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Superconductivity in single-crystalline aluminum- and gallium-hyperdoped germanium

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Since the discovery of superconductivity in diamond [1] with a boron content above the equilibrium solid solubility (ESS), many studies have been performed to find new “superconducting semiconductors.” Such a materials class would enable the monolithic integration of quantum and conventional semiconductors was given in recent review articles [6–9]. A brief introduction into the research field of superconducting semiconductors was given in recent review articles [6–9].

The term “superconducting semiconductor” is a bit misleading, since in a semiconductor, the carriers necessary for superconductivity is impossible. Therefore, the semiconductor has to be heavily doped above the metal-insulator transition (MIT). It turned out that an acceptor concentration in excess of 1 at. % (i.e., above $5 \times 10^{20}$ cm$^{-3}$) is required to induce the superconductivity in germanium. Such concentration is higher than the ESS of typical acceptors in Ge. However, for a perfect monolithic integration of superconducting nanocircuits in semiconductor devices, a homogeneous and single-crystalline structure is desirable. It remains an unresolved question whether superconducting semiconductor films of sufficient quality can be fabricated at all
by today’s top-down selective doping technologies and which semiconductor-acceptor combination is most promising. Since the tendency for disorder and cluster formation by hyperdoping increases with the covalent bond strength of the semiconductor and decreases with growing acceptor solid solubility, the Ge:Ga system appears to be favorable compared to diamond:B and Si:B [14]. Previous studies demonstrated that conventional implantation doping of Ge with Ga enables a maximum hole concentration of $6.6 \times 10^{20} \text{cm}^{-3}$ after annealing at 450°C for 1 h [24]. Higher temperatures of conventional long-term annealing led to Ga clustering. In order to reduce acceptor diffusion and clustering, FLA in the millisecond range without layer melting is an appropriate method [25–28]. With this method hole concentrations up to $1.4 \times 10^{21} \text{cm}^{-3}$ and superconductivity at critical temperatures below 0.5 [4,15] and 2.0 K [29] have been achieved in Ge layers with about 6 and 8 at.% Ga content, respectively. Unfortunately, the layers are nanocrystalline [4,15], and the activation level of the Ga acceptors varies from sample to sample up to a factor of 2, which is due to the formation of Ga-rich nanocrystallites [29]. Single-crystalline Ge:Ga has been obtained by RTA [14]. However, in this case, a large amount of the Ga atoms accumulates as an amorphous film at the SiO$_2$/Ge interface. This interface layer becomes superconducting below 6 K, which is similar to the critical temperature of Ga clusters.

In this paper we show that an optimized FLA process can be used to fabricate single-crystalline, superconducting layers of hyperdoped $p$-type Ge. In addition to Ga doping, we also investigate Al doping. Similar to Ga, Al has high ESS but a higher diffusivity than Ga and is more difficult to activate. As shown recently, ion implantation of Al into Ge followed by conventional thermal annealing results in a maximum hole concentration of only $1 \times 10^{20} \text{cm}^{-3}$ [30]. Here we show that the maximum carrier concentration in Ga- and Al-implanted Ge followed by FLA exceeds $10^{21} \text{cm}^{-3}$. FLA suppresses the dopant diffusion and segregation. The recrystallized Ge is single crystalline with critical temperatures of $T_c \sim 0.5$ K. Moreover, first-principles investigation of superconductivity in Al-doped and Ga-doped Ge using $ab$ $initio$ calculations within the Eliashberg-McMillan theory reveals that the Ga:Al system behaves similar to a Ge:Ga covalent superconductor, where the critical temperature can be tuned by the carrier concentration.

II. EXPERIMENTAL

A. Sample fabrication

$N$-type (Sb-doped, $\rho > 10 \Omega \text{cm}$), (100)-oriented Ge wafers are used as substrates for acceptor implantation in order to electrically isolate the processed layer from the substrate by formation of a $p$-$n$ junction. First, a 30-nm-thick SiO$_2$ cover layer is sputter-deposited to protect the Ge surface during ion implantation and annealing. Then the wafers are implanted with Ga or Al ions with different fluencies of 1, 2, and $4 \times 10^{16} \text{cm}^{-2}$ and energies of 100 keV for Ga$^+$ and 50 keV for Al$^+$ ions. The implantation energies are chosen in such a way that the acceptor profiles are similar, with a maximum acceptor concentration at a depth of 60 nm, as predicted by the SRIM simulation code [31]. Figure 1(a) shows the calculated Ga and Al distributions implanted into the SiO$_2$/Ge wafers for an ion fluence of $2 \times 10^{16} \text{cm}^{-2}$. The peak concentration and the depth distribution of Al and Ga within Ge are different for the same ion fluence and similar projected ion range $R_p$. After implantation, a heavily doped amorphous surface layer of about 120 nm width with a relatively sharp interface to the single-crystalline Ge substrate was formed (see Supplemental Material, Fig. S1a) [15,32]. The presence of a sharp amorphous/crystalline interface is an important precondition for the explosive solid-phase epitaxy process which appears during millisecond-range FLA of the implanted layer [29].

The Al peak concentration is about 6 at.%, whereas the Ga concentration exceeds 10 at.%. This is due to different interactions of light (Al) and heavy (Ga) elements with germanium during the ion-implantation process. This is due to different stopping power and energy loss straggling for different ions within the solid. For heavier ions the stopping power and the energy loss straggling are higher, causing a smaller FWHM of the depth distribution of the implanted ions and, in consequence, a higher peak concentration for the same ion fluence.

In order to activate the dopants and recrystallize the implanted layer, we have used a strongly nonequilibrium thermal processing, i.e., flash lamp annealing. Implanted samples were annealed either from the front side (f-FLA) or from the rear side (r-FLA) with an energy density deposited to the sample surface in the range of 50–130 J cm$^{-2}$. The annealing time was 3, 6, or 20 ms. The influence of the annealing time on the recrystallization process of the implanted layer is presented in Supplemental Material (see Fig. S1b [15,32]). Figure 1(b) shows the temperature distribution within the implanted layer after front- and rear-side FLA for 20 ms. The f-FLA leads to a partial epitaxial regrowth of the implanted layer and to the formation of polycrystalline hyperdoped Ge at the surface [15]. Taking into account the wavelength spectrum of the Xe lamps in the FLA system (300–800 nm) and the optical properties of Ge, the main part of the flash light is absorbed by implanted Ge within 50 nm from the surface. This causes a temperature gradient within the implanted layer. For a short moment (in the submicrosecond range), the surface is much hotter than the amorphous/crystalline interface. Also, the threshold energy needed for crystalline seed nucleation is lower than the energy needed for the epitaxial regrowth [33]. Therefore, during f-FLA, the recrystallization of the implanted layer starts from the surface and a polycrystalline layer is formed. In order to avoid the formation of such a polycrystalline layer at the top of implanted Ge, we developed the rear-side FLA process [28]. In this case, the implanted sample is annealed from the rear side and the heat is transferred through the wafer to the implanted surface. Using r-FLA, the amorphous/crystalline interface is heated first. Therefore, before the surface temperature reaches the level needed for crystalline seed nucleation, the whole implanted layer is recrystallized due to the explosive solid-phase epitaxy [28]. We have found that using a 400-$\mu$m-thick Ge layer the optimal annealing time for rear-side annealing is 20 ms. RTA and pulsed laser annealing (PLA) are alternatives to FLA annealing techniques. During RTA, similar to the r-FLA process,
FIG. 1. The Ga and Al depth distribution in Ge obtained by the SRIM code for an ion-implantation fluence of $2 \times 10^{16}$ cm$^{-2}$ (a) and simulated temperature distribution at the implanted surface during 20 ms FLA from the front side (f-FLA, black curve) and from the rear side (r-FLA, red curve) (b).

B. Characterization techniques

The crystallization process of ion-implanted and annealed samples is studied using Rutherford backscattering-channeling spectrometry (RBS/C). The RBS/C measurements are performed on the samples before and after annealing using the 1.7 MeV He$^+$ beam. To investigate the microstructural properties of the implanted Ge layer, cross-sectional bright-field transmission electron microscopy (TEM) investigations are performed in a Titan 80-300 (FEI) microscope operated at an accelerating voltage of 300 kV. High-angle annular dark-field scanning transmission electron microscopy (HAADF-STEM) imaging and spectrum imaging based on energy-dispersive X-ray spectroscopy (EDXS) are performed at 200 kV with a Talos F200X microscope equipped with a Super-X EDXS detector system (FEI). Prior to TEM analysis, the specimen mounted in a high-visibility low-background holder was placed for 10 s into a model 1020 Plasma Cleaner (Fischione) to remove contaminations. The optical properties are investigated by micro-Raman spectroscopy. The phonon spectra were obtained in backscattering geometry in the range of 100–600 cm$^{-1}$ using a 532-nm Nd:YAG laser with a liquid-nitrogen-cooled CCD camera.

The concentration of carriers in the implanted and annealed samples is estimated from temperature-dependent Hall-effect measurements in van der Pauw configuration. The electrical properties of the annealed samples are measured at millikelvin temperatures in a dry dilution refrigerator (Triton 400 by Oxford Instruments), which allows sweeping temperature in the range from 10 mK to 30 K. Four-probe ac measurements were taken using the ac lock-in method with an excitation current of 10 nA and frequency 127 Hz. The existence of superconducting states in hyperdoped $p$-type Ge was predicted by $ab$ initio calculations within the Eliashberg/McMillan theory.

III. RESULTS AND DISCUSSION

A. Microstructure

The recrystallization process of ion-implanted and flash-lamp-annealed Ge is investigated using RBS random (RBS/R) and channeling (RBS/C) spectrometry. Since Al and Ga are lighter than Ge, they unfortunately cannot be measured directly by RBS. However, the ratio between the yields of the RBS/C and RBS/R spectra ($\chi_{\text{min}}$) is a measure of the crystalline quality of the sample. In our case, $\chi_{\text{min}}$ for Al- and Ga-hyperdoped Ge after FLA is in the range of (5 ± 1)% (see Fig. 5), which is slightly higher than $\chi_{\text{min}}$ for the virgin Ge. Moreover, the RBS/C spectra recorded from the as-implanted samples provide information about the thickness of the amorphized layer, which is needed to calculate the carrier density using Hall-effect measurements.

Figure 2 shows the RBS/R and RBS/C spectra obtained from the Al-doped sample before and after r-FLA, from the Ga-doped sample after r-FLA and from virgin Ge. As
FIG. 2. The RBS/R and RBS/C spectra obtained from Al- and Ga-hyperdoped Ge. The RBS/C spectrum recorded from virgin Ge is shown as well. The concentrations of Al and Ga in Ge are at the level of $2 \times 10^{21}$ cm$^{-3}$.

expected, the RBS/C spectrum obtained from the as-implanted sample reveals the formation about a 120-nm-thick amorphous surface layer. The thickness of the amorphous layer is calculated based on the RBS data using the RUMP Software. After 20-ms r-FLA with an energy density of 120 J cm$^{-2}$, the yield of RBS/C spectrum drops down to the level registered from the virgin Ge wafer. This behavior points to an epitaxial regrowth of the implanted layer during r-FLA. Moreover, we can conclude that Al atoms are incorporated into the lattice of Ge. Taking into account that the solid solubility of Al in Ge is in the range of $5 \times 10^{20}$ cm$^{-3}$, the investigated sample contains 4 times more Al in substitutional positions than the solid solubility limit. Such a gain is only possible due to the strongly nonequilibrium character of the process. The absence of significant dechanneling suggests that the formation of Al clusters is also suppressed by the millisecond-range r-FLA.

In the case of Ga-hyperdoped Ge after r-FLA, the RBS/C spectrum also reveals full incorporation of Ga into the Ge lattice. The yield of the RBS/C spectrum obtained from virgin Ge and the Ga-doped sample is at the same level, meaning that the Ga-implanted sample behaves the same way as Al-doped Ge after r-FLA. In both cases, the ESS limit has been overcome by 4 times.

In order to clarify the lattice position of Al and Ga within Ge, we performed particle-induced x-ray emission (PIXE) spectroscopy in the random and channeling direction. Figures 3(a) and 3(b) show the PIXE spectra obtained from the Al- and Ga-doped samples, respectively. The peaks are identified as the characteristic x-ray emissions of the Al K$_\alpha$ (1.78 keV), Ga K$_\alpha$ (9.27 keV), Ge K$_\alpha$ (9.85 keV), and Ge K$_\beta$ (10.98 keV) lines. Since in the PIXE channeling spectra the Al K$_\alpha$ and Ga K$_\alpha$ intensities drop down to the noise level, it can be concluded that both Al and Ga atoms are fully incorporated into Ge lattice sites, even with a concentration being 4 times higher than the ESS.

More insight into the microstructure is provided by TEM. Please note that the SiO$_2$ capping layer is still present for these samples. Figure 4(a) displays a cross-sectional bright-field TEM image taken from Al-doped Ge after annealing. In this case, both single dislocations within the implanted layer and end-of-range defects are detected [34]. Figure 4(b) shows the Ge, Al, and O distributions based on EDXS analysis from a representative surface region, as exemplarily marked by the white square in Fig. 4(a). Aluminum is quite evenly distributed within Ge, showing only few small agglomerates over the implantation depth, which is in good agreement with RBS and PIXE data.

Figures 4(c) and 4(d) show a cross-sectional bright-field TEM micrograph and the corresponding superimposed Ge, Ga, and O element distributions obtained from Ga-hyperdoped Ge. Here, the Ga is completely homogeneously distributed within the implanted layer. Moreover, in the case of the Ga-doped sample, even the end-of-range defects are not detected. For Al as well as Ga, the recrystallized Ge is single-crystalline. This is in contrast to our previous results, where front-side flash lamp annealing was used [15]. Applying
f-FLA, the implanted layer is composed of polycrystalline Ge with Ga clusters and an epitaxial layer which has a thickness of about 70% of the thickness of the implanted layer. Using r-FLA, we can fully suppress the formation of poly-Ge and Ga clusters for dopant concentrations much above the ESS.

B. Superconductivity

The established electrical parameters of the studied doped layers are summarized in Table I. The carrier concentration was estimated from the Hall-effect measurements. The thickness of the doped layer was determined by RBS measurements. The presented activation efficiency is a ratio between the total acceptor concentration and the carrier concentration estimated from the Hall-effect measurement at 3 K. The presented critical temperatures are taken from Fig. 5. We find that for diluted hyperdoped Ge the minimum carrier concentration needed for superconductivity is in the range of $1 \times 10^{21} \text{cm}^{-3}$. If we assume that the superconductivity in $p$-type Ge is phonon mediated, for the same doping level the critical temperature should be slightly higher for Al-doped Ge

<table>
<thead>
<tr>
<th>Sample</th>
<th>Dopant concentration</th>
<th>Carrier concentration at 3 K</th>
<th>Activation efficiency</th>
<th>Critical temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge:Al</td>
<td>$\sim 6%$</td>
<td>$10.7 \times 10^{20} \text{cm}^{-3}$</td>
<td>44.5%</td>
<td>$T_c \sim 0.15 \text{K}$</td>
</tr>
<tr>
<td>Ge:Ga</td>
<td>$\sim 10%$</td>
<td>$12.6 \times 10^{20} \text{cm}^{-3}$</td>
<td>31.5%</td>
<td>$T_c \sim 0.45 \text{K}$</td>
</tr>
</tbody>
</table>
than for the Ga-doped sample due to a stronger phonon coupling. Unfortunately, in hyperdoped Ge it is very challenging to control the carrier concentration keeping the same dopant concentration. In fact, for the doping level above the solid solubility, we are not able to activate 100% of the implanted element. Therefore experimental verification of theoretical predictions is very challenging. Here we decided to compare samples with similar hole concentrations.

It is worthy to note that we are able to show superconducting Ge hyperdoped with Al. The achieved hole concentration is the highest ever published for Al-doped samples. Figure 5(a) shows the temperature dependence of resistance for Al-hyperdoped Ge. The superconducting temperature is about 150 mK. This is much lower than that predicted by calculation ($T_C \sim 480$ mK, shown later) but also much below the critical superconducting temperature for Al thin film or Al clusters [35]. If we take into account the carrier concentration, which is roughly half of the Al concentration, the obtained $T_C$ is at a reasonable level. The temperature dependence of the resistance for the Ga-doped sample is shown in Fig. 5(b). In Ga-hyperdoped Ge, $T_C$ is about 400 mK. Present critical temperatures are obtained from samples annealed from the rear side with a flash energy density of 120 J cm$^{-2}$. The annealing at lower energy densities is not sufficient to recrystallize the implanted layer (no superconductivity), while annealing at higher energy densities activates the dopant diffusion and cluster formation, leading to the superconductivity driven by metallic clusters (see Supplemental Material Figs. 1(b) and 2) [15]. Consequently, the fraction of electrically active dopants, both Al and Ga, in the substitutional position is much smaller.

C. Model calculations for the electron-phonon coupling

According to the BCS theory the critical temperature of a homogeneous superconductor grows with increasing electron-phonon coupling strength and Debye temperature. Theoretical calculations demonstrate that in homogeneously doped semiconductors the critical temperature scales with their hole concentration [36,37]. The critical temperature in diamond can exceed 20 K for a hole concentration of 10% ($\sim 10^{22}$ cm$^{-3}$). Much lower critical temperatures ($<1$ K) have been predicted for Si and Ge. We used the supercell technique to model the hyperdoped Ge. For simulation we have used Ge doped with Al or Ga with the concentration of 6.25%, corresponding to the 2×2×2 supercell with one Ge atom substituted by an Al(Ga) atom. All calculations were performed within the plane-wave implementation of the local-density approximation (LDA) [38] to density functional theory (DFT) [38–40] in the QUANTUM-ESPRESSO package [41]. Norm-conserving pseudopotentials with a kinetic energy cutoff of 45 Ry were used to represent electron-ion interactions. The $k$-point sampling of the Brillouin zone was set to 6×6×6 during the structural relaxation and electronic structure calculations, while a dense 12×12×12 Monkhorst-Pack grid [32] was used for the phonon linewidth calculations. Phonon spectra and electron-phonon coupling constants were calculated using density functional perturbation theory [42] with a 3×3×1 mesh of $q$ points. For all calculations we have used an optimized lattice constant of Ge supercell of 11.234 Å. The hyperdoping of Ge with Al or Ga will lead to a lattice expansion by 0.2% (11.259 Å for Al-doped Ge) or lattice compression by 0.1% (11.226 Å for Ga doped Ge), respectively.

Figure 6 shows the electronic structure of Al- and Ga-hyperdoped Ge. According to our calculations for the same dopant concentration, the density of states $N(E_F)$ in Al-hyperdoped Ge ($\sim 2.82$ [states/eV/(supercell)]) is slightly higher than in Ga-hyperdoped Ge ($\sim 2.67$ [states/eV/(supercell)]). In both cases the hyperdoped Ge is strongly degenerate, with the Fermi level ($E_F$) located deep in the valence band ($\sim 0.67$ eV below the top of the valence-band maximum). The electronic states near the $E_F$ of Al- and Ga-hyperdoped Ge are very similar, and they originate from $p$ states of Ge and acceptor dopants. The hyperdoped Ge is an $sp^3$ covalent metal.

In a similar way to the electronic structure of hyperdoped Ge, we have calculated the phonon band structure of Al- and Ga-hyperdoped Ge (see Fig. 7). By solid blue circles we...
have marked the highest optical phonons at the $\Gamma$ point. In the case of Al-hyperdoped Ge we have found three optical $T_2$ modes located at 256, 267, and 364 cm$^{-1}$ and one $A_1$ optical mode with frequency around 188 cm$^{-1}$. The strongest electron-phonon coupling strength $\lambda_{vq}$ at the $\Gamma$ point is at $A_1$ with $\lambda_{vq} \sim 0.16$. The triple degeneracy also produces

FIG. 6. The electronic structure of Al- (a) and Ga-hyperdoped (b) Ge with the corresponding total density of electronic states $N(E_F)$.

FIG. 7. Phonon band structure with corresponding phonon DOS for Al-hyperdoped Ge (a) and for Ga-hyperdoped Ge (b). The unit of phonon DOS (PHDOS) is states/(cm$^{-1}$ x supercell). Blue solid circles indicate the partial electron-phonon coupling strength $\lambda_{vq}$ at the $\Gamma$ point. (c, d) show the Raman spectra obtained from Al- and Ga-hyperdoped Ge, respectively. The Raman spectrum of virgin undoped Ge is shown for comparison.
electron-phonon coupling strength of about \( \lambda_{\nu(q=q_1)} \sim 0.05 - 0.06 \). The phonon structure of Ga-hyperdoped Ge is very similar to Al-hyperdoped Ge. The \( A_1 \) optical mode should be located at 187 cm\(^{-1}\), with maximum electron-phonon coupling of about 0.12. The optical \( T_2 \) modes in Ga-hyperdoped Ge are located at 254, 259, and 266 cm\(^{-1}\), and the \( \lambda_{\nu(q=q)} \) is in the range of 0.04–0.07. The theoretical calculated phonon structure of hyperdoped Ge was verified using micro-Raman spectroscopy. The micro-Raman spectra were collected under 532-nm laser excitation, with a laser power of 3.2 mW and a focal diameter of about 1 \( \mu \)m. Figures 7(c) and 7(d) show the Raman spectra obtained from Al- and Ga-hyperdoped Ge, respectively. The transverse-optical phonon mode of Ge–Ge in intrinsic Ge is located at 300.5 cm\(^{-1}\). After hyperdoping the TO phonon mode of the Ge–Ge vibrational is shifted down to 288.2 cm\(^{-1}\) for the Al-doped sample and down to 281.9 cm\(^{-1}\) for the Ga-doped Ge. This is very close to the theoretically predicted values for the high frequency of the zone-center optical mode in hyperdoped Ge (about 278 cm\(^{-1}\)). The shift of the TO phonon mode in ultrahigh-doped Ge and the peak asymmetry is due to the phonon softening and the Fano effect [43–45]. Besides the TO phonon mode we can easily distinguish the \( A_1 \) phonon modes in both samples. The measured peak position of the \( A_1 \) mode located at 188 cm\(^{-1}\) fits well to the theoretically predicted phonon energy using density of states (DOS) calculation.

Due to the fact that the \( T_2 \) phonon mode positions are close to the strongest zone-center TO phonon mode it is difficult to distinguish them. But in the case of Al-hyperdoped Ge, the \( T_2 \) mode at 364 cm\(^{-1}\) is well visible [see Fig. 7(c)]. In Ga-hyperdoped Ge all three \( T_2 \) modes are overlapped with the TO phonon mode. Next we analyze the electron-phonon coupling. Figures 8(a) and 8(b) show the Eliashberg spectral function and the integrated electron-phonon coupling constant \( \lambda(\omega) \). The total \( \lambda \) calculated for Al- and Ga-doped Ge are similar and equal to 0.355 and 0.350, respectively. The calculated logarithmic phonon frequency \( \omega_{\log} \) is about 243.6 K for Al-doped Ge and about 245.1 K for Ga-doped Ge, which is much smaller than the \( \omega_{\log} \) in other group IV superconductors—for example, about 700 K for Si:B and about 1287 K for boron-doped diamond [45]. Finally, we have calculated the superconducting critical temperature for both samples. We found that the expected \( T_C \) for Al-doped Ge should be slightly higher than that for the Ga-doped sample, mainly due to slightly higher phonon-carrier coupling. The \( T_C \) for Ge:Al is 0.48 K and for the Ge:Ga system the \( T_C \) should be about 0.43 K. According to our calculation(s), the superconductivity in diluted \( p \)-type hyperdoped Ge should be phonon mediated.

Note that there is significant discrepancy between calculated \( T_C \) and the experimental values shown in Table I. The theoretical calculation cannot take into account all phenomena which may exist in real samples. In fact, we have a Gaussian distribution of the implanted elements introducing a kind of inhomogeneity into the doped layer which is not accounted for in our calculations and modeling. Next, although the thermal treatment is very short and we were not able to detect big metallic clusters within the implanted Ge layer, we cannot exclude the formation of Ga clusters with a diameter below the resolution limit of our TEM system. Moreover, the effective carrier concentration is lower than the nominal dopant concentration, which is not taken into account for simulation.

**IV. CONCLUSIONS**

We have fabricated single-crystalline Al- and Ga-doped superconducting Ge where the diffusion and clustering of dopants are suppressed by utilization of strongly nonequilibrium thermal processing. Using rear-side FLA, the implanted Ge layers recrystallize epitaxially due to the explosive solid-phase epitaxy. The theoretically predicted critical temperatures qualitatively agree with experimental values. With further optimizing dopant concentrations and annealing parameters, our work will pave the way for monolithic integration of superconducting nanocircuits in semiconductor devices.

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