International Workshop on FLOW COntrol by Tailored MAGnetic Fields

Open and closed-loop Control of Weakly Conducting Fluids

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Outline:

- **Part 1**
    - **2posittion p-controller**
- **Part 2**
    - **Nonlinear feedback control recipes**

Motivation

Given:

a validated mathematical model for a physical process.

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Model based control of the physical process

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Applications

Control of weakly conducting fluids(Sfb 609)

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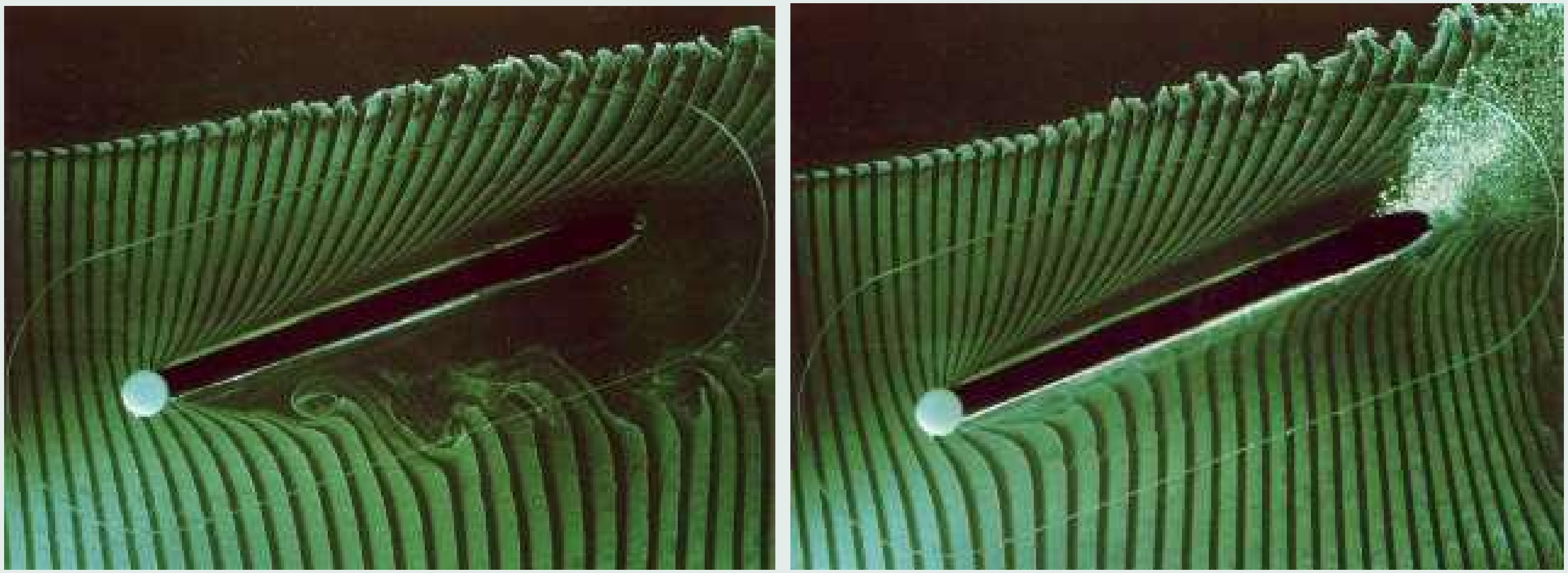

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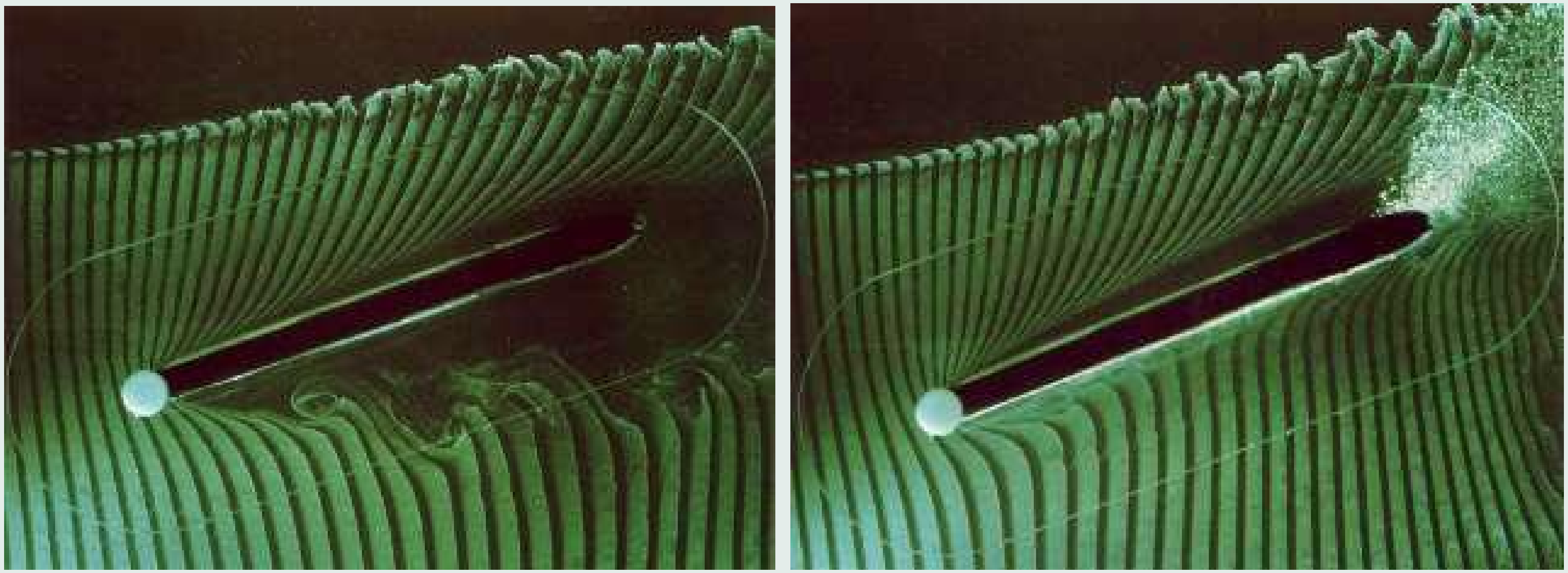

Develop methods for circular cylinder

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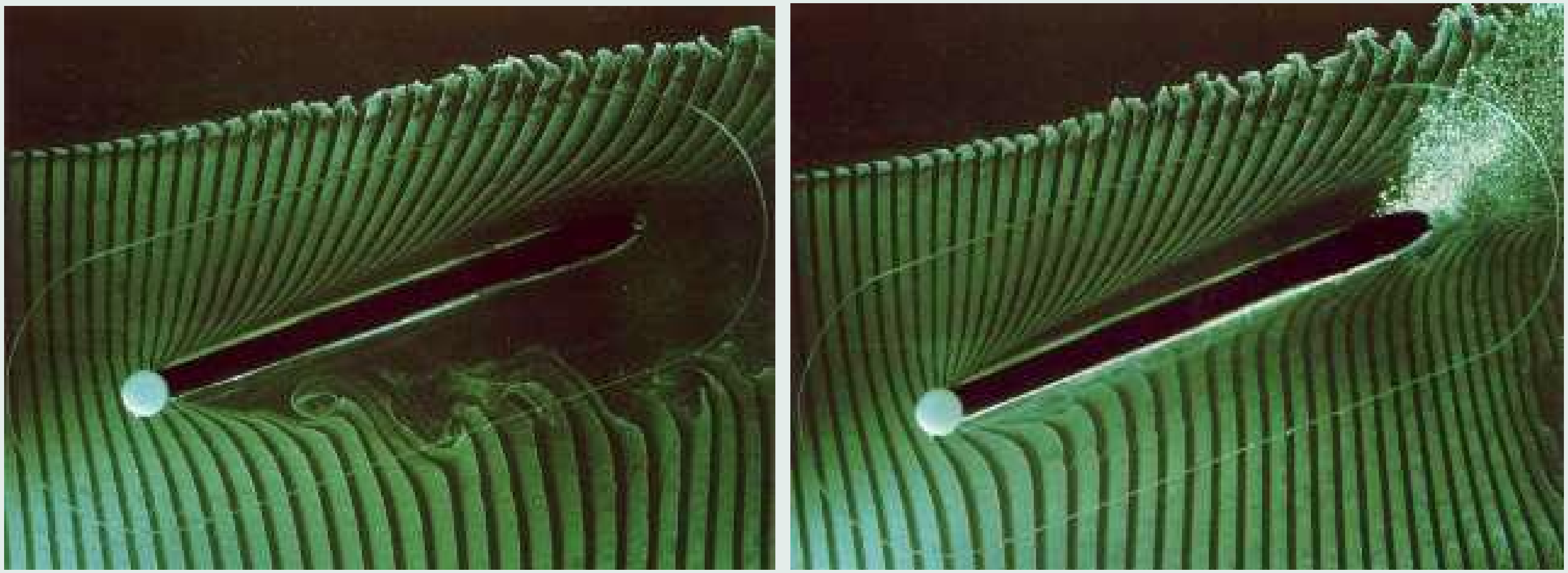

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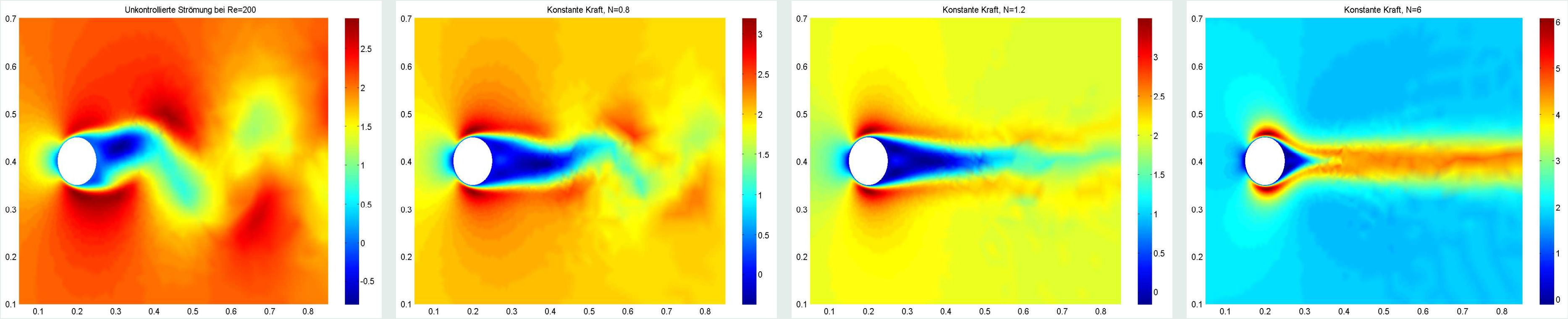

Control target: Re-attach flow utilizing near wall Lorentz forces

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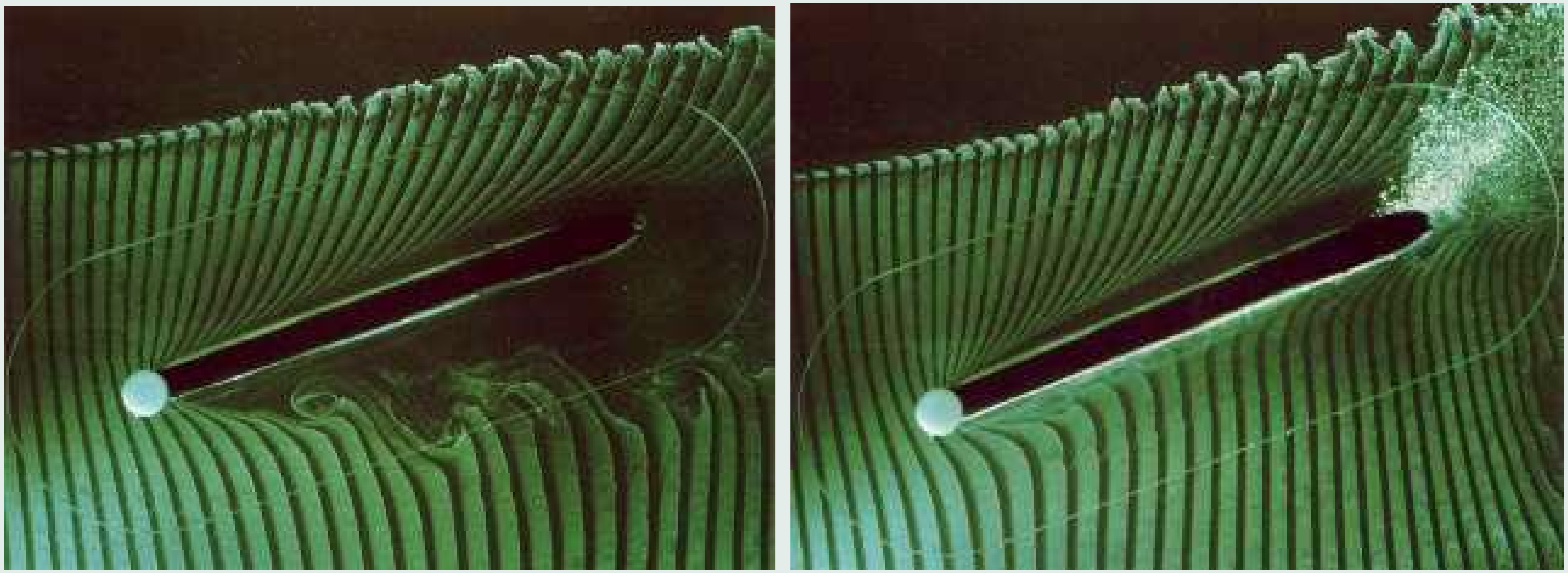

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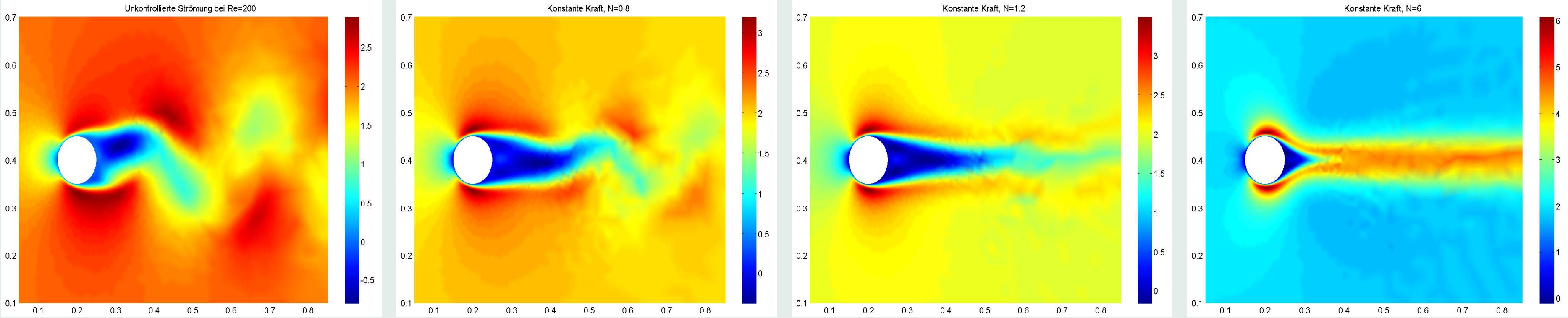

Control target: Re-attach flow utilizing near wall Lorentz forces

Desired: Closed-loop control strategy

Control strategies (problem depending)

- **Optimal (robust) open-loop control**
- **(robust) feedback control**

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3 concepts

- **Design of a two-position controller on the basis of the mathematical model**
- **model-predictive control, instantaneous control**
- **model-reduction and control**

Part 1: Control of the flow past a cylinder, conducted by the experiment.

On the surface of the cylinder a Lorentz force is generated with actuators. It decays exponentially in the flow (T. Berger et al., Phys. Fluids 2000).

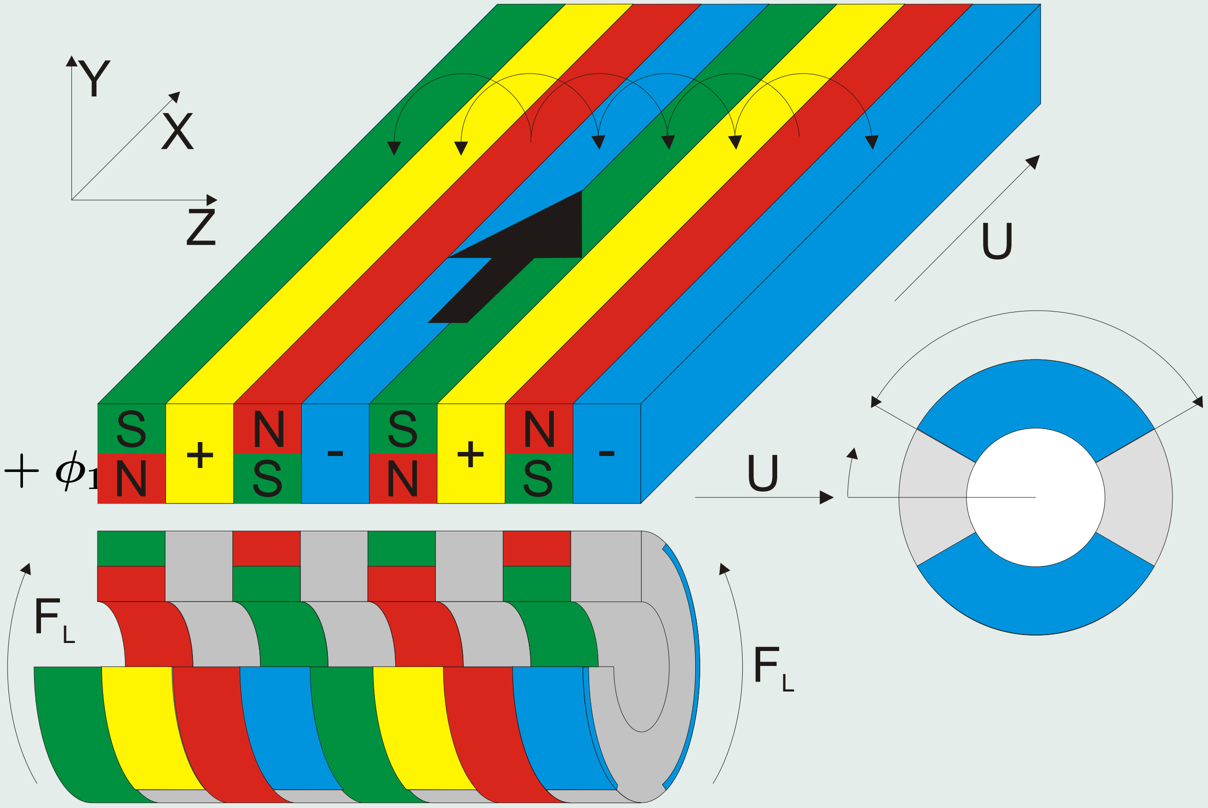
Generation of the Lorentz Force

On the surface of the cylinder a Lorentz force is generated with actuators. It decays exponentially in the flow (T. Berger et al., Phys. Fluids 2000). The Lorentz force has the form

$$\boldsymbol{F_L(x,y)} := \boldsymbol{e_\phi g(\phi) e^{-\frac{\pi}{a}\cdot dist[(x,y),\ cylinder]}},$$

$$\boldsymbol{g(\phi)} = \begin{cases} 1, & \phi_0 \le \phi \le \phi_1 \\ -1, & 180^\circ + \phi_0 \le \phi \le \ 180^\circ + \phi_1 \\ 0, & \boldsymbol{else} \end{cases},$$

$\boldsymbol{a}$ electrode-/magnet-spacing.



The direction of the Lorentz Force is defined by the arrangement of the magnets/electrodes.

Navier-Stokes-Equations

$$\boldsymbol{u}_t + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} + \boldsymbol{\nabla} p - \frac{1}{\mathrm{Re}} \boldsymbol{\nabla}^2 \boldsymbol{u} = N \boldsymbol{F}_L$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

plus initial and boundary conditions.
u denotes the flow velocity, p the pressure, and $\mathrm{Re} = \frac{U_\infty D}{\nu}$ the Reynolds number, with ν the constant kinematic viscosity, D the cylinder diameter and U_∞ the characteristic velocity. The Interaction parameter N describes the ratio of the electromagnetic and the inertial forces of the flow,

$$N = \frac{J_0 B_0 D}{\rho U_\infty^2} \quad ,$$

with J_0 the current density and B_0 magnetic induction.

The numerics are validated against the computations of R. Grundmann/O. Posdziech (TUD). Comparison with experiments by G. Gerbeth/T. Weier et al. (FZR) to be addressed.

Implementation in FEMLAB.

Spatial discretization: Taylor-Hood elements.

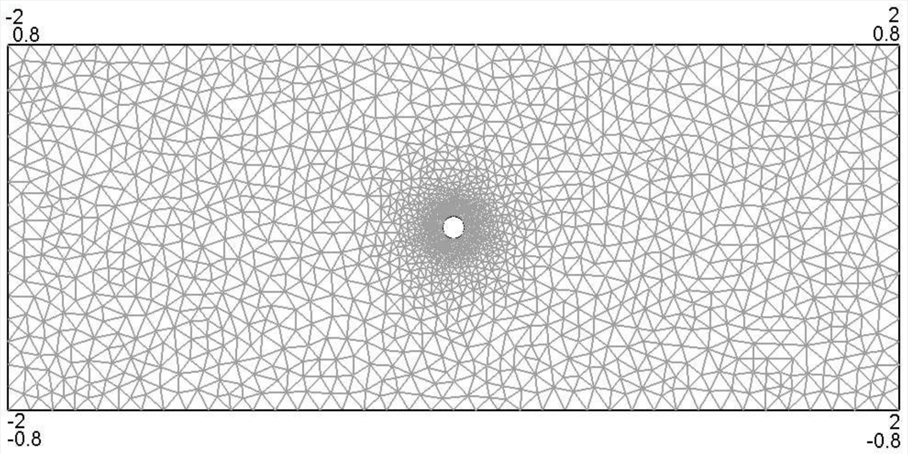
Integration schema: BDF (or other).

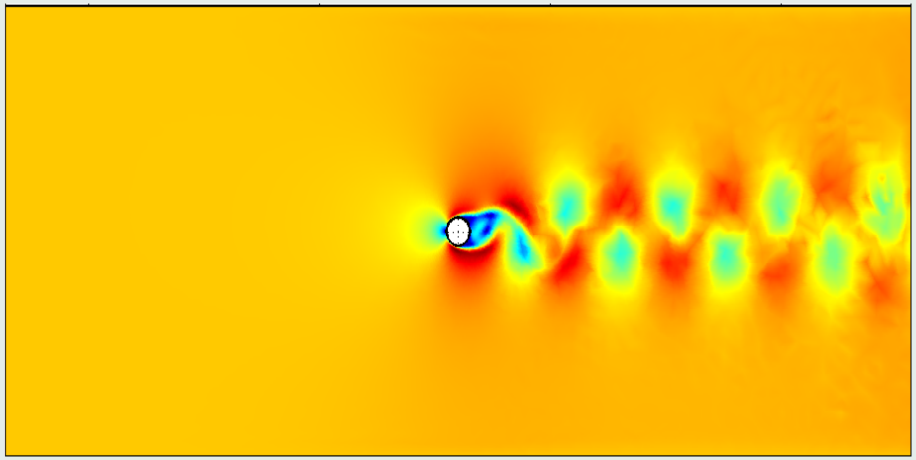
Inflow condition: Block/parabolic profile. Cylinder surface: no-slip.

Upper and lower boundary: slip/no-slip.

Outflow: do-nothing.

Computational domain:





The large (in relation to the cylinder) domain is needed in order to compute the correct drag coefficients.

The drag is defined as:

$$F_D = F_{D_f} + F_{D_p} + F_{D_{em}} = \underbrace{\int_{\partial cyl.} \rho\nu \frac{\partial u_t}{\partial n_y}\, dS}_{=F_{D_f}} - \underbrace{\int_{\partial cyl.} p n_x \, dS}_{=-F_{D_p}} - \underbrace{J_0 B_0 \frac{a}{\pi} D(\cos\phi_0 - \cos\phi_1)}_{=-F_{D_{em}}},$$

in which u_t is the tangential velocity and n_i denote the components of the normal vector. With F_D the drag coefficient C_D is given by:

$$C_D = \frac{2F_D}{\rho U_\infty^2 D} \quad , C_{D_f} = \frac{2F_{D_f}}{\rho U_\infty^2 D}, \quad C_{D_p} = \frac{2F_{D_p}}{\rho U_\infty^2 D}, \quad C_{D_{em}} = \frac{2F_{D_{em}}}{\rho U_\infty^2 D} \quad .$$

To determine the Strouhal Number the frequency of vortex separation f is used,

$$St = \frac{fD}{U_\infty}$$

St is specified by the Lift force:

$$F_l = -\int_{\partial cyl.} \rho\nu \frac{\partial u_t}{\partial n_x} + p n_y \, dS$$

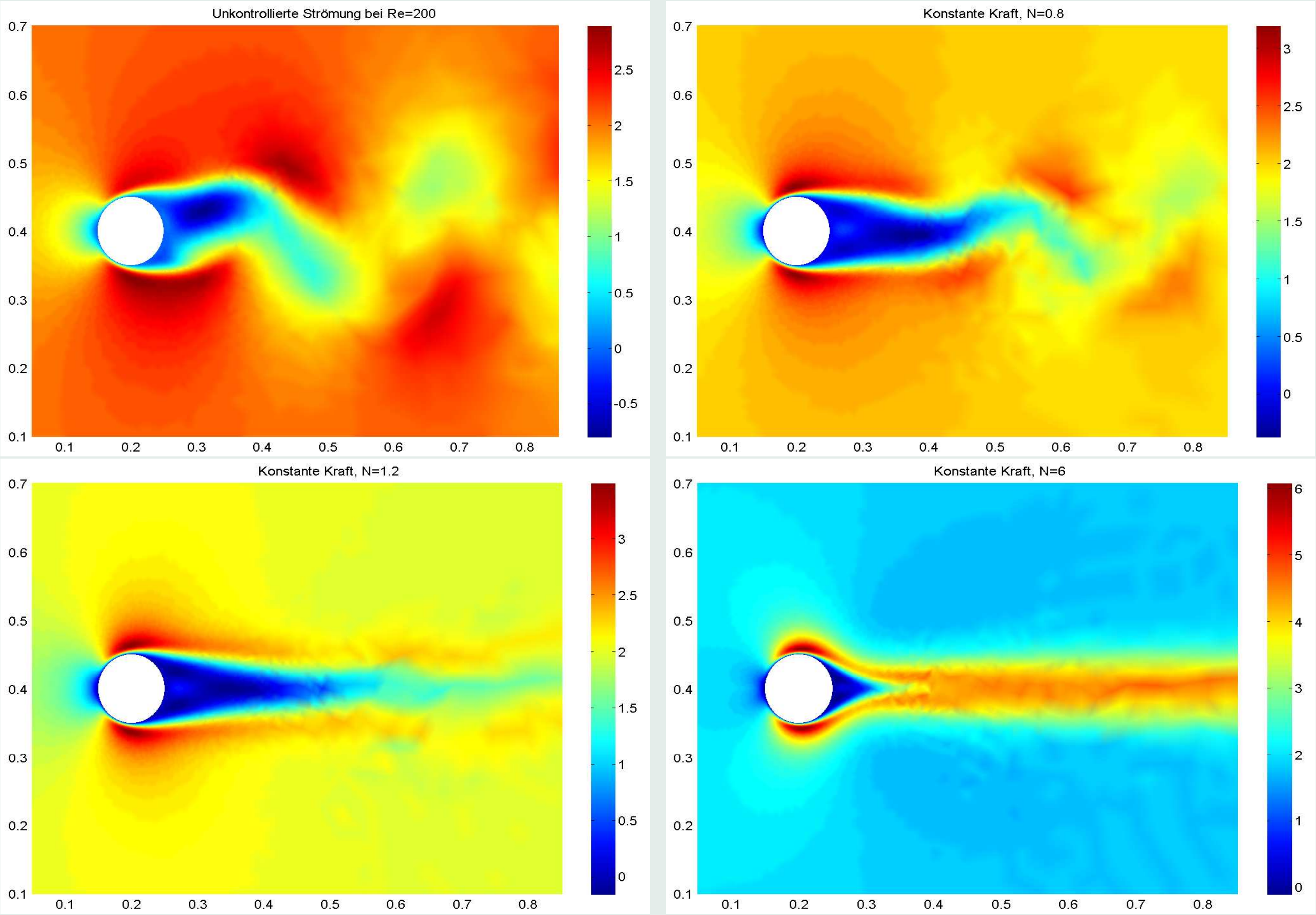
The lift coefficient is given by:

$$C_L = \frac{2F_L}{\rho U_\infty^2 D} \quad .$$

Constant Forcing

Re $= 200$, stepsize $\Delta t = 0.01$

The formation of the vortex street is suppressed by application of a tangential Lorentz force:



Unkontrollierte Strömung bei Re=200

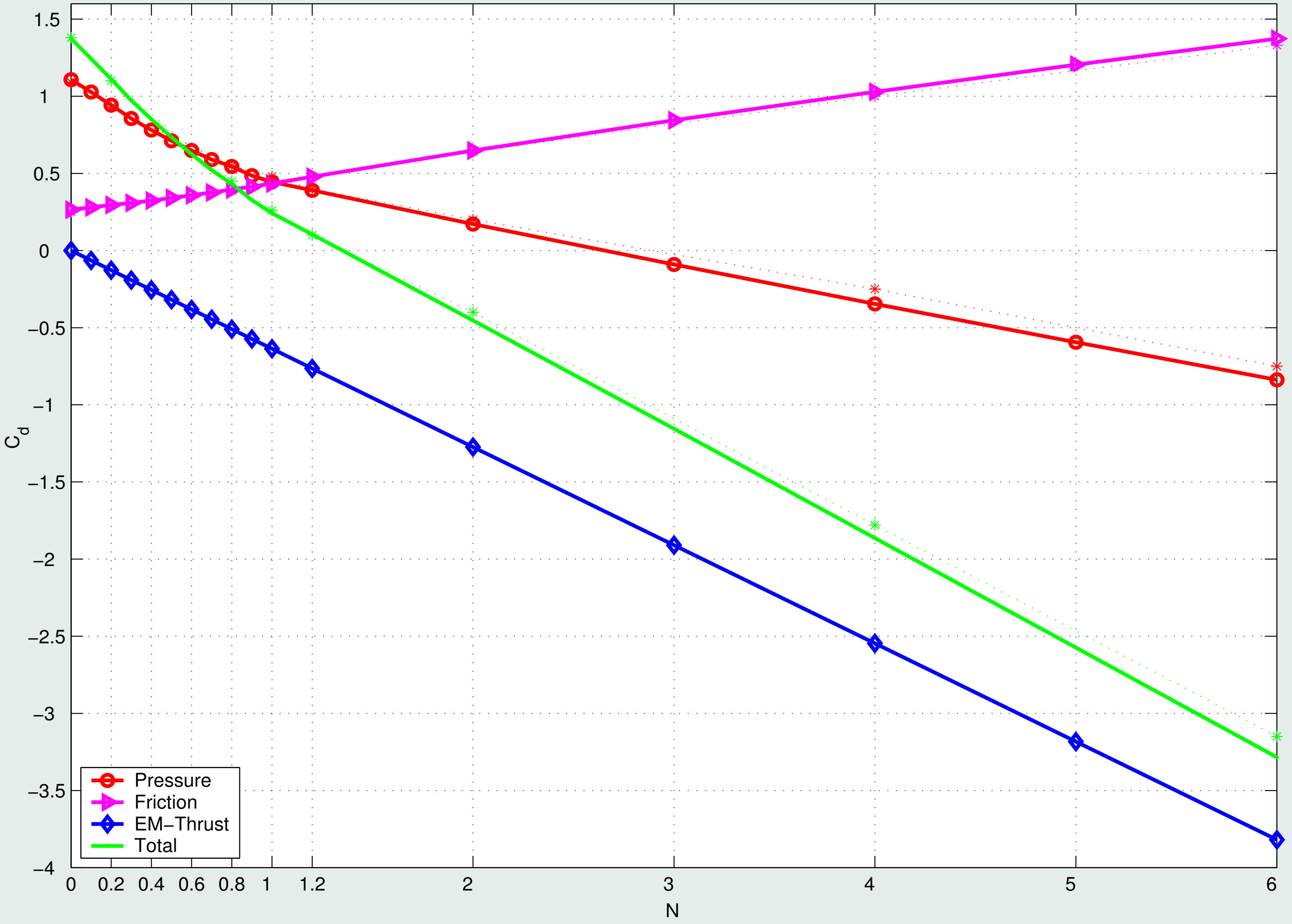
Konstante Kraft, N=0.8

Konstante Kraft, N=1.2

Konstante Kraft, N=6

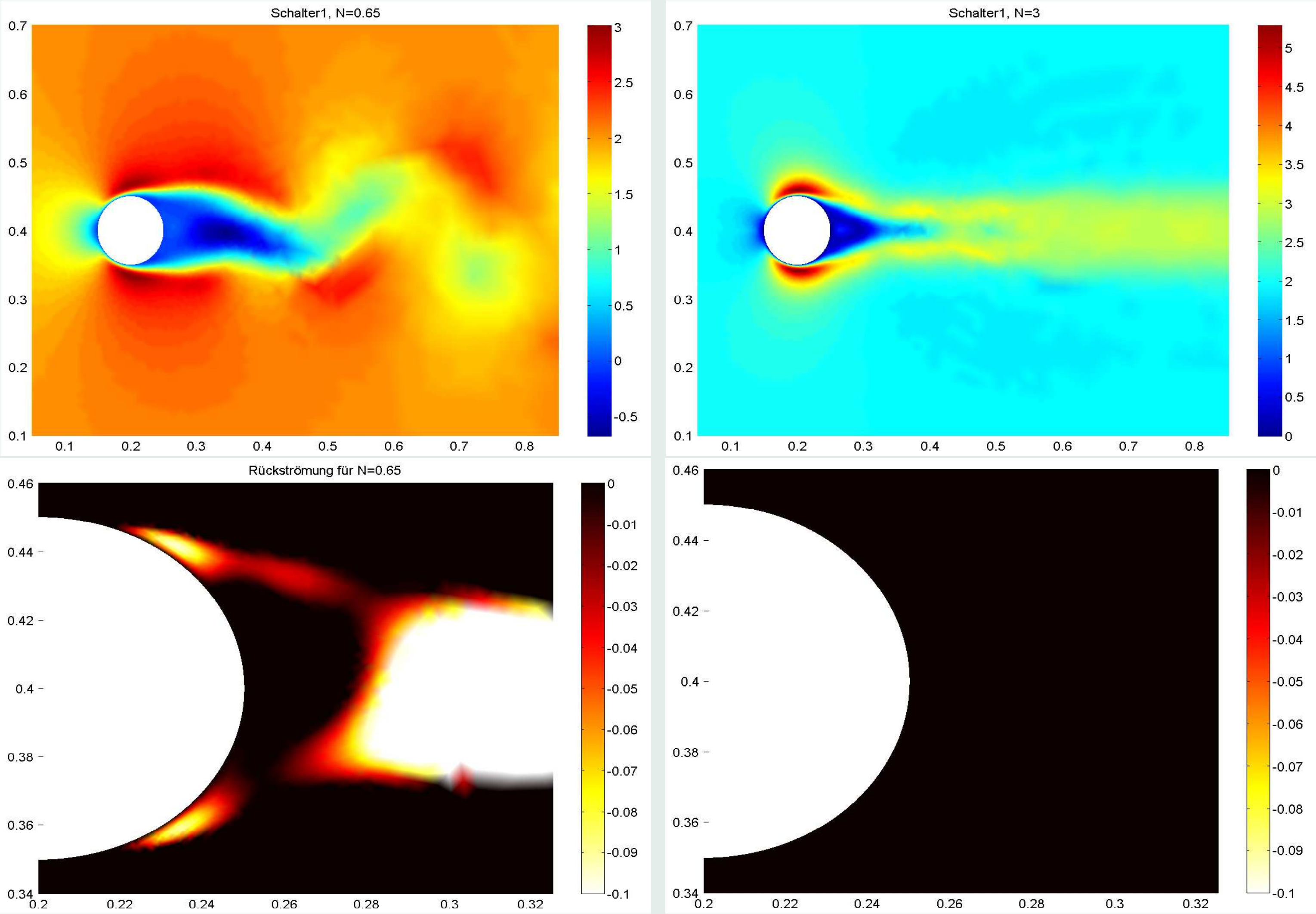
Drag Coefficients:

Figure 8: Dependency of the drag coefficients on the interaction parameter at Re=200
Comparison with Grundmann & Posdziech´s results



2 Point Proportional Controller

The Lorentz force is switched on, if the fluid flows back in a 10° sector around the rear stagnation point. Cf. Zhihua Chen's Dissertation ***Electro-Magnetic Control of Cylinder Wake***



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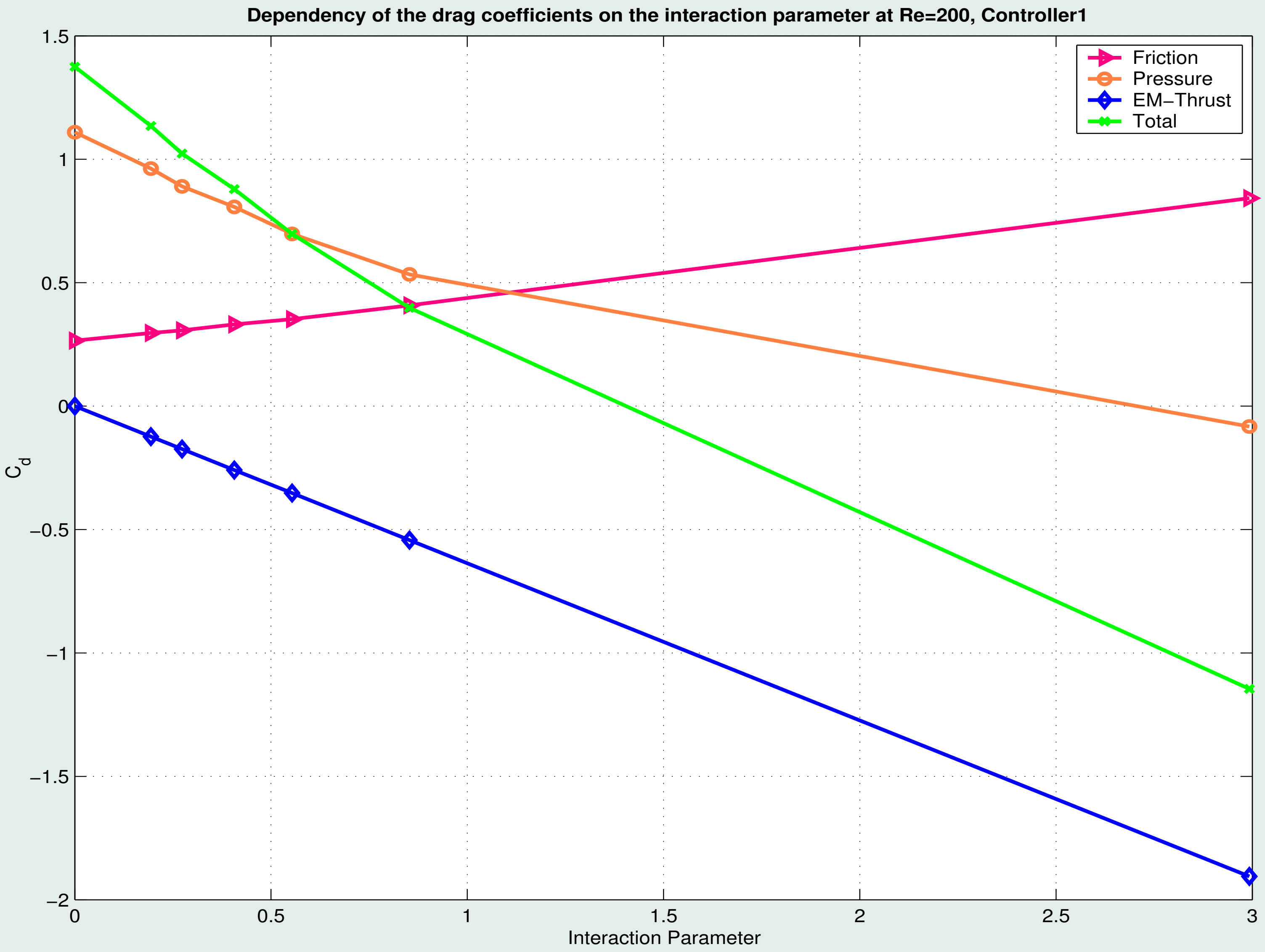
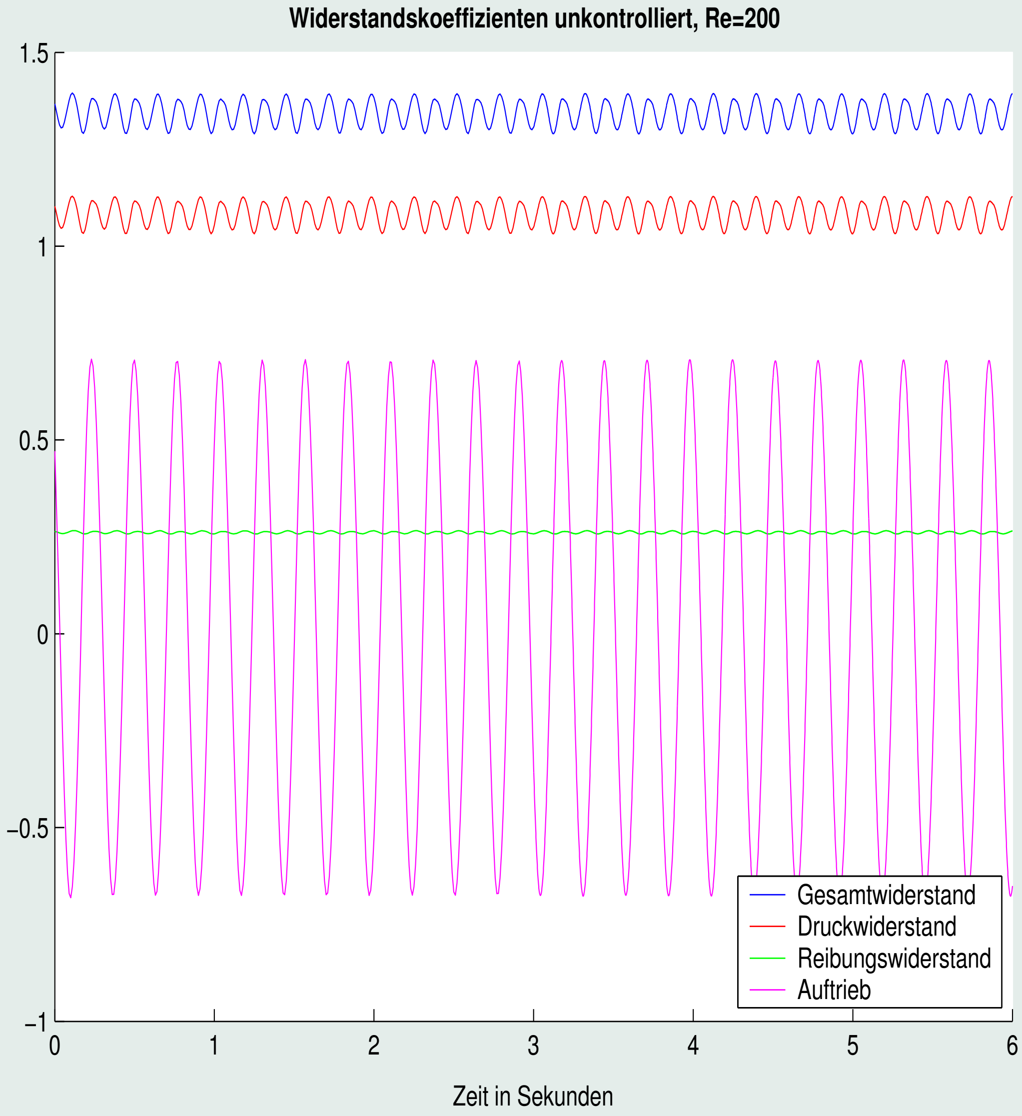
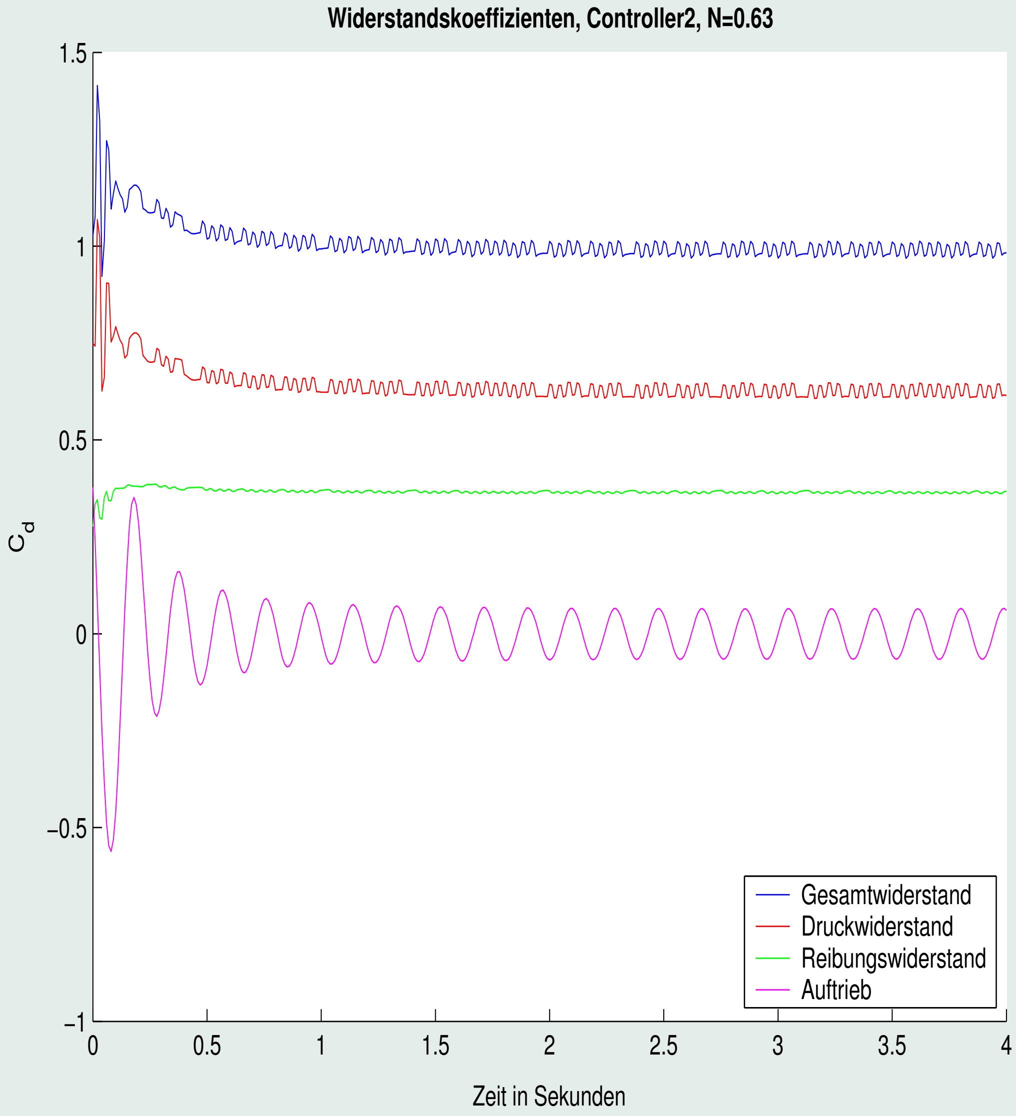


Time-dependent drag coefficients:



Widerstandskoeffizienten unkontrolliert, Re=200



Widerstandskoeffizienten, Controller2, N=0.63

Part2: Nonlinear feedback control recipes

Model predictive control, instantaneous control

Receding horizon control and instantaneous control

Idea of receding horizon approaches

Receding horizon control and instantaneous control

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1 At time t_k compute an optimal time discrete control strategy $u_{k+1}, \dots, u_{k+l}$.

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In applications: Real-time, fast control required $\longrightarrow$ fast answers, decisions.

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Suboptimal variant: instantaneous control: Apply only one gradient step to the approximate computation of the optimal strategy for $l = 1$.

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Control target

Given some initial state x_0, find a control law $\mathcal{B}u(t) = K(x(t))$ which steers the state $x(t)$ towards a given trajectory $\bar{x}$:

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Mathematical model:

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Mathematical model:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) + \boldsymbol{A}\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{b}(\boldsymbol{x}, \boldsymbol{t})$$

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$$\boldsymbol{\dot{x}(t) + Ax(t) = b(x,t) + \mathcal{B}u(t)} \text{ state,}$$

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Mathematical model:

$$\begin{aligned} \dot{x}(t) + Ax(t) &= b(x,t) + \mathcal{B}u(t) \text{ state,} \\ y(t) &= \mathcal{C}x(t) \text{ observation,} \\ x(0) &= x_0 \end{aligned}$$

Here

- $\bar{x}$ desired stationary state, or
- $\bar{x}$ a reference trajectory obtained from open loop optimal control.

From now onwards: $\mathcal{B} = ID$ und $\mathcal{C} = ID$.

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Discretization

Discretize the state equation w.r.t. time

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$$\boldsymbol{(I + hA)x^{k+1} \quad = \quad x^k + hb^k}$$

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and minimize at every time step an instantaneous version of the cost

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$$(\boldsymbol{P}_k) \quad \begin{cases} \min \boldsymbol{J}(\boldsymbol{u}^{k+1}) = \frac{\gamma}{2}|\boldsymbol{u}^{k+1}|^2 + \frac{1}{2}|\boldsymbol{x}^{k+1} - \bar{\boldsymbol{x}}^k|^2 \\ \text{s.t.} \\ (\boldsymbol{I} + h\boldsymbol{A})\boldsymbol{x}^{k+1} \quad = \quad \boldsymbol{x}^k + h\boldsymbol{b}^k \end{cases}$$

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$$(P_k) \quad \left\{ \begin{array}{l} \min J(u^{k+1}) = \frac{\gamma}{2}|u^{k+1}|^2 + \frac{1}{2}|x^{k+1} - \bar{x}^k|^2 \\ \text{s.t.} \\ (I + hA)x^{k+1} \quad = \quad x^k + hb^k + u^{k+1}. \end{array} \right.$$

Time discretization with implicit Euler.

Receding horizon control and instantaneous control

Feedback oracle

1. **Set $x^0 = \phi$, $k = 0$ and $t_0 = 0$.**

Receding horizon control and instantaneous control

Feedback oracle

1. Set $x^0 = \phi$, $k = 0$ and $t_0 = 0$.

2. Given an initial control u_0^k, set

$$u^{k+1} = \mathrm{RECIPE}(u_0^k, x^k, \bar{x}^k, t_k)$$

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3. Solve

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4. Set $t_{k+1} = t_k + h$, $k = k + 1$. If $t_k < T$ goto 2.

Receding horizon control and instantaneous control

Instantaneous control

For instantaneous control the oracle RECIPE is given by

$$\boldsymbol{u = \mathrm{RECIPE}(v, x^k, z, t_k)}$$

Receding horizon control and instantaneous control

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iff

- **Solve $\boldsymbol{(I + hA)x = x^k + hb(x^k, t_k) + v}$,**

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$$\boldsymbol{u} = \text{RECIPE}(\boldsymbol{v}, \boldsymbol{x}^k, \boldsymbol{z}, \boldsymbol{t}_k)$$

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- Solve $(\boldsymbol{I} + \boldsymbol{h}\boldsymbol{A})\boldsymbol{x} = \boldsymbol{x}^k + \boldsymbol{h}\boldsymbol{b}(\boldsymbol{x}^k, \boldsymbol{t}_k) + \boldsymbol{v}$,

- solve $(\boldsymbol{I} + \boldsymbol{h}\boldsymbol{A})\boldsymbol{\lambda} = -(\boldsymbol{x} - \boldsymbol{z})$,

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- **determine** $\boldsymbol{\rho} > 0$**,**

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- determine $\boldsymbol{\rho} > 0$,
- set RECIPE $= \boldsymbol{v} - \boldsymbol{\rho}\boldsymbol{d}$

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$$u = \text{RECIPE}(v, x^k, z, t_k)$$

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- Solve $(I + hA)x = x^k + hb(x^k, t_k) + v$,
- solve $(I + hA)\lambda = -(x - z)$,
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This oracle realizes steepest descent for problem (P_k).

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Model predictive control

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- **Solve the optimality system for $\boldsymbol{u}$**

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(\boldsymbol{I} + \boldsymbol{hA})\boldsymbol{\lambda} &= \boldsymbol{z} - \boldsymbol{x} \\
\boldsymbol{\gamma u} - \boldsymbol{\lambda} &= 0 \quad (\boldsymbol{\gamma u} - \boldsymbol{\lambda} \geq 0) \text{ in case of constraints.}
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- set RECIPE $= \boldsymbol{u}$

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- **set RECIPE $= \boldsymbol{u}$**

This oracle realizes solution of problem $(\boldsymbol{P}_k)$.

Receding horizon control and instantaneous control

Feedback operators

$\boldsymbol{u}_0^{j+1} = \mathbf{0}.$

- **Instantaneous control rewritten**

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Here

$$B \;:=\; (I + hA)^{-1} \text{ and } S := \gamma(BB + \gamma I)^{-1}BB.$$

Receding horizon control and instantaneous control

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Receding horizon control and instantaneous control

Stability of the closed-loop control operators, distributed control

Receding horizon control and instantaneous control

Stability of the closed-loop control operators, distributed control

Let h be small, $\bar{x}, x_0$ sufficiently smooth.

The discrete controllers K_I^d and K_O^d generate iterates $\{x^k\}$ which satisfy

$$|x^k - \bar{x}(t_k)|_{0,1} \;\leq\; c\,\kappa^k$$

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The continuous controllers K_I^d and K_O^d steer the system exponentially fast to $\bar{x}$:

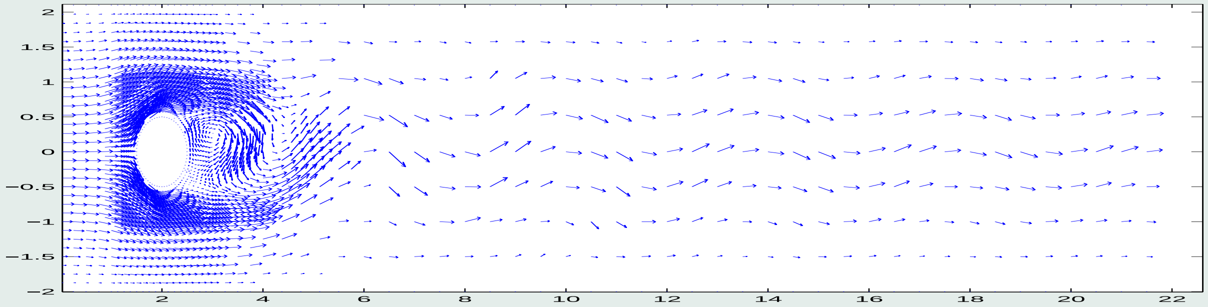
$$|x(t) - \bar{x}(t)|_{0,1} \;\leq\; c\exp(-\frac{c}{h}t).$$

Control with near wall Lorentz force

Control: $u(t,x) = \sum_{i=1}^{n} u_i(t) f_i(x)$, $f_i(x) = e^{-|x-x_i|^2}$, x_i **point on cylinder surface.**

Desired flow $\bar{x}$**: Stokes flow.**

Amplitudes



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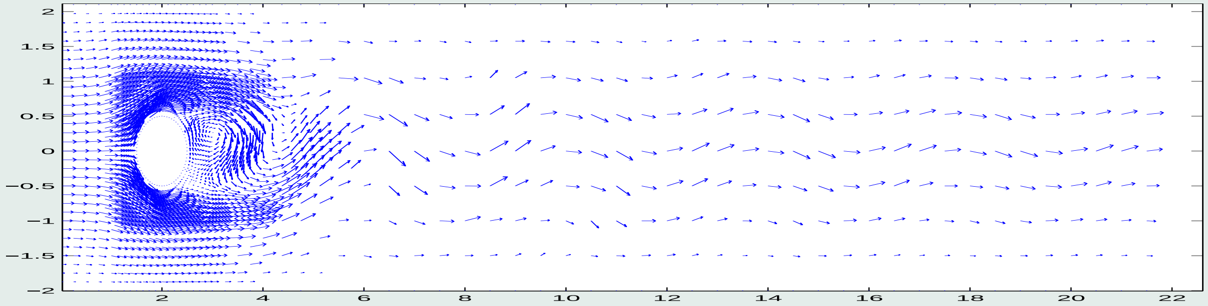
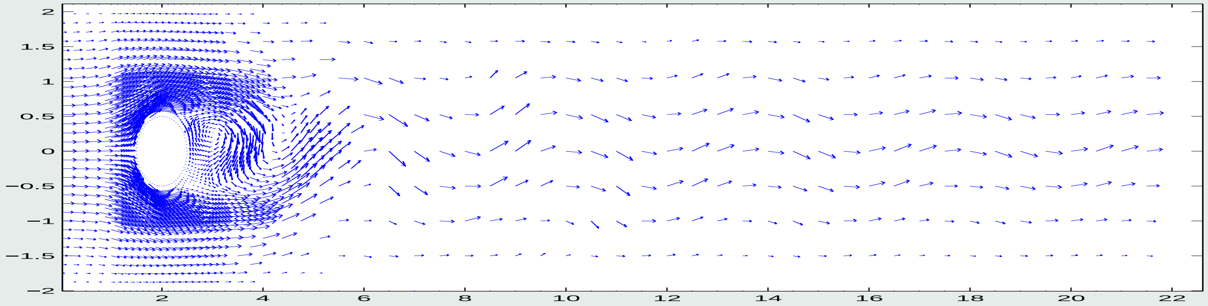


Controlled flow



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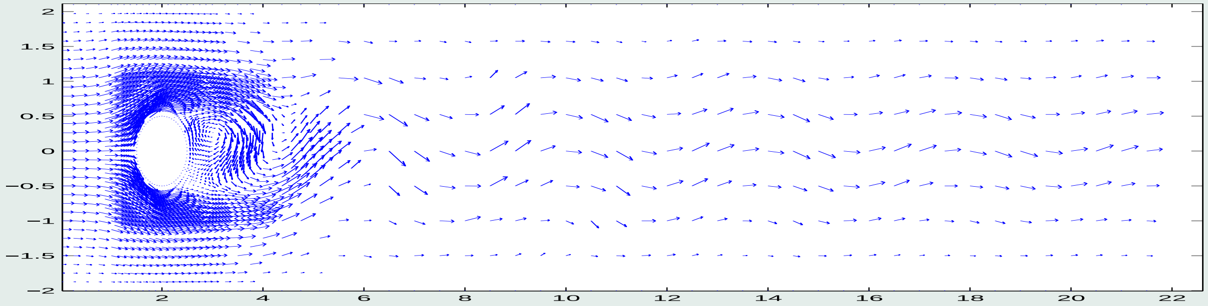
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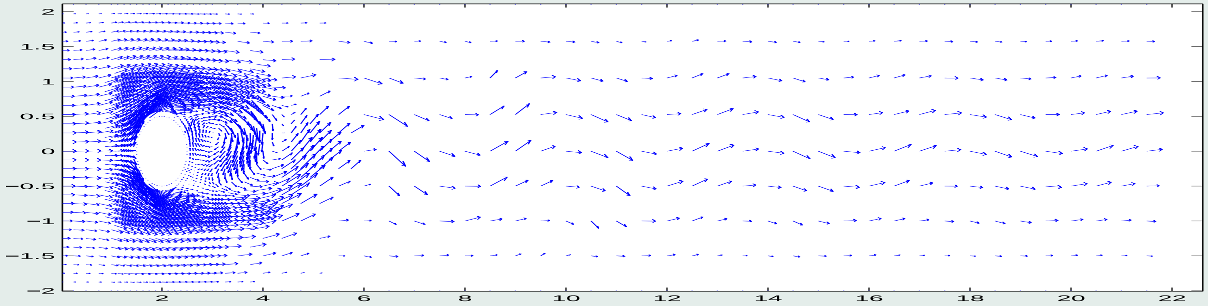
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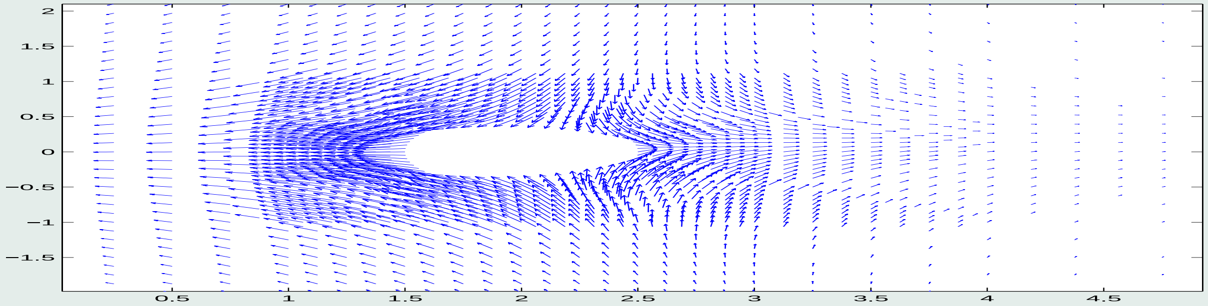
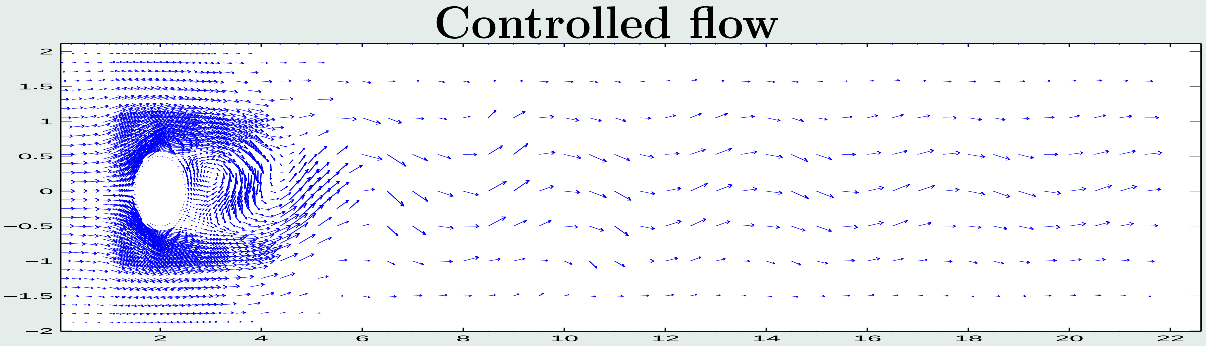

Lorentz force

Control with near wall Lorentz force

Control: $u(t,x) = \sum_{i=1}^{2} u_i(t) e_\phi f_i(x),\ f_i(x) = g(\phi) e^{-\frac{\pi}{a}\cdot dist[(x,y),\ cylinder]}$

Desired flow $\bar{x}$ **with** $x_1 = 1$.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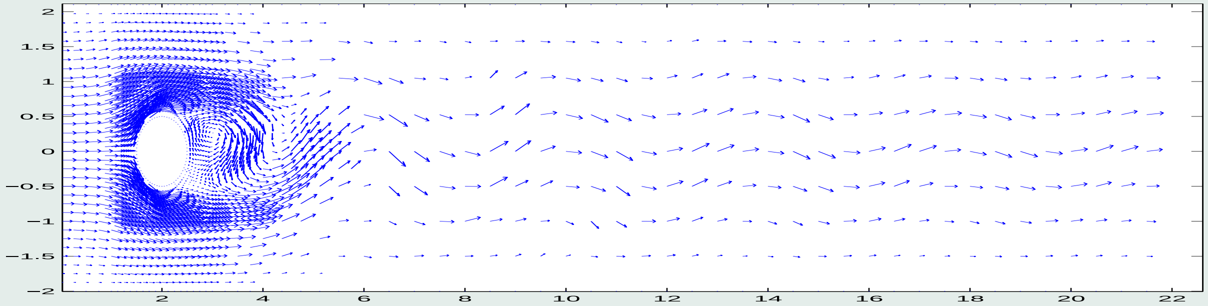

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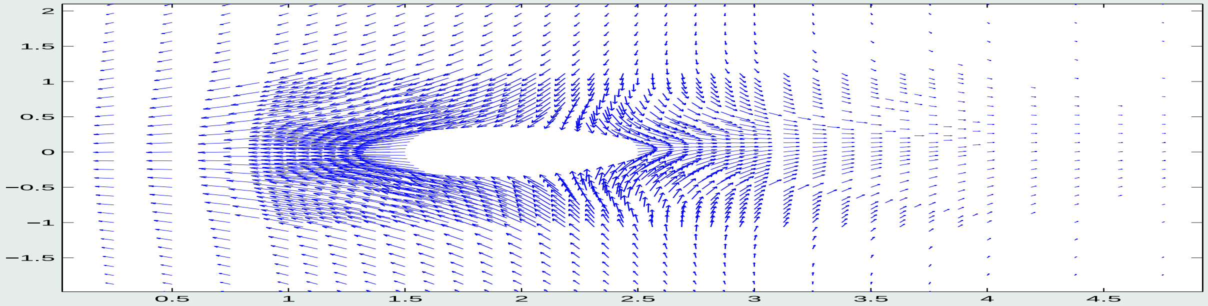

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CPU time needed to compute the instantaneous control strategy =2,5 times CPU time needed for one forward solve.

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This amounts to 1-2 % of the CPU time needed to compute the optimal control trajectory.

Conclusions and future work

2position proportional controller, conducted by experiment

General construction recipe of feedback control operators for pde systems

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- Instantaneous control

- Model predictive control

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2position proportional controller, conducted by experiment

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- **Instantaneous control**
- **Model predictive control**
- **Stability of corresponding controllers (distributed control)**

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Apply methods to more complex configurations.

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Extend analysis to boundary control and/or observation

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Extend analysis to boundary control and/or observation

Provide numerical analysis.

Further informations on the topics

http://www.math.tu-dresden.de/˜hinze