Non-Perturbative Aspects of QCD: Condensates

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$$\rho, \omega \sim \mathbf{\bar{u}u \mp \bar{d}d} : \quad \mathbf{m_q \langle \bar{q}q \rangle}, \quad \textbf{...} \quad \textbf{HADES}$$

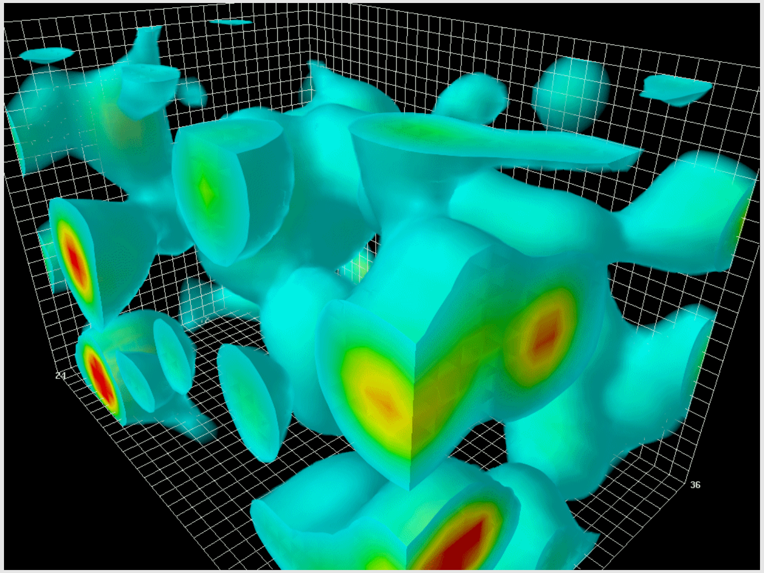
$$\mathbf{D \sim \bar{c}d} : \quad \mathbf{m_c \langle \bar{q}q \rangle}, \quad \mathbf{\langle \bar{c}c \rangle = ?} \quad \textbf{...} \quad \textbf{CBM @ FAIR}$$

$$\langle \frac{\alpha_\mathbf{s}}{\pi} \mathbf{G}_{\mu\nu} \mathbf{G}^{\mu\nu} \rangle$$

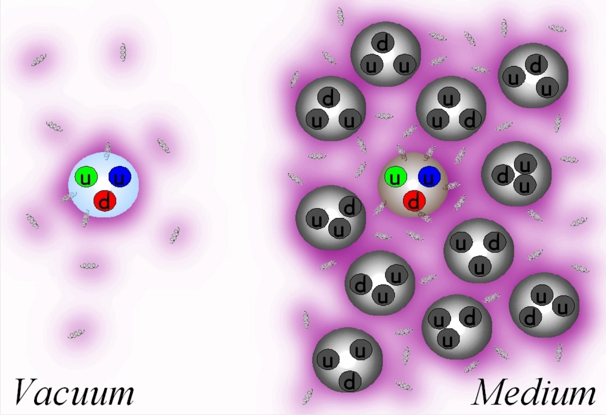
Current-Current Correlation Function

$$\mathbf{\Pi(q) = i \int d^4x e^{iqx} \langle \Omega | T[j(x)\bar{j}(0)] | \Omega \rangle}$$

- $\mathbf{a|\Omega\rangle \neq 0}$
- **state of minimum energy**



[Leinweber:http://www.physics.adelaide.edu.au/ dleinweb/]



particle	interpolating field $\mathbf{j(x)}$
$\mathbf{D^+}$-meson	$\mathbf{i\bar{d}(x)\gamma_5 c(x)}$
$\mathbf{D^-}$-meson	$\mathbf{i\bar{c}(x)\gamma_5 d(x)}$
ρ-meson	$\mathbf{\frac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right)}$
ω-meson	$\mathbf{\frac{1}{2}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right)}$
nucleon	$\mathbf{\epsilon^{abc}\left[u_a^T C\gamma_\mu u_b\right]\gamma_5\gamma^\mu d_c}$

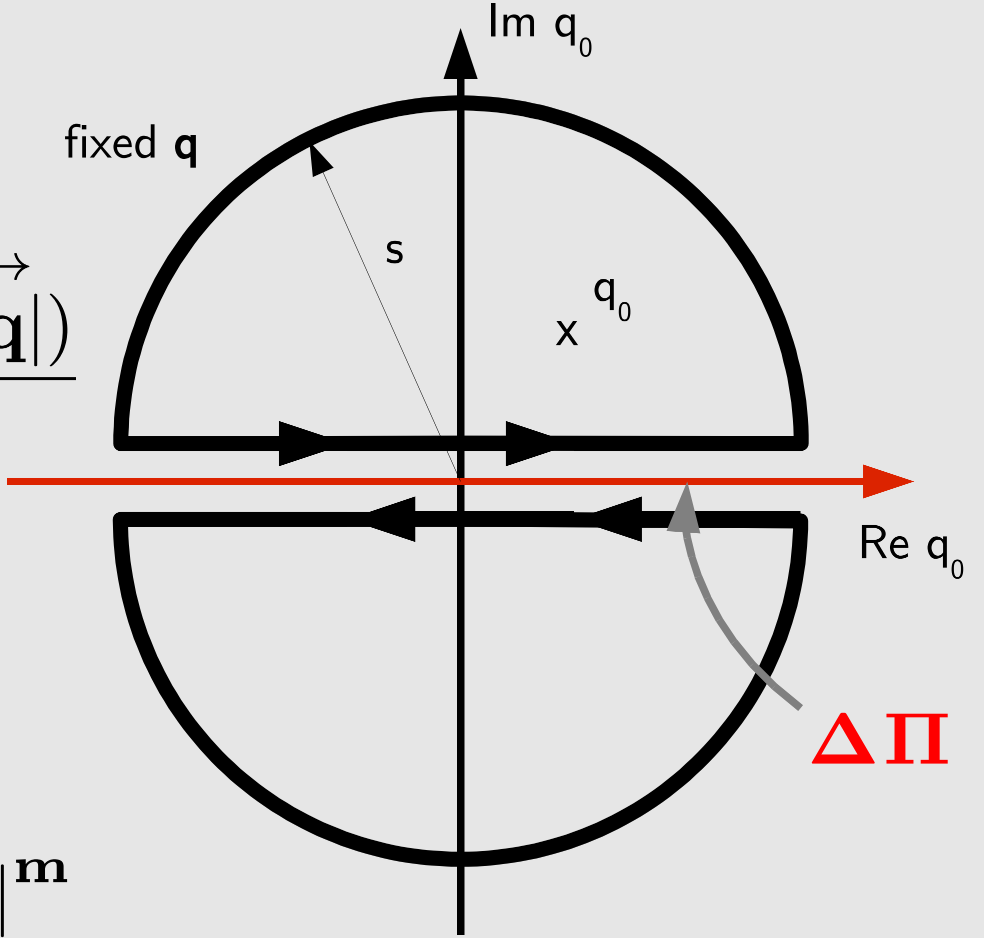
Analytic Properties of $\Pi(q)$

Lehmann-representation: poles at the entire real axis

in-medium: $\mathbf{\Pi(q_\mu, v_\nu) = \Pi(q^2, v^2, q \cdot v) \equiv \Pi(q_0, |\vec{q}|)}$

dispersion relation:

$$\mathbf{\Pi(q_0, |\vec{q}|) = \frac{1}{\pi} \int_{-\infty}^{+\infty} ds \frac{\Delta\Pi(s, |\vec{q}|)}{s - q_0} + polynomials}$$



restriction: $\mathbf{|\Pi(q_0)| \overset{|q_0| \to \infty}{\leq} |q_0|^m}$

for some arbitrary but finite and fixed m

Operator Product Expansion (OPE)

$$\mathbf{T[A(x)B(y)] = \sum_i C_i(x-y)O_i}$$

expansion at operator level! $\Rightarrow$ **state independent Wilson coefficents**

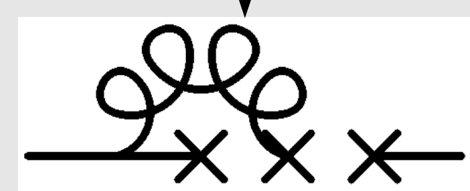
$$\mathbf{\langle\Omega|T[A(x)B(y)]|\Omega\rangle = \sum_i C_i(x-y)\langle\Omega|O_i|\Omega\rangle}$$

condensates: parameters characterizing QCD

$$\mathbf{\Pi(q) \Rightarrow \Pi_{OPE}(q) = \sum_i \tilde{C}_i(q)\langle O_i\rangle}$$

$\mathbf{O_i = 1}$, $\quad \bar{\mathbf{\Psi}}\mathbf{\Psi}$, $\quad \frac{\alpha_s}{\pi}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}$, $\quad \bar{\mathbf{\Psi}}\sigma^{\mu\nu}\mathbf{G}^{\mathbf{A}}_{\mu\nu}\mathbf{t}^{\mathbf{A}}\mathbf{\Psi}$





$+$ **additional condensates in medium, e.g.** $\langle\mathbf{\Psi}^{+}\mathbf{\Psi}\rangle$

QCD Sum Rules*

$$\Pi(q_0, |\vec{q}|) = \frac{1}{\pi}\int_{-\infty}^{+\infty} ds \frac{\Delta\Pi(s, |\vec{q}|)}{s - q_0}$$

QCD structure via OPE

splitting in hadronic parts

$$\Pi_{OPE}(q_0, |\vec{q}|) = \frac{1}{\pi}\left(\overbrace{\underbrace{\int_{s_0}^{\infty} + \int_{-\infty}^{-s_0}} + \int_{-s_0}^{s_0}}\right) ds \frac{\Delta\Pi(s, |\vec{q}|)}{s - q_0}$$

semi-local quark hadron duality

OPE ←

hadronic properties via optical theorem

$\Delta\Pi = \mathrm{Im}\Pi^{\mu}_{\mu} \to$ observable,

e.g. Dilepton emission rate or $R = \frac{\sigma_{e^+e^- \to hadrons}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \propto \mathrm{Im}\Pi^{\mu}_{\mu}(s)$

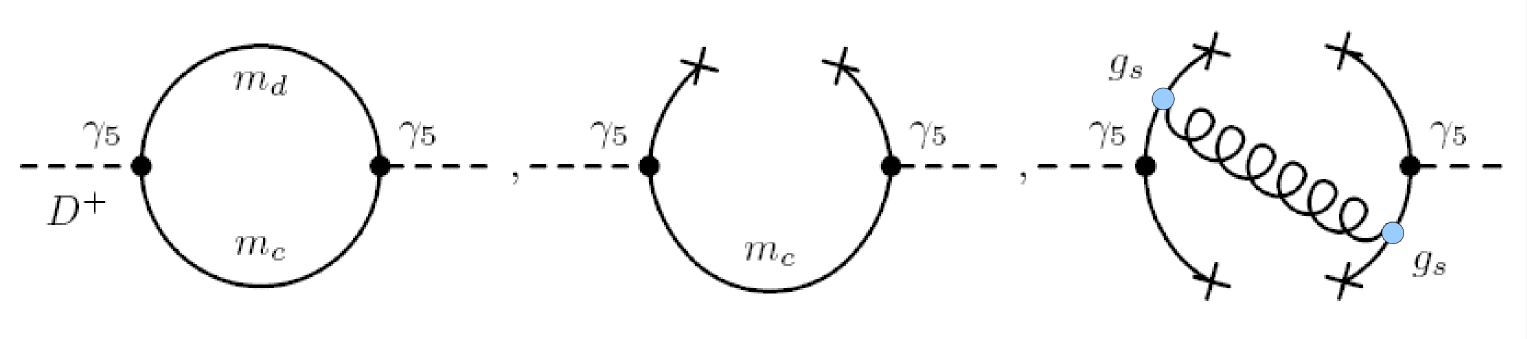
[*Shifman, Vainshtein, Zakharov: Nucl.Phys.B147(1979)]

Methods to perform an OPE

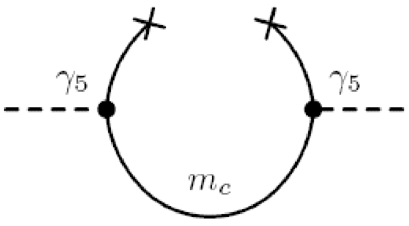
- **plane-wave method: evaluate a certain Wilson coefficient by calculating matrix elements using appropriate states, e.g.**
  $\langle \mathbf{G}|\mathbf{T}\left[\mathbf{j}(\mathbf{x})\bar{\mathbf{j}}(\mathbf{0})\right]|\mathbf{G}\rangle = \langle \mathbf{G}|\sum_{\mathbf{i}} \mathbf{C_i}(\mathbf{x})\mathbf{O_i}|\mathbf{G}\rangle = \mathbf{C_{G^2}}\langle \mathbf{G}|\mathbf{G^2}|\mathbf{G}\rangle$

- **Fock-Schwinger method: perturbative calculation of Wilson coefficients by using a perturbative quark propagator in a weak background field $\mathbf{A}_\mu$**

  $\rightarrow$ applying Wick's theorem to the correlation function: $\mathbf{\Pi_{D^+}(q) = \Pi^{(0)}(q) + \Pi^{(2)}(q) + \Pi^{(4)}(q)}$



Expansion and Projection



- **e.g.** D^+**-meson:** $\mathbf{\Pi^{(2)}(q)}$ **contains the following structure**

$$\mathbf{\Pi^{(2)}(q) = i \int d^4x e^{iqx} \langle \Omega | : \bar{d}(x) \gamma_5 S_c^{per}(x,0) \gamma_5 d(0) : | \Omega \rangle}$$

- $\langle : \bar{\mathbf{d}} \gamma_\mu \mathbf{D}_\nu \mathbf{d} : \rangle, \quad \langle : \mathbf{G}^{\mathbf{a}}_{\mu\nu} \mathbf{G}^{\mathbf{b}}_{\alpha\beta} : \rangle, \quad \langle : \bar{\mathbf{d}} \mathbf{D}_\mu \mathbf{D}_\nu \mathbf{d} : \rangle, \quad \ldots$

vacuum **+ medium**

$$\frac{\delta^{ab}}{96} \left(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha} \right) \langle : G^2 : \rangle$$

$$- \frac{\delta^{ab}}{24} \Bigg(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha} - 2 \Bigg(g_{\mu\alpha} \frac{v_\nu v_\beta}{v^2} + g_{\nu\beta} \frac{v_\mu v_\alpha}{v^2}$$

$$- g_{\nu\alpha} \frac{v_\mu v_\beta}{v^2} - g_{\mu\beta} \frac{v_\nu v_\alpha}{v^2} \Bigg) \Bigg) \langle : \frac{(vG)^2}{v^2} - \frac{G^2}{4} : \rangle$$

Fock-Schwinger Method

- **Fock-Schwinger Gauge:** $\mathbf{x^\mu A_\mu(x) = 0}$

$$\rightarrow \mathbf{A}_\mu(\mathbf{x}) = \sum_{\mathbf{k=0}}^{\infty} \frac{\mathbf{x}^\rho}{\mathbf{k!(k+2)}} \prod_{\mathbf{m=1}}^{\mathbf{k}} \mathbf{x}^{\alpha_\mathbf{m}} \left[\prod_{\mathbf{n=1}}^{\mathbf{k}} \mathbf{D}_{\alpha_\mathbf{n}} \mathbf{G}_{\rho\mu} \right]_\mathbf{0}$$

$$\text{and} \quad \boldsymbol{\Psi}(\mathbf{x}) = \sum_{\mathbf{k=0}}^{\infty} \frac{\mathbf{1}}{\mathbf{k!}} \prod_{\mathbf{m=1}}^{\mathbf{k}} \mathbf{x}^{\alpha_\mathbf{m}} \left[\prod_{\mathbf{n=1}}^{\mathbf{k}} \mathbf{D}_{\alpha_\mathbf{n}} \boldsymbol{\Psi} \right]_\mathbf{0}$$

- **quark propagator in background field:**

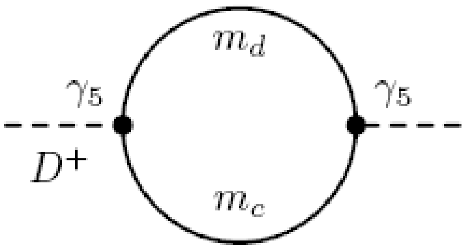
$$\mathbf{S}^{\mathbf{pert}}(\mathbf{p};\mathbf{A}) = \sum_{\mathbf{k=0}}^{\infty} \mathbf{S}^{(\mathbf{free})}(\mathbf{p}) \prod_{\mathbf{n=0}}^{\mathbf{k}} \left[\left(\gamma \tilde{\mathbf{A}} \right) \mathbf{S}^{(\mathbf{free})}(\mathbf{p}) \right]^{\mathbf{n}}$$

$$\text{with} \quad \tilde{\mathbf{A}}_\mu = \sum_{\mathbf{k=0}}^{\infty} \frac{(-\mathbf{i})^{\mathbf{k+1}}\mathbf{g}}{\mathbf{k!(k+2)}} \left[\prod_{\mathbf{n=0}}^{\mathbf{k}} \mathbf{D}_{\alpha_\mathbf{n}} \mathbf{G}_{\rho\mu} \right]_\mathbf{0} \delta^\rho \prod_{\mathbf{m=0}}^{\mathbf{k}} \delta^{\alpha_\mathbf{m}}$$

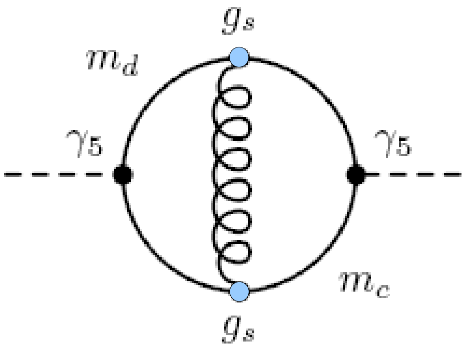
Mass-Logarithms for Heavy-Light Systems

- applying Wick's theorem to the correlation function:

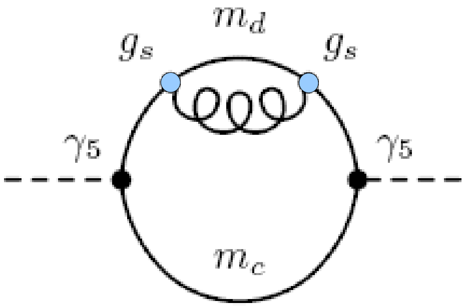
$$\mathbf{\Pi_{D^+}(q) = \Pi^{(0)}(q) + \Pi^{(2)}(q) + \Pi^{(4)}(q)}$$



$$\mathbf{i \int \frac{d^4p}{(2\pi)^4} \langle Tr \left[\gamma_5 S_c^{pert}(p+q)\gamma_5 \tilde{S}_d^{pert}(p)\right]\rangle}$$



- mass logarithms $\left(\ln \mathbf{m^2}\right)$ of light quarks appear
- remnants of large distance behaviour
- to perform a consistent seperation of scales $\rightarrow$ absorption into condensates



Absorption of Divergences

- def. of physical condensate:

$$\langle\boldsymbol{\Omega}|\bar{\boldsymbol{\Psi}}\mathbf{O}\left[\mathbf{D}_\mu\right]\boldsymbol{\Psi}|\boldsymbol{\Omega}\rangle = \langle\boldsymbol{\Omega}|{:}\bar{\boldsymbol{\Psi}}\mathbf{O}\left[\mathbf{D}_\mu\right]\boldsymbol{\Psi}{:}|\boldsymbol{\Omega}\rangle$$

$$-\mathbf{i}\int \mathbf{d^4p}\langle\boldsymbol{\Omega}|\mathbf{Tr}\left[\mathbf{O}\left(-\mathbf{ip}_\mu - \mathbf{i}\tilde{\mathbf{A}}_\mu\right)\mathbf{S^{pert}}(\mathbf{p})\right]|\boldsymbol{\Omega}\rangle$$

- dimensional regularization + renormalization in MS-scheme $\rightarrow$ cancelation of mass-logarithms
- Wick-theorem leads to normal ordered condensates
- divergences are absorbed into condensates

Physical Condensates

results in $\overline{\text{MS}}$-scheme up to $O(\alpha_s)$ for heavy and light quarks

$$\langle\bar{q}q\rangle = \langle:\bar{q}q:\rangle + \frac{3}{4\pi^2}m_q^3\left(\ln\frac{\mu^2}{m_q^2}+1\right) - \frac{1}{12m_q}\langle:\frac{\alpha_s}{\pi}G^2:\rangle + \ldots$$

$$\langle\bar{q}g\sigma G^A t^A q\rangle = \langle:\bar{q}g\sigma G^A t^A q:\rangle - \frac{1}{2}m_q\ln\frac{\mu^2}{m_q^2}\langle:\frac{\alpha_s}{\pi}G^2:\rangle + \ldots$$

$$\langle\bar{q}\gamma_\mu iD_\nu q\rangle = \langle:\bar{q}\gamma_\mu iD_\nu q:\rangle + \frac{9}{4\pi^2}m_q^4 g_{\mu\nu}\left(\ln\frac{\mu^2}{m_q^2}+\frac{5}{12}\right) - \frac{g_{\mu\nu}}{48}\langle:\frac{\alpha_s}{\pi}G^2:\rangle$$
$$+\frac{1}{18}\left(g_{\mu\nu}-4\frac{v_\mu v_\nu}{v^2}\right)\left(\ln\frac{\mu^2}{m_q^2}-\frac{1}{3}\right)\langle:\frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2}-\frac{G^2}{4}\right):\rangle + \ldots$$

$$\langle\bar{q}iD_\mu iD_\nu q\rangle = \langle:\bar{q}iD_\mu iD_\nu q:\rangle - \frac{m_q^5}{2\pi^2}g_{\mu\nu}\left(\ln\frac{\mu^2}{m_q^2}+1\right) - \frac{m_q}{16}g_{\mu\nu}\left(\ln\frac{\mu^2}{m_q^2}-\frac{1}{3}\right)\langle:\frac{\alpha_s}{\pi}G^2:\rangle$$
$$+\frac{m_q}{36}\left(g_{\mu\nu}-4\frac{v_\mu v_\nu}{v^2}\right)\left(\ln\frac{\mu^2}{m_q^2}+\frac{2}{3}\right)\langle:\frac{\alpha_s}{\pi}\left(\frac{(vG)^2}{v^2}-\frac{G^2}{4}\right):\rangle + \ldots$$

→ mixing of condensates

→ additional terms for in-medium case

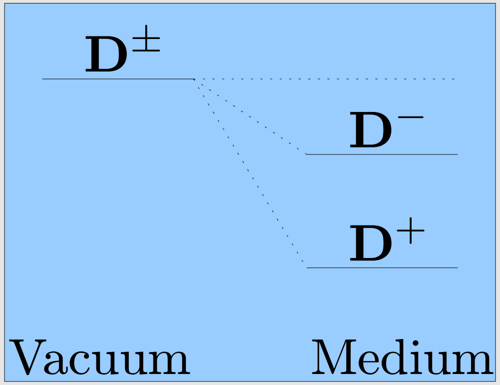
[Wilson coeff. cf. Morath/Weise (2001), Hayashigaki (2000), Zschocke (2005)]

Unification of light and heavy quark physics

- $\rightarrow \langle \bar{c}c \rangle = -\frac{g^2}{48\pi^2 m_c}\langle G^2 \rangle - \frac{g^3}{1440\pi^2 m_c^3}\langle G^3 \rangle - \ldots$

  no new independent parameter for heavy quarks

- next steps: complete OPE
  Borel transformation
  continuum asymmetry
  $D^+ - D^-$, $D^0 - \bar{D}^0$ pattern



[a la Morath/Weise]

Summary

- **dispersion relation + OPE = QCD Sum Rules**
- **medium modification → condensates → OPE**
  → physical ground state → hadrons: $\mathbf{D}^{\pm}$
- **consistent separation of scales leads to**
  mixing of condensates
- **heavy quark condensates (e.g. $\langle \bar{c}c \rangle$) not new**
- **further details of in-medium sum rules of D-mesons**
  under investigation (my diploma thesis)