

## Experimental and numerical analysis of the complex permittivity of open-cell ceramic foams

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1 Experimental and numerical analysis of the complex permittivity of open-cell  
2 ceramic foams

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15

16 **Abstract**

17 Open-cell ceramic foams are promising materials in the field of microwave heating. They  
18 can be manufactured from susceptor materials and can, therefore, be used as selective  
19 heating elements. In this study, the complex permittivities of ceramic foam materials,  
20 including silicon-infiltrated silicon carbide (SiSiC), pressureless sintered silicon carbide  
21 (SSiC), silicate bonded silicon carbide (SBSiC), and cordierite were determined. The  
22 dielectric properties of the foams were determined by the cavity perturbation technique  
23 using a TE<sub>104</sub> WR340 waveguide resonator at 2.45 GHz. Samples were preheated in a  
24 tubular furnace, enabling temperature-dependent permittivity measurements up to 200 °C.  
25 The effective dielectric constant and effective loss factor were found to depend on the  
26 porosity and material composition of the foam. The SiSiC material had a higher effective  
27 dielectric constant than the SSiC and SBSiC ceramics. The effective dielectric constant of  
28 the foams showed a trend of gradual increase with increasing temperature. Some selected  
29 dielectric mixing relations were then applied to describe the effective permittivity of the  
30 foams and compare them with predictions from finite element simulations performed using  
31 the CST Studio Suite. The foam morphologies and simple block inclusions were used in the  
32 simulations.

33 **Keywords:** A. microwave processing, C. dielectric properties, D. silicon carbide, D.  
34 cordierite.

## 35 1 Introduction

36 Open-cell solid foams are interconnected solid networks (also referred to as skeletons),  
37 through which fluids can pass (see Figure 1A). They exhibit characteristics that make them  
38 extremely attractive for various thermal applications, such as heat exchangers, porous  
39 burners, electric heaters [1], and catalyst supports [2,3].

40 **Figure 1.** (A) Image of the SiSiC foam morphology obtained by SEM; micro X-ray  
41 tomography ( $\mu$ CT) cross-sectional images of the SiSiC foams (B) 30 ppi and (C) 45 ppi.

42 Microwave heating has been increasingly applied to many industrial processes and can also  
43 be considered as a promising approach for the selective heating of foams. The microwave  
44 absorbance and the heating characteristics of a material are commonly described by its  
45 complex permittivity, which is constituted of real and imaginary parts ( $\epsilon = \epsilon' - j\epsilon''$ ), i.e.,  
46 the dielectric constant ( $\epsilon'$ ) and dielectric loss factor ( $\epsilon''$ ), respectively. The effective  
47 permittivity ( $\epsilon_{\text{eff}}$ ) is the dielectric permittivity of a system or object that is made of two or  
48 more materials. In the case of open-cell foams, which can be manufactured from any  
49 sinterable material (e.g., ceramics),  $\epsilon_{\text{eff}}$  is a function of the permittivities of the fluid ( $\epsilon_{\text{air}}$ )  
50 and solid ( $\epsilon_{\text{ceramic}}$ ) phases.

51 Silicon carbide (SiC) is a ceramic base material that is suitable for the manufacturing of  
52 open-cell solid foams. Its high-temperature stability, hardness, and corrosion resistance  
53 make it an outstanding candidate for many engineering applications. Due to its excellent

54 microwave susceptibility (i.e., the degree of absorption of the electromagnetic energy and  
55 its conversion to heat), SiC is used to improve the heating efficiency in microwave  
56 applications, either by blending low-loss materials with SiC particles or by placing those  
57 materials close to or in contact with the bulky susceptor portions [4]. The temperature-  
58 dependent complex permittivity of SiC powder (bulk density of  $1.89 \pm 0.06 \text{ g/cm}^3$ ) was  
59 measured at 2.45 GHz by Fernandez et al. [5], as shown in Table 1. However, the  
60 permittivity can be significantly influenced by doping or impurities. For example, boron-  
61 doping increases both the dielectric constant and loss factor, from  $\epsilon = 6.70 - j0.02$  to  $\epsilon =$   
62  $65.44 - j32.40$  (at 2.45 GHz) [6].

63 **Table 1.** Complex permittivity of SiC powder and cordierite.

64 The porosity ( $P$ ) of solid foams is defined as the ratio of the volume of voids to the total  
65 volume of the foam. As  $P$  increases, the effective permittivity decreases. For example, an  
66 SiC foam with  $P = 83.7 \pm 1.0 \%$  and a linear pore density of 50 pores per inch (ppi), has an  
67  $\epsilon_{\text{eff}} = 4.7 - j0.33$  at 2.45 GHz (at room temperature of approximately 20 °C) [6], which is  
68 lower than the  $\epsilon_{\text{eff}}$  for  $P = 41.1 \pm 0.18 \%$  (SiC powder) [5], as reported in Table 1.

69 The objective of this study was the analysis of the relationship between the morphology and  
70 effective permittivities (at 2.45 GHz) of SiC-based and cordierite-based open-cell solid  
71 foams. A low-loss magnesium-aluminum-cyclosilicate ( $\text{Mg}_2\text{Al}_3(\text{Si}_5\text{AlO}_{18})$ ) ceramic  
72 material known as cordierite was compared with the SiC-based foams; the permittivity of  
73 cordierite bulk as indicated in the literature is provided in Table 1. Thus, a systematic study  
74 of the dielectric parameters was performed for ceramic-based solid foams with different

75 porosities, pore densities, and compositions. Furthermore, the effective permittivities that  
76 were predicted by some mixing relations were compared against those derived from  
77 performed electromagnetic simulations.

## 78 **2 Experimental methods and materials**

### 79 **2.1 Experimental dielectric measurements**

80 Different techniques can be used to measure the dielectric properties of solid materials [9].  
81 Among them, the cavity perturbation technique compares the resonance frequency ( $f_1$ ) and  
82 quality factor ( $Q_1$ ) of a cavity without perturbation against the resonance frequency ( $f_2$ ) and  
83 quality factor ( $Q_2$ ) of the same cavity after perturbation by a small material sample. The  
84 complex permittivity of the perturbation sample is then calculated according to

$$\varepsilon' = \frac{V_c (f_1 - f_2)}{V_s A f_2} + 1, \text{ and} \quad (1)$$

$$\varepsilon'' = \frac{V_c}{B * V_s} \left( \frac{1}{Q_1} - \frac{1}{Q_2} \right), \quad (2)$$

85 where  $V_c$  is the cavity volume and  $V_s$  is the sample volume [10]. The coefficients  $A$  and  $B$   
86 can be determined by calibration, either using samples of an identical size and well-known  
87 permittivities that are similar to the material of interest or by employing a numerical  
88 calibration procedure. The latter is based on full-wave 3D electromagnetic simulations of  
89 the forward problem. By variation of the sample permittivity within the anticipated interval,

90 these simulations allow the correlation of the calculated frequency shift ( $f_1 - f_2$ ) and the  
91 changes in the quality factor ( $1/Q_1 - 1/Q_2$ ) with a given permittivity. The complex  
92 permittivity is then calculated from the measured scattering parameters ( $S$ -parameters)  
93 using this correlation [10]. The transmission factor  $S_{21}$ , which is the ratio of the transmitted  
94 power (at output port 2) to the incident power (at input port 1) characterizes the resonance  
95 behavior.

96 **Figure 2.** Schematic of the dielectric measurement system based on the cavity perturbation  
97 technique.  $S_{21}(f)$  is the frequency-dependent  $S$ -parameter response at port 2 to the signal at  
98 port 1.

99 A schematic of the experimental system used to acquire the complex permittivity of the  
100 samples is shown in Figure 2. A more detailed description of the dielectric measurement  
101 system developed by Soldatov et al. can be found in [11]. The resonator was based on a  
102 standard rectangular WR-340 waveguide cavity, and the length of the cavity was chosen to  
103 be resonant at a frequency of 2.45 GHz in the  $TE_{104}$  mode. Thus, the electric field standing  
104 wave pattern had four maxima along the longest side of the cavity. The microwave signals  
105 were coupled using inductive coupling ports with a 10 mm width and 43.18 mm height on  
106 both ends. The cavity was connected to a vector network analyzer (VNA Agilent E5071C)  
107 with coaxial-to-waveguide adaptors and coaxial cables to measure the  $S_{21}$  parameter. Two  
108 small opposing holes were made in the walls of the cavity at the position of one of the four  
109 electric field maxima to enable the positioning of the sample inside the cavity (within a

110 quartz tube sample holder). A pyrometer (CellaTemp-PA) at the opposite end measured the  
111 sample surface temperature inside the resonator. A resistive tubular furnace (Carbolite  
112 MFT) was placed in front of the waveguide, allowing the sample to be heated to 1200 °C  
113 before entering the cavity. In this study, the temperature range for the characterization was  
114 30 °C–200 °C. The parameters  $S_{21}$  and the temperature were recorded by a computer, which  
115 provided automatic control of the entire data acquisition process as well as predefined  
116 heating scenarios. The acquired data (i.e., resonance frequency shifts and  $Q$ -factors) were  
117 then post-processed to obtain the real and imaginary parts of the effective material  
118 permittivity (at different temperatures), via Equations (1) and (2).

## 119 **2.2 Ceramic foams**

120 Very high temperatures or pressures (or both) are required to densify pure SiC powder by  
121 sintering due to its covalent bond characteristics [12]. Therefore, the use of additives to  
122 make the sintering process easier is preferred. In this study, the following types of SiC  
123 ceramics were investigated [13]:

- 124   ▪ Silica-bonded silicon carbide (SBSiC) is an oxide-bonded SiC ceramic manufactured  
125    from coarse and medium-grained SiC powders by sintering at standard pressure, during  
126    which amorphous silica or cristobalite (or both) bond the SiC particles.
  
- 127   ▪ Pressureless sintered silicon carbide (SSiC) is produced using very fine SiC powder that  
128    contains sintering additives.

129   ▪ Silicon-infiltrated silicon carbide (SiSiC) is produced by the reaction of molten silicon  
130   with compact SiC powder, containing a high amount of free carbon. Initially, the carbon  
131   reduces the native silica layer on the SiC particles. **The infiltrating melted silicon then**  
132   **reacts with the remaining free carbon to produce secondary SiC, leading to SiC bonding**  
133   **between the SiC grains.** The remaining melted silicon fills the inter-particle voids  
134   creating a somewhat dense microstructure.

135   The compositions and structures of SBSiC, SSiC, and SiSiC are shown in Table 2 and Figure  
136   3, respectively.

137   **Figure 3.** Schematic structure of (A) SBSiC, (B) SSiC, and (C) SiSiC.

138   **Table 2.** Material compositions.

139   Solid foams have a macro-porous open-cell structure composed of stiff vertices  
140   interconnected with struts, which form a continuous grid. Images example of an SiSiC solid  
141   foam and its pore structure, obtained by scattering electron microscopy (SEM) and X-ray  
142   micro-computed tomography ( $\mu$ CT), is presented in Figure 1. The struts have small cavities  
143   inside them, which are isolated from the exterior. As the pore density increases, the pore  
144   size decreases. The foam structure results from the replication technique, wherein the  
145   polymeric template defines the cell size and shape. The open-cell polymer foams were  
146   impregnated with a ceramic slurry. The excess slurry was then removed by squeezing and  
147   centrifugation, resulting in the uniform coating of the polymer foam struts. During the

148 subsequent heat treatment, the polymeric template was decomposed (by pyrolysis), and the  
149 ceramic material became sintered. The same SiC powders were used to manufacture the  
150 SSiC and SiSiC test samples, which were used to identify the influences of the various  
151 compositional materials and binding types (see Table 2). These powders were fine and  
152 reactive, with a median grain diameter  $d_{50} \approx 2 \mu\text{m}$ . The SBSiC also included a second,  
153 coarser SiC powder ( $d_{50} \approx 20 \mu\text{m}$ ) to achieve a fine to coarse ratio of 2:3.

154 Cylindrical samples of the bulk material and the open-cell foams having nominal pore  
155 densities of 30 ppi, 45 ppi, and 60 ppi and porosities in the range of 86.9 % to 96.5 % were  
156 produced (IKTS Fraunhofer, Germany) for dielectric characterization based on the cavity  
157 perturbation technique. The average cell diameters, densities, porosities, and dimensions of  
158 bulk and open-cell ceramic foam samples are summarized in Table 3. The average cell  
159 diameters of the foams were determined by optical 2D-image analysis of light micrographs  
160 using a Pore!Scan-System from Goldlücke GmbH [40]. The density of the foam strut  
161 material was lower than the maximum feasible density because no subsequent compaction,  
162 excepting the eventual sintering shrinkage, can be applied to open-cell ceramic foams. **The**  
163 **same suspension was used for both sample types to identify the structure-related differences**  
164 **between the foam and bulk samples.**

165 **Table 3.** Bulk and foam sample properties.

## 166 3 Calculations

### 167 3.1 Mesh construction for the numerical simulations

168 Two morphology classes, i.e., foams and blocks, were analyzed to compare their influences  
169 on the effective complex permittivity. The foam morphology chosen for the simulation was  
170 obtained from numerous 2D digital cross-sectional images of a rectangular 45 ppi SiSiC  
171 foam sample (size: 40 mm × 40 mm × 25 mm, porosity: 85 %) recorded by  $\mu$ CT. The back-  
172 projection algorithm [14] was used for the 3D sample reconstruction. The spatial resolution  
173 of the reconstructed tomographs was 56  $\mu$ m per voxel. The foam stack was first cut to a  
174 smaller representative cubic volume with a 15 mm edge length to reduce the computational  
175 burden of the simulations. A representative cube is a rectangular cuboidal representation  
176 that utilizes a particular volume of the foam that has the same porosity as the sample.

177 Additional meshes with lower porosities were produced using a 3D dilatation filter to study  
178 the porosity over a wide range (see Section 4.3) [15]. This filter dilates the cross-sectional  
179 structure of the foam using a spherical kernel diameter of 3 voxels, which increases the  
180 skeleton volume, and, consequently, decreases the porosity. The mesh reconstruction (see  
181 Figure 4) was then achieved by applying the marching cubes algorithm [16], following  
182 which the small non-connected structures were eliminated. Further, mesh simplification  
183 (i.e., the transformation of a given polynomial mesh into another with fewer faces, edges,  
184 and vertices), mesh smoothing (i.e., the removal of noise from the mesh surface), and

185 uniform mesh resampling (to optimize the point distribution) were performed [39]. A  
186 detailed description of the mesh construction can be found in [17].

187 **Figure 4.** Foam morphology reconstruction and meshing.

188 Finally, the meshes were imported into the CST Microwave Studio Suite and incorporated  
189 as inclusions in a box. The outer box represented the air and the inclusions for the ceramic  
190 foams. The resulting boxes with the inclusions had porosities of 11 %, 30 %, 50 %, 70 %,  
191 and 85 %, based on the ceramic and air domains.

192 For the block morphology inclusions, a series of blocks were used instead. The blocks were  
193 generated in the directions of the box corners, resulting in an arrangement of eight blocks.  
194 The block size was selected to obtain porosities ranging from 0 % to 100 % at 10 %  
195 increments. Additionally, the porosities were adjusted to facilitate direct comparison with  
196 the porosities of the foam morphologies mentioned previously. Two approaches were used  
197 for the **block model** generation, wherein the properties of the blocks and matrix were  
198 interchanged. The first approach involved block-shaped ceramic inclusions in an air matrix,  
199 while the second used block-shaped air inclusions in a ceramic matrix. These arrangements  
200 facilitated the comparison of the effects of the non-connected and connected ceramic  
201 structures to those of real foam structures. **Examples** of the foam and block inclusions are  
202 shown in Figure 5.

203 **Figure 5.** Examples of the foam morphology inclusions (upper row), ceramic block  
204 inclusions (middle row), and air block inclusions (lower row), within the boxes. Ceramic  
205 and air materials are color-coded in red and blue, respectively.

### 206 **3.2 Numerical modeling and complex permittivity calculation**

207 Electromagnetic wave propagation was simulated at 2.45 GHz using the CST Microwave  
208 Studio Suite and the model shown in Figure 6 to estimate the effective permittivities of the  
209 different foams. Simulations of the hexagonal meshes representing the foam morphologies  
210 (minimum 1 000 000 cells) and block morphologies (minimum 300 000 cells) were  
211 performed using a transient domain solver. The boundary conditions were defined as  
212 follows: the pairs of normal faces in the  $x$ ,  $y$ , and  $z$  directions were taken as the electric  
213 walls, magnetic walls, and free space boundaries with no reflections (i.e., open walls),  
214 respectively. The input and output microwave ports were defined at reference planes shifted  
215 to the normal faces in the  $z$ -direction of the models. The bulk permittivity from the  
216 effective-medium approximation of the SiSiC was used for the ceramic inclusions (foams  
217 and blocks). Air properties were set for the waveguide domain. Finally, the scattering  
218 parameters were acquired by numerical simulation. The simulation setup and sequential  
219 procedure are visually summarized in Figure 6.

220 **Figure 6.** Sequential procedure for the foam morphology (upper row) and block  
221 morphology (lower row) inclusions used for computing the scattering parameters using  
222 numerical simulations.

223 In the next step, the effective permittivities were determined using the  $S$ -parameters [18].  
 224 The  $S$ -parameter retrieval method [19,20] was used to calculate the permittivity from the  
 225 reflected ( $S_{11}$ ) and transmitted scattering signals ( $S_{21}$ ). Initially, the impedance ( $z$ ) was  
 226 obtained as

$$z = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}. \quad (3)$$

227 The refractive index ( $n$ ) was then calculated by applying the obtained impedance and  
 228 Equation (4):

$$n = \frac{1}{k_0 L} \left[ \text{Im} \left\{ \ln \left( \frac{S_{21}}{1 - \frac{(z-1)S_{11}}{(z+1)}} \right) \right\} + 2m\pi - i \cdot \text{Re} \left\{ \ln \left( \frac{S_{21}}{1 - \frac{(z-1)S_{11}}{(z+1)}} \right) \right\} \right], \quad (4)$$

229 where  $k_0$  is the wavenumber,  $L$  is the sample length in the wave propagation direction  
 230 between ports 1 and 2,  $m$  is the fundamental branch of the sinusoidal function periodicity  
 231 (in this work,  $m = 0$  because  $L < \lambda_{foam}/4$ ), and  $\lambda_{foam}$  is the wavelength in the foam at  
 232 2.45 GHz. The effective permittivity was then calculated as

$$\epsilon_{eff} = \frac{n}{z}. \quad (5)$$

233

## 234 4 Results and discussion

### 235 4.1 Determination of the complex permittivity

236 Figure 7 provides the effective dielectric constants ( $\epsilon'_{\text{eff}}$ ) and the effective loss factors ( $\epsilon''_{\text{eff}}$ )  
237 of the cordierite and SiC-based samples obtained from the temperature dependence  
238 measurements using the cavity perturbation method. Notably, the bulk did not necessarily  
239 refer to samples having a 100 % theoretical density (as listed in Table 3) because of the  
240 previously described preparation technique. In contrast to the SiSiC, the other bulk  
241 materials had significant residual porosities (SBSiC: 29.1 %, SSiC: 8.7 %, cordierite:  
242 29.3 %). The permittivity of the bulk SiSiC sample is not shown because it was detected  
243 uncertainty regarding its composition and porosity. Further, the high losses of the bulk  
244 SiSiC, SSiC, and SBSiC samples resulting from excessive perturbation caused the solution  
245 of the inverse problem associated with the cavity perturbation technique to provide  
246 unreliable results. Thus, the results for these bulk materials are also not shown. For all  
247 materials, the samples had a range of complex permittivity values (represented by the error  
248 bars in Figure 7), which was primarily caused by the spread in the **sample geometry** (see  
249 Table 3).

250 No clear trend could be observed for the relationship of the permittivity to the skeleton  
251 structure or pore size (in terms of the pore density). Notably, the pore density was provided  
252 as the mean ppi-value and is a measure of the interface density [21], which corresponded to  
253 the inverse of the mean chord length. The unit of ppi is not recommended for use in

254 scientific papers because the inch is not a unit recognized in the international system of  
255 units. However, it is used in various industrial standards for foam production [22] (as was  
256 the case in this study). Further, it can be observed that, in most of the samples, the  
257 permittivity decreased with increasing porosity because the contribution of the air to the  
258 effective permittivity increased ( $\epsilon_{\text{air}} < \epsilon_{\text{ceramic}}$ ).

259 **Figure 7.** Effective dielectric constants (left) and effective loss factors (right) of cordierite  
260 (first row), SiSiC (second row), SBSiC (third row), and SSiC (fourth row) at a frequency of  
261 2.45 GHz.

262 As seen in Figure 7, the permittivity of the SiSiC differed significantly from that reported  
263 for the SSiC and SBSiC, highlighting the impact of the free silicon infiltration. The  
264 dielectric properties of SiC ceramic foams depend strongly on the characteristics of their  
265 doping, additives, or impurities.

266 From Figure 7, it is evident that the SBSiC and SSiC had similar effective dielectric  
267 constants. In comparison, the SiSiC exhibited a significantly higher effective dielectric  
268 constant, presumably related to the free silicon that infiltrated the SiSiC foam structure  
269 (Figure 3). Given that silicon is a semiconductor with a dielectric constant of approximately  
270 11.9 [23] at 2.45 GHz, the resulting effective dielectric constant was higher as compared to  
271 the SiC foams that did not contain free silicon.

272 **The effect of the temperature on the loss factor** was considerable for the SSiC but not the  
273 other foams, presumably related to the comparably higher fraction of silicon carbide and  
274 the excess carbon that was not completely consumed by the native silica layer of the SiC  
275 particles during the sintering process (see Table 2). When adding carbon to SiC (C-SiC)  
276 [24], its conductivity changes from *n*-type to *p*-type, so a greater amount of carbon would  
277 increase the conductivity and, therefore, its loss factor. Furthermore, **increasing**  
278 **temperatures result** in increased carbon-based conductivity and permittivity [25]. Moreover,  
279 after the manufacturing process is complete, some additive materials used in the  
280 manufacturing process may remain as reacted materials or binders. In this study, boron was  
281 employed as the sintering aid. Boron is a trivalent element that creates *p*-type electron  
282 vacancies and, thus, further increases the electrical conductivity in the SSiC.

283 Highly porous materials always contain some water in their pores due to the ambient  
284 atmosphere. Being exposed to elevated temperature causes water to leave the material, thus  
285 leading to a decrease in its effective permittivity. This effect, however, coincides with the  
286 change of permittivity in the skeleton material of the foam. The effect of water depletion on  
287 the effective permittivity competes in foams made from typical dielectrics like oxides,  
288 nitrides, or carbides, which have increasing trends in permittivity versus temperature.  
289 Semiconducting additives, such as silicon and carbon (in this case conducting), in SiSiC  
290 and SSiC foams, respectively, could cause **increased effective loss factors**, which correlates  
291 with the SiC skeleton temperature behavior and inversely with water depletion. **The**  
292 **interplay of these three factors that define the resulting temperature behavior of the**  
293 **permittivity of the foams in this study is clearly different between SiSiC and SSiC.**

294 Compared to SSiC, SBSiC has no carbon or boron source added, and includes alumina  
295 ( $\text{Al}_2\text{O}_3$ ) and silica ( $\text{SiO}_2$ ). Fused amorphous silica ( $\text{SiO}_2$ ) has a dielectric constant of  
296 approximately  $\epsilon'_{\text{SiO}_2} \sim 3.8$  throughout the frequency range of  $1 \times 10^{-5}$  to 40 GHz [26], and  
297 is known for its very low loss factor of  $\epsilon''_{\text{fused silica}} < 1 \times 10^{-3}$  [27]. Thus, the competition  
298 of only two substances, i.e., SiC and water, could be assumed. Interestingly, the water  
299 depletion resulted in a declining trend of the loss factor with temperature, but an increasing  
300 trend with the dielectric constant. This is likely a consequence of the relatively higher  
301 contribution of the water loss factor than its dielectric constant.

302 The notable increase of the cordierite permittivity for  $P = 29\%$  is a result of its domination  
303 of the skeleton material composition. In contrast, at  $P \sim 85\%$ , the water depletion **almost**  
304 fully compensated for the trend of increasing permittivity. As seen in Figure 7, only the loss  
305 factor of SSiC increased significantly with temperature. Thus, the microwave absorbance of  
306 SiSiC and cordierite foams could be considered to be constant within the studied  
307 temperature range.

308 The assessment of whether the foams behaved as homogeneous effective media is also  
309 important. Foam can be considered to behave as an effective medium when the wavelength  
310 of the incident electromagnetic radiation  $\lambda$  is much larger than the mean cell size.  
311 Therefore, the criterion used to determine whether the system behaves as an effective  
312 medium is based on its inclusion size parameter, which is calculated as  $x = 2\pi r/\lambda$ , where  $r$   
313 represents the inclusion or particle size. Mishchenko et al. proposed a threshold value of  
314  $x < 0.15$  [37].

315 The permittivity of a medium affects the wavelength according to  $\lambda = \lambda_0/\sqrt{\varepsilon_{eff}}$ , where  $\lambda_0$   
316 is the wavelength in a vacuum. In this study, the cells are considered as spheres, and their  
317 diameters ranged between  $\sim 1.46$  mm (30 ppi) and  $< 1$  mm (60 ppi). As shown in Table 4,  
318 inclusion size parameters  $x < 0.15$  were obtained for all foam types; i.e., they can be  
319 treated as effective homogeneous media.

320 **Table 4.** Maximum inclusion size parameter ( $x$ ) of foams; 30 ppi foams were used for the  
321 calculations because they have the largest cell diameter.

## 322 **4.2 Permittivity mixing relations**

323 As in the effective-medium approximations and empirical mixing relations (see Table 5),  
324 two-phase mixing relations were used to describe the effective permittivity of the ceramic  
325 foams in terms of their porosities and bulk permittivities. The bulk permittivities of the  
326 SiSiC, SBSiC, and SSiC were not determined and, accordingly, were considered as  
327 parameters in the mixing relations that were fitted to the experimental data using iterative  
328 least-squares estimation.

329 When the porosity reaches 100 %, the effective permittivity should approach the  
330 permittivity of the material occupying the pores, in this case, air;  $\varepsilon'_{eff}(P = 1) = \varepsilon'_{air} \approx$   
331 1.00, which corresponds to  $\varepsilon'_{air}$  at 25 °C and is almost constant within the studied  
332 temperature range. In contrast, if the porosity is 0 %,  $\varepsilon'_{eff}(P = 0) = \varepsilon'_{cer}$  represents the

333 dielectric constant of the ceramic material of the skeleton or the fully dense ceramic  
334 samples.

335 **Table 5.** Mixing relations for the description of effective permittivities. (<sup>1</sup> **The exponential**  
336 **relation proposed by Pabst and Gregorová [29] is not identical to the simple exponential**  
337 **relation promoted by Rice [36]).**

#### 338 **4.2.1 Effective-medium approximations**

339 The most popular effective-medium approximations [28–30,35] were used for simultaneous  
340 data fitting: the Maxwell-Garnett approximation (Maxwell type relation), Bruggeman-  
341 Landauer approximation (self-consistent relation), Differential Effective Medium (DEM,  
342 differential relation), the exponential relation (from Rice [35]), and the Pabst-Gregorová  
343 exponential relation [29] (see Table 5), as shown in Figure 8. The dimensional parameter  
344 ( $d$ ) for the spheres was set as  $d = 3$ . The adjustable parameter ( $B$ ) of the Pabst-Gregorová  
345 exponential relation was set to  $B = 2$  for the spherical pores [29], and  $B$  in the exponential  
346 relation (from Rice [35]) was estimated from experimental data fitting.

347 **Figure 8.** Comparison of the measured and predicted effective dielectric constants using  
348 various effective medium approximation relation for SiSiC (upper-row, left), SBSiC  
349 (upper-row, right), SSiC (lower-row, left) and cordierite (lower-row, right) at 2.45 GHz and  
350 100 °C. The SSiC permittivities from power extrapolated to 100 °C powder\* [5] and  
351 foam\*\* [6] were used for comparison.

352 The estimated effective dielectric constants of the fully dense materials were:  $\epsilon'_{\text{SiSiC}} =$   
353  $95.05 \pm 0.31$ ,  $\epsilon'_{\text{BSiC}} = 19.34 \pm 0.10$ ,  $\epsilon'_{\text{SSiC}} = 20.26 \pm 0.40$ , and  $\epsilon'_{\text{cord}} = 5.35 \pm 0.21$ .  
354 Among the compared relations, the best agreement was obtained using the Maxwell-  
355 Garnett relation for the SiC-based foams. The Maxwell-Garnett approach is a non-  
356 symmetric relation wherein the sphere inclusions are well-separated from each other.  
357 Because  $\epsilon'_{\text{air}} < \epsilon'_{\text{cer}}$ , the Maxwell-Garnett approach corresponds to the Hashin-Shtrikman  
358 upper bound, as shown in Figure 8.

359 Additionally, there was some agreement between the extrapolated permittivities (at 100 °C)  
360 of the SiC powder [5] and foam [6] with boron impurities and the Maxwell-Garnett  
361 approach relation for the SSiC, despite these materials having a slightly different  
362 composition. Further, the dielectric constant of fully dense SSiC ( $\epsilon'_{\text{SSiC}} = 20.26 \pm 0.40$ )  
363 agrees with the extrapolated value of  $\epsilon'_{\text{SSiC}} = 20.3 \pm 0.3$  (at 100 °C) from a fully dense tile  
364 sample [38] of SiC with boron impurities.

#### 365 **4.2.2 Fixed-parameter weighted mean relations**

366 Fixed-parameter weighted mean relations [28], such as those based on the arithmetic,  
367 harmonic, and geometric means (see Table 5) were also used for simultaneous data fitting.  
368 The Hashin-Shtrikman (HS) bounds were used along the fixed-parameter weighted mean  
369 relations because the foams were considered to be anisotropic. The results are shown in  
370 Figure 9.

371 **Figure 9.** Comparison of the measured and predicted effective dielectric constants using  
372 the weighted mean relations with Hashin-Shtrikman bounds for SiSiC (upper-row, left),  
373 SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite (lower-row, right) at 2.45  
374 GHz and 100 °C.

375 The estimated effective dielectric constants of the fully dense materials for the fixed-  
376 parameter weighted mean relations were:  $\varepsilon'_{\text{SiSiC}} = 91.07 \pm 0.31$ ,  $\varepsilon'_{\text{SBSiC}} = 19.73 \pm 0.10$ ,  
377  $\varepsilon'_{\text{SSiC}} = 20.63 \pm 0.40$ , and  $\varepsilon'_{\text{cord}} = 5.69 \pm 0.21$ , which agreed exceptionally well with the  
378 values estimated by the effective-medium approximations.

379 For the SiC-based foams, the weighted mean relations were practically reduced to the fitted  
380 upper bound because the data were primarily concentrated at the right side of the graph  
381 ( $P > 0.8$ ). Thus, the lines for the upper Wiener or HS bounds ( $\Psi_{\text{arithm}} \approx \Psi_{\text{harm}} \approx$   
382  $\Psi_{\text{geom}} \approx 0$ ) nearly overlapped. Conversely, the measured data for the cordierite was  
383 between the HS bounds. All weighted mean relations agreed well with the measured data.  
384 The weight parameters estimated for cordierite using the HS bounds were  
385  $\Psi_{\text{geom/arithm/harm}} \approx 0.25$  and  $\varepsilon'_{\text{cord}} = 5.69$ .

### 386 **4.2.3 General sigmoidal mean relations**

387 The  $N$ -parameter ( $N$ ) in the general sigmoidal mean relation [28] denotes the type of  
388 volume-weighting (which is subharmonic for  $N < -1$ , harmonic for  $N = -1$ , geometric  
389 for  $N = 0$ , arithmetic for  $N = 1$ , and superarithmetic for  $N > 1$ ). Figure 10 shows the

390 comparison of the measured and predicted effective dielectric constants for different  $N$ -  
391 parameters; one parameter was fitted to the experimental data with the condition of  $N \neq 0$ ,  
392 and the other was fitted by setting  $N = 0$ .

393 **Figure 10.** Comparison of the measured and predicted effective dielectric constants using  
394 the general sigmoidal mean relations for different  $N$ -parameters and bounds for SiSiC  
395 (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite (lower-  
396 row, right) at 2.45 GHz and 100 °C.

397 The estimated effective dielectric constants of the fully dense materials for the general  
398 sigmoidal mean relation were essentially equal to those obtained from the **weighted** mean  
399 relation, which is probably related to both relations using the HS bounds. Good agreement  
400 was obtained for the non-zero  $N$ -parameter ( $N \neq 0$ ) for all materials  $N \geq 60.83$ , which  
401 corresponds with the volume-weighted superarithmetic mean relations coinciding with the  
402 Hashin-Shtrikman upper bound.

#### 403 **4.2.4 General power mean relations**

404 Similar to the previous relation, the  $n$ -parameter in the general power mean relation [28]  
405 denotes the type of volume-weighting, which is harmonic for  $n = -1$ , geometric for  $n =$   
406  $0$ , and arithmetic for  $n = 1$ . The comparison of the predicted and measured effective  
407 dielectric constants is shown in Figure 11.

408 **Figure 11.** Comparison of predicted and measured effective dielectric constants using the  
409 general power mean relations (solid line:  $n = 0$ , dashed line:  $n \neq 0$ ); and bounds for the  
410 SiSiC (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite  
411 (lower-row, right) at 2.45 GHz and 100 °C.

412 Sigmoidal relations did not well represent the effective dielectric constant of the SiC-based  
413 materials because the data were mainly located at the right side of the graphs, which caused  
414 unrealistically high values of the fully dense effective dielectric constant ( $\epsilon'_{\text{SiSiC}} \gg 1000$ )  
415 and the  $n$ -parameters to be almost zero. In contrast,  $n = 0.327$  was determined for the  
416 cordierite to ensure satisfactory agreement between the experimental and predicted results.  
417 This value was close to the  $n$ -parameter value ( $n = 1/3$ ) predicted by Landau-Lifshitz and  
418 Looyenga [33–34] for the formula  $(\epsilon'_{\text{eff}})^{1/3} = (1 - P)(\epsilon'_{\text{cer}})^{1/3} + P(\epsilon'_{\text{air}})^{1/3}$ . Interestingly,  
419 this formula was developed for the finely dispersed mixing of isotropic particles, which  
420 would seem a reasonable assumption for cordierite foams as well. The dielectric constant  
421 estimated for the fully dense cordierite was  $\epsilon'_{\text{cord}} = 6.038$ .

### 422 **4.3 Complex permittivity calculated by the numerical simulations**

423 The effective permittivities were also calculated from the scattering parameters obtained  
424 via electromagnetic simulations for the SiSiC foam inclusions (porosities of 11 %, 30 %, 50  
425 %, 70 %, and 85 %) and their corresponding block morphologies at 2.45 GHz. The  
426 effective permittivities of the fully dense SiSiC that were obtained from the effective  
427 medium approximations were used. Predictions from the simulations in comparison with

428 the effective medium approximation relations are shown in Figure 12. The simulated  
 429 effective dielectric constants of the foam morphologies demonstrated a tendency similar to  
 430 the Bruggeman-Landauer relation. Further, the effective dielectric constants of the block  
 431 morphologies with ceramic and air inclusions behaved similarly to the DEM-complement  
 432 relation [35] and Maxwell-Garnett relations, respectively; the DEM-complement relation  
 433 refers to the mixing wherein the phases and their volume fractions are interchanged, as is  
 434 the case for ceramic inclusions:  $\frac{\epsilon'_{\text{eff}} - \epsilon'_{\text{cer}}}{\epsilon'_{\text{air}} - \epsilon'_{\text{cer}}} \left( \frac{\epsilon'_{\text{air}}}{\epsilon'_{\text{eff}}} \right)^{1/d} = P$ .

435 **Figure 12.** Comparison of the effective dielectric constants for the SiSiC open-cell foams  
 436 and blocks at 2.45 GHz and 100 °C obtained via simulations with the experimental data and  
 437 effective medium approximation relations.

438 The Bruggeman-Landauer relation handles each phase symmetrically, making it invariant  
 439 to the simultaneous interchanges of the phase permittivities and porosities. The host  
 440 medium or inclusions are not required to be defined as ceramic or air for the entire porosity  
 441 range. Therefore, phase-inversion symmetry of the material is allowed, as is expected from  
 442 the foam morphology models. However, the relation predicts a percolation threshold at  $P =$   
 443  $2/3$  (because air is an insulator), which results in an inflection at this point. This does not  
 444 hold completely in the case of the simulated foam with  $P \geq 0.70$  (see Table 9), indicating  
 445 that the struts of the ceramic material were still connected at even high porosity values. The  
 446 permittivity of the simulated foam further explains the apparent agreement in the trends

447 with  $P = 0.50$ , which matches the bi-continuous [30] segment ( $1/3 < P < 2/3$ ) of the  
448 relation.

449 The Maxwell-Garnett relation is derived from the effective-medium approximation  
450 homogenization theory for approximately spherical dielectric particles that are uniformly  
451 distributed on a background host medium [26] and provides better prediction accuracy  
452 when large numbers of inclusions are present. Further, the inclusions are not required to be  
453 the same size as long as they are much smaller than the wavelength of the operating field.  
454 Additionally, the effective permittivity depends on the degree of polydispersivity of the  
455 spherical particles [30], which leads to a percolation threshold of  $P = 1$ , i.e., the ceramic  
456 phase is no longer connected when everything is entirely composed of air. This holds for  
457 the air block inclusions because the ceramic remains interconnected throughout the porosity  
458 range and thus, agrees well with the Maxwell-Garnett relation, which approaches the upper  
459 HS bound because  $\epsilon'_{\text{cord}} > \epsilon'_{\text{air}}$ . In comparison, the cells of the simulated foams tend to not  
460 be as well-represented by spherical particles isolated in a host medium because they are  
461 interconnected through open pores, resulting in less agreement with the Maxwell-Garnett  
462 relation (see Table 9). Moreover, **the measured data were better represented** by the air block  
463 inclusions, possibly due to the semiconductive silicon effectively connecting the entire  
464 skeleton.

465 The DEM and DEM-complement relations are non-symmetric. These relations always  
466 address the spherical inclusion phase as a dilute concentration with respect to the effective

467 medium, which is the best approximation of the ceramic inclusions embedded in a  
468 continuous medium, i.e., the ceramic block inclusions.

469 For the block inclusions, non-equal contributions generally arise due to the significant  
470 difference between their dielectric properties. When the host medium was air ( $\epsilon'_{\text{air}}/\epsilon'_{\text{SiSiC}} =$   
471 0.058), charge carrier accumulation near the interphase boundaries was promoted, which  
472 caused the mixing to be a non-conducting network. Conversely, when the host medium was  
473 the SiSiC ( $\epsilon'_{\text{SiSiC}}/\epsilon'_{\text{air}} = 17.20$ ), the mixing is a conducting network [34], which increases  
474 the leak current and results in increased dielectric loss. Consequently, the dielectric loss  
475 factor of the air block inclusions (SiSiC host medium) was higher than the loss factor of the  
476 ceramic block inclusions (air host medium), as shown in Table 6.

477 **Table 6.** Comparison of the effective dielectric permittivities of the SiSiC, as predicted by  
478 the Bruggeman-Landauer, Maxwell-Garnett, and DEM-complement relations, with the  
479 results of the block and foam simulations.

## 480 **5 Conclusions and outlook**

481 In this study, the dielectric properties of SiC-based solid foams, namely SBSiC, SSiC, and  
482 SiSiC, as well as cordierite were analyzed. The complex permittivity was of particular  
483 interest because it provides the imaginary part or loss factor, which is a key parameter for  
484 predicting and designing systems heated by microwaves. Both the effective dielectric  
485 constant and the effective loss factor of the SiSiC and SBSiC foams had remarkably

486 different values as compared to the pure SiC, which could be attributed to the additives or  
487 dopants used during the sintering of the foams. The primary conclusions from this study  
488 are:

- 489     ▪ The dielectric properties of open-cell solid foams based on SiC were strongly  
490       correlated with the foam porosity. In contrast, no correlation was observed for the  
491       pore density.
  
- 492     ▪ Satisfactory agreement was obtained for cordierite foams by applying the mixing  
493       relations. Notably, the general power mean relation (fitted to the experimental data)  
494       returned an exponent  $n \approx 1/3$ , which agrees with that of the well-known Landau  
495       mixing model prediction exponential value of  $n = 1/3$ .
  
- 496     ▪ The effective permittivities predicted by the Bruggeman-Landauer relation  
497       reasonably agreed with the numerical simulations that were based on the real foam  
498       morphology because this model accounted for the phase-inversion symmetry of the  
499       foam skeleton as its porosity increased. By comparison, the Maxwell-Garnett and  
500       DEM-complement relations were suggested to predict the numerical simulation  
501       results based on simple block non-interconnected (air and ceramic) inclusions,  
502       respectively.

503 Future work will focus on numerical simulations and experimental predictions of the  
504 dielectric properties of other foam materials and the disclosure of SiSiC, SBSiC, and SSiC

505 fully dense permittivities. Furthermore, the effects of dopants and sintering additives on the  
506 dielectric properties of SiC foams will be studied systematically.

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#### 511 **Conflict of interest**

512 The authors declare that they have no known competing financial interests or personal  
513 relationships that could have appeared to influence the work reported in this paper.

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635 **Figure captions**

636 **Figure 1.** (A) Image of the SiSiC foam morphology obtained by SEM; micro X-ray  
637 tomography ( $\mu$ CT) cross-sectional images of the SiSiC foams (B) 30 ppi and (C) 45 ppi.

638 **Figure 2.** Schematic of the dielectric measurement system based on the cavity perturbation  
639 technique.  $S_{21}(f)$  is the frequency-dependent  $S$ -parameter response at port 2 to the signal at  
640 port 1.

641 **Figure 3.** Schematic structure of (A) SBSiC, (B) SSiC and (C) SiSiC.

642 **Figure 4.** Foam morphology reconstruction and meshing.

643 **Figure 5.** Examples of the foam morphology inclusions (upper row), ceramic block  
644 inclusions (middle row), and air block inclusions (lower row), within the boxes. Ceramic  
645 and air materials are color-coded in red and blue, respectively.

646 **Figure 6.** Sequential procedure from foam morphology (upper row) and block morphology  
647 (lower row) inclusions towards computational modeling for computing scattering  
648 parameters using numerical simulations.

649 **Figure 7.** Effective dielectric constants (left) and effective loss factors (right) of cordierite  
650 (first row), SiSiC (second row), SBSiC (third row) and SSiC (fourth row) at a frequency of  
651 2.45 GHz.

652 **Figure 8.** Comparison of measured and predicted effective dielectric constants using  
653 various effective medium approximation relations for SiSiC (upper-row, left), SBSiC  
654 (upper-row, right), SSiC (lower-row, left) and cordierite (lower-row, right) at 2.45 GHz and

655 100 °C (SSiC permittivities from power extrapolated to 100°C powder\* [5] and foam\*\* [6]  
656 were used for comparison).

657 **Figure 9.** Comparison of the measured and predicted effective dielectric constants using the  
658 weighted mean relations with Hashin-Shtrikman bounds for SiSiC (upper-row, left), SBSiC  
659 (upper-row, right), SSiC (lower-row, left), and cordierite (lower-row, right) at 2.45 GHz  
660 and 100 °C.

661 **Figure 10.** Comparison of the measured and predicted effective dielectric constants using  
662 the general sigmoidal mean relations for different  $N$ -parameters and bounds for SiSiC  
663 (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite (lower-  
664 row, right) at 2.45 GHz and 100 °C.

665 **Figure 11.** Comparison of predicted and measured effective dielectric constants using the  
666 general power mean relations (solid line:  $n = 0$ , dashed line:  $n \neq 0$ ); and bounds for the  
667 SiSiC (upper-row, left), SBSiC (upper-row, right), SSiC (lower-row, left), and cordierite  
668 (lower-row, right) at 2.45 GHz and 100 °C.

669 **Figure 12.** Comparison of the effective dielectric constants for the SiSiC open-cell foams  
670 and blocks at 2.45 GHz and 100 °C obtained via simulations with the experimental data and  
671 effective medium approximation relations.

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675 **Table captions**

676 **Table 2.** Complex permittivity of SiC powder and cordierite.

677 **Table 2.** Material compositions.

678 **Table 3.** Bulk and foam sample properties.

679 **Table 4.** Maximum inclusion size parameter ( $x$ ) of foams; 30 ppi foams were used for the  
680 calculations because they have the largest cell diameter.

681 **Table 5.** Mixing relations for the description of effective permittivities. (<sup>1</sup> The exponential  
682 relation proposed by Pabst and Gregorová [29] is not identical to the simple exponential  
683 relation promoted by Rice [36]).

684 **Table 6.** Comparison of the effective dielectric permittivities of the SiSiC, as predicted by  
685 the Bruggeman-Landauer, Maxwell-Garnett, and DEM-complement relations, with the  
686 results of the block and foam simulations.

687

688 **Table 3.** Complex permittivity of SiC powder and cordierite.

Material	Temperature (°C)	Frequency (GHz)	Complex permittivity ( $\epsilon$ )	Reference
SiC powder	30	2.45	$11.45 - j2.00$	[5]
	50		$11.72 - j1.65$	
	70		$11.87 - j1.36$	
Cordierite	~20	$1 \times 10^{-6}$	$6 - j0.06$	[31]
	~20	8.52	$4.77 - j0.008$	[32]
	20	10	$4.30 - j8.6 \times 10^{-4}$	[7]
	~20	18.3	$4.89 - j11.6 \times 10^{-4}$	[8]

689

690 **Table 2.** Material compositions.

Material	Ingredients
SBSiC	Bi-modal SiC powder composition, silica sources and alumina powder as inorganic binder matrix.
SSiC	Fine SiC powder, organic temporary binder acting as carbon source after pyrolysis, and boron as a sintering aid.
SiSiC	Fine SiC powder, organic temporary binder acting as carbon source after pyrolysis to react with the infiltrated silicon.
Cordierite	Composition of cordierite chamotte, bentonite, alumina, clay, and soapstone.

691

692 **Table 3.** Bulk and foam sample properties.

Material	Description	Averaged cell diameter ( $\mu\text{m}$ )	Density ( $\text{g}/\text{cm}^3$ )	Porosity (%)	Diameter / height of samples (mm)
SBSiC	Bulk	-	2.14	29.1	7.9 / 10.0
	30 ppi	$\approx 1460$	0.18 - 0.28	90.7 - 94.0	$7.2 \pm 0.7 / 9.3 \pm 0.7$
	45 ppi	$\approx 890$	0.16 - 0.22	92.6 - 94.7	
	60 ppi	$\approx 600$	0.22	92.7	
SSiC	Bulk	-	2.88	8.7	7.9 / 10.0
	30 ppi	$\approx 1460$	0.19 - 0.21	93.3 - 94.1	$6.9 \pm 1.0 / 9.0 \pm 1.0$
	45 ppi	$\approx 890$	0.11 - 0.15	95.2 - 96.4	
	60 ppi	$\approx 600$	0.11 - 0.17	94.7 - 96.5	
SiSiC	Bulk	-	2.69	0.0	7.9 / 10.0
	30 ppi	$\approx 1460$	0.27 - 0.35	88.0 - 88.9	$7.4 \pm 0.5 / 9.5 \pm 0.5$
	45 ppi	$\approx 890$	0.20 - 0.22	90.9 - 91.8	
	60 ppi	$\approx 600$	0.18 - 0.20	92.2 - 93.0	
Cordierite	Bulk	-	1.80	29.3	7.9 / 10.0
	30 ppi	$\approx 1460$	0.33 - 0.37	85.3 - 87.2	$7.4 \pm 0.5 / 9.5 \pm 0.5$
	45 ppi	$\approx 890$	0.27 - 0.49	80.7 - 89.5	
	60 ppi	$\approx 600$	0.27	89.5	

693

694 **Table 4.** Maximum inclusion size parameter ( $x$ ) of foams; 30 ppi foams were used for the  
 695 calculations because they have the largest cell diameter.

Foams	SBSiC	SSiC	SiSiC	Cordierite
$\epsilon'_{eff,30ppi}$	1.85	1.67	7.10	1.45
$\lambda$ (mm)	90.7	99.3	46.4	101.32
$x_{30ppi}$	0.05	0.05	0.10	0.05

696

697 **Table 5.** Mixing relations for the description of effective permittivities. (<sup>1</sup> The exponential  
698 relation proposed by Pabst and Gregorová [29] is not identical to the simple exponential  
699 relation promoted by Rice [36]).

Effective-medium approximations	
Maxwell-Garnett approximation (Maxwell-type relation non-symmetric) [30]	$\frac{\epsilon'_{\text{eff}} - \epsilon'_{\text{cer}}}{\epsilon'_{\text{eff}} + (d-1)\epsilon'_{\text{cer}}} = (P) \frac{\epsilon'_{\text{air}} - \epsilon'_{\text{cer}}}{\epsilon'_{\text{air}} + (d-1)\epsilon'_{\text{cer}}}$
Bruggeman-Landauer approximation (self-consistent relation symmetric) [30, 35]	$(1-P) \frac{\epsilon'_{\text{cer}} - \epsilon'_{\text{eff}}}{\epsilon'_{\text{cer}} + (d-1)\epsilon'_{\text{eff}}} = -(P) \frac{\epsilon'_{\text{air}} - \epsilon'_{\text{eff}}}{\epsilon'_{\text{air}} + (d-1)\epsilon'_{\text{eff}}}$
Differential Effective Medium (DEM) approximation (Bruggeman relation non-symmetric) [30, 35]	$\frac{\epsilon'_{\text{air}} - \epsilon'_{\text{eff}}}{\epsilon'_{\text{air}} - \epsilon'_{\text{cer}}} \left( \frac{\epsilon'_{\text{cer}}}{\epsilon'_{\text{eff}}} \right)^{1/d} = 1 - P$
Exponential relation [36]	$\frac{\epsilon'_{\text{eff}}}{\epsilon'_{\text{cer}}} = \exp(-B \cdot P)$
Pabst-Gregorová <sup>1</sup> exponential relation [29]	$\frac{\epsilon'_{\text{eff}}}{\epsilon'_{\text{cer}}} = \exp\left(-B \frac{P}{1-P}\right)$
Empirical mixing relations	
Weighted arithmetic mean of the upper and lower bounds [28]	$\epsilon'_{\text{eff}} = (1 - \Psi_{\text{arithm}})\epsilon_B^+ + \Psi_{\text{arithm}}\epsilon_B^-$
Weighted harmonic mean of the upper and lower bounds [28]	$\epsilon'_{\text{eff}} = \left[ \frac{(1 - \Psi_{\text{harm}})}{\epsilon_B^+} + \frac{\Psi_{\text{harm}}}{\epsilon_B^-} \right]^{-1}$
Weighted geometric mean of the upper and lower bounds [28]	$\epsilon'_{\text{eff}} = \exp\left[(1 - \Psi_{\text{geom}}) \ln(\epsilon_B^+) + \Psi_{\text{geom}} \ln(\epsilon_B^-)\right]$
General sigmoidal mean of the upper and lower bounds [28]	$\epsilon'_{\text{eff}} = (1 - \delta_{0,N})[(1 - P)(\epsilon_B^+)^N + P(\epsilon_B^-)^N]^{\frac{1}{N}} + \delta_{0,N}\{\exp[(1 - P) \ln(\epsilon_B^+) + P \ln(\epsilon_B^-)]\}$
	where $\delta_{0,N} = \begin{cases} 1 & \text{if } N = 0 \\ 0 & \text{otherwise} \end{cases}$
General power mean of the upper and lower dielectric constants [28]	$\epsilon'_{\text{eff}} = (1 - \delta_{0,n})[(1 - P)(\epsilon'_{\text{cer}})^n + P(\epsilon'_{\text{air}})^n]^{\frac{1}{n}} + \delta_{0,n}\{\exp[(1 - P) \ln(\epsilon'_{\text{cer}}) + P \ln(\epsilon'_{\text{air}})]\}$
	where $\delta_{0,n} = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$
Wiener bounds ( $\epsilon_B = \epsilon_W$ )	Hashin-Shtrikman bounds ( $\epsilon_B = \epsilon_{SH}$ )
Upper Wiener bound [28] $\epsilon_W^+ = (1 - P)\epsilon'_{\text{cer}} + P\epsilon'_{\text{air}}$	Upper Hashin-Shtrikman bound [28] $\epsilon_{HS}^+ = (1 - P)\epsilon'_{\text{cer}} + P\epsilon'_{\text{air}} - \frac{(1 - P)P(\epsilon'_{\text{cer}} - \epsilon'_{\text{air}})^2}{3\epsilon'_{\text{cer}} - (1 - P)(\epsilon'_{\text{cer}} - \epsilon'_{\text{air}})}$
Lower Wiener bound [28] $\epsilon_W^- = \left[ \frac{(1 - P)}{\epsilon'_{\text{cer}}} + \frac{P}{\epsilon'_{\text{air}}} \right]^{-1}$	Lower Hashin-Shtrikman bound [28] $\epsilon_{HS}^- = (1 - P)\epsilon'_{\text{cer}} + P\epsilon'_{\text{air}} - \frac{(1 - P)P(\epsilon'_{\text{cer}} - \epsilon'_{\text{air}})^2}{3\epsilon'_{\text{air}} + P(\epsilon'_{\text{cer}} - \epsilon'_{\text{air}})}$

700 **Table 6.** Comparison of the effective dielectric permittivities of the SiSiC, as predicted by  
 701 the Bruggeman-Landauer, Maxwell-Garnett, and DEM-complement relations, with the  
 702 results of the block and foam simulations.

Porosity	Effective dielectric permittivity $\epsilon'_{\text{eff}}$ (-)					
	Simulation with foam morphology	Simulated blocks – air inclusions	Simulated blocks –ceramic inclusions	Bruggeman-Landauer	Maxwell-Garnett	DEM-complement
0.11	79.83 – $j$ 2.99	78.30 – $j$ 3.94	20.74 – $j$ 0.25	79.62 – $j$ 4.27	80.41 – $j$ 4.32	55.59 – $j$ 2.40
0.30	51.03 – $j$ 1.43	56.80 – $j$ 2.96	7.99 – $j$ 0.04	53.12 – $j$ 2.82	58.37 – $j$ 3.12	19.35 – $j$ 0.43
0.50	26.42 – $j$ 0.72	37.49 – $j$ 1.97	4.29 – $j$ 0.01	25.85 – $j$ 1.28	38.74 – $j$ 2.05	6.65 – $j$ 0.05
0.70	8.96 – $j$ 0.39	21.62 – $j$ 1.09	2.49 – $j$ 0.00	5.11 – $j$ 0.09	21.98 – $j$ 1.13	2.75 – $j$ 0.01
0.85	5.40 – $j$ 0.01	0.00 – $j$ 0.00	1.64 – $j$ 0.00	1.75 – $j$ 0.00	10.95 – $j$ 0.54	1.71 – $j$ 0.00

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