39].

One can show (see Sec. S.II) that the total energy (1)

is minimized by a stationary solution $\mathbf{m} = \sin\Theta\mathbf{e}_s + \cos\Theta\mathbf{n}$, where function $\Theta(s) \in \mathbb{R}$ is determined by equation

$$\Delta_s\Theta - \sin\Theta\cos\Theta\Xi + r'r^{-1}(d - 2k_2)\sin^2\Theta = \mathcal{H}'. \quad (3)$$

Here and below all distances are considered dimensionless and they are measured in units of the magnetic length ℓ , the prime denotes the derivative with respect to the natural parameter s – the arc length along γ . The radial part of the Laplace operator reads $\Delta_s f = r^{-1}(rf)'$. The function $r(s)$ unambiguously determines the surface and its curvilinear properties, see Sec. S.I. The dimensionless DMI constant $d = D/\sqrt{AK}$ is the only material parameter, which controls the system, and $\Xi = 1 + r^{-2}r'^2 - k_2^2 + d\mathcal{H}$.

It is important to note that any solution of Eq. (3) and its energy (S8) are invariant with respect to the transformation $\Theta \rightarrow \Theta + \pi$, i.e. any solution is doubly degenerate with respect to the replacement $\mathbf{m} \rightarrow -\mathbf{m}$ [40]. Consequently, one can fix the boundary condition $\Theta(0) = \pi$ at the bump center without loss of generality and consider different boundary conditions at the infinity: $\Theta(\infty) = n\pi$ with $n \in \mathbb{Z}$. The same invariance takes place for the transformation $k_1 \rightarrow -k_1$, $k_2 \rightarrow -k_2$, $d \rightarrow -d$, $\Theta \rightarrow 2\pi - \Theta$. This property is reflected in the symmetry of the diagram of skyrmion states, see Fig. 3.

Following Ref. 36 one can show that topological charge (mapping degree to S^2) of such a radially symmetrical solution on a localized bump reads (see Sec. S.III) $Q = \frac{1}{2}[\cos\Theta(\infty) - \cos\Theta(0)]$. It means that only values $Q = 0$ (for odd n) or $Q = 1$ (for even n) are possible. A state with $Q = -1$ appears under the transformation $\mathbf{m} \rightarrow -\mathbf{m}$ applied to the state with $Q = 1$.

Due to the presence of the right-hand-part driving term in Eq. (3) the trivial solutions $\Theta \equiv 0, \pi$ (i.e. $\mathbf{m} = \pm\mathbf{n}$) are generally not possible. It means that even for large anisotropy the magnetization vector deviates from the normal direction, except surfaces with $\mathcal{H} = \text{const}$, e.g. planar films, spherical and minimal surfaces. Such a prediction was previously made in Ref. 34. An analogous driving appears in 1D curvilinear wires and results in curvature induced domain wall motion along the curvature gradient [41]. Thus, Eq. (3) makes one expect a leading role of the mean curvature gradient in the analogous curvature induced skyrmion motion.

In the planar film limit $k_1 = k_2 \equiv 0$, $\mathcal{H} \equiv 0$ and $r(s) = s$. In this case Eq. (3) is transformed into the well-known [2, 26, 29, 33] chiral skyrmion equation. Such a planar system is controlled by the only parameter d . There is the critical value $d_0 = 4/\pi$, which separates two ground states, namely the uniform state $\mathbf{m} = \mathbf{n}$ for the case $|d| < d_0$, and helical periodical state for $|d| > d_0$ [2, 26, 29, 33]. For the case $|d| < d_0$ the planar form of Eq. (3) has a stable topological ($Q = 1$) solution – a skyrmion, which has the following features: (i) for

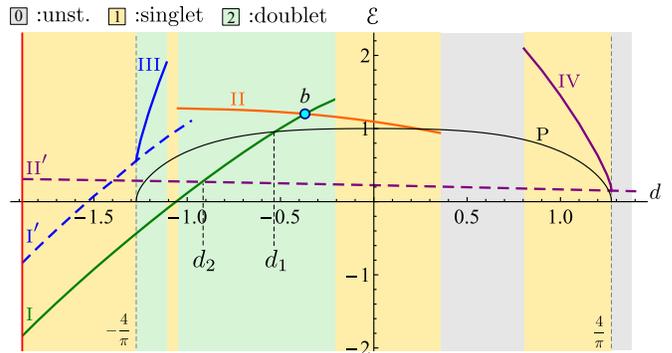


FIG. 2. **Energies of different solutions.** Solid lines I–IV and dashed lines I', II' show energies (S8) of topological non-trivial (skyrmion) and trivial states, respectively for the bump with $\mathcal{A} = 2$ and $r_0 = 1$. Energy of the planar skyrmion is shown by the thin line P. States I, II, I', II' are similar to the same name states in Fig. 1. States III and IV correspond to skyrmions whose radius much exceeds the lateral bump size, see Figs. S3, S6. The background filling corresponds to the number of stable skyrmion states, see also Fig. 3.

a given value of d the skyrmion solution is unique; (ii) the skyrmion energy is always higher than energy of the uniform perpendicular state, i.e. the planar skyrmion is an excitation of the ground state. As we show below, these well-known properties are violated in the general case of the curvilinear defect.

Gaussian bump.—As an example, we consider a class of localized curvilinear defects in form $z(r) = \mathcal{A}e^{-r^2/(2r_0^2)}$. Here amplitudes $\mathcal{A} > 0$ and $\mathcal{A} < 0$ correspond to bumps that are convex or concave, respectively, and r_0 determines the bump width. In Figs. 1(a) we demonstrate stable equilibrium solutions of Eq. (3) for certain values of parameters. There is a number of principal differences as compared to the planar case:

- (i) Topological ($Q = 1$) as well as trivial ($Q = 0$) solutions are generally not unique: for given values of geometrical and material parameters a set of equilibrium magnetization states can appear with a ladder of energy levels. This makes the curvilinear defect conceptually similar to a quantum well with a finite number of discrete energy levels. However, in contrast to the quantum systems the transitions between levels with the same Q are only allowed. Such a transitions are expected to be accompanied by emission or absorption of magnons.
- (ii) The lowest energy level can be topological non-trivial ($Q = 1$). It is remarkable that this effect appears due to the local interactions only. As a consequence, curvilinear defects arranged in a periodical lattice generate a zero-field skyrmion lattice as a ground state of the system, see Fig. 1(c).

Let us consider skyrmions of small and big radii, which are shown in the Fig. 1 as states I(“1”) and II(“0”), respectively. Their radii [42] are close to extrema points of the Gauß curvature $\mathcal{K} = k_1k_2$, which plays an important

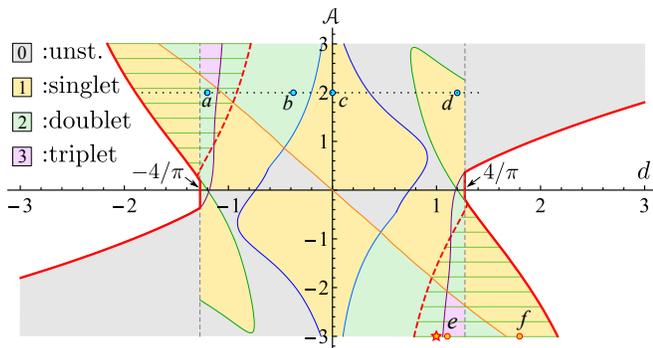


FIG. 3. **Diagram of skyrmion states for Gaussian bump with $r_0 = 1$.** In the white area the skyrmion solutions does not exist. Number of any other area (see legend) coincides with the number of stable skyrmion solutions. At least one skyrmion solution exists within the gray area ‘0’, however the bump center is a position of unstable equilibrium for it. Within the other areas the corresponding number of skyrmion are pinned at the bump center. The horizontal dashed shows areas where the lowest energy level is skyrmion one. Star marker shows parameters of Fig. 1. The solutions spectra for points a - f are presented in Sec. S.V. Dotted horizontal line $A = 2$ corresponds to Fig. 2.

role in a coupling between topological defects and curvature [43, 44]. On the other hand, the radius of skyrmion II is of one order of magnitude smaller than radius of the skyrmion, which is stabilized by the intrinsic DMI in a planar film for the same value of d . Thus, the small radius skyrmion is stabilized mostly by the curvature effects [36, 45–47], while the big radius skyrmion is stabilized due to the simultaneous action of the intrinsic DMI and curvature. Structures similar to the big radius skyrmions were previously observed experimentally in Co/Pd and Co/Pt multilayer films containing an array of curvilinear defects in form of spherical concavities [48, 49] as well as convexes [50]. The topologically trivial state I' can be treated as a joint state of small and big radii skyrmions, which compensate topological charges of each other. And the state II' is an intermediate one between uniform $\mathbf{m} = -\mathbf{e}_z$ and normal $\mathbf{m} = -\mathbf{n}$ states, what reflects the competition between exchange and anisotropy interactions. Note that states I and I' as well as states II and II' differ in presence or absence of the small-radius skyrmion at the bump center. In Fig. 1(a) we show only stable solutions with $\Delta\Theta = |\Theta(\infty) - \Theta(0)| \leq \pi$. Solutions with the larger phase incursion, so called skyrmioniums [29, 51] or target skyrmions [10, 27, 33, 52, 53], are in principle also possible.

The appearance of skyrmions of type I (big radius) and type II (small radius) is a common feature of the considered curvilinear defects, and takes place for concave as well as for convex geometries. In order to illustrate the last statement we show the energies $\mathcal{E}(d)$ for all equilibrium states, which appear for a convex bump, see

Fig. 2. For the given geometrical parameters we found numerically all solutions of Eq. (3) with $\Delta\Theta \leq \pi$ for each value d . Then a stability analysis (see Sec. S.IV) was applied for each of the solutions. Finally, four stable topological (skyrmion) solutions (lines I-IV) and two stable non-topological solutions (lines I' and II') are found. The magnetization distributions, that correspond to all of these solutions, are shown in Sec. S.V. Lines I and II correspond to the considered above big (“1”) and small (“0”) radius skyrmions, respectively. Remarkably these states can have equal energies – point b in Fig. 2. This makes the proposed application for the storing of a bit of information more practically relevant: switching between states “0” and “1” can be easily controlled by application of pulse of magnetic field directed along or against the vertical axis.

As well as for the concave geometry (Fig. 1) the big radius skyrmion on a convex bump can have the lowest energy in the system (the range $d < d_2$). It is important to note that there is a range of parameters $-4/\pi < d < d_1$ where a skyrmion on a bump has lower energy than a planar skyrmion for the same d . This implies that flexible enough planar films can spontaneously undergo a skyrmion induced deformation. Such a soliton-induced magnetic film deformation was earlier predicted for cylindrical geometries [54–58].

In order to systematize possible skyrmion solutions, that can appear on Gaussian bumps, we build a diagram of skyrmion states, see Fig. 3. We apply the same method as for the case of Fig. 2, but restricting ourselves with skyrmion solutions. The following general features can be established: (i) The range of skyrmions existence widens with increasing of the bump amplitude. (ii) For a wide range of parameters (gray area ‘0’) the skyrmion centered on the bump experiences a displacement instability because the bump center is a position of unstable equilibrium. (iii) In the vicinity of the critical value $d = \pm 4/\pi$ there is a wide area of parameters (the dashed area), where the skyrmion state has the lowest energy in the class of radially symmetrical solutions.

Conclusions.— We have generalized the skyrmion equation for the case of an arbitrary surface of rotation. Considering specifically a Gaussian bump we have shown that its skyrmion solution is generally not unique — a discrete ladder of equilibrium skyrmion states appears. We propose to use a suitably shaped curvilinear defect with a doubly degenerate skyrmion state as carrier of a bit of information. We also predict the effect of spontaneous deformation of an elastic magnetic film under skyrmion influence. Finally, we found a wide range of parameters, where a skyrmion pinned on the bump has lower energy than other possible states. This feature can be used for generating of a ground state zero-field skyrmion lattice.

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